The University of Texas at Austin
Oden Institute for Computational Engineering and Sciences

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PyDMD - DYNAMIC MODE DECOMPOSITION IN PYTHON

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Main contributors and maintainers

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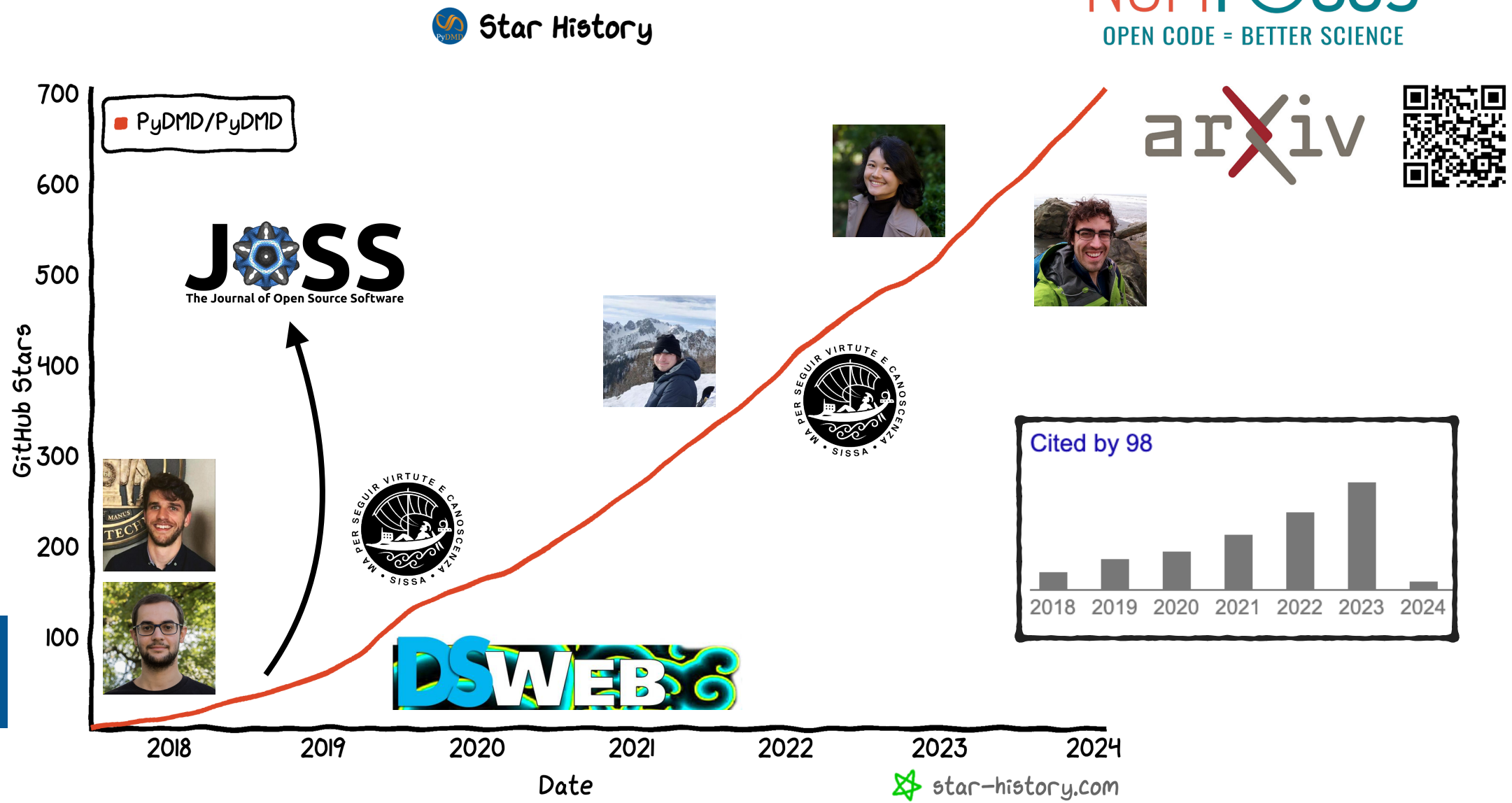
Francesco Andreuzzi

