In[∘]:=

```
(* Trapazoidal Rule *)
f[x_] := x^3;
Xmin = 0;
Xmax = 1;
n = 100;

h = (Xmax - Xmin) / n;
x[0] = Xmin;
Do[x[i + 1] = x[i] + h, {i, 0, n}];

y[0] = f[x[0]];
Do[y[i] = f[x[i]], {i, 1, n}];

Trapzo = 0.5 h (y[0] + y[n] + 2 Sum[y[i], {i, 1, n - 1}])
```

Out[∘]= 0.250025

```
(* Gauss Quadrature *)
f[x_] := 2 x^3 - 3 x^2 + 4 x - 5;
a = -2;
b = 4;

I1 = f[1 / Sqrt[3]] + f[-1 / Sqrt[3]];
I2 = 0.5 (b - a) (Simplify[f[0.5 * (a + b + (b - a) x)] /. x → (1 / Sqrt[3])] +
     Simplify[f[0.5 * (a + b + (b - a) x)] /. x → (-1 / Sqrt[3])]);
Switch[{a, b} == {-1, 1}, True, I1, False, I2]
```

Out[∘]= 42.

In[∘]:=

```
(* Laguerre Polynomials *)
L[x_, n_] := Simplify[(Exp[x] / n!) D[Exp[-x] x^n, {x, n}]];
Table[L[x, n], {n, 0, 10}] // TableForm
```

Out[∘]//TableForm=

$$1$$

$$1-x$$

$$\frac{1}{2}\left(2-4x+x^2\right)$$

$$1-3x+\frac{3x^2}{2}-\frac{x^3}{6}$$

$$1-4x+3x^2-\frac{2x^3}{3}+\frac{x^4}{24}$$

$$1-5x+5x^2-\frac{5x^3}{3}+\frac{5x^4}{24}-\frac{x^5}{120}$$

$$1-6x+\frac{15x^2}{2}-\frac{10x^3}{3}+\frac{5x^4}{8}-\frac{x^5}{20}+\frac{x^6}{720}$$

$$1-7x+\frac{21x^2}{2}-\frac{35x^3}{6}+\frac{35x^4}{24}-\frac{7x^5}{40}+\frac{7x^6}{720}-\frac{x^7}{5040}$$

$$1-8x+14x^2-\frac{28x^3}{3}+\frac{35x^4}{12}-\frac{7x^5}{15}+\frac{7x^6}{180}-\frac{x^7}{630}+\frac{x^8}{40320}$$

$$1-9x+18x^2-14x^3+\frac{21x^4}{4}-\frac{21x^5}{20}+\frac{7x^6}{60}-\frac{x^7}{140}+\frac{x^8}{4480}-\frac{x^9}{362880}$$

$$1-10x+\frac{45x^2}{2}-20x^3+\frac{35x^4}{4}-\frac{21x^5}{10}+\frac{7x^6}{24}-\frac{x^7}{42}+\frac{x^8}{896}-\frac{x^9}{36288}+\frac{x^{10}}{3628800}$$

In[∘]:=
```
(* Legendre Polynomials *)
P[x_, n_] := Simplify[D[(x^2 - 1)^n, {x, n}] / (n ! 2^n)];
Table[P[x, n], {n, 0, 10}] // TableForm
```
Out[∘]//TableForm=

1

x

$\frac{1}{2}\left(-1+3x^2\right)$

$\frac{1}{2}x\left(-3+5x^2\right)$

$\frac{1}{8}\left(3-30x^2+35x^4\right)$

$\frac{1}{8}x\left(15-70x^2+63x^4\right)$

$\frac{1}{16}\left(-5+105x^2-315x^4+231x^6\right)$

$\frac{1}{16}x\left(-35+315x^2-693x^4+429x^6\right)$

$\frac{1}{128}\left(35-1260x^2+6930x^4-12\,012x^6+6435x^8\right)$

$\frac{1}{128}x\left(315-4620x^2+18\,018x^4-25\,740x^6+12\,155x^8\right)$

$\frac{1}{256}\left(-63+3465x^2-30\,030x^4+90\,090x^6-109\,395x^8+46\,189x^{10}\right)$

In[∘]:=
```
(* Hermite Polynomials *)
H[x_, n_] := Simplify[((-1)^n) Exp[x^2] D[Exp[-x^2], {x, n}]];
Table[H[x, n], {n, 0, 10}] // TableForm
```
Out[∘]//TableForm=

1

$2x$

$-2+4x^2$

$4x\left(-3+2x^2\right)$

$4\left(3-12x^2+4x^4\right)$

$8x\left(15-20x^2+4x^4\right)$

$8\left(-15+90x^2-60x^4+8x^6\right)$

$16x\left(-105+210x^2-84x^4+8x^6\right)$

$16\left(105-840x^2+840x^4-224x^6+16x^8\right)$

$32x\left(945-2520x^2+1512x^4-288x^6+16x^8\right)$

$32\left(-945+9450x^2-12\,600x^4+5040x^6-720x^8+32x^{10}\right)$

In[ ]:=
```
(* Euler Method *)
f[x_, y_] := x + y;
x[0] = 0;
y[0] = 1;
n = 20;
h = 0.05;

Do[x[i + 1] = x[i] + h, {i, 0, n}];
Do[y[i + 1] = y[i] + h f[x[i], y[i]], {i, 0, n}];

Table[{x[i], y[i]}, {i, 0, n, 2}] // TableForm
```

