VOLATILITY CLUSTERING, NEW HEAVY-TAILED DISTRIBUTION AND THE STOCK MARKET RETURNS IN SOUTH KOREA

Yoon Hong, PhD, Research Fellow
Department of Economics
Hanyang University, South Korea
Ji-chul Lee, PhD, Assistant Professor
Department of Economics
Dongseo University, South Korea
Guoping Ding, PhD, Assistant Professor
School of Business
Nanjing University, China

Introduction

Since the establishment of the Republic of Korea, the economy of South Korea has received great success. The country emerged from a very poor underdeveloped economy to one of the most developed economies. Some South Korean brands become internationally famous with products sold across the nations. The stock market is one of the crucial components of the nation's economy. The South Korea Stock Exchange is the 14th largest exchange over the world with market capitalization is $1.33 Trillion adjusted US Dollars as of March 2017. The market capitalization to GDP ratio is 110.64%, which indicates the importance of the financial market. In this paper, we make use of the Korea Composite Stock Price Index (KOSPI). KOSPI is the index of all common stocks traded on the South Korea Exchange. It is a very popular market index and the representative stock market index of South Korea.

During the past several decades, two stylized facts for asset returns have been frequently presented by researchers: heavy tails and volatility clustering. The heavy tails indicate that historical distribution of the asset returns has heavier tails than the standard normal distribution, and the volatility clustering says that high-volatility events tend to cluster in time. In the following, we re-investigate the two stylized facts, heavy tails and volatility clustering, but are interested in the stock market returns on the South Korea Exchange. Our framework is similar as in Guo (2017a, 2017b), and we compare empirical performance of the two different types of heavy-tailed distribution, Student's *t*, and normal reciprocal inverse Gaussian (NRIG), within the generalized autoregressive conditional heteroskedasticity (GARCH) framework for the daily KOSPI returns. We are particularly interested in the risk management aspect. Our results indicate two important findings: i) the daily KOSPI returns exhibit conditional heavy tails even after volatility clustering effect has been accounted for; and ii) the NRIG distribution has a better in-sample performance than the Student's *t* distribution.

Literature Review

Since the economy miracle of South Korea, its stock market has also received considerable attentions from the academia. Many researchers have applied GARCH based models in analyzing the stock market in South Korea. However, most of the studies are either focusing on the effects of some special events or the inter-linkage with stock markets in some other countries. Sola et al. (2002) proposed a bivariate Marrkov-GARCH model to analyze volatility links between three different emerging markets. Choudhry (2000) investigated the day of the week effect on seven emerging Asian stock markets returns and conditional variance (volatility), including South Korea, using the GARCH model. Choudhry (2000) found that there was significant presence of the day of the week effect on both stock returns and volatility, though the result involving both the return and volatility are not identical in all seven cases. Lee (2009) examines the volatility spillover effects among six Asian country stock markets, including the stock market in South Korea, using bivariate vector auto-regression generalized autoregressive conditional heteroskedasticity (VAR-GARCH) model. Lee (2009) found that there were statistically significant volatility spillover effects within the stock markets of these countries. Fang (2002) used stock daily data for South Korea and other four Asian countries, and showed that currency depreciation affected adversely stock returns and/or increases market volatility over the period of the Asian crisis 1997–1999.

In this paper, we re-consider the GARCH framework but focus on risk management of the stock market returns in South Korea. We focus on two different types of heavy-tailed distributions: the Student's *t* distribution and the normal reciprocal inverse Gaussian distribution. In 1987, Bollerslev introduced the Student's *t* distribution and the GARCH model with the Student's *t* distribution could capture dynamics of a variety of foreign exchange rates and stock price indices returns. Politis (2004) introduced the truncated standard normal distribution into the ARCH model and showed the empirical performance of the new type of heavy-tailed distribution on three real datasets. Tavares et al. (2008) model the heavy tails and asymmetric effect on stocks returns volatility into the GARCH framework, and showed the Student's t and the stable Paretian with (α < 2) distribution clearly outperform the Gaussian distribution in fitting S&P 500 returns and FTSE returns. Su and Hung (2011) provides a comprehensive analysis of the possible influences of jump dynamics, heavy-tails, and skewness with regard to Value at Risk (VaR) estimates through the assessment of both accuracy and efficiency. Su and Hung (2011) consider a range of stock indices across international stock markets during the period of the U.S. Subprime mortgage crisis, and show that the GARCH model with normal, generalized error distribution (GED) and skewed normal distributions provide accurate VaR estimates.

