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ETAS modeling of seismicity - background and current applications

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ISM

Outlines

- The meaning of conditional intensity seismicity (time-varying seismicity rate)
- From the Omori-Utsu formula to Ogata's ETAS model
- Space-time ETAS model, estimation and stochastic declustering
- Relative quiescence and test hypothesis related to earthquake clustering
- Simulation and forecasting

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1. Functions related to waiting times (1)

- Notations.
  - N: point process in timeunder consideration
  - t: a certain temporal location.
  - u: waiting time to next event from time t.
  - $\mathcal{H}_t$: observations up to time t but not include t.
- Cumulative probability distribution function (c.p.d.f.).
$$F_t(u) = \Pr\{\text{from } t \text{ onwards, waiting time to next event} \leq u \mid \mathcal{H}_t\}.$$
- Probability density function (p.d.f.).
$$f_t(u)\,\mathrm{d}u = \Pr\{\text{from } t \text{ onward, waiting time is between } u \text{ and } u + \mathrm{d}u \mid \mathcal{H}_t\}.$$
- Survival function.
$$S_t(u) = \Pr\{\text{from } t \text{ on, waiting time} > u \mid \mathcal{H}_t\} = \Pr\{\text{no event occurs between } t \text{ and } t+u \mid \mathcal{H}_t\}.$$
- Hazard function.
$$\begin{aligned} h_t(u)\,\mathrm{d}u \quad &= \quad \Pr\{\text{next event occurs between } t+u \text{ and } t+u+\mathrm{d}u \\ &\qquad \mid \text{it does not occur between } t \text{ and } t+u, \mathcal{H}_t.\}. \end{aligned}$$

Functions related to waiting times (2)

- Relations among c.p.d.f., p.d.f., survival and hazard functions.

$$
\begin{aligned}
S_t(u) &= 1 - F_t(u) = \int_u^\infty f_t(s)\,\mathrm{d}s = \exp\left[-\int_0^u h_t(s)\,\mathrm{d}s\right], &(6)\\
F_t(u) &= 1 - S_t(u) = \int_0^u f_t(s)\,\mathrm{d}s = 1 - \exp\left[-\int_0^u h_t(s)\,\mathrm{d}s\right], &(7)\\
f_t(u) &= \frac{\mathrm{d}F_t}{\mathrm{d}u} = -\frac{\mathrm{d}S_t}{\mathrm{d}u} = h_t(u)\,\exp\left[-\int_0^u h_t(s)\,\mathrm{d}s\right], &(8)\\
h_t(u) &= \frac{f_t(u)}{S_t(u)} = -\frac{\mathrm{d}}{\mathrm{d}u}[\log S_t(u)] = -\frac{\mathrm{d}}{\mathrm{d}u}[\log(1 - F_t(u))]. &(9)
\end{aligned}
$$

- (*Prove the above equations.*)
- Properties of the hazard function.
  - Additivity. For a point process N consisting of two independent subprocess, N_1 and N_2. Denoting the hazard function of the subprocesses by $h_t^{(i)}$, with $i = 1$ or 2. Then, $h_t(u) = h_t^{(1)}(u) + h_t^{(2)}(u)$.
  - Exchangeability between t and u. If there is no event occurring between t and $t + u$ and $v \geq u \geq 0$, then $h_t(v) = h_{t+u}(v - u)$.

Conditional intensity

- $h_t(0)$. If an event occurs at t, then $h_t(0)$ has two possible explanations:
  - Hazard function at $t - t_{prev}$ from the occurrence time t_{prev} of the last previous event, denoted by $h_{t_-}(0)$;
  - Hazard function at 0 from the event occurring at t, denoted by $h_{t_+}(0)$;
  - Conventionally, the first explanation is used for $h_t(0)$, i.e., $h_t(0) = h_{t_-}(0)$.
- Conditional intensity. $\lambda(t) \equiv h_t(0)$.
  Also frequently written as $\lambda(t|\mathcal{H}_t)$, where $\mathcal{H}_t$ represents the observation history up to time t but not including t.
- Other definition.

$$\lambda(t)\,\mathrm{d}t = \Pr\{\text{one or more events occur in } [t, t+dt) \mid \mathcal{H}_t\}. \tag{10}$$

When then $\Pr\{N[t, t+\mathrm{d}t) > 1 \text{ for all } t\} = o(\mathrm{d}t)$,

$$\begin{aligned}
\mathbf{E}\left[N[t, t+\mathrm{d}t) \mid \mathcal{H}_t\right] &= \sum_{n=0}^{\infty} n \Pr\{N[t, t+\mathrm{d}t) = n \mid \mathcal{H}_t\} \\
&\approx \Pr\{N[t, t+\mathrm{d}t) = 1 \mid \mathcal{H}_t\} + o(\mathrm{d}t) \\
&\approx \lambda(t)\,\mathrm{d}t + o(\mathrm{d}t).
\end{aligned}$$

Likelihood of point process (relation between conditional intensity and Janossy intensity)

Given an observed dataset of a point process N, say $\{t_1, t_2, \cdots, t_n\}$, in a given time interval $[S, T]$. The likelihood function L is the joint probability density of waiting times:

$$
\begin{aligned}
& L(N; S, T)\,\mathrm{d}t_1\,\mathrm{d}t_2 \cdots \mathrm{d}t_n \\
= \; & \Pr\{\text{The waiting time from } S \text{ is in } (t_1 - S, t_1 - S + \mathrm{d}t_1)\} \\
& \times \Pr\{\text{The waiting time from } t_1 \text{ is in } (t_2 - t_1, t_2 - t_1 + \mathrm{d}t_2) \mid \text{what happens before } t_1\} \\
& \times \cdots \\
& \times \Pr\{\text{The waiting time from } t_n \text{ is greater than } T - t_n \mid \text{what happens before } t_n\} \\
= \; & f_S(t_1 - Sd)\,\mathrm{d}t_1 \times f_{t_1}(t_2 - t_1)\,\mathrm{d}t_2 \times \cdots \times f_{t_{n-1}}(t_n - t_{n-1})\,\mathrm{d}t_n \times S_{t_n}(T - t_n) \\
= \; & h_S(t_1 - S) \exp\left[-\int_S^{t_1} h_S(u - S)\,\mathrm{d}u\right] \mathrm{d}t_1 \\
& \times \prod_{i=1}^{n-1} \left\{ h_{t_i}(t_{i+1} - t_i) \exp\left[-\int_{t_i}^{t_{i+1}} h_{t_i}(u - t_i)\,\mathrm{d}u\right] \mathrm{d}t_i \right\} \exp\left[-\int_{t_n}^{T} h_{t_n}(u - t_n)\,\mathrm{d}u\right] \\
= \; & \left[\prod_{i=1}^{n} h_{t_i}(0)\,\mathrm{d}t_i\right] \exp\left[-\int_S^T h_u(0)\,\mathrm{d}u\right] = \left[\prod_{i=1}^{n} \lambda(t_i)\,\mathrm{d}t_i\right] \exp\left[-\int_S^T \lambda(u)\,\mathrm{d}u\right].
\end{aligned} \tag{11}
$$

We usually write the above formula as its logarithm, i.e.,

$$
\log L(N; S, T) = \sum_{i=1}^{n} \log \lambda(t_i) - \int_S^T \lambda(u)\,\mathrm{d}u. \tag{12}
$$

