HDDA Tutorial: Matrices and Factor Analysis

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Tutorial 9

Consider the factor model

$$\mathbf{y} = \mathbf{\Lambda f} + \boldsymbol{\xi}$$

where $\mathbf{y}$ is a $p \times 1$ vector of observed variables, $\mathbf{f}$ is an $r \times 1$ vector of latent factors with $r < p$, $\mathbf{\Lambda}$ is a matrix of loadings and $\boldsymbol{\xi}$ is a $p \times 1$ vector of idiosyncratic errors variables. Also assume that $\mathbf{f} \sim N(\mathbf{0}, \mathbf{I})$ and $\boldsymbol{\xi} \sim N(\mathbf{0}, \mathbf{\Psi})$.

1. What are the dimensions of $\mathbf{\Lambda}$?
2. What is the expected value of $\mathbf{y}$?
3. Derive the expected variance covariance matrix of $\mathbf{y}$. Hint, you can use a rule of matrices that $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$