Sara Ichinaga

FAST COMPUTING

Historical perspective



NUMFOCUS
OPEN CODE = BETTER SCIENCE

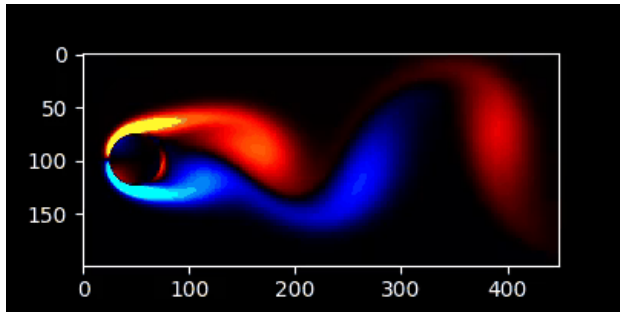
JOSS
The Journal of Open Source Software

arXiv

PyDMD

Dynamic mode decomposition at a glance

Collect data

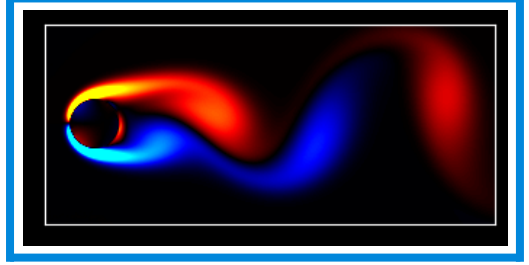


Build data matrices

$$X = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix}$$

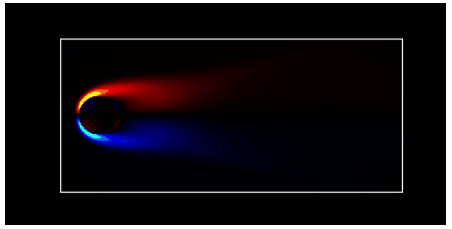
$$X' = \begin{bmatrix} | & | & & | \\ x_2 & x_3 & \cdots & x_m \\ | & | & & | \end{bmatrix}$$

$t = t_2$

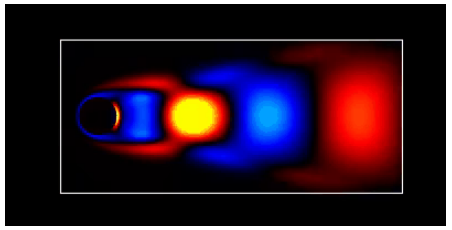


$$\mathrm{A} = X'X^{\dagger}$$
$$\mathrm{A}\Phi = \Phi\Lambda$$

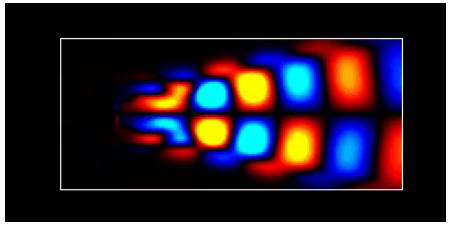
Obtain diagnostics



$f_1 = 0$
(λ_1, φ_1)



$f_2 = 0.208$
(λ_2, φ_2)



$f_4 = 0.416$
(λ_4, φ_4)

$$x_{k+1} = \sum_{j=1}^{r} \varphi_j \lambda_j^k b_j = \Phi \Lambda^k b$$

Schmid, JFM 2010.

Rowley, Mezic, et al., JFM 2009.

Tu, Rowley, et al., JCD 2014.

Kutz, Brunton, et al., SIAM 2016.

DMD algorithm

1. Collection of the snapshots

- Creation of two matrices: snapshots (**X**) and time-shifted snapshots (**Y**)

2. Eigenvalues and modes computation

- Approximation of the operator **A** $\mathbf{Y} \approx \mathbf{A}\mathbf{X}$
- SVD of the snapshots matrix $\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$
- Reduction of the operator $\tilde{\mathbf{A}} = \mathbf{U}^*\mathbf{Y}\mathbf{V}\mathbf{\Sigma}^{-1}$
- Eigendecomposition $\tilde{\mathbf{A}}\mathbf{W} = \mathbf{W}\mathbf{\Lambda}$
- Computation of the modes $\mathbf{\Phi} = \mathbf{Y}\mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{W}$ $\mathbf{\Phi} = \mathbf{U}\mathbf{W}$