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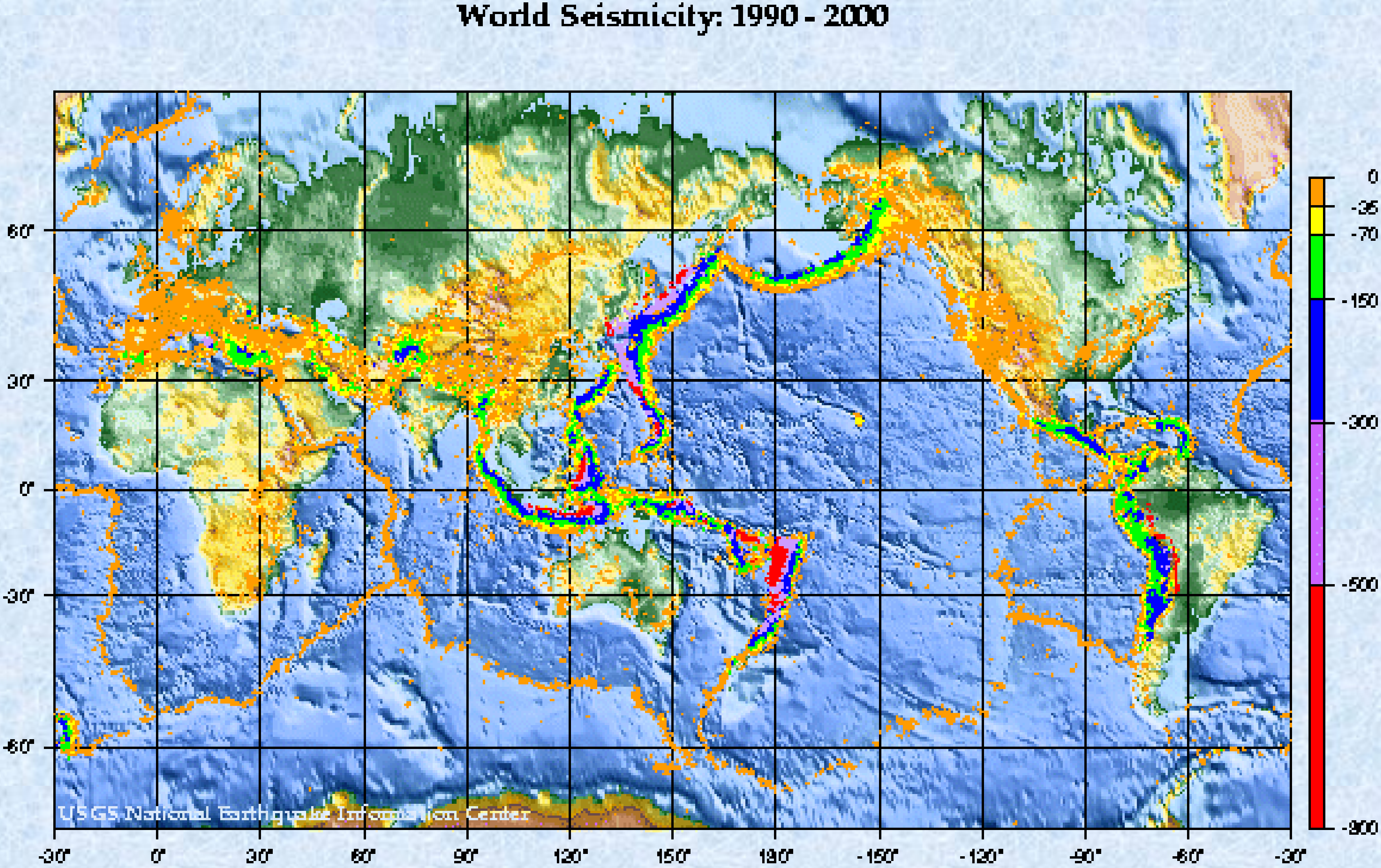
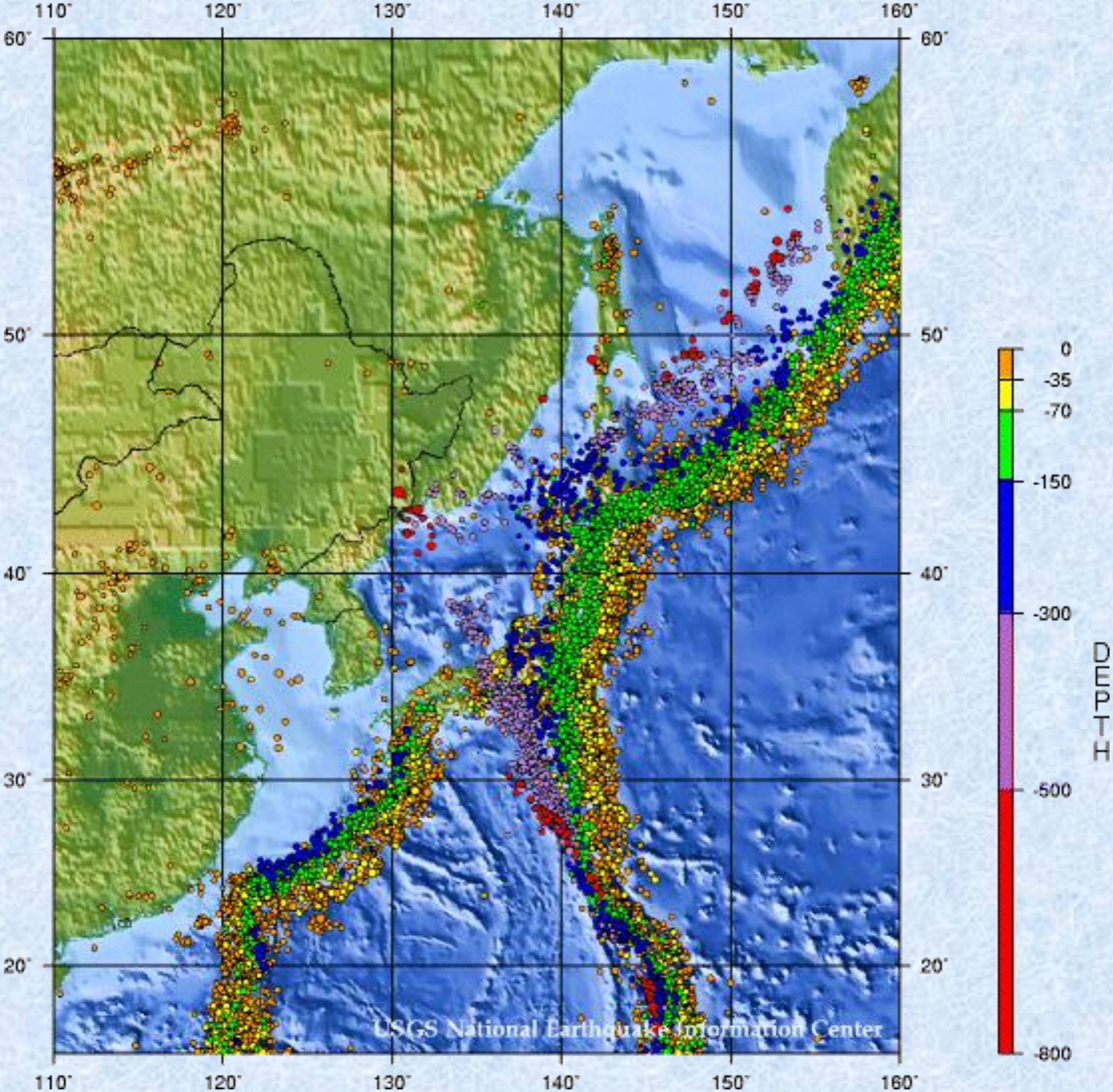
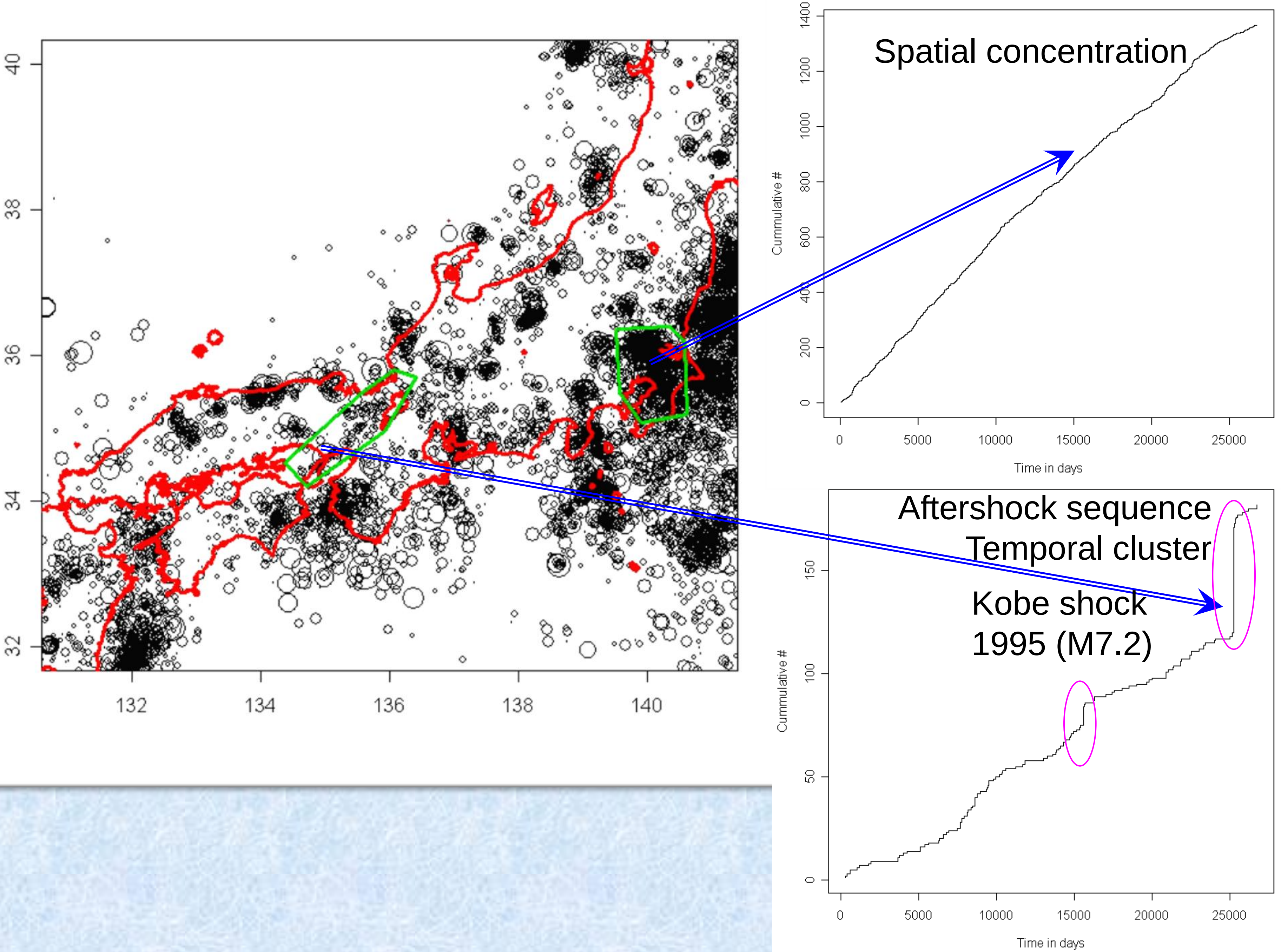
Features of Seismicity (USGS)



Seismicity of Japan and Kuril Islands: 1990 - 2000




Spatial concentration
Aftershock sequence
Temporal cluster
Kobe shock
1995 (M7.2)
Cummulative #
Time in days
40
38
36
34
32
132
134
136
138
140
0
200
400
600
800
1000
1200
1400
0
50
100
150
0
5000
10000
15000
20000
25000

4. Omori-Utsu formula (Reasenberge-Jones model)

- Omori (1894): the rate of felt aftershocks of the 1891 M_s8.0 Nobi earthquake,

$$n(t) = K(t+c)^{-1}, \tag{26}$$

t: the time from the mainshock.
K and c: constants.

- Utsu (1957): the decay of the aftershock numbers could vary. (Utusu-Omori formula)

$$n(t) = K(t+c)^{-p} \tag{27}$$

p: ranges between 0.6 and 2.5 with a median of 1.1.

