

J. Hankel Transform and Its Application to
Theory of Seismology



Fast Hankel Transform and Its Application to Seismic Modeling

by

Lydia Deng

**Center for Wave Phenomena
Colorado School of Mines
Golden, Colorado 80401
(303)273-3557**

Fast Hankel Transform and Its Application to Seismic Modeling

Lydia Deng

ABSTRACT

The Hankel(Fourier-Bessel) transform is a useful tool for seismologists, but its computational cost can be frustratingly high.

Noting that the Hankel transform is equivalent to the cascade of an Abel transform and a 1-D Fourier transform, this paper presents a fast algorithm for computing the Hankel transform. Using the state-variable model for a linear shift-variant filter, the Abel transform can be computed efficiently. The total computation cost for the fast Abel transform followed by an FFT is proportional to $3KN + 2N \log N$ where N is the number of samples to be transformed and K is the order of the Hankel transform approximation. $K = 9$ is used for this application.

This paper describes this fast Hankel transform algorithm and discusses its efficiency, accuracy and its possible extensions. The paper also shows the application of this technique to the modeling of wavefields in vertically inhomogeneous media. Compared the use of the direct integration and that of the Fourier-Bessel series summation, the use of this fast Hankel transform significantly reduces the computation time for such modeling.

INTRODUCTION

Three-dimensional (3-D) seismic waves propagating in a vertically inhomogeneous medium are circularly symmetric about a vertical axis in both the space and wavenumber domains. The modeling of such waves is commonly performed in the wavenumber domain using a cylindrical coordinate system. Defining each depth slice of the modeled seismic wavefield in the space domain requires numerical evaluation of the Hankel transform, the equivalent of a 2-D Fourier transform of a circularly symmetric func-

tion represented in a circular coordinate system.

The costs of the direct evaluation of the Hankel transform and of its contained Bessel functions are proportional to N^2 , where N is the number of samples to be transformed. In the modeling of 3-D seismic wavefields, the number of wavenumbers, N , in the Hankel transform is large. The modeling of such wavefields, as well as many other scientific applications, require that the numerical evaluation of the Hankel transform be repeated many times, which makes the cost of direct evaluation prohibitively expensive. Because of this high cost, many methods have been developed for efficient numerical evaluation of the Hankel transform.

Dietrich (1990) noted that a reduction in the number of samples to be evaluated is obtained by expressing the Hankel transform as a Fourier-Bessel series (e.g., Watson, 1952). Although the number of input samples is reduced to a smaller value, M (the number of zeros of the Bessel function), the overall costs of evaluating the Hankel transform and its contained Bessel functions are still proportional to M^2 .

The numerical evaluation of the Bessel functions is eliminated by representing the Hankel transform as an Abel transform (e.g., Bracewell, 1965) followed by a Fourier transform. Efficient algorithms for the numerical evaluation of Fourier transforms, with costs that are proportional to $N \log N$, already exist. However, the cost of direct evaluation of the Abel transform is still proportional to N^2 . Therefore, this approach of evaluating the Hankel transform will be more efficient only if the cost of evaluating the Abel transform is reduced.

The Abel transform can be thought of as a shift-variant filter. Bracewell (1965) and Hansen (1978) showed that under the proper non-linear coordinate transformations, the Abel transform can be converted to a shift-invariant filter. Although the cost of an Abel transform is reduced approximately to the cost of a fast Fourier transform, the method requires the over-sampling of the new coordinate to as many as either N^2 or $N \ln N$ samples, depending on the particular coordinate transformation.

Hansen (1985) subsequently developed an algorithm that implements the Abel transform by a recursive filter at a cost that is proportional to N . Hansen's method requires no resampling. The overall cost of numerically evaluating a Hankel transform thus becomes comparable to that of an FFT.

In this paper, I restate Hansen's algorithm (noting some discrepancies between his and my expressions), extend the algorithm to the Hankel transform of the first-order, and discuss the efficiency and accuracy of this algorithm. I then use Hansen's fast Hankel transform filter in Korn's method (1987) for modeling total seismic wavefield in a vertically inhomogeneous medium. I demonstrate the significant computational savings that can be obtained by evaluating Hankel transforms using Hansen's method instead of using the Fourier-Bessel method or direct integration.

