```
(* Lagrange's Equation of motion *)
OperatorL[q_, t_] := ( ∂t (∂∂tq #) - ∂q # ) &

(*Examples

 1. Harmonic Oscillator *)
L = (1/2) * (m (∂t x[t])^2 - k x[t]^2);

Print["Equation of motion is..."]
OperatorL[x[t], t][L] == 0

Print["Solution is..."]
DSolve[OperatorL[x[t], t][L] == 0, x[t], t]
```

Equation of motion is...

Out[]= $k\,x[t] + m\,x''[t] == 0$

Solution is...

Out[]= $\left\{\left\{x[t] \to c_1 \operatorname{Cos}\left[\frac{\sqrt{k}\ t}{\sqrt{m}}\right] + c_2 \operatorname{Sin}\left[\frac{\sqrt{k}\ t}{\sqrt{m}}\right]\right\}\right\}$

```
(* 2. Shortest distance in plane *)
L = Sqrt[1 + (∂x y[x])^2];

Print["Equation is..."]
OperatorL[y[x], x][L] == 0

Print["Solution is..."]
DSolve[OperatorL[y[x], x][L] == 0, y[x], x]
```

Equation is...

Out[]= $-\frac{y'[x]^2\, y''[x]}{\left(1 + y'[x]^2\right)^{3/2}} + \frac{y''[x]}{\sqrt{1 + y'[x]^2}} == 0$

Solution is...

Out[]= $\{\{y[x] \to c_1 + x\, c_2\}\}$

```
(* 3. Pendulum *)
L = (1/2) m (l ∂t θ[t])^2 + m g l Cos[θ[t]];

Print["Equation is..."]
OperatorL[θ[t], t][L] == 0
```

Equation is...

Out[]= $g\, l\, m \operatorname{Sin}[\theta[t]] + l^2\, m\, \theta''[t] == 0$

```
(* 4. Atwood machine *)
L = (1/2) (m1 + m2) (∂t x[t])^2 + m1 g x[t] + m2 g (l - x[t])

Print["Equation of motion is..."]
Simplify[OperatorL[x[t], t][L] == 0]

Print["Solution is..."]
Simplify[DSolve[OperatorL[x[t], t][L] == 0, x[t], t]]
```

Out[ ]= $g\, m2\, (l - x[t]) + g\, m1\, x[t] + \frac{1}{2} (m1 + m2)\, x'[t]^2$

Equation of motion is...

Out[ ]= $g\, (m1 - m2) == (m1 + m2)\, x''[t]$

Solution is...

Out[ ]= $\left\{\left\{x[t] \to \frac{g\, (m1 - m2)\, t^2}{2\, (m1 + m2)} + c_1 + t\, c_2\right\}\right\}$

```
(* 5. Central force proble having 2 DOF *)
L = (m/2) ((∂t r[t])^2 + (r[t] × ∂t θ[t])^2) - k/r[t];

OperatorsL = MapThread[OperatorL, {{r[t], θ[t]}, {t, t}}];
Print["Equations of motion are..."]
Do[Print[OperatorsL[[i]][L] == 0], {i, 1, Length[Eqns]}]
```

Equations of motion are...

$-\frac{k}{r[t]^2} - m\, r[t]\, \theta'[t]^2 + m\, r''[t] == 0$

$2\, m\, r[t]\, r'[t]\, \theta'[t] + m\, r[t]^2\, \theta''[t] == 0$

```
(* Pendulum on Oscilator, 2 DOF *)
L = ((M + m)/2) (∂t x[t])^2 + (m/2) (l ∂t θ[t])^2 +
   m l Cos[θ[t]] (∂t x[t]) (∂t θ[t]) + m g l Cos[θ[t]]

OperatorsL = MapThread[OperatorL, {{x[t], θ[t]}, {t, t}}];
Print["Equations of motion are..."]
Do[Print[Simplify[OperatorsL[[i]][L] == 0]], {i, 1, Length[Eqns]}]
```

Out[ ]= $g\, l\, m\, \mathrm{Cos}[\theta[t]] + \frac{1}{2} (m + M)\, x'[t]^2 + l\, m\, \mathrm{Cos}[\theta[t]]\, x'[t]\, \theta'[t] + \frac{1}{2} l^2\, m\, \theta'[t]^2$

Equations of motion are...

$$l\, m \,\mathrm{Sin}[\theta[t]]\, \theta'[t]^2 == (m + M)\, x''[t] + l\, m\, \mathrm{Cos}[\theta[t]]\, \theta''[t]$$

$$l\, m \left(g\, \mathrm{Sin}[\theta[t]] + \mathrm{Cos}[\theta[t]]\, x''[t] + l\, \theta''[t]\right) == 0$$