```
(* Example - 1 L2H *)
L = (1/2) m (l ∂_t θ[t])^2 + m g l Cos[θ[t]];
Reduce[Pθ == ∂_{∂_t θ[t]} L, ∂_t θ[t]]
```

Out[61]= $(P\theta == 0 \,\&\&\, m == 0) \;||\; \left(l\, m \neq 0 \,\&\&\, \theta'[t] == \frac{P\theta}{l^2\, m}\right) \;||\; (P\theta == 0 \,\&\&\, m \neq 0 \,\&\&\, l == 0)$

In[62]:=
```
(* Peak relevent solution *)
H = (Pθ ∂_t θ[t] - L) /. {θ'[t] -> Pθ/(l^2 m)}
```

Out[62]= $\frac{P\theta^2}{2\, l^2\, m} - g\, l\, m \operatorname{Cos}[\theta[t]]$

```
(* Example - 2 L2H *)
```

In[63]:=
```
L = (1/2) m (∂_t q[t])^2 - (λ/2) q[t] (∂_t q[t])^2;
Reduce[P == ∂_{∂_t q[t]} L, ∂_t q[t]]
```

Out[64]= $(P == 0 \,\&\&\, m == \lambda\, q[t]) \;||\; \left(m - \lambda\, q[t] \neq 0 \,\&\&\, q'[t] == \frac{P}{m - \lambda\, q[t]}\right)$

In[66]:=
```
(* Peak relevent solution *)
H = Simplify[(P ∂_t q[t] - L) /. {q'[t] -> P/(m - λ q[t])}]
```

Out[66]= $\frac{P^2}{2\, m - 2\, \lambda\, q[t]}$

In[67]:=
```
(* Example - 3 L2H *)
L = (1/2) m (∂_t q[t])^2 + q[t] (∂_t q[t]);
Reduce[P == ∂_{∂_t q[t]} L, ∂_t q[t]]
```

Out[68]= $(P == q[t] \,\&\&\, m == 0) \;||\; \left(m \neq 0 \,\&\&\, q'[t] == \frac{P - q[t]}{m}\right)$

In[69]:=
```
(* Peak relevent solution *)
H = Simplify[(P ∂_t q[t] - L) /. {q'[t] -> (P - q[t])/m}]
```

Out[69]= $\frac{(P - q[t])^2}{2\, m}$

In[70]:=

```
(* Hamiltonian to Lagrangian *)
(**)
(**)
(* Example - 1 *)
```

$$H = \frac{P\theta^2}{2\,l^2\,m} - g\,l\,m\,\mathrm{Cos}[\theta[t]];$$

$$\mathrm{Reduce}[\partial_t \theta[t] == \partial_{P\theta} H,\ P\theta]$$

Out[71]= $P\theta == l^2\,m\,\theta'[t]\ \&\&\ l\,m \neq 0$

In[72]:=

```
(* Peak relevent solution *)
```

$$L = \left(P\theta\,\partial_t \theta[t] - H\right)\ /.\ \left\{P\theta \to l^2\,m\,\theta'[t]\right\}$$

Out[72]= $g\,l\,m\,\mathrm{Cos}[\theta[t]] + \frac{1}{2}\,l^2\,m\,\theta'[t]^2$

In[73]:=

```
(* Example - 2 *)
```

$$H = \frac{P^2}{2\,m - 2\,\lambda\,q[t]};$$

$$\mathrm{Reduce}[\partial_t q[t] == \partial_P H,\ P]$$

Out[74]= $P == \left(m - \lambda\,q[t]\right)\,q'[t]\ \&\&\ m - \lambda\,q[t] \neq 0$

In[76]:=

```
(* Peak relevent solution *)
```

$$L = \mathrm{Simplify}\left[\left(P\,\partial_t q[t] - H\right)\ /.\ \left\{P \to \left(m - \lambda\,q[t]\right)\,q'[t]\right\}\right]$$

Out[76]= $\frac{1}{2}\,\left(m - \lambda\,q[t]\right)\,q'[t]^2$

In[77]:=

```
(* Example - 3 *)
```

$$H = \frac{\left(P - q[t]\right)^2}{2\,m};$$

$$\mathrm{Reduce}[\partial_t q[t] == \partial_P H,\ P]$$

Out[78]= $P == q[t] + m\,q'[t]\ \&\&\ m \neq 0$

In[81]:=

```
(* Peak relevent solution *)
```

$$L = \mathrm{Simplify}\left[\left(P\,\partial_t q[t] - H\right)\ /.\ \{P \to q[t] + m\,q'[t]\}\right]$$

Out[81]= $\frac{1}{2}\,q'[t]\,\left(2\,q[t] + m\,q'[t]\right)$