Perseption in Robotics
Term 3, 2022

PS 2

A (15)

1) Write the value for the covariance Q of the noise added to the observation function, knowing that the parameter <u>bearing_std</u> is its standard deviation.

```
'--bearing_std',
type='float'
action='store'
help='Diagonal of Standard deviations of Observations noise Q (rad)'
default=0.35)
```

$\Rightarrow$ bearing_std $= 0.35$ radians

Hence $Q = [\text{bearing\_std}^2] = [0.35^2] = \underline{[0,1225]}$

2) Write the equation for the covariance R_t of the noise added to the transition function and their corresponding numeric values for the initial robot command $u = [\delta_{rot1}, \delta_{trans}, \delta_{rot2}]^T = [0, 10, 0]^T$. Find out the default values of alpha.

$$R_t = \begin{bmatrix} \alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{tran}^2 & 0 & 0 \\ 0 & \alpha_3 \delta_{tran}^2 + \alpha_4 (\delta_{rot1}^2 + \delta_{rot2}^2) & 0 \\ 0 & 0 & \alpha_1 \delta_{rot2}^2 + \alpha_2 \delta_{tran}^2 \end{bmatrix}, \quad \begin{aligned} \alpha_1 &= 0{,}05^2 \\ \alpha_2 &= 0{,}001^2 \\ \alpha_3 &= 0{,}05^2 \\ \alpha_4 &= 0{,}01^2 \end{aligned}$$

$$R_t \Big|_{\substack{\alpha_1, \alpha_2, \alpha_3, \alpha_4 \\ u}} = \begin{bmatrix} 0{,}05^2 \cdot 0^2 + 0{,}001^2 \cdot 10^2 & 0 & 0 \\ 0 & 0{,}05^2 \cdot 10^2 + 0{,}01^2 (0^2 + 0^2) & 0 \\ 0 & 0 & 0{,}05^2 \cdot 0^2 + 0{,}001^2 \cdot 10^2 \end{bmatrix} = \begin{bmatrix} 0{,}0001 & 0 & 0 \\ 0 & 0{,}25 & 0 \\ 0 & 0 & 0{,}0001 \end{bmatrix}$$

3) Then derive the equations for the Jacobians G_t, V_t, H_t and evaluate them at the initial mean state $\mu_1 = [x, y, \theta]^T = [180, 50, 0]^T$ as it is considered in <u>run.py</u>

$$G_t = \frac{\partial g(x_{t-1}, \mu_t)}{\partial x_{t-1}} \Big|_{\mu_{t-1}} = \begin{bmatrix} 1 & 0 & -\delta_{tran} \sin(\theta + \delta_{rot1}) \\ 0 & 1 & \delta_{tran} \cos(\theta + \delta_{rot1}) \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow G_{t=1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V_t = \frac{\partial g(x_{t-1}, m_t)}{\partial m_t} = \begin{bmatrix} -\delta_{trans} \cdot \sin(\theta + \delta_{rot1}) & \cos(\theta + \delta_{rot1}) & 0 \\ \delta_{tran} \cdot \cos(\theta + \delta_{rot1}) & \sin(\theta + \delta_{rot1}) & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$V_{t=1} = \begin{bmatrix} 0 & 1 & 0 \\ 10 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} ; \quad \cancel{H_t = \frac{\partial h(x_t)}{\partial x_t}\bigg|_{m_t} = \left[\frac{m_y - y}{q}, -\frac{m_x - x}{q}, -1 \right]}$$