DEVELOPMENT OF THE ALGORITHM

The definition of a Hankel transform (e.g., Bracewell 1965), $F(k)$, of a function, $f(r)$, is

$$F(k) \equiv \int_0^\infty f(r) J_0(kr) r dr, \quad (1)$$

and its inverse transform is

$$f(r) = \frac{1}{2\pi} \int_0^\infty F(k) J_0(kr) k dk. \quad (2)$$

$F(k)$ is equivalent to a 2-D Fourier transform of the circularly symmetric function $f(r)$. The numerical calculations of the Hankel transform could be done by a 2-D FFT. However, because of the cylindrical coordinate system used in our applications, the data in terms of Cartesian coordinate system is not given nor is it desired.

The Abel transform (e.g., Bracewell, 1965) is defined as,

$$g(y) \equiv \int_{|y|}^\infty \frac{2f(r)}{\sqrt{1 - (y/r)^2}} dr. \quad (3)$$

It can be shown that a Hankel transform is the Fourier transform of an Abel transform; that is,

$$F(k) = \int_{-\infty}^\infty g(y) e^{iky} dy. \quad (4)$$

By changing variables and interpolating the data to either an exponential (Hansen, 1978) or a square-root grid (Bracewell, 1965; Mook, 1983; Wilson, 1986), the integral is transformed into a convolution, which is easily performed as a multiplication after an FFT. However, to avoid aliasing, the number of samples required for the square-root and exponential grids to avoid aliasing are N^2 and $N \ln N$, respectively, where N is the number of original samples. The increase in the number of samples required for the Fourier transform decreases the expected efficiency of the computation.

Hansen (1982) developed a method to realize a shift-variant filter through use of state variables. Using this method, the Abel transform can be implemented by a linear recursive filter (Hansen, 1985) for which the computational cost is proportional to N and which requires no resampling to a nonuniform grid.

Hansen's state-variable model of the Abel transform

The state-variable method is an alternative to convolution for realizing a linear filter. Any linear filter can be described by a differential equation with coefficients that characterize its impulse response. The filter then can be realized by introducing the intermediate state, $x(t)$, and parameterizing the impulse response into state matrices

$A(t), B(t), C(t), D(t)$ of the appropriate dimension. The state-variable model of a general linear filter is

$$\begin{aligned} x'(t) &= A(t)x(t) + B(t)f(t) \\ y(t) &= C(t)x(t) + D(t)f(t), \end{aligned} \quad (5)$$

which is illustrated in Figure 1. The state matrices A, B, C, D of the shift-invariant filter are constants, while those of shift-variant filter are not.

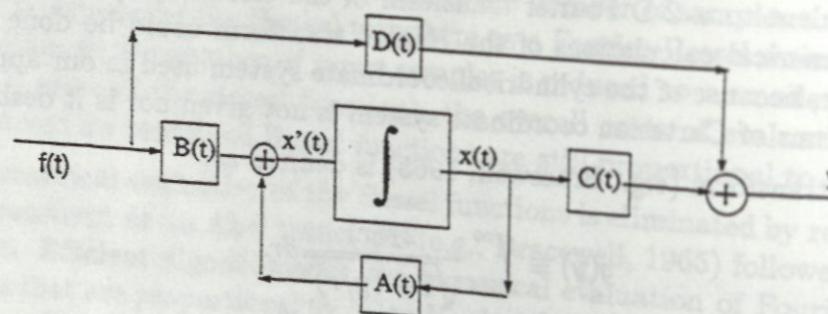


FIG. 1. The state variable realization of a linear filter

An Abel transform as shown in equation (3) can be interpreted as a linear filtering of input, $f(r)$, to obtain output, $g(y)$, with shift-variant impulse response, $h(y/r) = U(1 - (y/r)^2)[1 - (y/r)^2]^{-1/2}$, where $U(\cdot)$ is the Heaviside step function. In the state-variable model of the Abel transform filter, the parameters A, B, C , and D are chosen to approximate its impulse response, $h(y/r)$.

