Eqn 16(c) of Losa paper:

$$2\int_{R_0}^{R} \Phi(R,r)\, \underbrace{h_n f(r)}_{=f(R_0)\ (ZOH)}\, dr$$

$$= 2h f(R_0) \int_{R_0}^{R} \left(\frac{r}{R}\right)^{\lambda} dr$$

$$= 2h f(R_0) \frac{R^{\lambda+1} - R_0^{\lambda+1}}{(\lambda+1) R^{\lambda}}$$

$$= 2h f(R_0) \frac{R\left[1 - \left(R_0/R\right)^{\lambda+1}\right]}{(\lambda+1)}$$

Substitute $R_0 = (N-n)\Delta$, $R = (N-n-1)\Delta$

$$\Rightarrow 2h f(R_0) \frac{(N-n-1)\Delta}{\lambda+1}\left[1-\left(\frac{N-n}{N-n-1}\right)^{\lambda+1}\right]$$

For the inverse Abel, there's an additional factor of $\frac{1}{\pi r}$ in front of the $g'(r)$. The integral becomes

$$2h\, g'(R_0) \int_{R_0}^{R} \left(\frac{r}{R}\right)^{\lambda} \frac{1}{\pi r}\, dr$$

$$= \frac{2h\, g'(R_0)}{\pi R} \int_{R_0}^{R} \left(\frac{r}{R}\right)^{\lambda-1} dr$$

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which is why, when $\lambda = 0$, you get a log result.

$$\int_{R_0}^{R} \left(\frac{r}{R}\right)^{\lambda - 1} dr = R \ln r \Big|_{R_0}^{R}$$

$$= R \ln(R/R_0)$$

$$\Rightarrow \frac{2h\, g'(R_0)}{\pi} \ln\left(\frac{R}{R_0}\right) \qquad \lambda = 0$$

and, for $\lambda \neq 0$, same integral as forward case,

$$2h\, g'(R_0) \frac{\cancel{R}\left[1 - (R_0/R)^{\lambda}\right]}{\pi \cancel{R} \lambda}$$

$$= 2h\, g'(R_0) \frac{1 - (R_0/R)^{\lambda}}{\pi \lambda} \quad \ldots$$

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