Non-Linear Systems

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This is a python function which takes as parameters, the initial population x_0, the growth parameter r, and the number of generations N, and returns an array containing the N successive population values. This will be done using the May equation, and display the results on a graph. This graph will display 50 successive population values for two different initial population values, which we will then use a while-loop determine after how many generations the populations diverge by 20%.

In a population model, successive populations are related by the May equation:

$$X_{i+1} = x_i e^{r(1-x_i)}$$

In this particular population model, our initial growth parameter r is set to 3.0 and the number of genertions N is set to 50.

The first population will have an initial population of $X_0 = 2.00000$. The second population will have an initial population of $X_0 = 2.00001$.

- Input function to determine number of steps required
- Input intial conditions
- Calculate the populations using the May equation
- Calculate the number of populations at which they diverge by 20%
- Output number of generations
- Output a plot of population 1 and population 2 on a graph

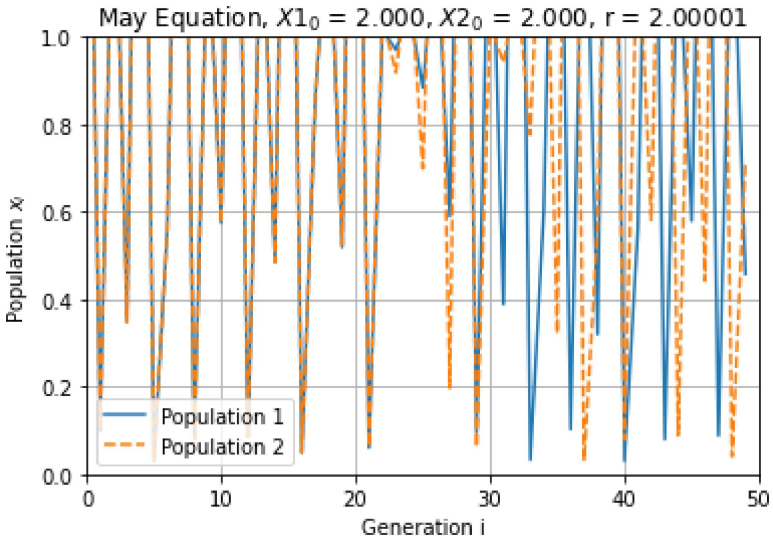
In [3]:
```
# Function to calculate N generations of the May equation
# with growth parameter r and initial population X_0
# Returns an array X containing the populations

def logistic(X_0, r, N):

    import numpy as np
    X = np.zeros(N)
```

```
    X[0] = X_0

    for i in range(N - 1):
        X[i+1] = X[i] * np.exp(r*(1 - X[i]))

    return X
```

In [5]:

```
def main():

    import matplotlib.pyplot as plt

    X1_0 = 2.00000   # set the initial population of Population 1
    X2_0 = 2.00001   # set the initial population of Population 2
    r = 3.0          # set the growth parameter
    N = 50           # set the number of generations

    # Call the function which calculates the populations of Population 1
    X1 = logistic(X1_0, r, N)

    # Call the function which calculates the populations of Population 2
    X2 = logistic(X2_0, r, N)

    # Determine after how many generations the populations diverge by 20%
    i = 0
    while abs(((X1[i] - X2[i]) / X1[i])) < 0.2:
        i += 1
    print("Population diverges by 20% after " +  str(i) + " generations")

    # Plot population against generation
    plt.plot(range(N), X1)
    plt.plot(range(N), X2, linestyle = 'dashed')
    plt.xlim(0, N)
    plt.ylim(0, 1.0)
    plt.legend(['Population 1', 'Population 2'], loc ="lower left")
    plt.xlabel("Generation i")
    plt.ylabel("Population $x_i$")
    plt.title("May Equation, $X1_0$ = {0:5.3f}, $X2_0$ = {0:5.3f}, r = {1}".format(X1_0, X2_0, r))
    plt.grid()

main()
```

```
Population diverges by 20% after 25 generations
```


May Equation, $X1_0$ = 2.000, $X2_0$ = 2.000, r = 2.00001
Population x_i
Generation i
Population 1
Population 2
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50