Since the system is shift-variant, the coefficient matrices are not constants. By letting $\xi = -\ln y$ and $\eta = -\ln r$, the Abel transform is reduced to a shift-invariant filter that can be implemented by convolution,

$$\bar{g}(\xi) = \int_{-\infty}^{\infty} -2\bar{f}(\eta)e^{-\eta}\bar{h}(\xi - \eta)d\eta = \{-2\bar{f}(\xi)e^{-\xi}\} * \bar{h}(\xi), \quad (6)$$

where $\bar{f}(\xi) = f(y(\xi))$, $\bar{h}(\xi - \eta) = h(y(\xi)/r(\eta))$, $\bar{g}(\xi) = g(y(\xi))$.

The shift-invariant filter, $\bar{h}(\xi)$, can be approximated by an exponential series of order K ,

$$\bar{h}(\xi) \approx \sum_{k=1}^K h_k e^{\lambda_k \xi}, \quad (7)$$

where the parameters h_k and λ_k are obtained by a non-linear least-squares method described later in this paper. Once h_k and λ_k are determined, after inverse coordinate transformation, the state-variable model of the Abel transform is expressed as

$$x'(r) = -\frac{1}{r}\bar{A}x(r) - 2\bar{B}f(r) \quad (8)$$

$$g(y) = \bar{C}x(|y|), \quad (9)$$

where the state matrices are given by

$$\bar{A} = \text{diag}(\lambda_1, \dots, \lambda_K); \quad \bar{B} = [h_1, \dots, h_K]^T; \quad \bar{C} = [1, \dots, 1].$$

Recursive filter for the Abel transform

With the state-variable model of the Abel transform in the previous section, the realization of the digital filter is obtained by numerically solving differential equation (8).

Discretizing equation (8) yields a difference equation represented by an intermediate state vector $\vec{x}(r)$ and a state transition matrix $\Phi(\cdot, \cdot)$,

$$\vec{x}(r) = \Phi(r, r_0)\vec{x}(r_0) - 2 \int_{r_0}^r \Phi(r, \xi)\bar{B}f(\xi)d\xi, \quad (10)$$

where

$$\Phi(r, r_0) = \text{diag}\left[\left(\frac{r_0}{r}\right)^{\lambda_1}, \dots, \left(\frac{r_0}{r}\right)^{\lambda_K}\right].$$

Here, $\vec{x}(r) = [x_1(r), \dots, x_K(r)]$ is the current state, $\vec{x}(r_0)$ is the previous state, and K is the order of approximation in equation (7). (In following Hansen's derivation, I encountered some discrepancies. Hansen's expression for $\vec{x}(r_i)$ has a plus sign rather than the minus sign shown in equation (10)).

Using linear interpolation between grid points, equation (10) becomes a linear recursion which begins at $r = r_{\max}$ proceeds towards $r = 0$. Then using the initial assumptions that $f(r) = 0$ when $r > r_{\max}$ and $\vec{x}(r_{\max}) = 0$, and letting $\Delta = r_{\max}/N$, $f_n = f(n\Delta)$, $g_n = g(n\Delta)$, and $\vec{x}_n = [x_{n,1}, \dots, x_{n,K}]^T = \vec{x}(n\Delta)/\Delta$, we get the $N - 1$ recursions for the state vector,

$$\vec{x}_{n-1} = \Phi(n)\vec{x}_n + B_0(n)f_n + B_1(n)f_{n-1}, \quad n = N, N-1, \dots, 2. \quad (11)$$

Finally, the Abel transform is given by

$$g_n = \Delta \sum_{k=1}^K x_{n,k}. \quad (12)$$

The precomputed parameters of the recursive filter in equation (11) are

$$\begin{aligned} \Phi(n) &= \text{diag}\left[\left(\frac{n}{n-1}\right)^{\lambda_1}, \dots, \left(\frac{n}{n-1}\right)^{\lambda_K}\right]; \\ B_0(n) &= [h_1\beta_0(n, \lambda_1) \dots h_K\beta_0(n, \lambda_K)]^T; \\ B_1(n) &= [h_1\beta_1(n, \lambda_1) \dots h_K\beta_1(n, \lambda_K)]^T; \\ \beta_0(n, \lambda) &= \frac{2(n-1)}{(\lambda+1)(\lambda+2)}[(n-1) + (\lambda-n+2)\left(\frac{n}{n-1}\right)^{\lambda+1}]; \\ \beta_1(n, \lambda) &= \frac{-2(n-1)}{(\lambda+1)(\lambda+2)}[(\lambda+n+1) - n\left(\frac{n}{n-1}\right)^{\lambda+1}]. \end{aligned} \quad (13)$$

(Note that the equations for β_0 and β_1 differ from those in Hansen's paper(1985).)

