Pump Similarity Laws Model

1   OVERVIEW

This model predicts the performance of a pump under different operating conditions using its **characteristic curves** and the **similarity laws**.

It estimates the **volumetric flow rate**, **mass flow rate**, and **shaft power** of a pump based on user inputs such as pressure, temperature, speed, and working fluid.

The aim of the model is to extrapolate the pump behavior at off-design conditions using known performance curves at a reference speed and fluid.

2   INPUTS, OUTPUTS, PARAMETERS

Three different mode exists for this model depending on the available inputs.
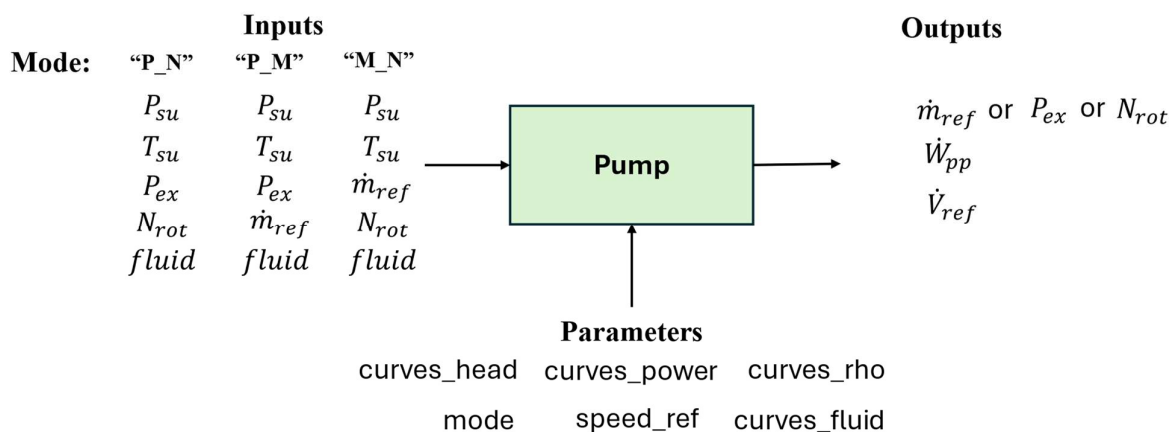

Figure 1: Indicator diagram of the pump model.

Where:

- curves_head: Data points from the pump's head-flow characteristic curve provided in the datasheet.
- curves_power: Data points from the pump's power-flow characteristic curve provided in the datasheet.
- curves_fluid: Reference fluid used for generating the characteristic curves
- curves_rho: The density at which the characteristic curves were measured or specified.
- speed_ref: The rotational speed at which the characteristic curves were measured or specified.

Example:

In Figure 2, the points taken from the characteristic curve are shown.

```
curves_head = [(0, 66), (0.5, 56), (1, 46), (1.5, 36), (2, 26), (3, 6), (3.3, 0)] # ([m^3/h], [m])
```

```
curves_power = [(0, 1030), (1, 900), (1.5, 830), (2, 760), (3, 620), (3.5, 550)]  # ([m^3/h], [W])
```

All of those points are for a speed reference of 2900 RPM and a water temperature of 20°C.
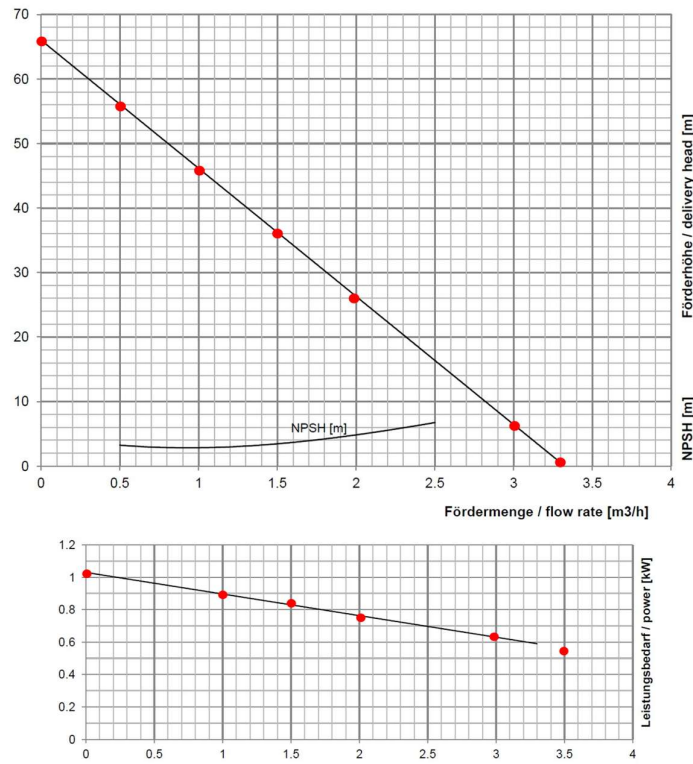
Figure 2: Example of a characteristic curve provided by the manufacturer.