- Reasenberg-Jones model

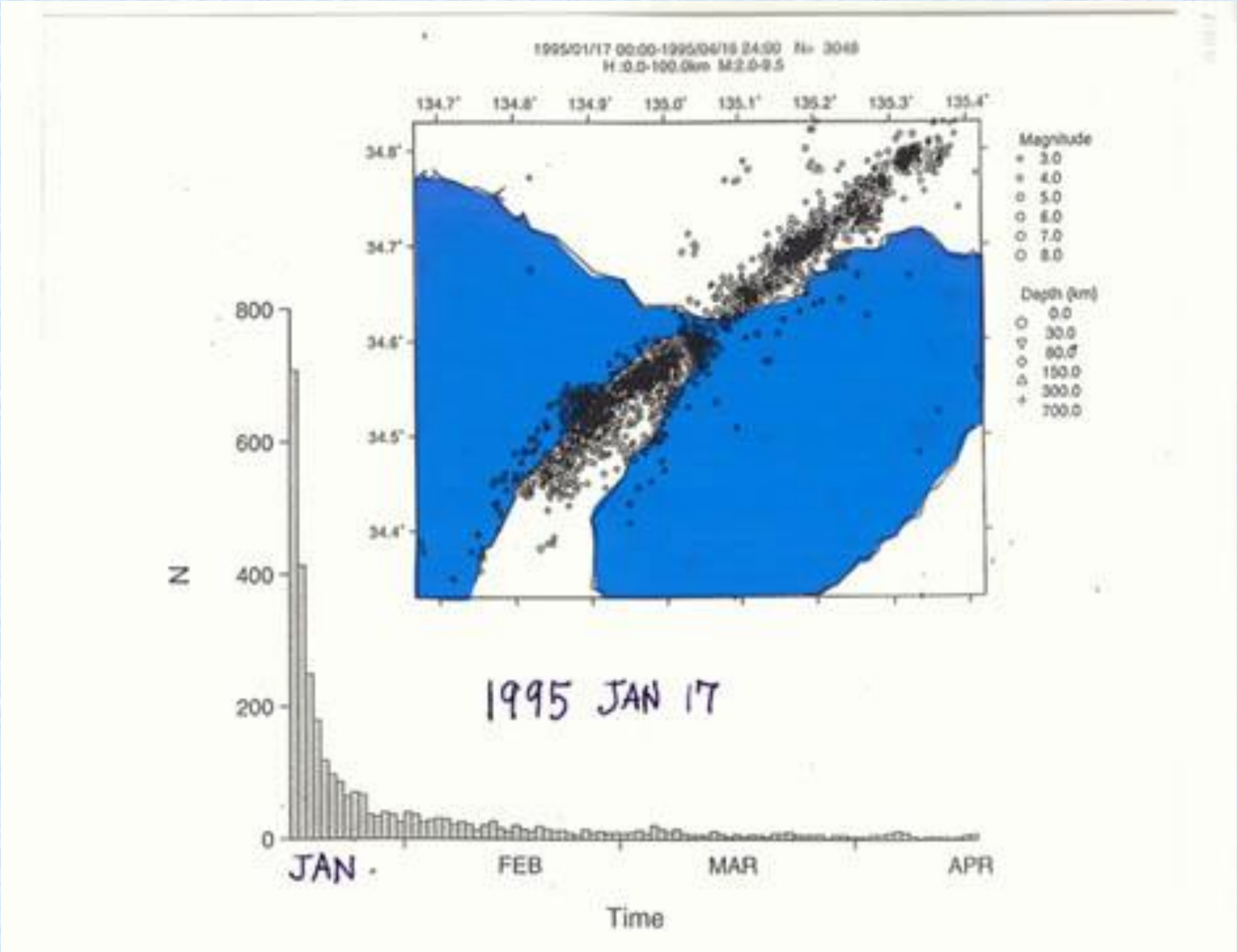
$$\lambda(t,m) = \frac{K\, s(m)}{(t+c)^p}, \tag{28}$$

$s(m)$: the magnitude probability density function.

- Multiple Omori-Utsu formula: Not only mainshocks trigger aftershocks, but also large aftershocks may trigger their own aftershocks.

$$\lambda(t) = K/(t-t_0+c)^{-p} + \sum_{i=1}^{N_T} \frac{K_i H(t-t_i)}{(t-t_i+c_i)^{-p_i}}, \tag{29}$$

t_0: the occurrence time of the mainshock;
t_i, $i = 1, \ldots N_T$: the occurrence times of the triggering aftershocks;
H: Heaviside function.



$$n(t) = \frac{K}{(t+c)^p}$$

Omori-Utsu Formula (Utsu 1956)

Utsu 1970

In many cases, an aftershock sequence is more complex than the modified Omori formula. The complexity arises from further clusters within an aftershock sequence, which become conspicuous as the threshold magnitude of the data become small. This is a schematic figure for such a sequence.

Utsu showed that the secondary aftershocks again obey the modified Omori formula as shown here.

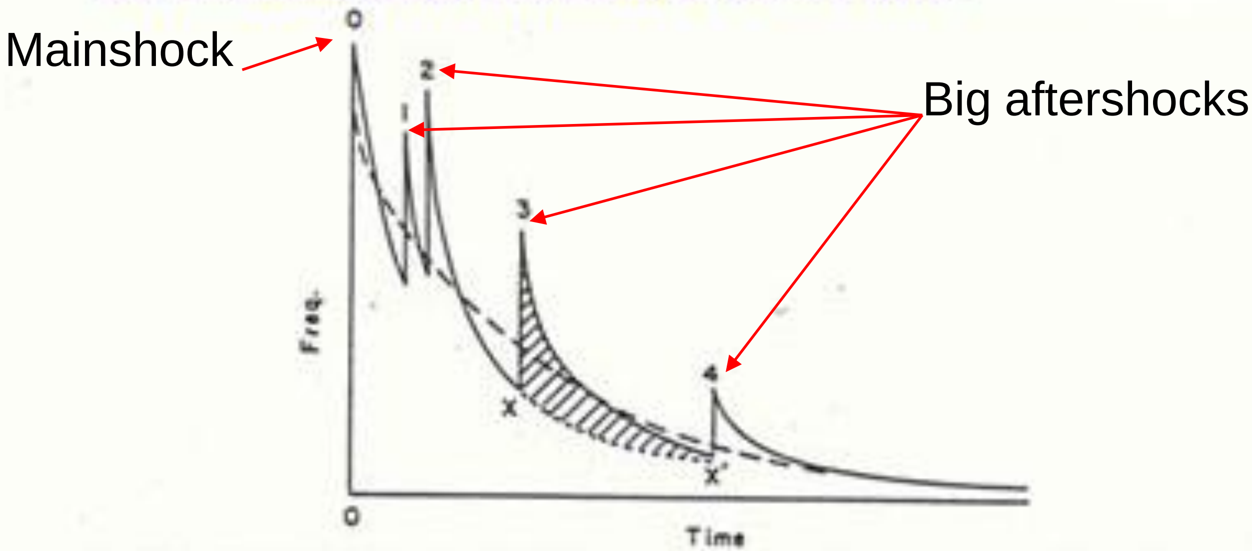
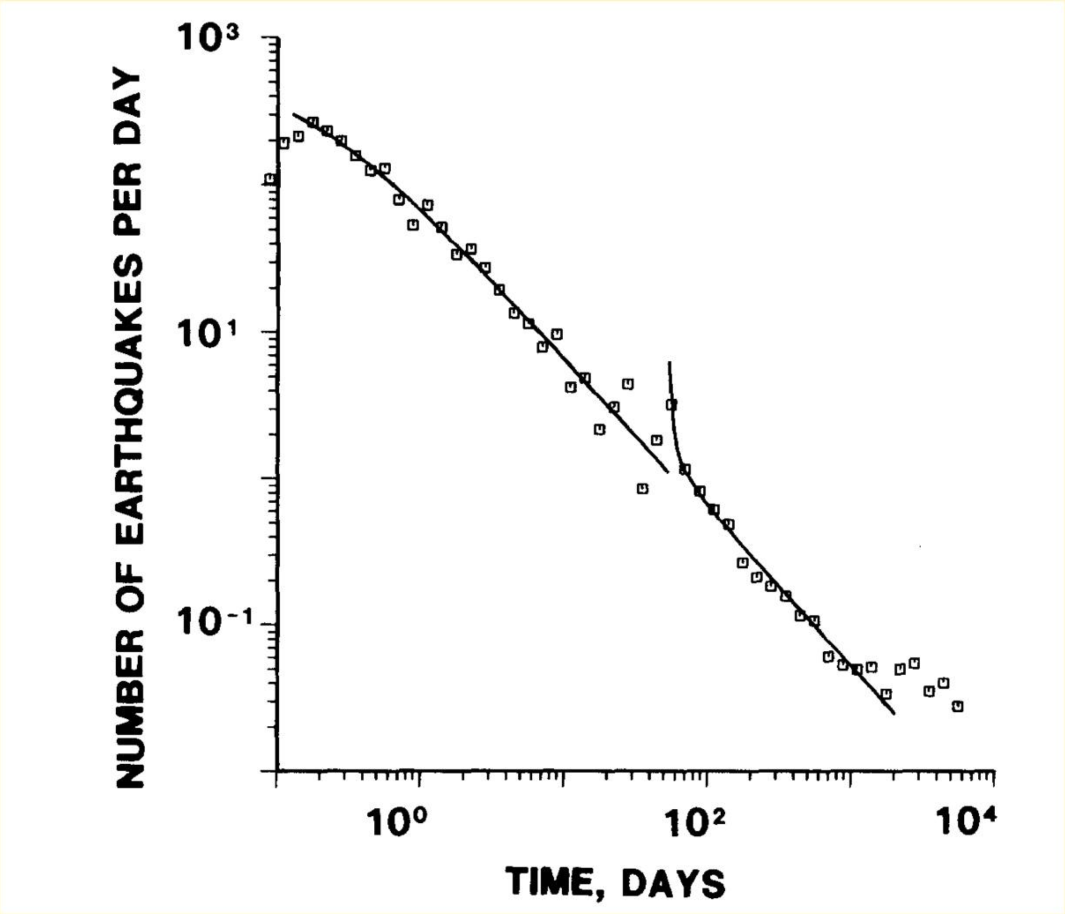


Fig. 105. A schematic graph of Type 1-C aftershock sequence. The borken line represents the curve for the modified Omori formula fitting the whole sequence. The shaded area indicate the secondary aftershocks t.iggered by shock 3.

The 1965 Rat Islands earthquake of $M_W 8.7$ and its aftershocks (Ogata and Shimazaki, 1984)

Here the AIC criteria selects the model where $c = c_1$ and $p = p_1$ as the best model among the class of models with $N_T = 1$.



An ordinary log-log plot of the number of aftershocks of the Rat island event per day against time (Ogata 1984). The curve represents the intensity function fitted to observed data points marked by the squares.

Temporal ETAS model

- Conditional intensity

$$\lambda(t) = \mu + \sum_{i:t_i<t} \kappa(m_i) g(t - t_i)$$

1. Direct productivity: $$\kappa(m) = Ae^{\alpha(m-m_C)}, \quad m \geq m_C$$

2. Time p.d.f (Omori-Utsu): $$g(t) = (p-1)\left(1 + t/c\right)^{-p}/c, \quad t > 0$$

- Likelihood function $$\log L = \sum_{t_i \in [0,T]} \log \lambda(t_i) - \int_0^T \lambda(t)dt$$

(Ogata, 1988)

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General Features of Seismicity

- Background seismicity is non-homogenous
- Earthquake clusters are complex, different in sizes, duration times, spatial distributions and, of most importance, multi-stage.
- Clusters overlap with each other and also with the background seismicity, spatially and temporally.