Since a singularity of the state-variable model occurs at the origin, the output at $r = 0$ is evaluated by a direct approximation of the Abel transform,

$$g_0 = f_0 + 2 \sum_{i=1}^{N-1} f_i.$$

In the recursion formula of the Abel transform filter, the sampling interval Δ appears explicitly only in the final scaling. Since all the parameters of the filter are independent of Δ , they can be pre-computed and stored in a table. The cost of the Abel transform is $3K + 1$ multiplies and $3(K - 1)$ adds per recursion step.

Since the Abel transform is given only for $y \geq 0$, it needs be extended to an even function about the origin in order to perform the subsequent Fourier transform. The total cost of the Hankel transform by this method is proportional to $3KN + 2N \log N$, where $K = 9$ is used.

The parameters of the Abel transform filter

The parameters of the state-variable model of the Abel transform in equation(13) are computed by approximating the impulse response of the coordinate transformed filter $\bar{h}(\xi) = (1 - e^{-2\xi})^{-1/2}$ with constants $\{h_k, \lambda_k, k = 1, \dots, K\}$. This is a problem of approximating a function by exponentials with unknown eigenvalues. The standard least-squares method can be used to compute the constants.

Hansen gave a set of coefficients for a ninth-order approximation, $K = 9$. I used a double-precision, non-linear least-squares problem solver in the MINPACK package to compute the constants to the same order, with Hansen's coefficients as an initial guess. Forcing $h_1 = 1$ and $\lambda_1 = 0$ to economize the calculation, I computed the parameters $(h_i, \lambda_i), i = 1, \dots, 9$, as listed in Table 1.

Hankel transform of high-order

The Hankel and Abel transforms of order n are defined as

$$F_n(k) \equiv \int_0^\infty f(r) J_n(kr) dr, \quad (14)$$

and

$$g_n(y) \equiv \int_{|y|}^\infty \frac{2T_n(y/r)f(r)}{\sqrt{1 - (y/r)^2}}, \quad (15)$$

respectively, where $T_n(x) = \cos(n \cos^{-1}(x))$ is the Chebyshev polynomial of the first kind. The state-variable approach presented above is, in principle, applicable to the higher orders, though it is difficult to find the constants that approximate the impulse response in equation (7), $\bar{h}(\xi)$. Only the first-order transform is discussed here.

k	h_k	λ_k
1	1.000	0.0
2	0.611	-2.084
3	0.895	-5.793
4	1.341	-14.63
5	2.025	-35.1
6	3.181	-83.3
7	5.909	-210.4
8	77.6009	-6674
9	528.222	-34898

Table 1. The approximate parameters of the Abel transform filter

For the first-order Abel transform, $n = 1$ and $T_1(x) = x$. All the derivations are essentially the same as above, except for an exponential multiplier in the impulse response,

$$g_1(y) = \frac{y}{r} g_0(y), \quad h_1(\xi) = e^\xi h_0(\xi).$$

Kim(1989) set up a first-order Abel transform filter by slightly modifying the parameters of the zero-order transform filter. Here, I present a simple way of computing the first-order Abel transform using the existing zero-order filter. That is,

$$g_1(y) = y \hat{g}_0(y),$$

where $\hat{g}_0(y)$ is the zero-order Abel transform of function $f(r)/r$.

The first-order Hankel transform is obtained by extending the output $g_1(y)$ to an odd function followed by a 1-D FFT.