3. Reconstruction and future state predictions

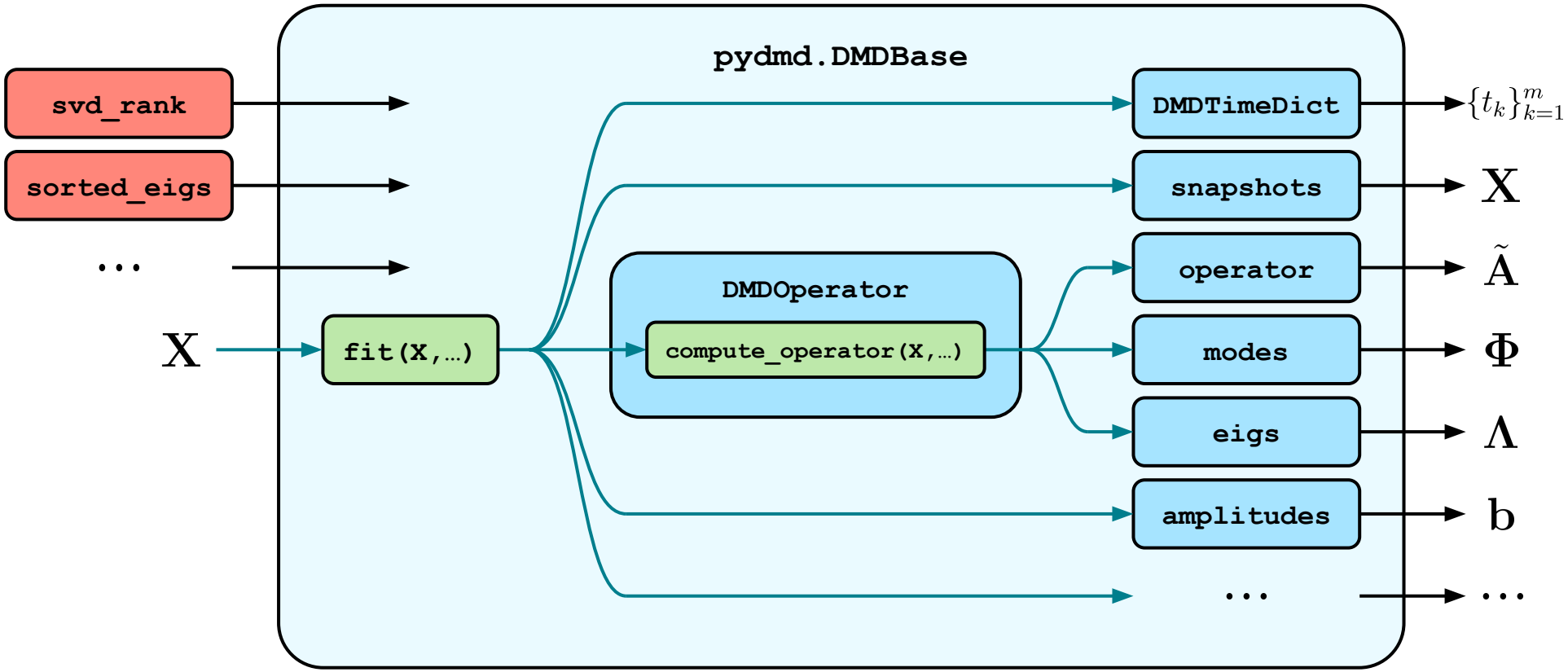
- Computation of the amplitudes **b** $\mathbf{b} = \mathbf{\Phi}^{\dagger}\mathbf{x}_0$
- Forecasting $\mathbf{x}_k = \mathbf{\Phi}\mathbf{\Lambda}^k\mathbf{b}$