Assumptions of the model

- Background seismicity is non-homogenous
- Each earthquake produces its own cluster independently, with the cluster size depending on its magnitude. Bigger events produce more.

Empirical Formulae

- Gutenberg-Richter law (Gutenberg and Richter, 1956) for magnitudes

$$\log(N \geq M) = a - bM$$

In probability language, p.d.f of magnitudes

$$s(m) = \beta e^{-\beta(m-m_c)}, \quad m \geq m_c$$

- Omori-Utsu law for aftershock times

$$n(t) = \frac{K}{(t+c)^p}$$

Space-time ETAS model

- Conditional intensity or stochastic intensity

$$\lambda(t,x,y,m)=\frac{\mathbf{E}\left[N(dt\times dx\times dy\times dm)\,|\,\mathcal{H}_t\right]}{dt\times dx\times dy\times dm}$$

$$\lambda(t,x,y,m)=s(m)\left[\mu(x,y)+\sum_{i:t_i<t}\kappa(m_i)g(t-t_i)f(x-x_i,y-y_i)\right]$$

1. Magnitude p.d.f (G-R): $s(m)=\beta e^{-\beta(m-m_C)},\quad m\geq m_C$
2. Direct productivity: $\kappa(m)=Ae^{\alpha(m-m_C)},\quad m\geq m_C$
3. Time p.d.f (Omori-Utsu): $g(t)=(p-1)(1+t/c)^{-p}/c,\quad t>0$
4. Location p.d.f: $f(x,y\,|\,m)=\dfrac{q-1}{\pi De^{\gamma(m-m_c)}}\left(1+\dfrac{x^2+y^2}{De^{\gamma(m-m_c)}}\right)^{-q}$

- -
  - $f(x,y;m)$: p.d.f.s the location of an offspring from an ancestor of magnitude m;
    - Model 1
$$f(x,y;m)=\frac{1}{2\pi D^2}e^{-\frac{x^2+y^2}{2D^2}}; \tag{41}$$
    - Model 2
$$f(x,y;m)=\frac{1}{2\pi D^2 e^{\alpha(m-m_0)}}e^{-\frac{x^2+y^2}{2D^2e^{\alpha(m-m_0)}}}, \tag{42}$$
where the parameter α is the same one as in $\kappa(m)$d;
    - Model 3
$$f(x,y;m)=\frac{q-1}{\pi D^2}\left(1+\frac{x^2+y^2}{D^2}\right)^{-q}; \tag{43}$$
    - Model 4
$$f(x,y;m)=\frac{q-1}{\pi D^2 e^{\alpha(m-m_0)}}\left(1+\frac{x^2+y^2}{D^2 e^{\alpha(m-m_0)}}\right)^{-q}; \tag{44}$$
    - Model 5
$$f(x,y;m)=\frac{q-1}{\pi D^2 e^{\gamma(m-m_0)}}\left(1+\frac{x^2+y^2}{D^2 e^{\gamma(m-m_0)}}\right)^{-q}. \tag{45}$$

Space-time ETAS model

- Conditional intensity

$$\lambda(t,x,y,m) = s(m)\left[\mu(x,y) + \sum_{i:t_i<t} \kappa(m_i)\, g(t-t_i)\, f(x-x_i, y-y_i)\right]$$

- Likelihood function

$$\log L = \sum_{(t_i,x_i,y_i,m_i)\in[0,T]\times A\times M} \log \lambda(t_i, x_i, y_i, m_i) - \int_0^T \iint_A \int_M \lambda(t,x,y,m)\,dt\,dx\,dy\,dm$$

Space-time ETAS model

- Time varying seismicity rate (conditional intensity or stochastic intensity)

$$\lambda(t,x,y,m) = s(m)\left[\mu(x,y) + \sum_{i:t_i<t} \kappa(m_i)g(t-t_i)f(x-x_i,y-y_i)\right]$$

Contribution from background seismicity

$$\boxed{\frac{s(m)\mu(x,y)}{\lambda(t,x,y,m)}}$$

Contribution from the *i*-th event

$$\boxed{\frac{s(m)g(t-t_i)f(x-x_i,y-y_i)}{\lambda(t,x,y,m)}}$$

Space-time ETAS model

- Time varying seismicity rate (conditional intensity or stochastic intensity) at event *j*

$$\lambda(t_j, x_j, y_j, m_j) = s(m_j)\left[\mu(x_j, y_j) + \sum_{i:t_i<t} \kappa(m_i) g(t_j - t_i) f(x_j - x_i, y_j - y_i)\right]$$

Contribution from background seismicity

$$\frac{s(m_j)\mu(x_j, y_j)}{\lambda(t_j, x_j, y_j, m_j)}$$

Contribution from the *i*-th event

$$\frac{s(m_j) g(t_j - t_i) f(x_j - x_i, y_j - y_i)}{\lambda(t_j, x_j, y_j, m_j)}$$

Space-time ETAS model

- For each event j

Contribution from background seismicity

$$\boxed{\frac{s(m_j)\mu(x_j, y_j)}{\lambda(t_j, x_j, y_j, m_j)}}$$

Contribution from the i-th event

$$\boxed{\frac{s(m_j)g(t_j - t_i)f(x_j - x_i, y_j - y_i)}{\lambda(t_j, x_j, y_j, m_j)}}$$

Space-time ETAS model

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$$\boxed{\frac{s(m_j)g(t_j - t_i)f(x_j - x_i, y_j - y_i)}{\lambda(t_j, x_j, y_j, m_j)}}$$

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Space-time ETAS model

- For each event j

$$\boxed{\varphi_j = \frac{s(m_j)\mu(x_j, y_j)}{\lambda(t_j, x_j, y_j, m_j)}}$$

$$\boxed{\rho_{ij} = \frac{s(m_j)g(t_j - t_i)f(x_j - x_i, y_j - y_i)}{\lambda(t_j, x_j, y_j, m_j)}}$$

Space-time ETAS model

- For each event j

Pr{event j is from background}

$$\varphi_j = \frac{s(m_j)\mu(x_j, y_j)}{\lambda(t_j, x_j, y_j, m_j)}$$

Pr{event j is from i}

$$\rho_{ij} = \frac{s(m_j)g(t_j - t_i)f(x_j - x_i, y_j - y_i)}{\lambda(t_j, x_j, y_j, m_j)}$$

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Thinning method

- For each event j

Pr{event j is from background} $\varphi_j = \dfrac{s(m_j)\mu(x_j, y_j)}{\lambda(t_j, x_j, y_j, m_j)}$

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Stochastic declustering: Set event j to be a background event or a child of event 1, 2, …, according to probabilities φ_j or $\rho_{1j}, \rho_{1j}, \ldots, \rho_{j-1,j}$ respectively

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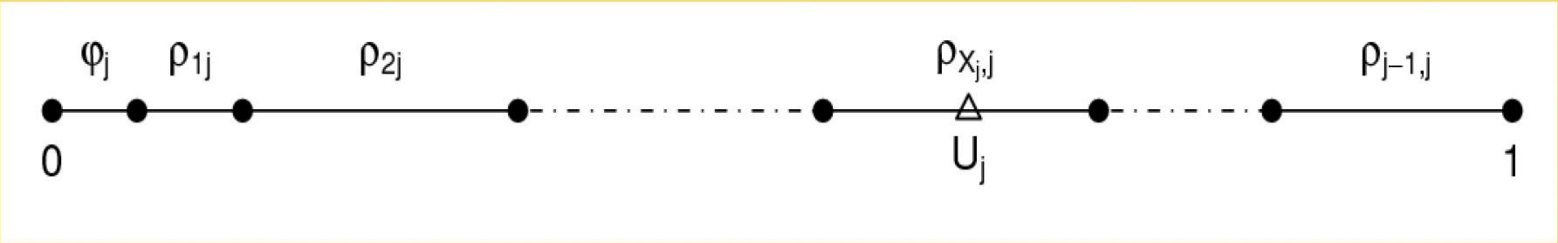
Stochastic declustering method



Algorithm: Generate a uniform random number U_j on [0, 1], set K satisfy

$$\varphi_j + \sum_{i=1}^{K} \rho_{ij} \le U_j < \varphi_j + \sum_{i=1}^{K+1} \rho_{ij}$$

Space-time ETAS model

- Time varying seismicity rate (conditional intensity or stochastic intensity)

$$\lambda(t,x,y,m) = s(m)\left[\mu(x,y) + \sum_{i:t_i<t} \kappa(m_i)\, g(t-t_i)\, f(x-x_i, y-y_i)\right]$$

Contribution from background seismicity

$$\boxed{\frac{s(m)\mu(x,y)}{\lambda(t,x,y,m)}}$$

Contribution from the *i*-th event

$$\boxed{\frac{s(m)\, g(t-t_i)\, f(x-x_i, y-y_i)}{\lambda(t,x,y,m)}}$$

Estimation problems

- Time varying seismicity rate (conditional intensity or stochastic intensity)

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How to estimate time-free total seismicity $\lambda(x,y)$?

How to estimate background seismicity?

How to estimate clustering parameters?

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How to estimate time-free total seismicity $\lambda(x,y)$?

Kernel, spline, tessellation, histogram, …

How to estimate background seismicity?

How to estimate clustering parameters?

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- Time varying seismicity rate (conditional intensity or stochastic intensity)

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How to estimate background seismicity?

How to estimate clustering parameters?

Maximum likelihood estimate if background seismicity $\boldsymbol{\mu}$ is know

Estimation problems

- Time varying seismicity rate (conditional intensity or stochastic intensity)

$$\lambda(t,x,y,m) = s(m)\left[\mu(x,y) + \sum_{i:t_i<t} \kappa(m_i)g(t-t_i)f(x-x_i,y-y_i)\right]$$

How to estimate time-free total seismicity $\lambda(x,y)$?

Kernel, spline, tessellation, histogram, …

How to estimate background seismicity?

?

How to estimate clustering parameters?