Discussion on the accuracy of the fast Hankel transform algorithm

We have already seen that this fast Hankel transform algorithm has a cost that is proportional to $3KN + 2N \log N$, where N is the number of samples to be transformed and K is the order of approximation to the impulse response $\bar{h}(\xi)$ of the shift-invariant filter of equation (7). As discussed earlier, this algorithm is much more efficient than the other transform methods that we cited above. The accuracy of this method depends on the nature of the function to be transformed.

The errors associated with this algorithm are caused by approximating the shift-invariant filter $h(\xi)$ to an exponential series (equation (7)) and by the linear interpolation of the data required to numerically solving differential equation (8). Following Hansen, I used a ninth-order approximation, $K = 9$. Figure 2 compares two known analytical Hankel transforms and their numerical evaluations using Hansen's fast

algorithm. Because linear interpolation is more accurate for the smooth Gaussian function than for the oscillating Jinc function, the Hankel transform in Figure 2a is more accurate than that in Figure 2c. It can be seen that the ninth-order approximation has very good accuracy.

For other applications requiring higher accuracy, a higher order of approximation K may be required. The parameters of the Abel transform filter can be obtained by the non-linear least-squares method stated earlier in this paper. Higher order approximation of the filter increases the accuracy of this fast Hankel transform algorithm, but can increase the computation time as well. However, because the cost of the Abel transform increases linearly with the order of approximation, improved accuracy can be obtained at a reasonable(linear) cost.

THE APPLICATION TO SEISMIC MODELING

In the study of the seismic waves propagation in vertically inhomogeneous media, the total wavefield response to a point source is needed to model reflections from an arbitrary reflector. The frequency-domain finite-difference method(Korn, 1988) is an accurate and efficient method to compute the total wavefield, especially for thinly-layered media. An inverse Hankel transform is needed, for every depth slice and frequency component, to transform the wavenumber representation back to the space domain. Therefore, efficiency in computation of the Hankel transform is an important issue for this modeling technique.

Korn(1987) uses the Fourier-Bessel series expansion of the Hankel transform (Watson, 1952) to represent the horizontal dependence of the wave motion,

$$f(r) = \frac{2}{r_0^2} \sum_{i=1}^{\infty} F(k_i) \frac{J_0(k_i)}{[J_1(k_i r_0)]^2}, \text{ where } J_0(k_i r_0) = 0, \quad (16)$$

where $r = r_0$ is the largest lateral distance from the source position in which we are interested. This distance can be considered as the distance of a reflector that is so far away from the source that no reflections from it will be recorded within the observation time.

Because data points are needed only at the zeroes of the Bessel function, the sampling interval decreases as the argument of the Bessel function increases. The minimum interval is $\Delta k_{min} = \pi/r_0$, which is the same as the maximum sampling interval required by Nyquist theorem, but Δk can be much larger than the Nyquist interval for the low wavenumbers and the near field(small r). Therefore, the efficiency of computing the discrete Hankel transform has been enhanced with the Fourier-Bessel method as compared to that of direct evaluation of the Hankel integral. According to Dietrich and my experiment shown in Figure 3, the ratio of the computation time of these two methods is approximately 1 : 2.

However, the Fourier-Bessel series summation requires evaluation of Bessel functions M times at each grid point, where $M < N$ and M is the number of wavenumber