DMDBase class

DMDBase

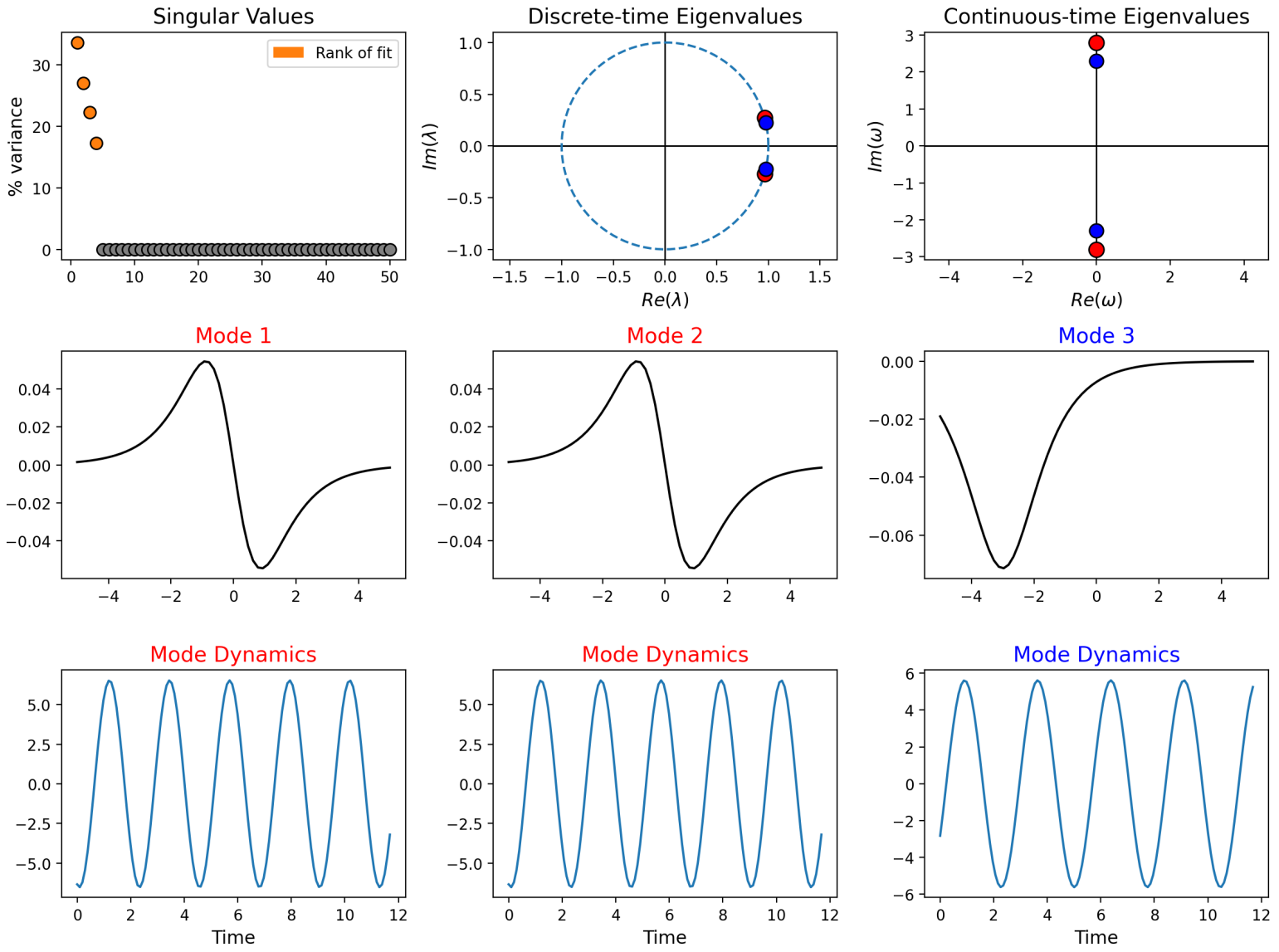
Base module for the DMD: *fit* method must be implemented in inherited classes

class **DMDBase**(*svd_rank=0, tlsq_rank=0, exact=False, opt=False, rescale_mode=None, forward_backward=False, sorted_eigs=False, tikhonov_regularization=None*) [source]