In this paper, we follow the model framework in Guo (2017a, 2017b) and are particularly interested in the NRIG distribution, a newly-developed heavy-tailed distribution. Our focuses are on their empirical performance in fitting the stock market returns in South Korea. The remainder of the paper is organized as follows. In Section 2, we discuss GARCH models and the heavy-tailed distributions. Section 3 summarizes the data. The estimation results are in Section 4. Section 5 concludes.

The Models

We consider a simple GARCH(1,1) process as:

$$\varepsilon_t = \mu + \sigma_t e_t, \tag{1}$$

$$\sigma_t^{\,2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\,2} + \beta_1 \sigma_{t-1}^{\,2}, \tag{2}$$

where the three positive numbers α_0, α_1 and β_1 are the parameters of the process and $\alpha_1 + \beta_1 < 1$. The assumption of a constant mean return μ is purely for simplification and reflects that the focus of the paper is on dynamics of return volatility instead of dynamics of returns. The variable e_t is identically and independently distributed (*i.i.d.*). Two types of heavy-tailed distributions are considered: the Student's *t* and the normal reciprocal inverse Gaussian (NRIG) distributions. The density function of the standard Student's *t* distribution with ν degrees of freedom is given by:

$$f(e_t \mid \psi_{t-1}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^{\,2}}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \ \nu > 4. \tag{3}$$

where ψ_{t-1} denotes the σ-field generated by all the available information up through time $t-1$.

The NRIG is a special class of the widely-used generalized hyperbolic distribution. The generalized hyperbolic distribution is specified as in Prause (1999):

$$f(e_t \mid \lambda, \mu, \alpha, \beta, \delta) = \frac{(\sqrt{\alpha^2 - \beta^2} / \delta)^{\lambda} K_{\lambda-1/2}(\alpha\sqrt{\delta^2 + (e_t - \mu)^2})}{\sqrt{2\pi}(\sqrt{\delta^2 + (e_t - \mu)^2} / \alpha)^{1/2-\lambda} K_{\lambda}(\delta\sqrt{\alpha^2 - \beta^2})} \exp(\beta(e_t - \mu)), \tag{4}$$

where $K_{\lambda}(\cdot)$ is the modified Bessel function of the third kind and index $\lambda \in \mathbb{R}$ and: $\delta > 0$, $0 \le |\beta| < \alpha$. When $\lambda = \frac{1}{2}$, we have the normalized NRIG distribution as:

$$f(\varepsilon_t \mid \psi_{t-1}) = \frac{\alpha K_0(\sqrt{(\alpha^2 - 1)^2 + \frac{\alpha^2 \varepsilon_t^{\,2}}{\sigma_t^{\,2}}})}{\pi \sigma_t} \exp(\alpha^2 - 1) \cdot \tag{5}$$

We also consider the normal distribution as the benchmark distribution:

$$f(e_t \mid \psi_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_t^{\,2}}} \exp(-\frac{e_t^{\,2}}{2\sigma_t^{\,2}}), \ \nu > 4. \tag{6}$$

Data and Summary Statistics

We explore empirical performance of GARCH models with heavy-tailed distribution by using the South Korea stock market returns series. Figure 1 is the daily KOSPI index prices.

Figure 1. Daily KOSPI prices

We collected the standardized KOSPI daily dividend-adjusted close returns from

Yahoo Finance for the period from July 2, 1997 to July 14, 2017, covering all the available data in Yahoo Finance. There are in total 4945 observations.

Figure 2 illustrates the dynamics of the KOSPI returns, and the figure exhibits significant volatility clustering.