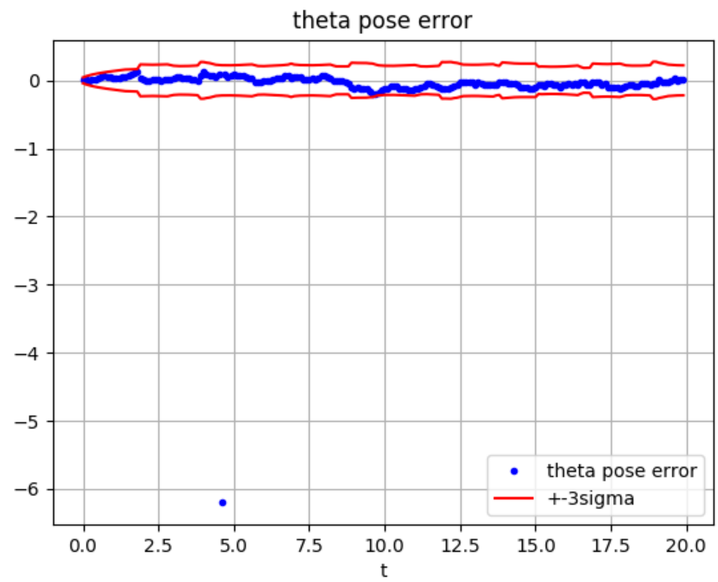
$$\cancel{q = (m_y - y)^2 + (m_x - x)^2}$$

$$H_t = \frac{\partial h(x_t)}{\partial x_t} = \left[\frac{m_y - y}{q} \quad -\frac{m_x - x}{q} \quad -1 \right] = \cancel{\left[\frac{m_y - 50}{(m_y - 50)^2 + (m_x - 180)^2} \right.}$$