Maximum likelihood estimate if background seismicity μ is know

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Kernel, spline, tessellation, histogram, …, with each event weighted by φ_j

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Kernel

Kernel with each event weighted by φ_j

Estimation problems

- Time varying seismicity rate (conditional intensity or stochastic intensity)

$$\lambda(t,x,y,m)=s(m)\left[\mu(x,y)+\sum_{i:t_i<t}\kappa(m_i)g(t-t_i)f(x-x_i,y-y_i)\right]$$

Kernel

Kernel with each event weighted by φ_j

$$\hat{\mu}(x,y)=\frac{1}{T}\sum_j \varphi_j h(x-x_j,y-y_j;d)$$

$$\hat{\lambda}(x,y)=\frac{1}{T}\sum_i h(x-x_i,y-y_i;d)$$

Solution—estimating parameters and background rate simultaneously

Algorithm:

1. Assume an initial background rate.
2. Using MLE to estimate parameters in the clustering structures.
3. Using the assumed background and estimated clustering parameters to evaluate φ_j .
4. Using φ_j to get a better background rate.
5. Update the background rate by this better one.
6. Repeat Steps 2 to 5 until results converge.

φ_j: Estimate of probability that event j is of background

Uses of stochastic declustering

- To estimate background rate
- To generate background catalogue

- For each event j, generate a uniform r.v. U in [0, 1].
- Select event j as background if $U < \varphi_j$.

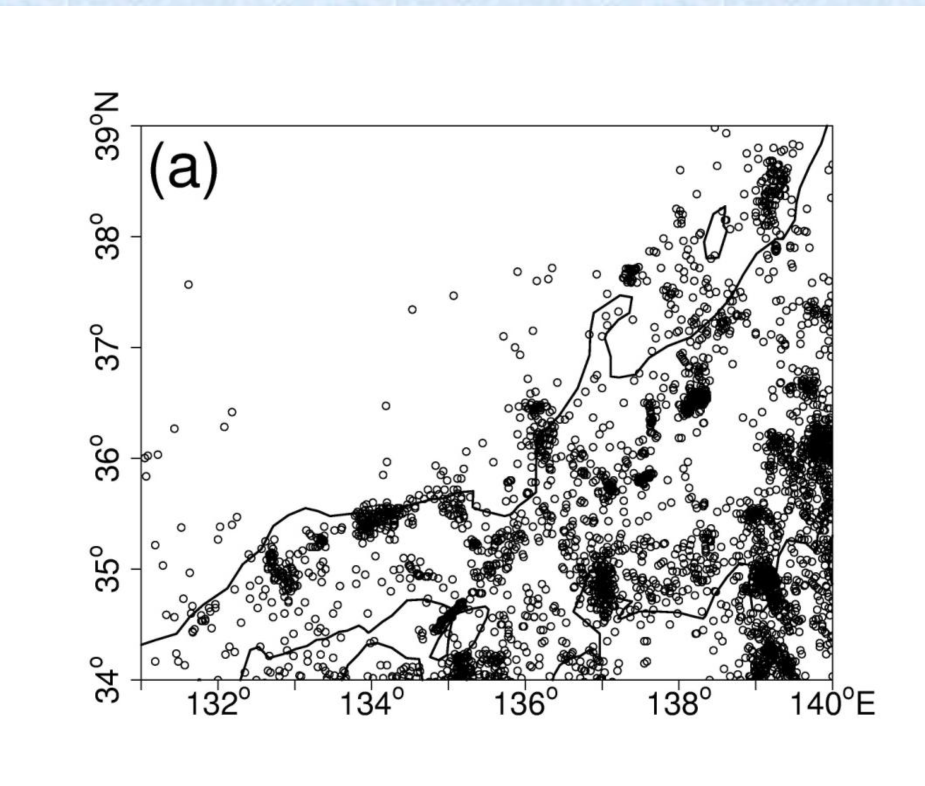
Uses of stochastic declustering

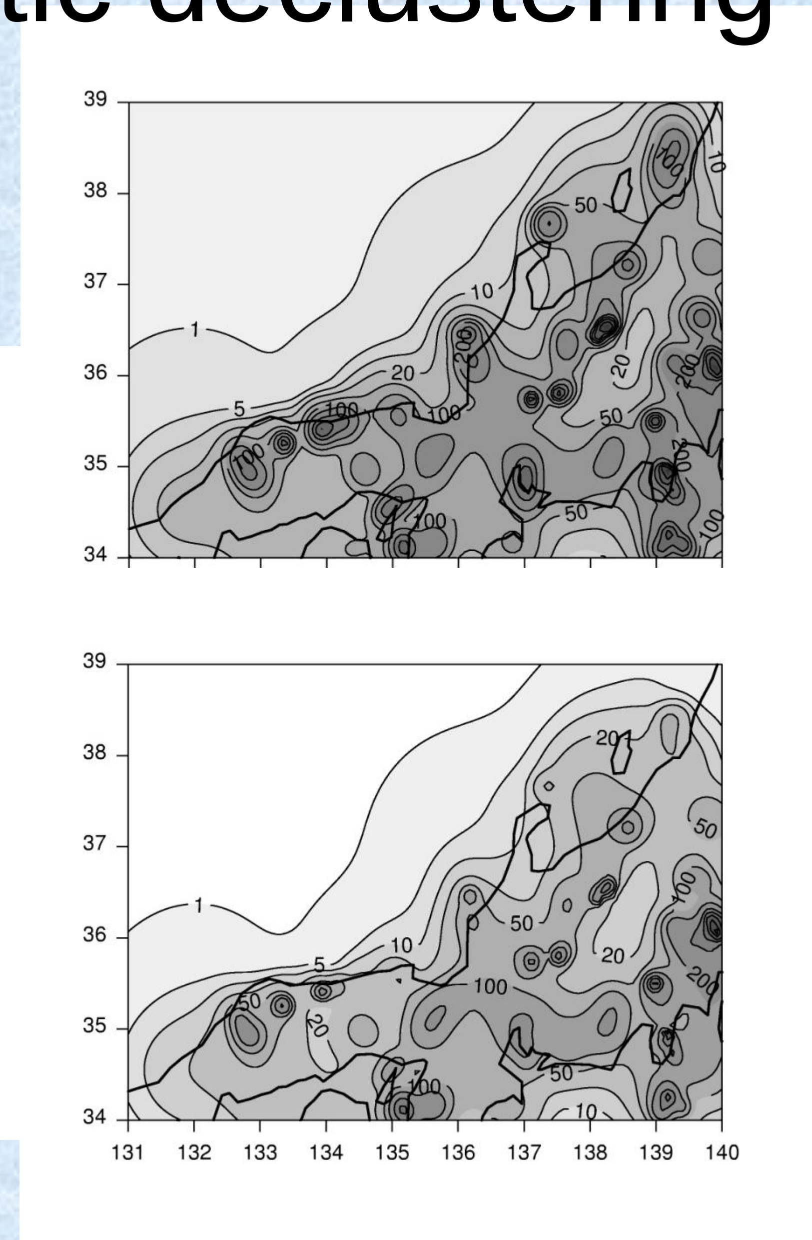
- To estimate background rate
- To generate background catalogue
- To separated clusters

- For each event j, generate a uniform r.v. U in [0, 1].
- Select event j as background if $U < \varphi_j$.
- Select event j to be children of K if K is the largest integer such that $U < \varphi_j + \sum_{i=1}^{K} \rho_{ij}$