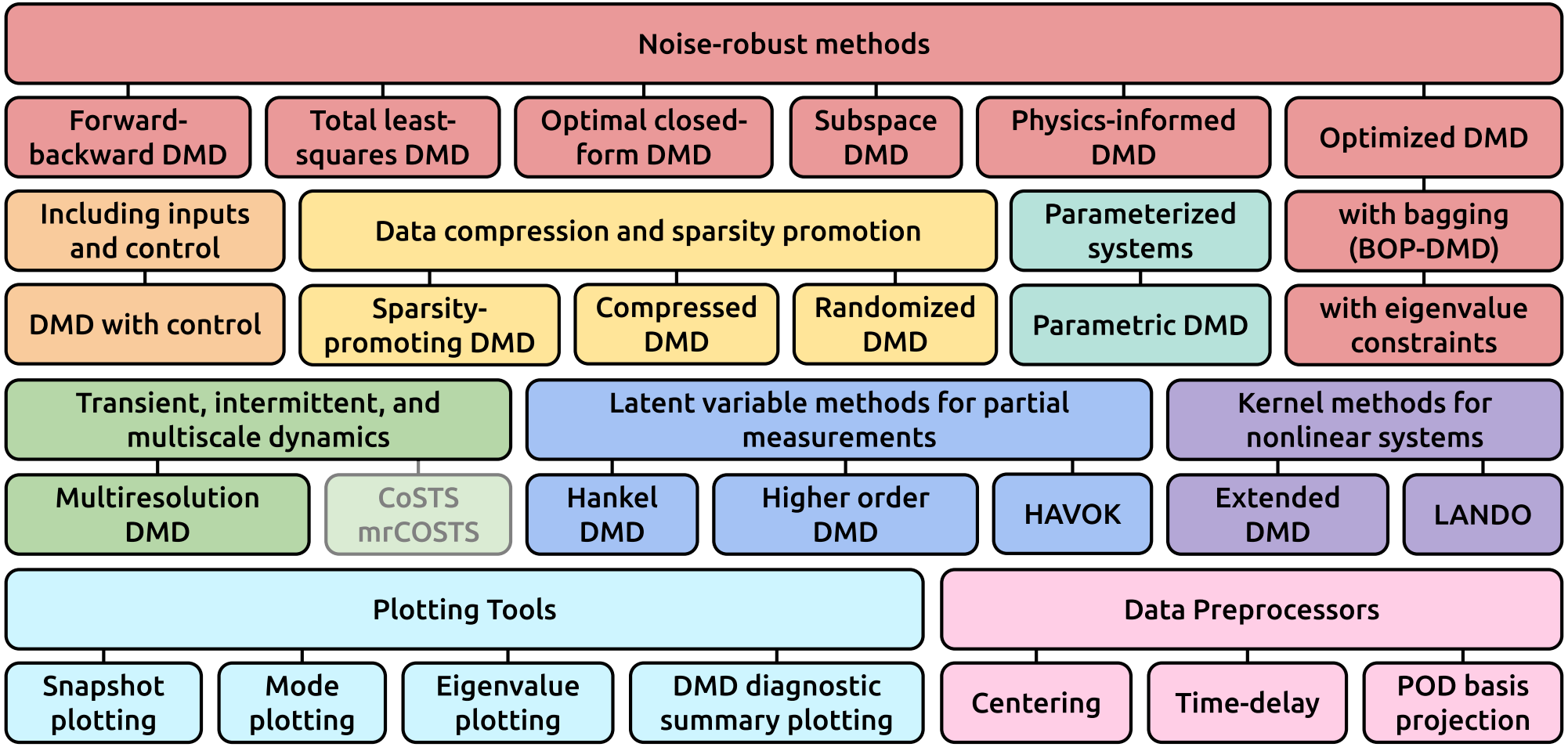


Plotting capabilities for diagnostics and data exploration

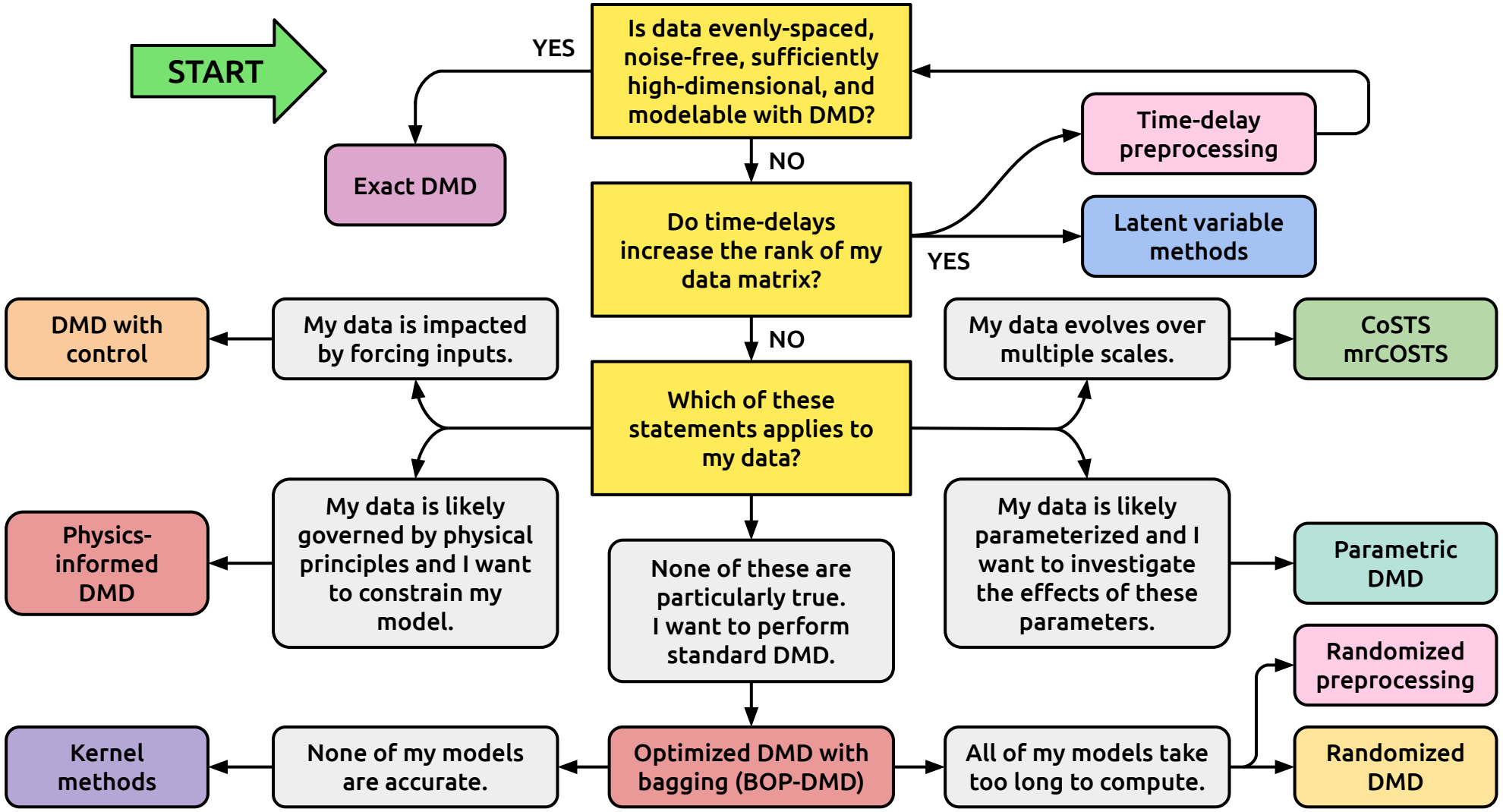
```
def plot_summary(
    dmd,
    *,
    x=None,
    y=None,
    t=None,
    d=1,
    continuous=False,
    snapshots_shape=None,
    index_modes=(0, 1, 2),
    filename=None,
```