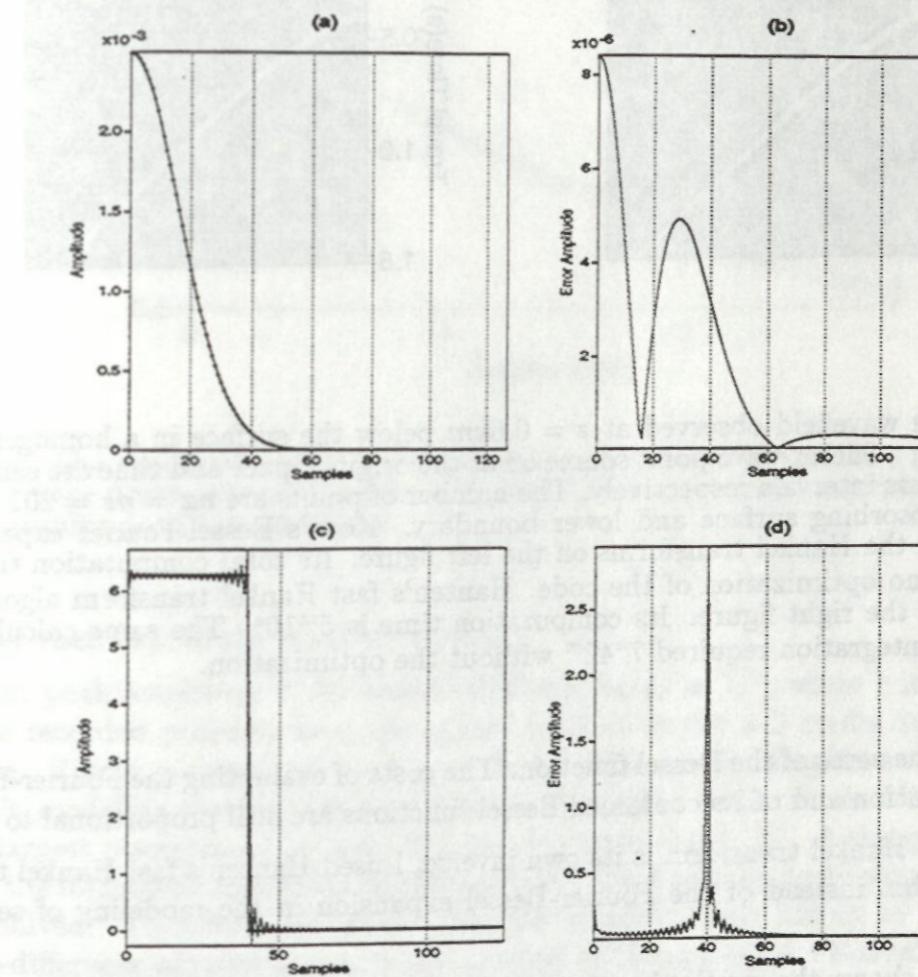


FIG. 2. Test of the fast Hankel transform algorithm and its accuracy. The processes are performed from $r_{max} = 1$ to $r = 0$ with 512 samples, but only 128 points are plotted in the figures. (a)Hankel transform of Gaussian $\exp(-\pi a^2 r^2)$ with $a = 20$. Its analytical transform $\exp(-k^2/(4\pi a^2))/a^2$ is compared to the numerical transform obtained by the fast (9th-order approximation) Hankel transform filter. (b) is the absolute error between these two results. The overall relative error is 0.06%. (c)Hankel transform of Jinc function $aJ_1(ar)/r$ with $a = 40\pi$. Its analytical transform $2\pi(U(r) - U(r-a))$ is compared to the numerical transform obtained by the fast (9th-order approximation) Hankel transform filter. (d) is the absolute error between these two results. The overall relative error is 1.2%.