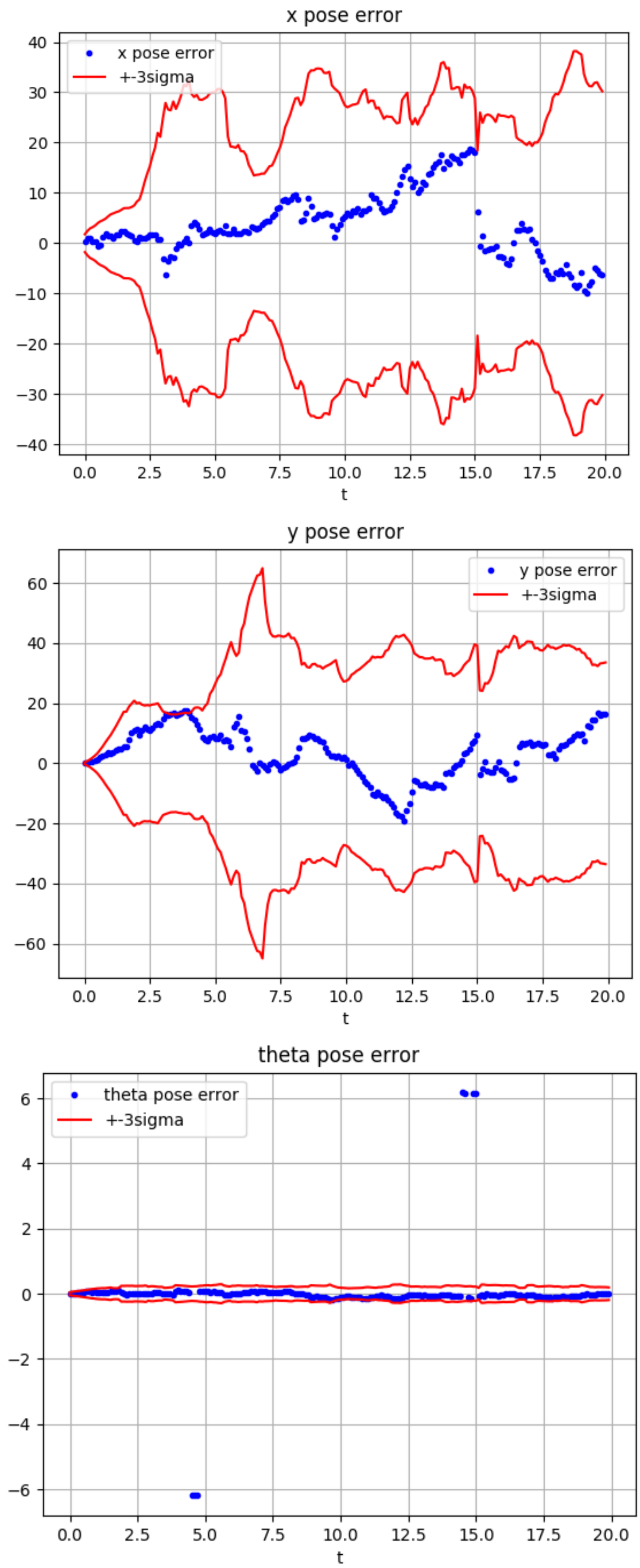
$$q = (m_y - y)^2 + (m_x - x)^2$$

$$H_t = \left[\frac{m_y - 50}{(m_y - 50)^2 + (m_x - 180)^2} \quad -\frac{m_x - 180}{(m_y - 50)^2 + (m_x - 180)^2} \quad -1 \right]$$