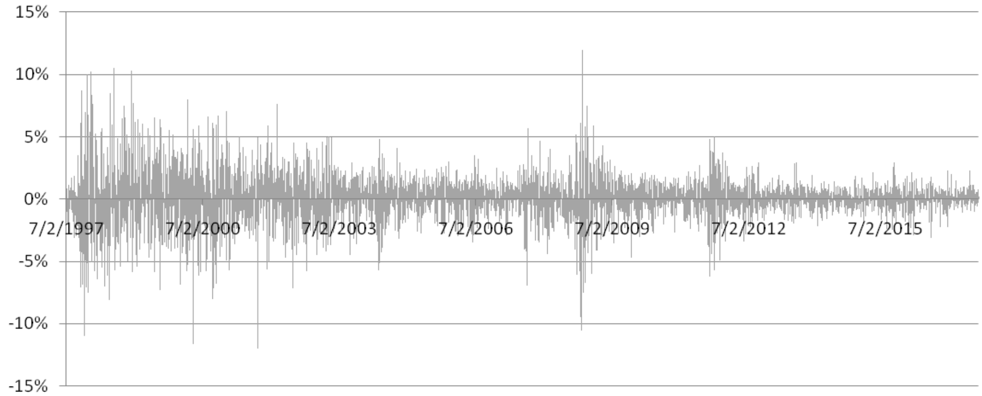

Figure 2. Daily KOSPI returns

Summary statistics of the data are reported in Table 1. The data present the standard set of well-known stylized facts of asset prices series: non-normality, limited evidence of short-term predictability and strong evidence of predictability in volatility. All series are presented in daily percentage growth rates/returns. The Bera–Jarque test conclusively rejects normality of raw returns in all series, which confirms our assumption that the model selected should account for the heavy-tail phenomenon. The smallest test statistic is much higher than the 5% critical value of 5.99. The market index is negatively skewed and has fat tails. The asymptotic SE of the skewness statistic under the null of normality is $\sqrt{6/T}$, and the SE of the kurtosis statistic is $\sqrt{24/T}$, where T is the number of observations. Almost all series exhibit statistically significant leptokurtosis, suggesting that accounting for heavy-tailedness is more pressing than skewness in modelling asset prices dynamics.

Table 1. Summary statistics

Series	*Obs.*	*Mean*	*Std.*	*Skewness*	*Kurtosis*	*BJ*	*Q(5)*	*$Q^{ARCH}(5)$*	*$Q^2(5)$*
KOSPI	4945	0.04%	1.79%	-0.026*	5.33**	76.1**	8.15*	3.22	31.24**

Note: and ** denote a skewness, kurtosis, BJ or Q statistically significant at the 5% and 1% level respectively*

BJ is the Bera-Jarque statistic and is distributed as chi-squared with 2 degrees of freedom, Q(5) is the Ljung-Box Portmanteau statistic, $Q^{ARCH}(5)$ is the Ljung-Box Portmanteau statistic adjusted for ARCH effects following Diebold (1986) and $Q^2(5)$ is the Ljung-Box test for serial correlation in the squared residuals. The three Q statistics are calculated with 5 lags and are distributed as chi-squared with 5 degrees of freedom.

We use the Ljung-Box portmanteau, or Q, statistic with five lags to test for serial correlation in the data, and adjust the Q statistic for ARCH models following Diebold (1986). The results that no serial correlation is found for almost all the series confirm our assumption of a constant mean return μ in Equation (1). The evidence of linear dependence in the squared demeaned returns, which is an indication of ARCH effects, is significant for all the series.

Estimation Results

We study the GARCH(1,1) model with the Student's *t* and the NRIG distributions is defined by maximizing the following log-likelihood function of equation:

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{T} \log(f(\varepsilon_t \mid \varepsilon_1, \cdots, \varepsilon_{t-1})) \ . \tag{7}$$

Table 2 reports estimation results of the GARCH(1,1) model with the two types of heavy-tailed distribution for all the KOSPI return series. All the parameters are significantly different from zero. There results show the NRIG distribution has better in-sample performance. Since the two distributions have the same number of parameters, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) also indicate the NRIG distribution has better empirical performance.

