Sensor Errors (for IMUs):

1) Bias: (nearly) constant offset from zero (additive)

2) Scale Factor: multiplicative "stretching"

3) Misalignment: Errors in orientation of sensor axes due to mounting + manufacturing

4) Nonlinearity: usually small - we will ignore it

5) Temperature: All of the above are temperature dependent due to thermal expansion / contraction.

6) Noise: Additive white/Gaussian noise

Linear Error Model:

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\text{sensor output}} = \underbrace{\begin{bmatrix} 1+s_1 & 0 & 0 \\ 0 & 1+s_2 & 0 \\ 0 & 0 & 1+s_3 \end{bmatrix}}_{\text{Scale Factor}} \underbrace{\begin{bmatrix} 1 & m_{12} & m_{13} \\ m_{21} & 1 & m_{23} \\ m_{31} & m_{32} & 1 \end{bmatrix}}_{\text{Misalignment}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\text{actual vector}} + \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{\text{bias}} + \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}_{\text{noise}}$$

$$= (I + T)x + b + v$$

$$T \approx \begin{bmatrix} s_1 & m_{12} & m_{13} \\ m_{21} & s_2 & m_{23} \\ m_{31} & m_{32} & s_3 \end{bmatrix}$$ (assuming small errors)

Data Sheet Specs:

- "Cross-Axis Sensitivity" → m_{ij}
- "Gain Error" or "Sensitivity Change" → s_i
- "Zero-rate Offset" → $\|b\|$

Noise:

- We typically like to work with Additive White Gaussian Noise (AWGN)
- AWGN is only ever an approximation to reality
- AWGN has zero mean:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} v_k = 0$$

Samples drawn from a Gaussian Distribution

- AWGN has a Covariance:

$$\lim_{N\to\infty} \frac{1}{N-1} \sum_{n=1}^{N} v_n v_n^T = Q$$

- We write this as $v_k \sim \mathcal{N}(0, Q)$

normal distribution

covariance Q

zero mean

- If Q is diagonal (elements of v are "uncorrelated") then the elements are the squares of the standard deviations:

$$Q = \begin{bmatrix} \sigma_{11}^2 & 0 & 0 \\ 0 & \sigma_{22}^2 & 0 \\ 0 & 0 & \sigma_{33}^2 \end{bmatrix}$$

- The integral of AWGN is a "random walk" or "Brownian Motion":

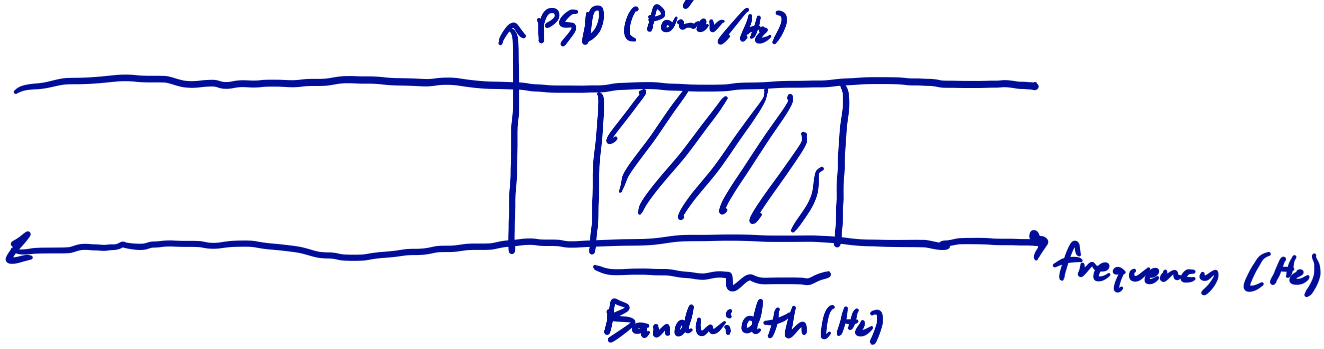
$$w_n = \sum_{k=1}^{n} v_k \sim N(0, nQ)$$

- The distance from the origin increases like $\sqrt{n}$

$$\|w_n\| = \sqrt{w_n^T w_n} \simeq \sqrt{n\,Tr(Q)}$$

- We model a slowly-varying sensor bias as a random walk.

* Noise Power Spectral Density



- The more bandwidth (the faster we sample) the more noise we get.

Another Look at Some Data Sheets:

Gyro Noise PSD $\approx 0.015^\circ/\text{sec}/\sqrt{\text{Hz}}$

$\Rightarrow$ at 10 Hz $\quad Q_{\omega\omega} = (0.015^\circ/\text{sec}/\sqrt{\text{Hz}})^2 \times 10\ \text{Hz}$

$$\approx 0.0022\ \text{deg}^2/\text{sec}^2$$

$$\Rightarrow V_n = \sqrt{0.0022}\ \underbrace{\text{randn}(3)}_{\text{samples drawn from } \mathcal{N}(0,1)}$$

- Angle Random Walk (ARW) is another way of specifying noise PSD for a gyro that you will see on gyro data sheets.

* Gyro Bias Stability $\approx 3^\circ/\text{hour}$
  - This is the std. dev. in b after ≈ 100 sec.

$$\Rightarrow Q_{bb} \approx \frac{(3^\circ/\text{hour})^2}{(3600)^2} \times \left(\frac{\text{Samp. period (s)}}{100\ \text{s}}\right)\ \text{deg}^2/\text{sec}^2$$

* These noise covariances based on the data sheet are only approximations and typically must be tuned in practice.