3   MODEL DESCRIPTION

3.1   Interpolation of Characteristic Curves

```
def _prepare_interpolators(self):
    """
```

The first step is to fit polynomials to the provided *curves_head* (flow, head) and *curves_power* (flow, power) points from the characteristic curves at the reference speed.
This is done using **NumPy's polyfit** and poly1d, resulting in continuous equations for head and power as a function of flow rate.

3.2   Mode = 'P_N'

If the mode is P_N, meaning that the exhaust pressure and rotational speed are known, the sequence for solving is the following:

1. Fluid Similarity Correction:

If the actual working fluid differs from the one used in the characteristic curves, a correction must be made to account for the change in **fluid density**:

Pressure difference:

$$\Delta P_{actual} = P_{ex} - P_{su} \tag{1}$$

$$\Delta P_{curve} = \Delta P_{actual} \cdot \frac{\rho_{curve}}{\rho_{actual}} \tag{2}$$

Head calculation:

$$H_{curve} = \frac{\Delta P_{curve}}{\rho_{curve} \cdot g} \tag{1}$$

1

Where:

- actual: stands for the fluid of the actual application
- curve: stands for the fluid used in the characteristic curve
- ρ = fluid density [kg/m³]
- g=9.81 $\frac{m}{s^2}$

This gives the corrected head to be used on the reference curve.

2. Flow Determination:

The volumetric mass flow rate corresponding to a given H_{curve} is obtained by solving the polynomial head curve for its flow rate variable. This is done using a numerical root-finding algorithm, specifically the bisection method.

3. Flow Similarity Law:

Using the similarity law, the flow is adjusted to the actual speed:

$$\dot{V}_{actual} = \dot{V}_{curve} \cdot \frac{N_{actual}}{N_{ref}} \qquad (5)$$

4. Power Similarity Law:

There are two corrections applied to the reference power:

- **Speed correction:**

$$\dot{W}_{actual} = \dot{W}_{curve} \cdot \left(\frac{N_{actual}}{N_{ref}}\right)^3 \qquad (6)$$

- **Fluid correction:**

$$\dot{W}_{actual} = \dot{W}_{curve} \cdot \frac{\rho_{actual}}{\rho_{curve}} \qquad (7)$$

These account for changes in both speed and fluid density.

3.3 Mode = 'P_M'

If the mode is P_M, meaning that the exhaust pressure and mass flow rate are known, the sequence for solving is the following:

1. Fluid Similarity Correction:

If the actual working fluid differs from the one used in the characteristic curves, a correction must be made to account for the change in **fluid density**. Same as for the precedent sequence, the head corresponding to the characteristic curve is found.

2. Flow Determination:

The volumetric mass flow rate corresponding to a given H_{curve} is obtained by solving the polynomial head curve for its flow rate variable. This is done using a numerical root-finding algorithm, specifically the bisection method.

3. Speed Similarity Law:

Using the similarity law, the actual speed is computed:

$$N_{actual} = N_{ref} \cdot \frac{\dot{V}_{actual}}{\dot{V}_{curve}} \qquad (5)$$

5. *Power Similarity Law:*

The actual power is computed based on the similarity laws.

3.4 Mode = 'M_N'

If the mode is M_N, meaning that the mass flow rate and the rotational speed are known, the sequence for solving is the following:

1. *Determine flow curve:*

Based on the volumetric mass flow rate, the volumetric mass flow rate from the curve can be found:

$$\dot{V}_{curve} = \dot{V}_{actual} \cdot \frac{N_{ref}}{N_{actual}} \qquad (5)$$

2. *Determine head curve:*

Based on the polynomial curve H_curve(V_dot_curve), the head can be found based on V_dot_curve.

3. *Determine pressure:*

Based on the head, and applying the similarity laws for speed:

$$\Delta P_{curve} = Head_{curve} \cdot \rho_{curve} \cdot g \cdot \frac{N_{actual}}{N_{ref}} \qquad (2)$$

And correcting for the density difference between both fluids:

$$\Delta P_{actual} = \Delta P_{curve} \cdot \frac{\rho_{actual}}{\rho_{curve}} \qquad (3)$$

4. *Determine power:*

The actual power is computed based on the similarity laws.

4   FUTURE ENHANCEMENTS

- Efficiency calculation
-