Task C. (20 pts) Extended Kalman Filter



Particle Filter

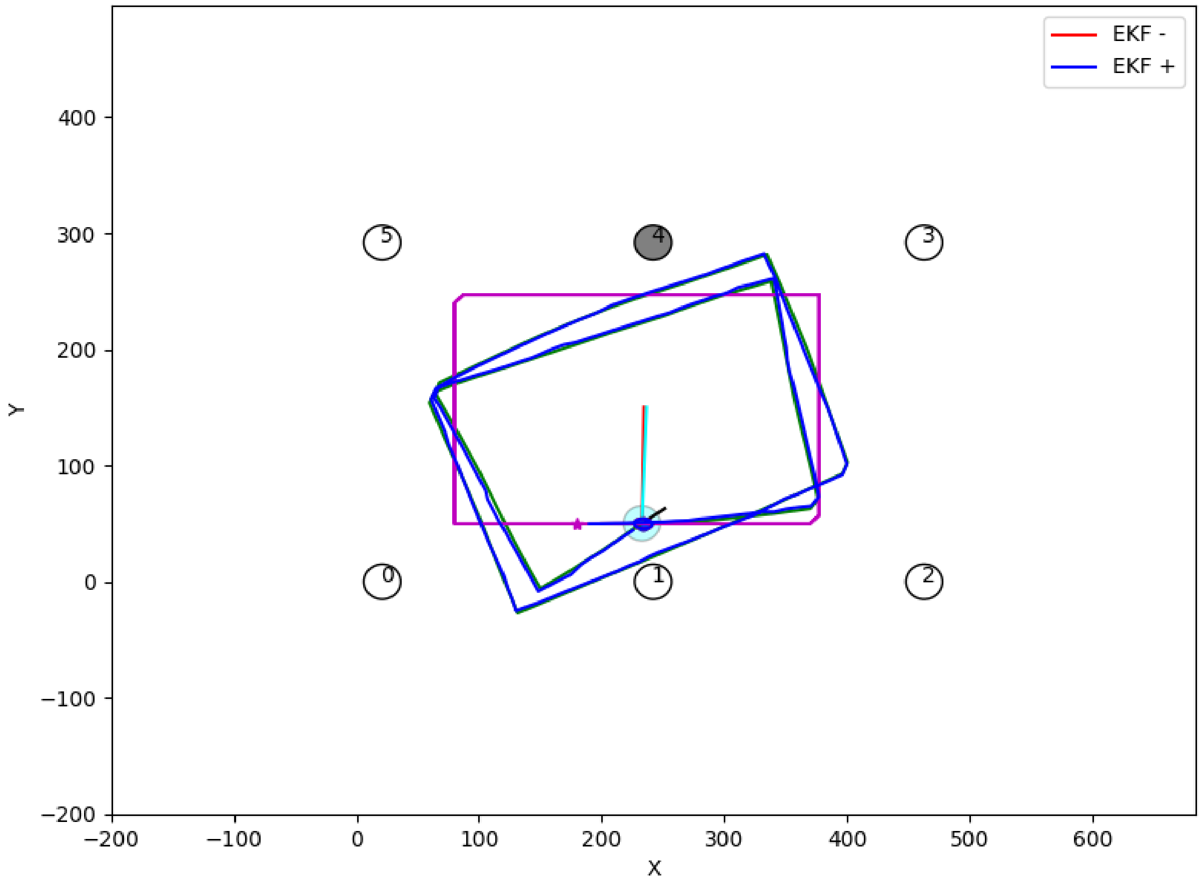


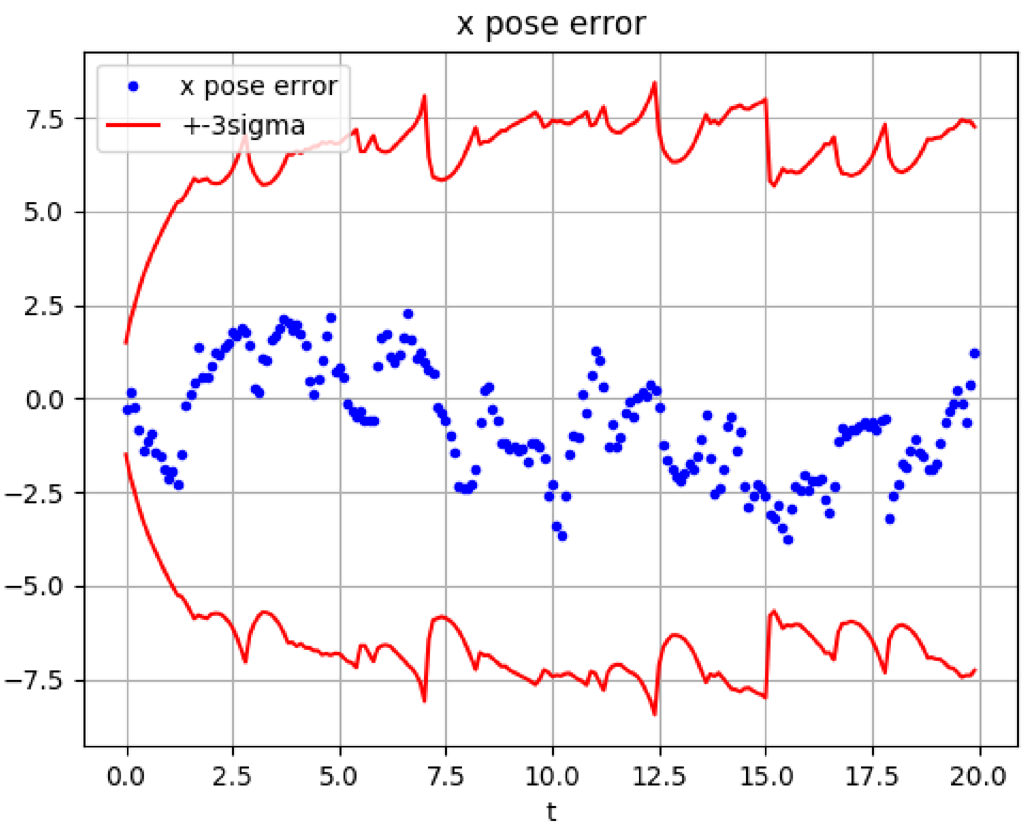
Seems like for both EKF and PF x and y state errors lie right in +-3 sigma uncertainty bounds. But for theta state errors we observe several outliers. Not that much, and this is an acceptable event (assuming Gaussian statistics).