Table 2. Estimation of the GARCH model with heavy-tailed innovations

	alpha1	beta1	1/nu (1/alpha)	log-likelihood	AIC	BIC
Normal	0.034**	0.923**		-9694	19391	19404
Student's t	0.043**	0.936**	0.171**	-9402	18809	18827
NRIG	0.041**	0.941**	0.682**	-9349	18704	18721

*Note: * and ** denote statistical significance at the 5% and 1% level respectively*

Conclusion

The empirical performances of normal reciprocal inverse Gaussian and the standard Student's *t* distributions under the GARCH framework are compared in fitting the South Korea stock market index returns. Our results illustrate the NRIG has better performance in capture the KOSPI returns dynamics. Guo (2017a) showed the NRIG distribution also performs well in risk management of the US stock return series. We believe the GARCH model with the NRIG distribution would also perform well in risk management of the South Korea stock return series. In addition, it would be interesting to consider some other asset classes as in Guo (2017c). These are all left for future research.

References

Bollerslev, T. (1987), "A conditional heteroskedastic time series model for security prices and rates of return data", *Review of Economics and Statistics*, vol. 69, pp.542-547.

Choudhry, T. (2000), "Day of the week effect in emerging Asian stock markets: evidence from the GARCH model", *Applied Financial Economics*, Vol. 10 No. 3, pp. 235-242.

Diebold, F. (1986), "Testing for serial correlation in the presence of ARCH", *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*, Vol. 3, pp. 323-328.

Fang, W. (2002), "The effects of currency depreciation on stock returns: evidence from five East Asian economies", *Applied Economics Letters*, Vol. 9 No. 3, pp. 195-199.

Guo, Z. (2017a), "Empirical Performance of GARCH Models with Heavy-tailed Innovations", Working paper, available at: https://www.econstor.eu/bitstream/10419/167626/1/GUO_NRIGMar2017.pdf (accessed August 25, 2017).

Guo, Z. (2017b), "GARCH models with the heavy-tailed distributions and the Hong Kong stock market returns", *International Journal of Business and Management*., forthcoming.

Guo, Z. (2017c), "Models with Short-Term Variations and Long-Term Dynamics in Risk Management of Commodity Derivatives", working paper, available at: https://www.econstor.eu/bitstream/10419/167619/1/22.%20Guo_EnergyFeb2017.pdf (accessed August 25, 2017).

Lee, S. (2009), "Volatility spillover effects among six Asian countries", *Applied Economics Letters*, Vol. 16, pp. 501-508.

Politis, D. (2004), "A heavy-tailed distribution for ARCH residuals with application to volatility prediction", *Annals of Economics and Finance*, Vol. 23, pp. 34-56.

Prause, K. (1999), *The generalized hyperbolic model: estimation, financial derivatives, and risk measures*, PhD Dissertation, University of Freiburg, Freiburg.

Sola, M., Spagnolo, F. and Spagnolo, N. (2002), "A test for volatility spillovers", *Economics Letters*, Vol. 76, pp. 77-84.

Su, J. and Hung, J. (2011), "Empirical analysis of jump dynamics, heavy-tails and skewness on value-at-risk estimation", *Economic Modelling*, Vol. 28 No. 3, pp. 1117-1130.

Tavares, A., Curto, J. and Tavares, G. (2008), "Modelling heavy tails and asymmetry using ARCH-type models with stable Paretian distributions", *Nonlinear Dynamics*, Vol. 51 No. 1, pp. 231-243.

VOLATILITY CLUSTERING, NEW HEAVY-TAILED DISTRIBUTION AND THE STOCK MARKET RETURNS IN SOUTH KOREA

Yoon Hong
Hanyang University, South Korea
Ji-chul Lee
Dongseo University, South Korea
Guoping Ding
Nanjing University, China

Abstract

As other developed economies over the world, the stock market plays a crucial role in facilitating the economic growth. In this paper, we compare two different types of heavy-tailed distribution, the Student's *t* distribution and the normal reciprocal inverse Gaussian distribution, within the generalized autoregressive conditional heteroskedasticity (GARCH) framework for the daily stock market returns of South Korea (KOSPI). Our results show two important findings: i) the daily KOSPI returns exhibit conditional heavy tails even after volatility clustering effect has been accounted for; and ii) the NRIG distribution has a better in-sample performance than the Student's *t* distribution.

Keywords: stock market, GARCH model, heavy-tailed distribution, KOSPI