Current capabilities

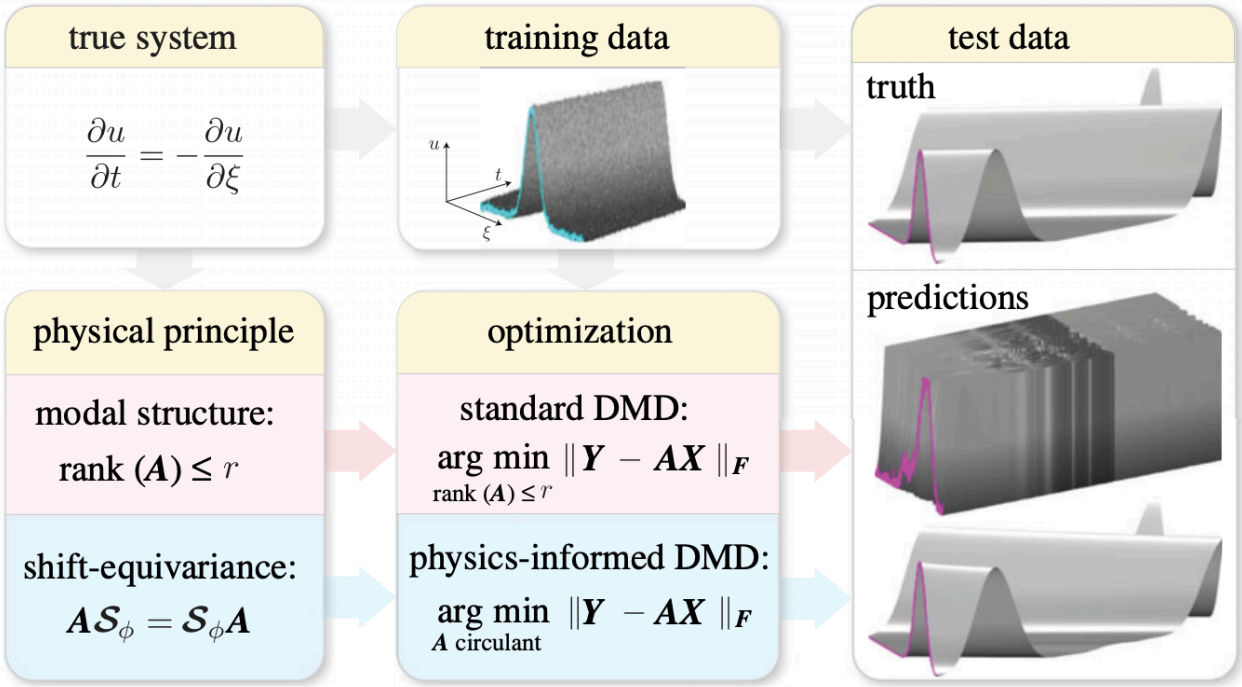


Which method should you use?