Task D. (15 pts)

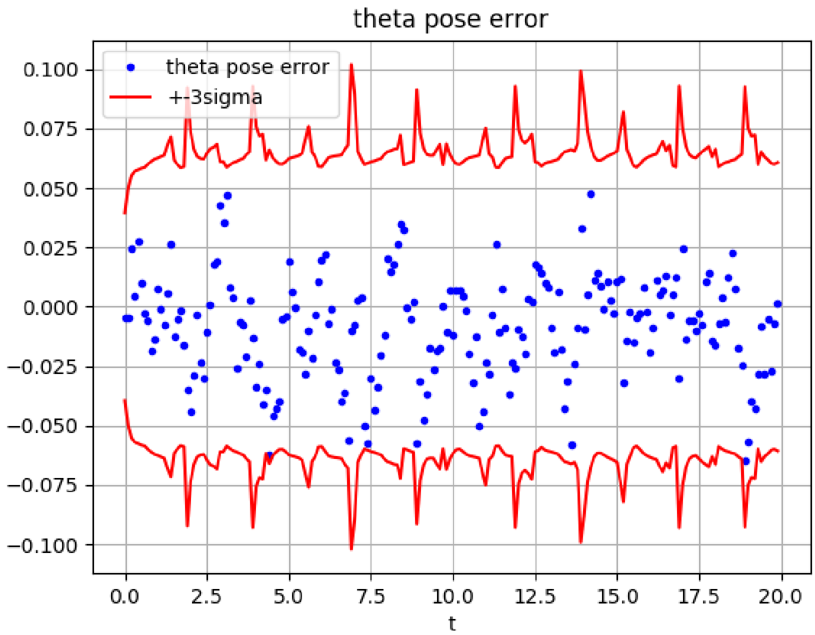
1) For EKF let's analyze two sources of noise: Sensor noise (Q matrix) and Motion noise (R matrix). We want to investigate filters behavior as the noise goes toward zero.

1. Sensor noise reduction (Q 100 times smaller)
   Noise reduction leads to the reduction of sensor uncertainty (the covariance ellipsoid collapses). So we consider the sensor to be more accurate. Hence we get more accurate precise results trajectory (no errors between Ground truth and EKF+). Robot finds out his true positions