Uses of stochastic declustering

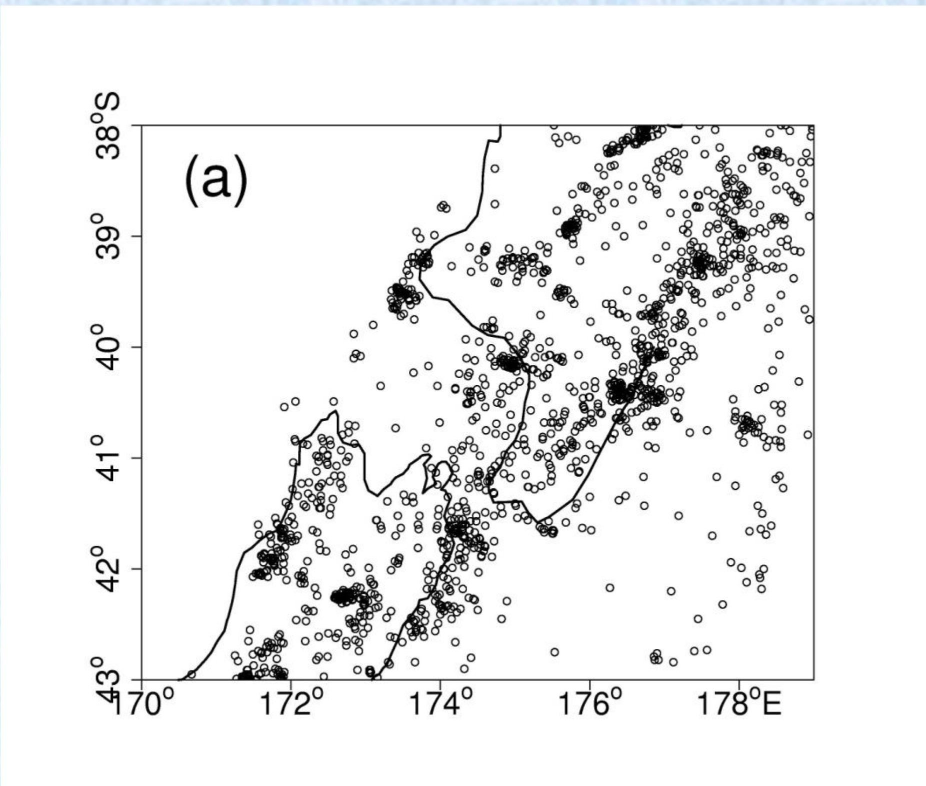
- To estimate background rate

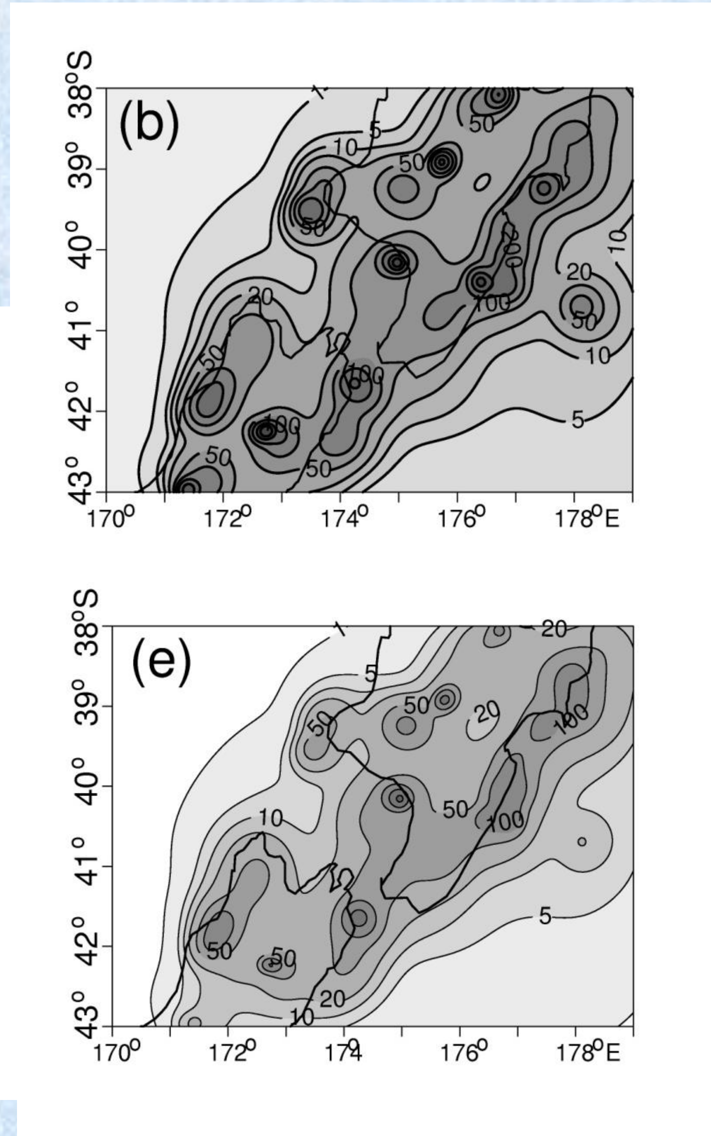




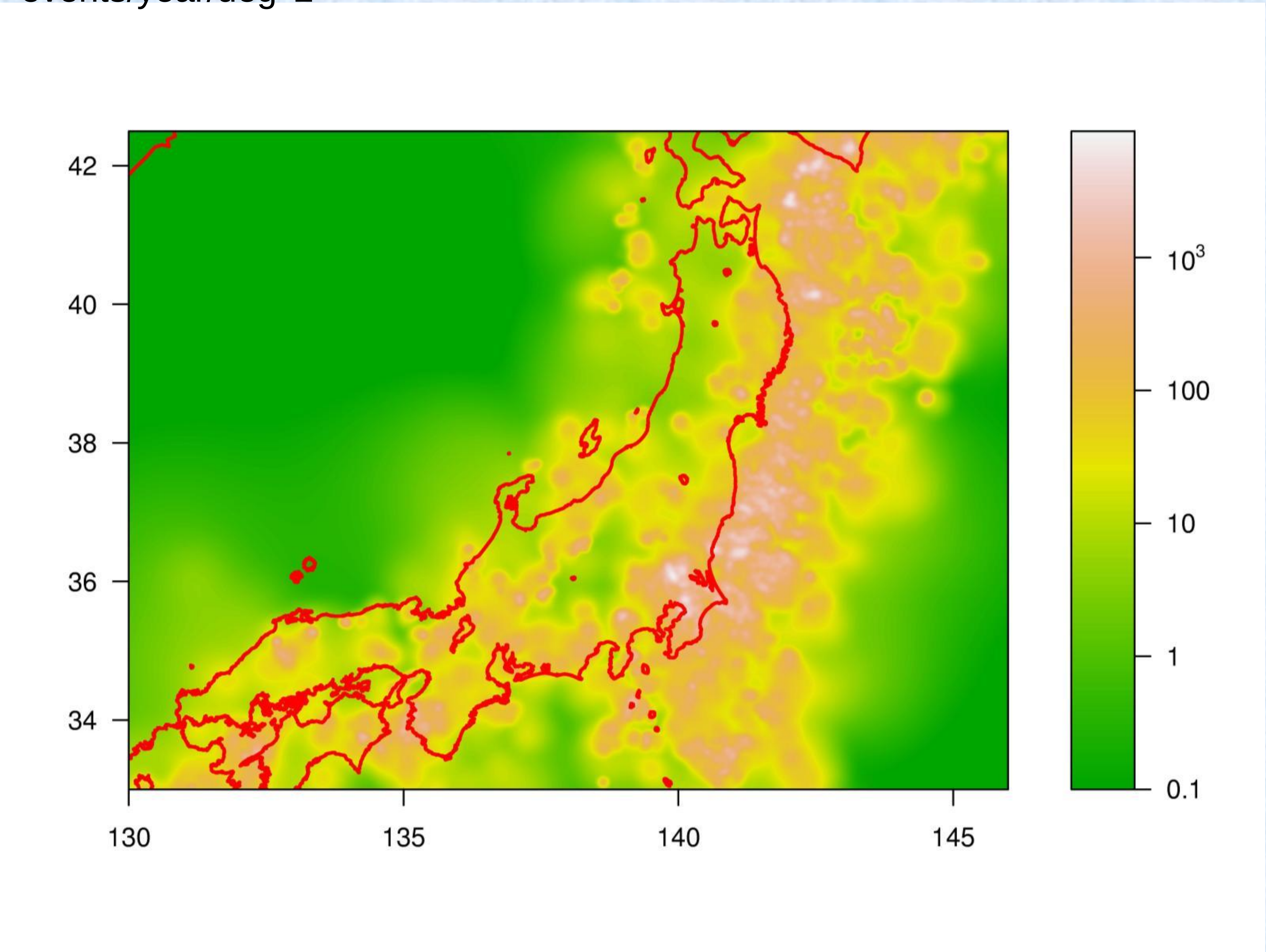
Uses of stochastic declustering

- To estimate background rate

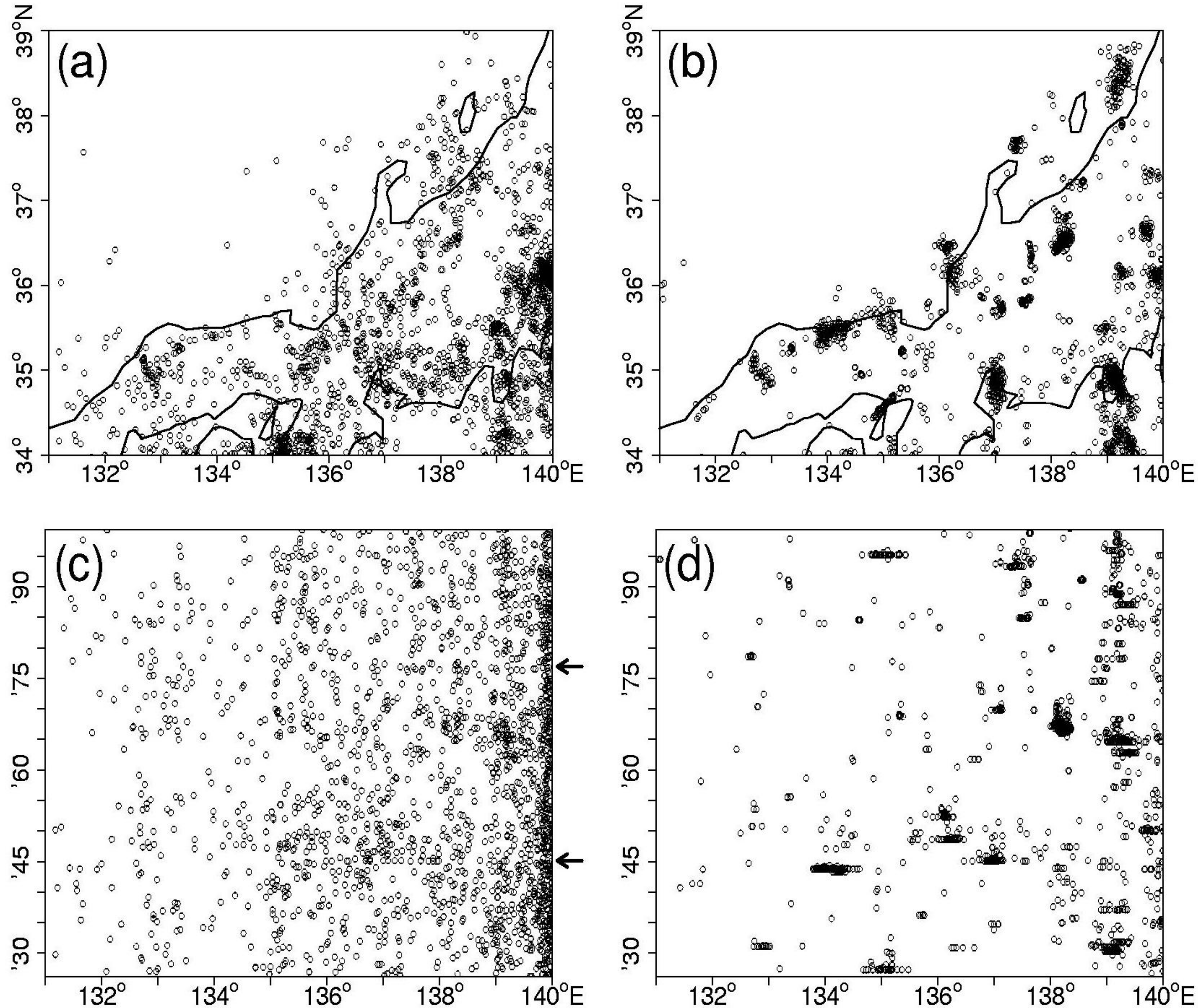




- Estimates of the background rate $\mu(x,y)$:
events/year/deg^2



Uses of stochastic declustering

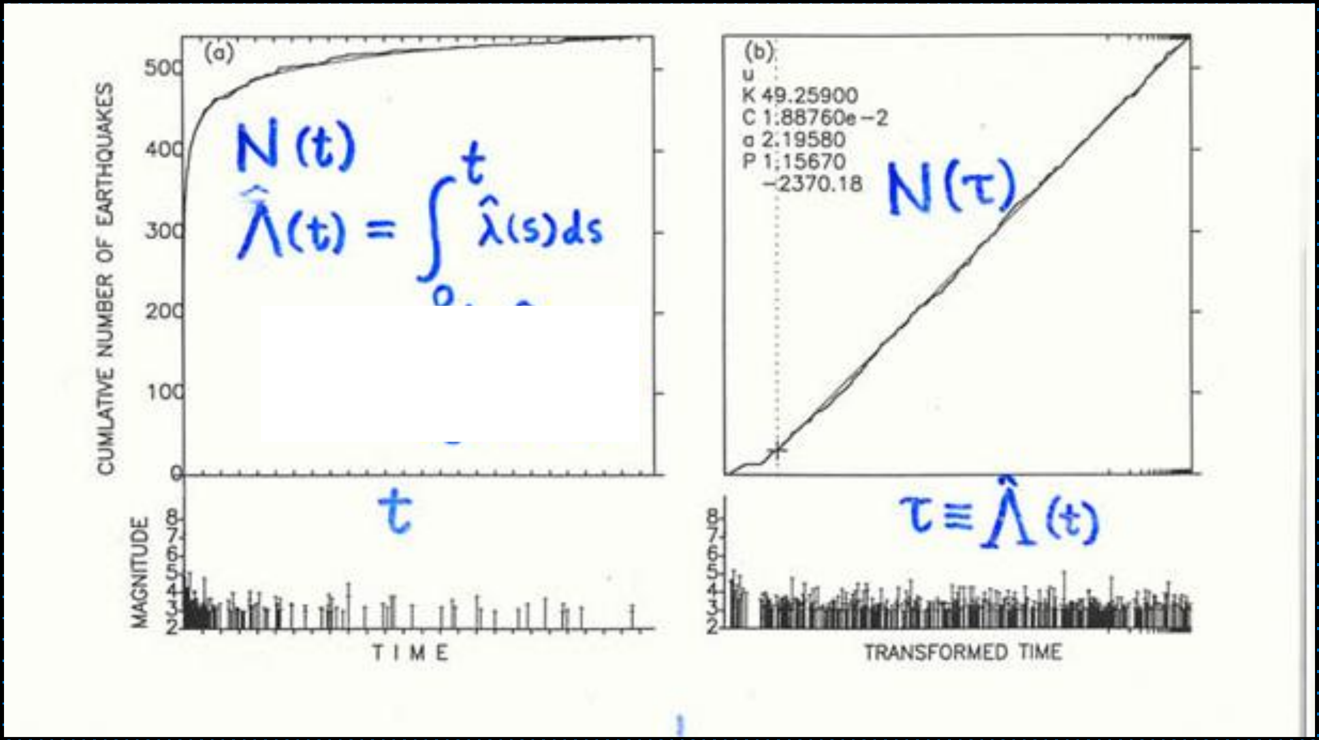


Outlines

- The meaning of conditional intensity seismicity (time-varying seismicity rate)
- From the Omori-Utsu formula to Ogata's ETAS model
- Space-time ETAS model, estimation and stochastic declustering
- Relative quiescence and test hypothesis related to earthquake clustering
- Simulation and forecasting

Combining quiescence with clustering

1. Transformed Time sequence (Ogata, 1992, JGR)



$$t_i \to \tau_i = \int_0^{t_i} \lambda(u)du$$

If $\{t_i\}$ is the observation of a process determined by conditional intensity $\lambda(t)$, the $\{\tau_i\}$ is a standard Poisson process.

Space-time ETAS model

- Time varying seismicity rate (conditional intensity or stochastic intensity) at event j

$$\lambda(t_j, x_j, y_j, m_j) = s(m_j)\left[\mu(x_j, y_j) + \sum_{i:t_i<t} \kappa(m_i)g(t_j - t_i)f(x_j - x_i, y_j - y_i)\right]$$

Contribution from background seismicity

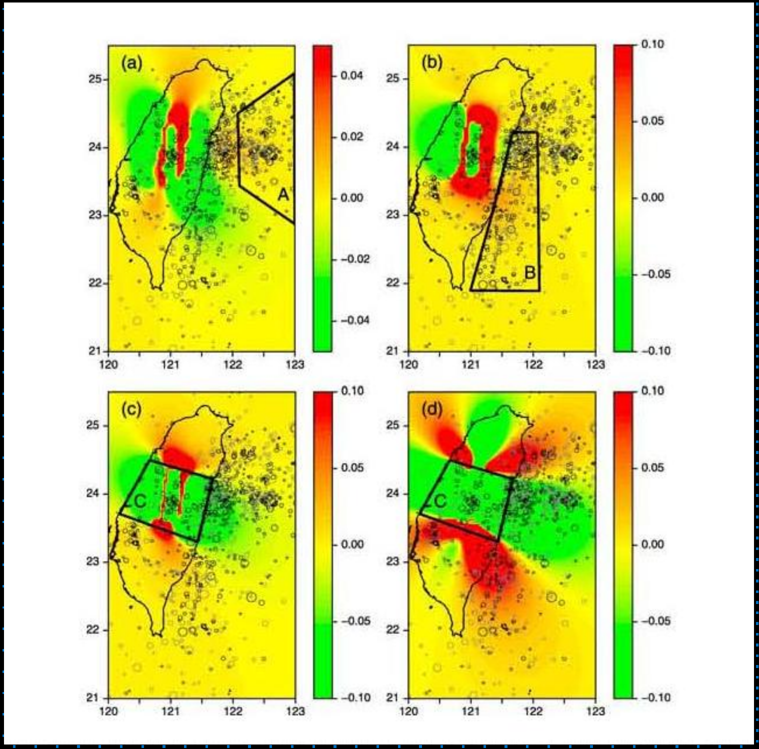
Contribution from the i-th event

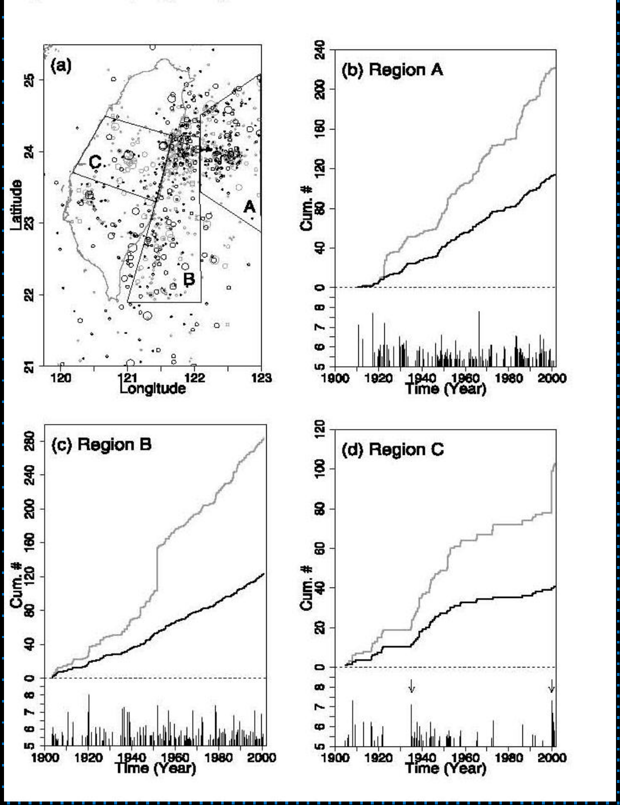
$$\boxed{\varphi_j = \frac{s(m_j)\mu(x_j, y_j)}{\lambda(t_j, x_j, y_j, m_j)}}$$

Combining quiescence with clustering

2. Quiescence in background seismicity (Zhuang et al., 2005)