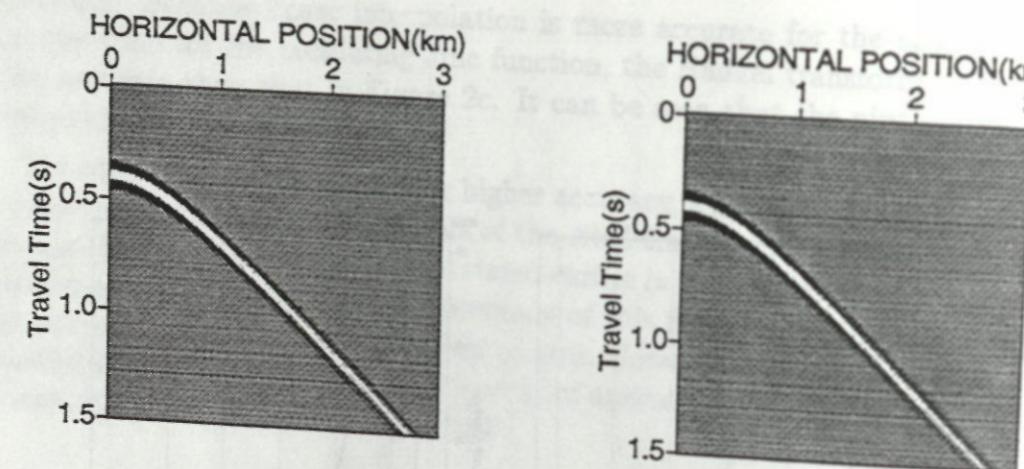


FIG. 3. The wavefield observed at $z = 0.6\text{ km}$ below the surface in a homogeneous medium with a Ricker wave point source on at the origin. Space and time are sampled at 15-m, 10-ms intervals respectively. The number of points are $nx = nz = 201$, $nt = 151$, with absorbing surface and lower boundary. Korn's Bessel-Fourier expansion was used for the Hankel transforms on the left figure. Its total computation time is 4^h44^m with no optimization of the code. Hansen's fast Hankel transform algorithm was used on the right figure. Its computation time is 5^m10^s . The same calculation with direct integration required 7^h42^m without the optimization.

samples at the zeros of the Bessel function. The costs of evaluating the Fourier-Bessel series summation and of its contained Bessel functions are still proportional to M^2 .

Since the Hankel transform is its own inverse, I used Hansen's fast Hankel transform algorithm instead of the Fourier-Bessel expansion in the modeling of seismic waves.

Figure 3 shows the synthetic seismogram generated by Korn's (1987) frequency-domain finite-difference method in a homogeneous medium, for a zero-phase Ricker wavelet point source. The Bessel-Fourier expansion, used in Korn's modeling on the left, has a computation time of 288min without any optimizing of the code. Hansen's fast Hankel transform, used on the right of the figure had a computation time about only 5min. (This fast Hankel transform code is optimized by Dr. Dave Hale and was in the standard CWP library.) For comparison, the same modeling done by the direct integration of the inverse Hankel transform took 462min.

While the efficiency of the Fourier-Bessel series and the direct integration of the inverse Hankel transform can be enhanced by optimizing the evaluation of the Bessel functions, because of its N^2 costs for a single Hankel transform and the large number of times it must be done, it is obvious that computation time, even after optimization of the Fourier-Bessel series summation or the direct integration will still be much

Point Source

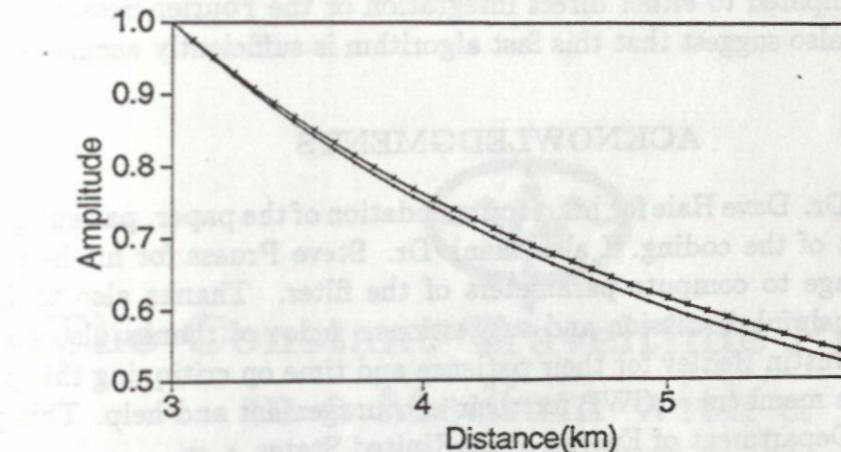


FIG. 4. The amplitude decay of with the distance of the far wavefield response to a Ricker wave point source. The line with stars is the $1/r$ curve, the other is the scaled peak amplitude of the wavefields.

greater than that of the Hansen's method.

The peak amplitude of far wavefield should decay as $1/r$, where r is the distance of the recorded position from the source position in the 3-D media due to a point source. Figure 4 compares the decay of the peak wavelet amplitude computed by Korn's modeling method (using Hansen's algorithm) with the ideal $1/r$ behavior. The largest discrepancy between the two curves is about 3%. Although not shown here, the maximum errors was significantly reduced when the depth sampling interval was halved. Therefore, the error seen here is attributable largely to errors in the finite-difference algorithm, not to the method for evaluating the Hankel transform.