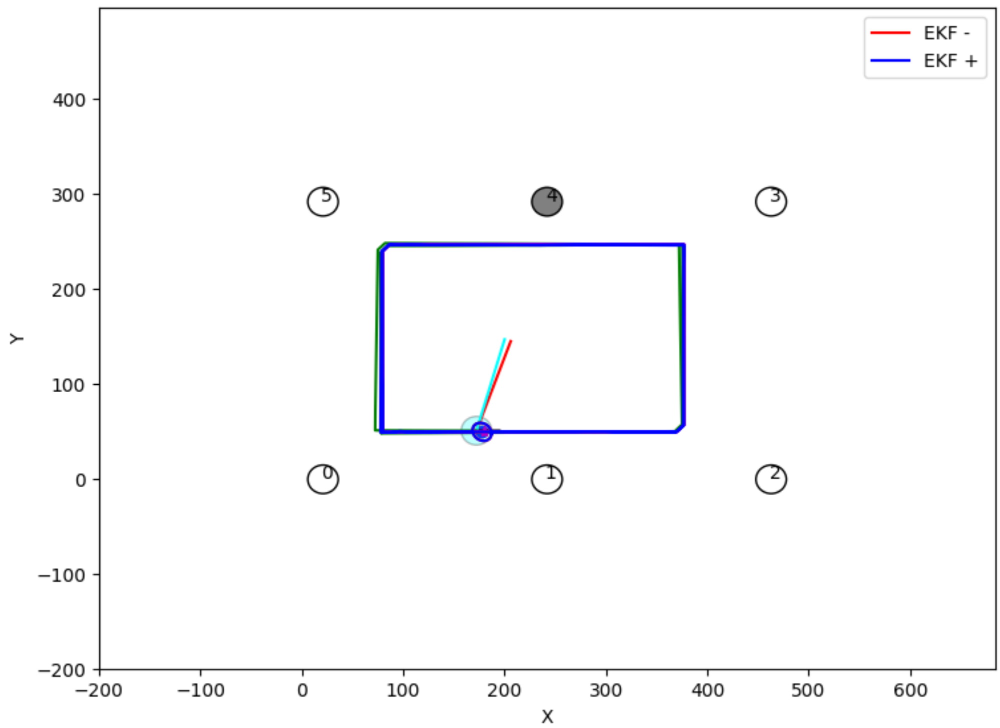


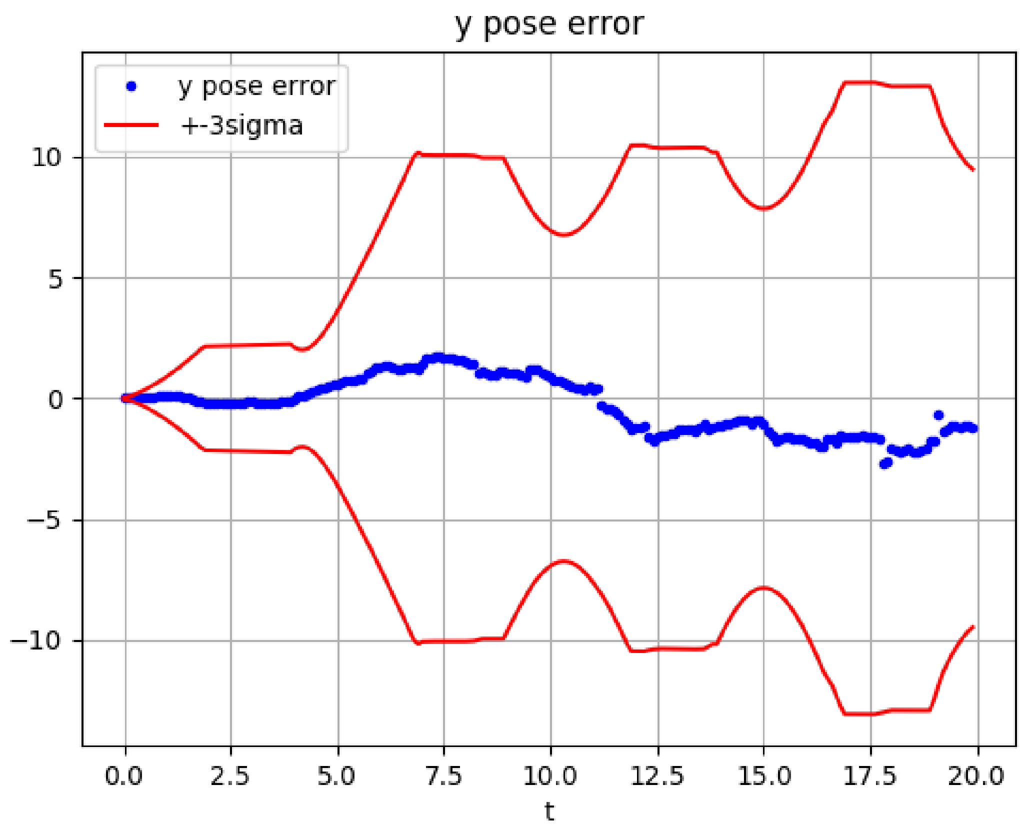

x pose error
x pose error
+-3sigma
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
t