Out[ ]//TableForm=

0	1
0.1	1.105
0.2	1.23101
0.3	1.38019
0.4	1.55491
0.5	1.75779
0.6	1.99171
0.7	2.25986
0.8	2.56575
0.9	2.91324
1.	3.3066

In[ ]:=
```
(* Runge Kutta method *)
f[x_, y_] := x - y;
x[0] = 0;
y[0] = 1;
n = 10;
h = 0.1;

k1[i_] := h f[x[i], y[i]];
k2[i_] := h f[x[i] + h / 2, y[i] + k1[i] / 2];
k3[i_] := h f[x[i] + h / 2, y[i] + k2[i] / 2];
k4[i_] := h f[x[i] + h, y[i] + k3[i]];

Do[x[i + 1] = x[i] + h, {i, 0, n}];
Do[y[i + 1] = y[i] + (1 / 6) (k1[i] + 2 k2[i] + 2 k3[i] + k4[i]), {i, 0, n}];

Table[{x[i], y[i]}, {i, 0, n}] // TableForm
```

Out[ ]//TableForm=

0	1
0.1	0.909675
0.2	0.837462
0.3	0.781637
0.4	0.740641
0.5	0.713062
0.6	0.697624
0.7	0.693171
0.8	0.698659
0.9	0.71314
1.	0.73576

In[ ]:=
```
(* Modified Euler method *)
f[x_, y_] := x^2 + y;
x[0] = 0;
y[0] = 1;
Xmax = 0.02;
samples = 2;
iteration = 2;

h = (Xmax - x[0]) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n}];

EMM[r_] :=
  Module[{n},
   y[r] = y[r - 1] + h f[x[r - 1], y[r - 1]];
   Do[y[r] = y[r - 1] + 0.5 h (f[x[r - 1], y[r - 1]] + f[x[r], y[r]]), {n, 1, iteration}];
   y[r]
  ];
Do[y[r] = EMM[r], {r, 1, samples}]

Table[{x[i], y[i]}, {i, 0, samples}] // TableForm
```

Out[ ]//TableForm=

0	1
0.01	1.01005
0.02	1.0202

In[106]:=
```
(* Simpson's 1/3rd rule *)
f[x_] := Sin[x];
Xmax = 2 Pi;
Xmin = 0;
n = 100;

h = (Xmax - Xmin) / n;
x[0] = Xmin;
Do[x[i + 1] = x[i] + h, {i, 0, n}];

y[0] = f[x[0]];
Do[y[i] = f[x[i]], {i, 1, n}];

Simps13 = (h / 3) (y[0] + y[n] + 4 Sum[y[i], {i, 1, n - 1, 2}] + 2 Sum[y[i], {i, 2, n - 2, 2}]) // N
```

Out[115]= 0.

In[304]:=
```
(* Simpson's 3/8rd rule *)
f[x_] := x^3;
Xmax = 1;
Xmin = 0;
n = 100;

h = (Xmax - Xmin) / n;
x[0] = Xmin;
Do[x[i + 1] = x[i] + h, {i, 0, n}];

y[0] = f[x[0]];
Do[y[i] = f[x[i]], {i, 1, n}];

Simps38 = (3 h / 8)
    (y[0] + y[n] + 3 Sum[y[i] + y[i + 1], {i, 1, n - 1, 3}] + 2 Sum[y[i], {i, 3, n - 1, 3}]) // N
```

Out[313]= 0.247538

In[314]:=
```
(* Bessel function *)
Table[Series[BesselJ[i, x], {x, 0, 10}], {i, 0, 5}] // TableForm
```

Out[314]//TableForm=

$$1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \frac{x^8}{147\,456} - \frac{x^{10}}{14\,745\,600} + O[x]^{11}$$

$$\frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} - \frac{x^7}{18\,432} + \frac{x^9}{1\,474\,560} + O[x]^{11}$$

$$\frac{x^2}{8} - \frac{x^4}{96} + \frac{x^6}{3072} - \frac{x^8}{184\,320} + \frac{x^{10}}{17\,694\,720} + O[x]^{11}$$

$$\frac{x^3}{48} - \frac{x^5}{768} + \frac{x^7}{30\,720} - \frac{x^9}{2\,211\,840} + O[x]^{11}$$

$$\frac{x^4}{384} - \frac{x^6}{7680} + \frac{x^8}{368\,640} - \frac{x^{10}}{30\,965\,760} + O[x]^{11}$$

$$\frac{x^5}{3840} - \frac{x^7}{92\,160} + \frac{x^9}{5\,160\,960} + O[x]^{11}$$