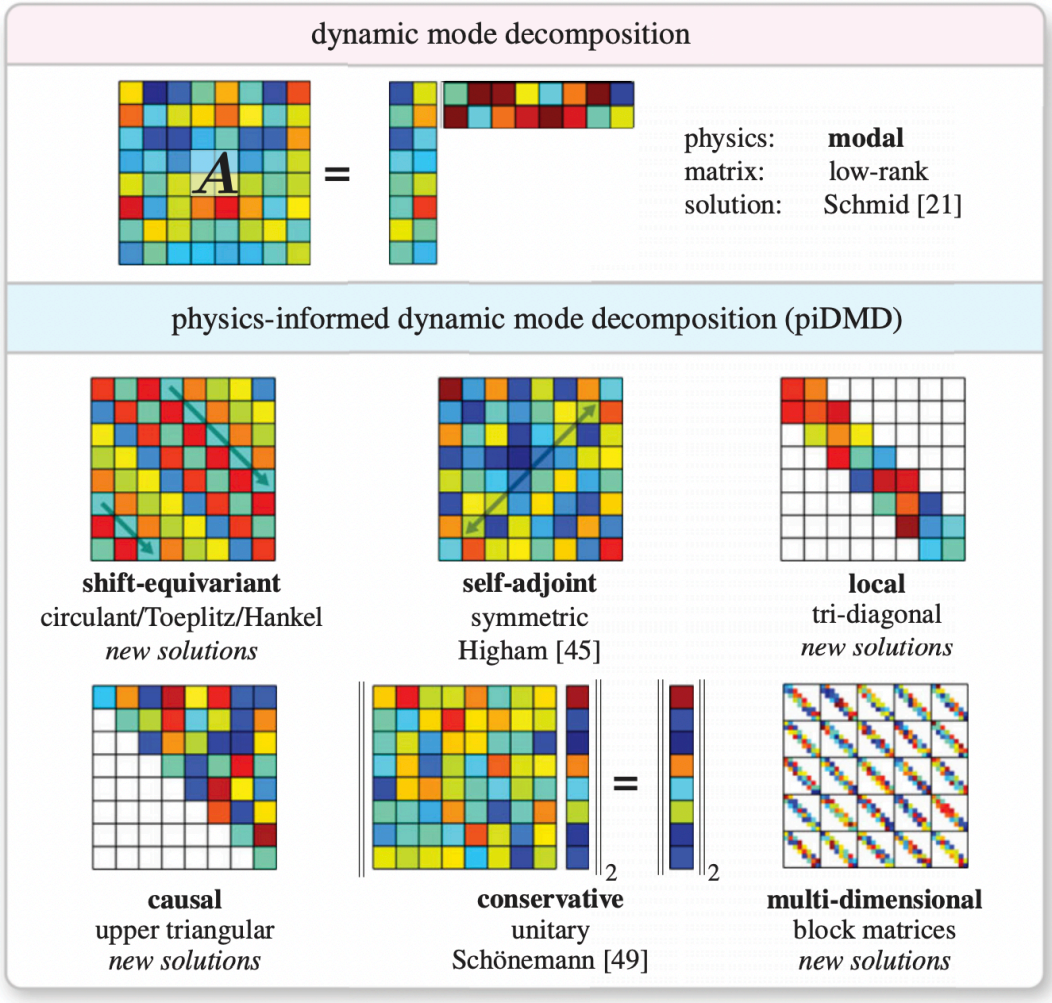


Physics-informed DMD

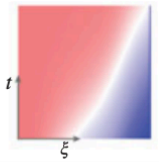
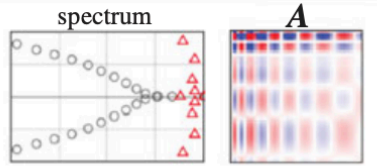
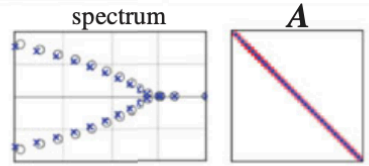
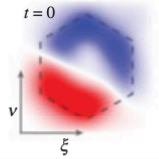
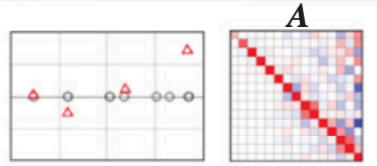
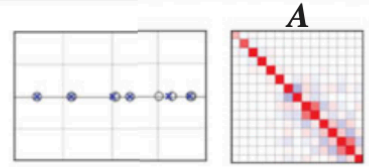
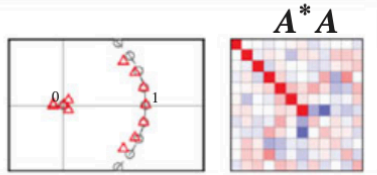
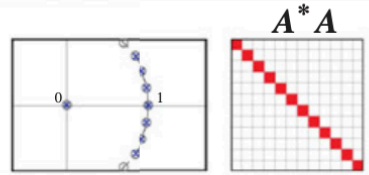
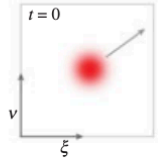
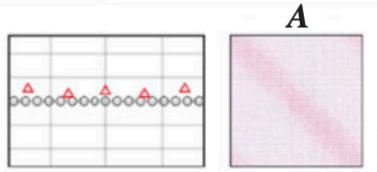
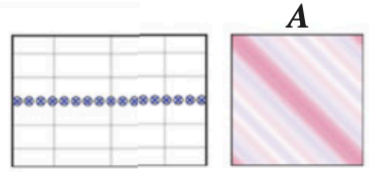
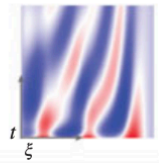
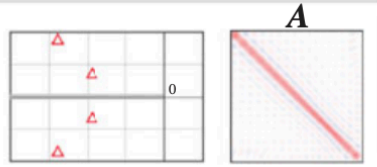
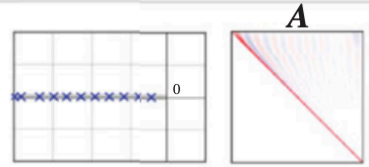
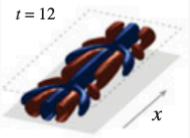
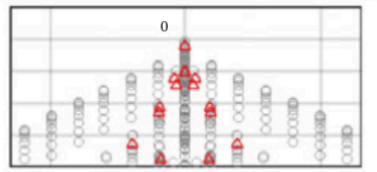
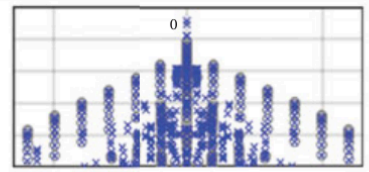
class **PiDMD**(*manifold, manifold_opt=None, compute_A=False, svd_rank=-1, tlsq_rank=0, opt=False*)



Baddoo et al., Proc. R. Soc. A, 2023



Different system models result in different matrix structures you can impose

system	physics (*matrix structure*)	data	standard DMD	physics-informed DMD
advection-diffusion $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial \xi^2} - v(\xi)\frac{\partial u}{\partial \xi}$	local (*tri-diagonal*)			
Schrödinger equation $i\hbar\frac{d}{dt}\lvert\psi(t)\rangle = \hat{H}\lvert\psi(t)\rangle$	self-adjoint (*Hermitian*)			
cylinder flow $\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = \frac{1}{Re}\nabla^2\boldsymbol{u} - \nabla p$	conservative (*unitary*)			