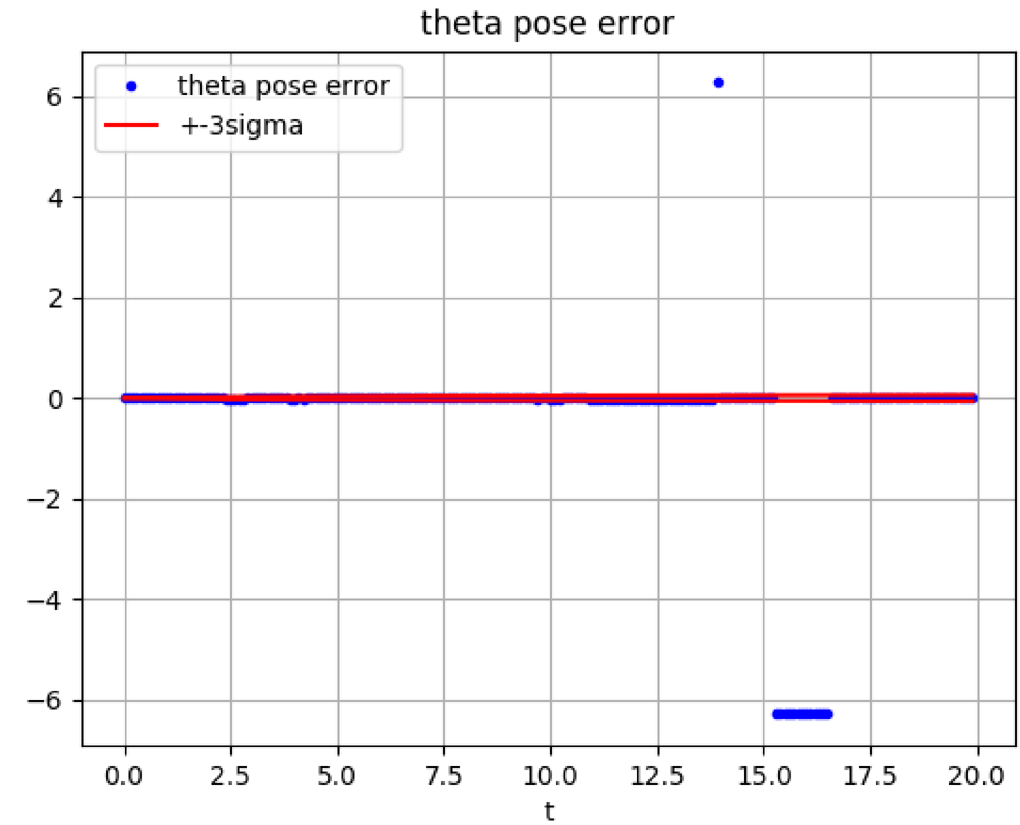

y pose error
y pose error
+-3sigma
8
6
4
2
0
−2
−4
−6
−8
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
t



2. Motion noise reduction (R 100 times smaller)
   Motion noise reduction means that a robot's motion should be executed in a more accurate way without any shifts. Reconstructed trajectory (EKF-) becomes much closer to the ground truth. And EKF+ itself becomes closer to the Ground truth (because of accurate motion measurements). Hence we get almost absolute overlap in ground truth, EKF- and EKF+ trajectories.