CONCLUSIONS

An important tool in seismology, the Hankel transform is equivalent to an Abel transform followed by a Fourier Transform. It can be computed by a linear recursive filter followed by a 1-D FFT. The cost is proportional to $3KN + 2N \log N$ for N data points and a K th-order state-variable model. The choice of $K = 9$ used here appears to be sufficiently accurate for the seismic modeling.

If higher accuracy is required, a nonlinear least-squares method can be used to determine the state-variable parameters. The cost of implementation increases only linearly with the order K of approximation.

High-order Hankel transforms can be computed using the fast method described here, but will require filter parameters which are difficult to compute. However, the

first-order Hankel transform can be computed using the existing zero-order Abel filter.

For seismic modeling of total wavefields, this algorithm significantly reduces computation time compared to either direct integration or the Fourier-Bessel expansion method. Studies also suggest that this fast algorithm is sufficiently accurate.

ACKNOWLEDGMENTS

Thanks go to Dr. Dave Hale for his recommendation of the paper, patient guidance and improvement of the coding. I also thank Dr. Steve Pruess for his help on the MINPACK package to compute parameters of the filter. Thanks also to Dr. M. Dietrich for his helpful discussion and suggestions. A lot of thanks also go to Dr. Ken Larner and Justin Hedley for their patience and time on critiquing this paper. I also thank all the members of CWP for their encouragement and help. This project is supported by Department of Energy of the United States.

REFERENCES

- Bracewell, R.N., 1965, The Fourier transform and its applications: McGraw-Hill, N.Y.
- Dietrich, M. 1990, An algorithm for the plane-wave decomposition of point-source seismograms: *Geophysics*, 55, 1380-1385.
- Hansen, E.W., 1978, Optical reconstruction from projections via circular harmonic expansion: *Opt. Commun.*, 24, 268-272.
- Hansen, E.W., 1982, State variable representation of a class of linear shift-variant systems: *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-30, 874-880.
- Hansen, E.W., 1985, Fast Hankel transform algorithm: *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-33, 666-671.
- Kim, H.J., 1989, A Fast algorithm for computing the Hankel transform of order 1, *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-37, 1291-1293.
- Korn, M. 1987, Computation of wavefields in vertically inhomogeneous media by a frequency-domain finite-difference method, and application to wave propagation in earth models with random velocity and density perturbations, *Geophysics J.R.Astr.Soc.* 88, 345-377.
- Mook, D.R. 1983, An algorithm for the numerical evaluation of the Hankel and Abel transforms: *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-31, 979-985.
- Watson, G.N. 1952, A treatise on the theory of Bessel functions: Cambridge, University Press, p. 576.
- Wilson, C.R. 1986, The Abel-Fourier method of Hankel transformation :Applications to seismic data, *Geophysical Prospecting* 545-568.



The Constant Traveltime Problem in a Medium with a Linear Velocity Gradient

by

Michel Dietrich and Jack K. Cohen

A constant traveltime curve in a medium with a linear velocity gradient is represented by a fourth-degree polynomial in x and z . This curve reduces to the well-known ellipse in the constant-velocity case, and to the circular waveform obtained by Slobnick in the coincident source-receiver case. For sufficiently large source-receiver separations and positive velocity gradients, the traveltime curve develops a secondary branch that corresponds to near-surface reflections. A parametric representation of the curve is given, characterizing the zero-offset ray in dip moveout processing is also given.

INTRODUCTION

The concept of constant traveltimes has been introduced with nearly migration methods based on geometrical principles. The sections used in seismic processing are the locus of all reflection points corresponding to a given traveltime T between a source site a receiver. In a homogeneous medium, the sections are ellipses for non-zero source-receiver separations, and circles in the coincident source-receiver case. The simplicity of these curves stems from the fact that the traveltimes can be directly converted into distances in a homogeneous medium.

The relationship between traveltimes and distances is more complex in heterogeneous media, because of the refraction of the rays. However, Slobnick (1989) showed that there is still a simple expression in the zero-offset case when the velocity is linearly increasing or decreasing with

Center for Wave Phenomena
Colorado School of Mines
Golden, Colorado 80401
(303)273-3557