two-dimensional advection $\frac{\partial u}{\partial t} = -\boldsymbol{c}\cdot\nabla u$	shift-equivariant (*block circulant*)			
Volterra integro-differential equation $\frac{\partial u}{\partial t} = \int_{-1}^{\xi} K(\xi, v)\, u\,(v, t)\, dv$	causal (*upper triangular*)			
channel flow $\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = \frac{1}{Re}\nabla^2\boldsymbol{u} - \nabla p$	shift-equivariant (*block circulant*)			

Baddoo et al., 2023

Code documentation

About

Python Dynamic Mode Decomposition

pydmd.github.io/PyDMD/

- DMDBase
- DMD Operator
- DMD
- BOPDMD: Optimized DMD and Bagging, Optimized DMD
- Compressed DMD
- DMD Modes Tuner
- DMD with control
- Kernelized extended DMD
- Forward-backward DMD
- Hankel DMD
- Havok
- Higher-order DMD
- Multi-resolution DMD

- OptDMD
- Parametric DMD
- Plotter
- Physics-informed DMD
- Randomized DMD
- Sparsity-promoting DMD
- Subspace DMD
- VarProDMD: Variable Projection for DMD
- Utilities

Tutorials

Tutorials for the users (almost 20)

Name	Description	PyDMD used classes
Tutorial1 [.ipynb, .py, .html]	Analyzing real, simple data sets with PyDMD	`pydmd.DMD` , `pydmd.BOPDMD`
Tutorial2 [.ipynb, .py, .html]	advanced features of standard DMD	`pydmd.DMD`
Tutorial3 [.ipynb, .py, .html]	multi-resolution DMD for transient phenomena	`pydmd.MrDMD`
Tutorial4 [.ipynb, .py, .html]	compress DMD for computation speedup	`pydmd.CDMD`

...

Tutorial for the developers

Name	Description	PyDMD used classes
Tutorial1 [.ipynb, .py, .html]	implementing a new version of DMD	`pydmd.DMDBase`