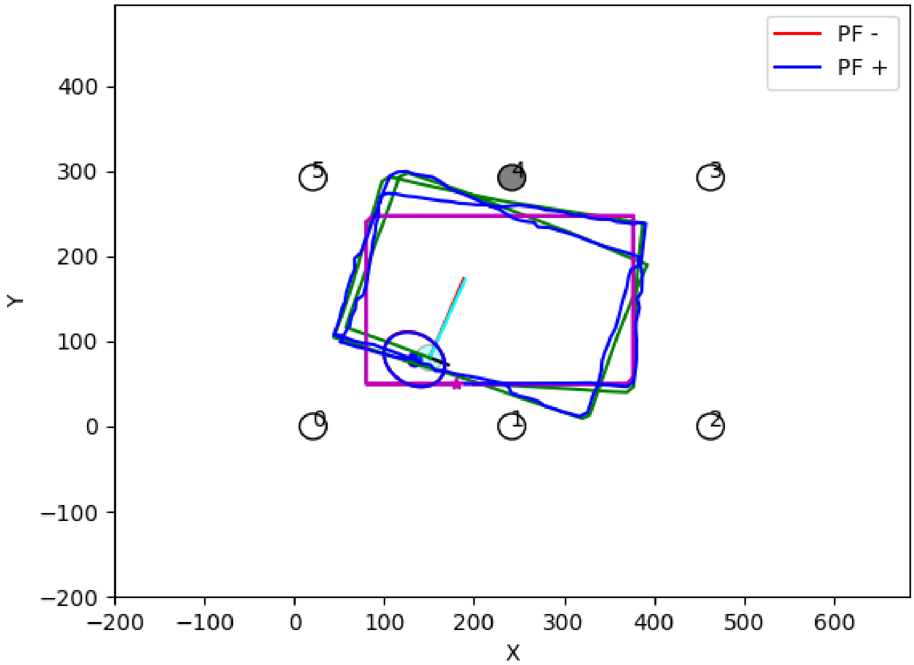



y pose error
y pose error
+-3sigma
10
5
0
−5
−10
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
t

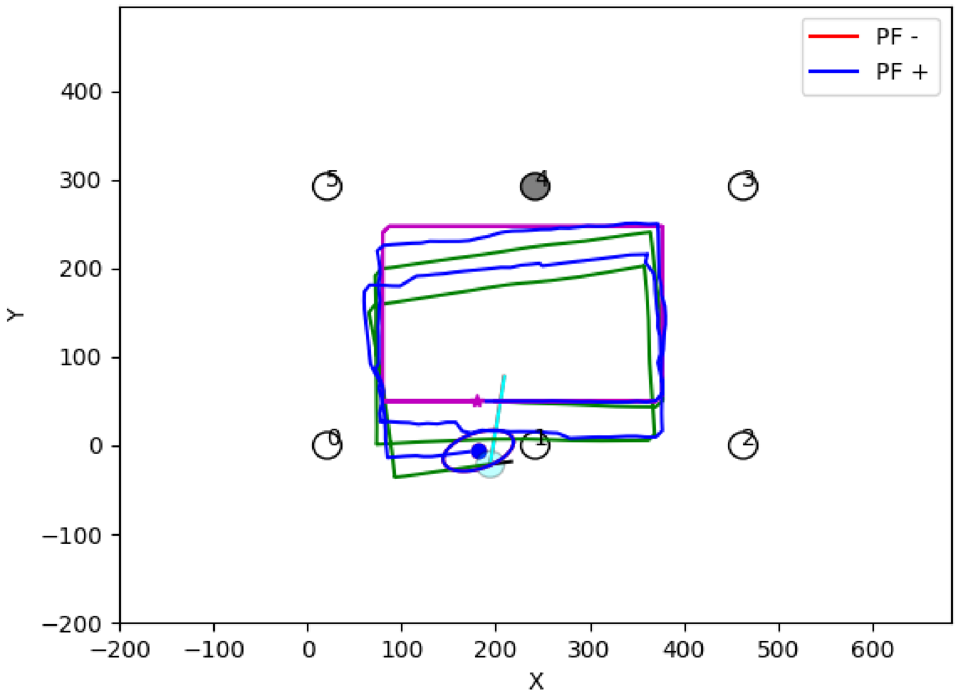

theta pose error
theta pose error
+-3sigma
6
4
2
0
−2
−4
−6
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
t

Thus, no matter what the noise is, its reduction leads to an increase in the accuracy of the EKF.

2) Let's investigate how do PF behave as the number of particles decrease:



N_particles = 400



N_particles=100

The more particles you have the better approximation you get. Less particles leads to worse precision of localization.

3) If the filter noise parameters underestimate or overestimate the true noise parameters? Please clarify what underestimation and overestimation of noise is?

Underestimated true noise parameters means that state errors are out of 3 sigma bounds of uncertainty. So filter become biased from true, since it is more certain and there are a lot of pretty close to true states out of bounds.
Overestimated true noise parameters means that we consider 3 sigma bounds to be much bigger than they should be. Less biased, but more state errors are inside of uncertainty bounds.

Both problems lead to the lack of accuracy in localization.