Background seismicity $$S(t) = \sum_{i:t_i<t} \varphi_i$$





Uses of stochastic reconstruction

- To inverse clustering features.
- Empirical functions (histograms) of weighted samples
  - ρ_{ij} : *i* triggers ρ_{ij} children, not 1 child
  - φ_i: we get φ_i background events, not 1 background event

Examples 1 – Short distance decay in space or long distance decay?

- Empirical p.d.f.s for offspring locations

$$\hat{f}_R(r) = \frac{\sum_{i,j} \rho_{ij} I(|r_{ij} - r| < \Delta r / 2)}{\Delta r \sum_{i,j} \rho_{ij}}$$

r, r_{ij} : Standardized distance between a parent and its children.

Space-time ETAS model

- Conditional intensity or stochastic intensity

$$\lambda(t,x,y,m) = \frac{\mathbf{E}\left[N(dt \times dx \times dy \times dm) \mid \mathcal{H}_t\right]}{dt \times dx \times dy \times dm}$$

$$\lambda(t,x,y,m) = s(m)\left[\mu(x,y) + \sum_{i:t_i<t} \kappa(m_i) g(t-t_i) f(x-x_i, y-y_i)\right]$$

1. Magnitude p.d.f (G-R): $s(m) = \beta e^{-\beta(m-m_C)}, \quad m \geq m_C$
2. Direct productivity: $\kappa(m) = A e^{\alpha(m-m_C)}, \quad m \geq m_C$
3. Time p.d.f (Omori-Utsu): $g(t) = (p-1)(1+t/c)^{-p}/c, \quad t > 0$
4. Location p.d.f: $f(x,y \mid m) = \dfrac{q-1}{\pi D e^{\gamma(m-m_c)}}\left(1 + \dfrac{x^2+y^2}{D e^{\gamma(m-m_c)}}\right)^{-q}$

Space-time ETAS model

- Conditional intensity or stochastic intensity

$$\lambda(t,x,y,m) = s(m)\left[\mu(x,y) + \sum_{i:t_i<t} \kappa(m_i)g(t-t_i)f(x-x_i, y-y_i)\right]$$

Standardized distance:

$$r_{ij} = \frac{(x_j - x_i)^2 + (y_j - y_i)^2}{De^{\gamma(m_i - m_c)}}$$

Location p.d.f:

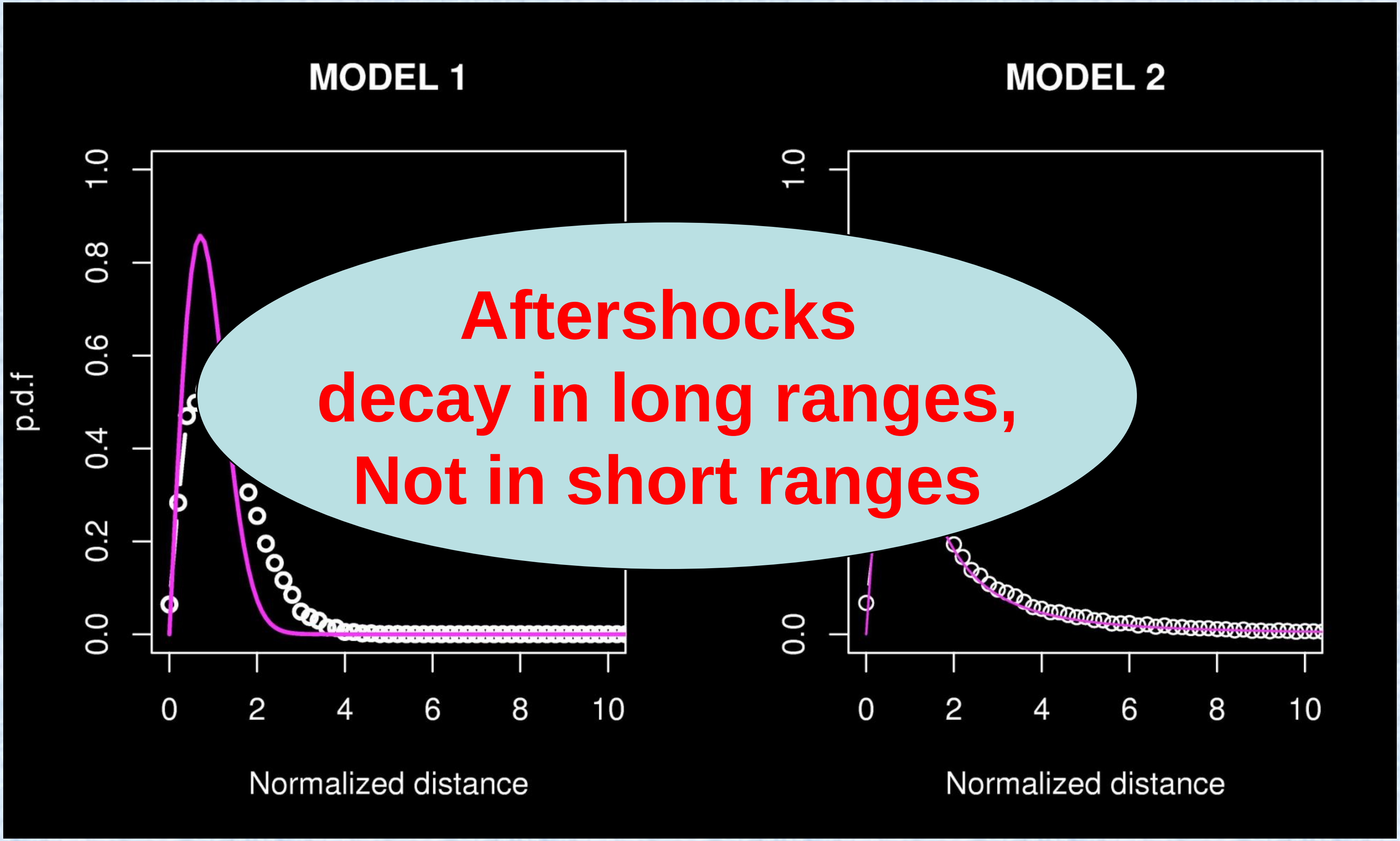
Model 1
$$f(x,y \mid m) = \frac{1}{2\pi De^{\alpha(m-m_c)}} \exp\left(-\frac{x^2+y^2}{2De^{\alpha(m-m_c)}}\right)$$

Model 2
$$f(x,y \mid m) = \frac{q-1}{\pi De^{\alpha(m-m_c)}} \left(1 + \frac{x^2+y^2}{De^{\alpha(m-m_c)}}\right)^{-q}$$

Results of location distributions for JMA catalog

Model 1: a short range decay (2-D gaussian => Rayleigh)

Model 2: a long range decay (inverse power => $Cr(1+r^2)^{-q}$)



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Simulation of temporal point processes

Simulate a point process of conditional intensity $\lambda(t)$ on $[S, T]$.

- *Method 1: inversion method*
  This method is based on the distribution of waiting times F:

$$1 - F_t(u) = \exp\left[-\int_t^{t+u} \lambda(v)\,\mathrm{d}v\right].$$

  1. Set $t = S$, and $i = 0$.
  2. Simulate a r.v. $U \sim Uniform(0,1)$. Find x by solving $U = \exp\left[-\int_t^x \lambda(u)\,\mathrm{d}u\right]$.
  3. Set $t_{i+1} = x$, $t = x$, $i = i + 1$, and return to step 2 until x exceed T.
  4. Return $t_1, t_2, \cdots, t_n$.

- *Method 2: thinning method*
  1. Set $t = S$, and $i = 0$.
  2. Find a positive M such that $\lambda(t) < M$ from t to the occurrence time of next event.
  3. Simulate a waiting time, x, according to a Poisson process of rate M, i.e., $x \sim Exp(M)$.
  4. Simulate a r.v. $U \sim Uniform(0,1)$.
  5. If $U \leq \lambda(t+x)/M$, set $t_{i+1} = t + x$, $t = t + x$ and $i = i + 1$. Otherwise, only set set $t = t + x$
  6. Return to step 2 until t exceed T.
  7. Return $t_1, t_2, \cdots, t_n$.

Requiring that step 2 is valid.

Probability forecast procedures

Suppose we are given the observation of earthquakes and precursors to a time t, and try to prediction the occurrence of at least one event in a given time interval $[t, t + dt)$ and a space-time window $S \times \Omega$ with a formulated model (Vere-Jones 1999).
Time T^* to the next event from the time t

$$\Pr\{T^* > \tau\} = \exp\left[-\int_0^\tau \lambda(t+u|\mathcal{H}_t)du\right]. \quad (48)$$

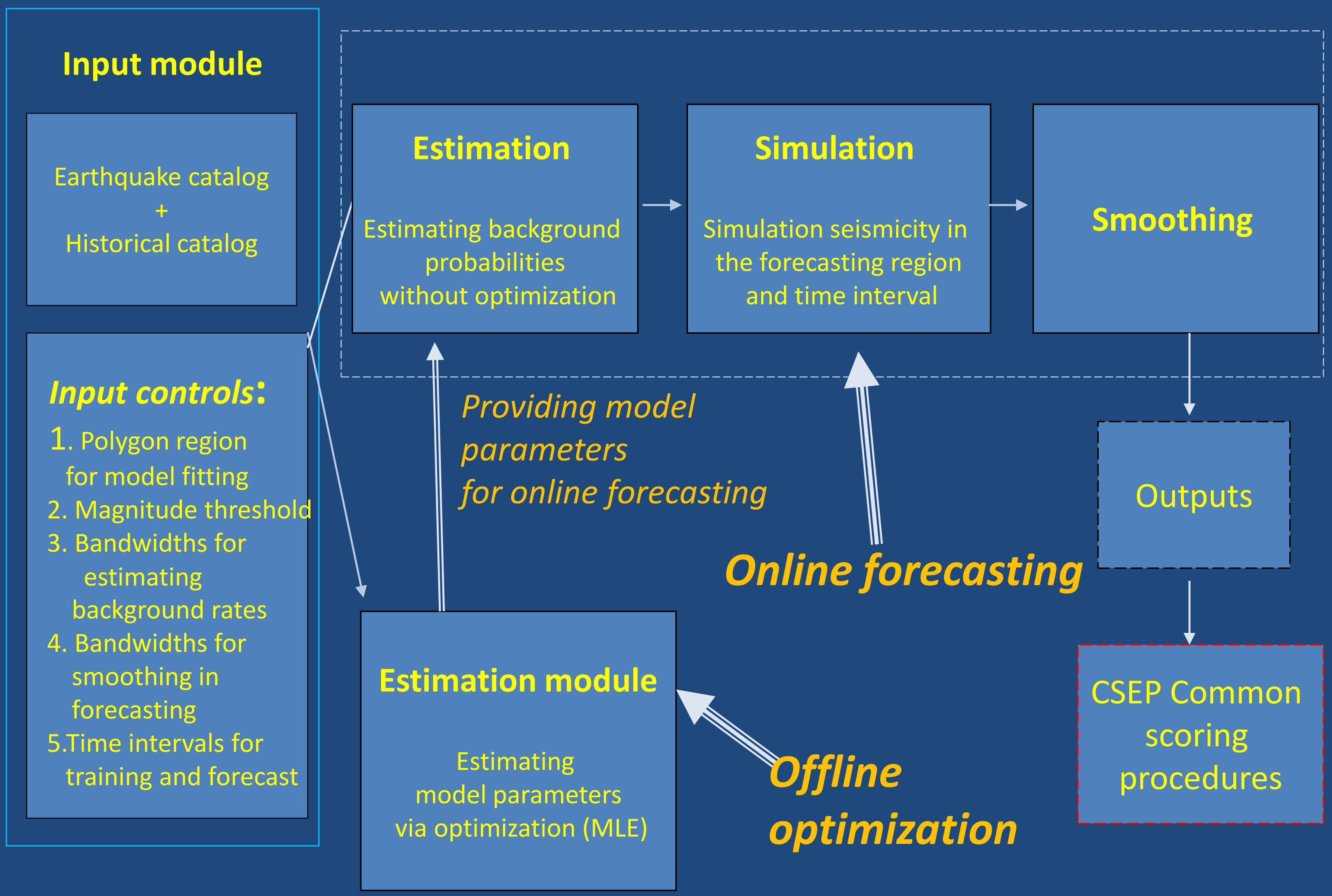
Forecasting based on simulations

1. Fit the model to the observation data up to the given time t to get the model parameters by using, e.g., maximum likelihood estimates.
2. Simulate a time τ to the next occurring event by using the $\lambda(t|\mathcal{H}_t)$, or simulate the occurrence time to the next occurring event and its location and magnitude by using the $\lambda(t, x, M|\mathcal{H}_t)$ if the conditional intensity has a space-time-magnitude form. The simulation method generally depend on the form of the conditional intensity function.
3. Add the new events to the history, and return to step (2), replacing t by $t + \tau$.
4. Continue until the end of the time interval for forecasting, $(t, t + \delta)$, has exceed.
5. Repeat the above simulation in Steps (2) to (5) for a large member, the proportion containing at least an event in the target prediction space-time-magnitude window can be determined and used as estimates of the required forecast probabilities.

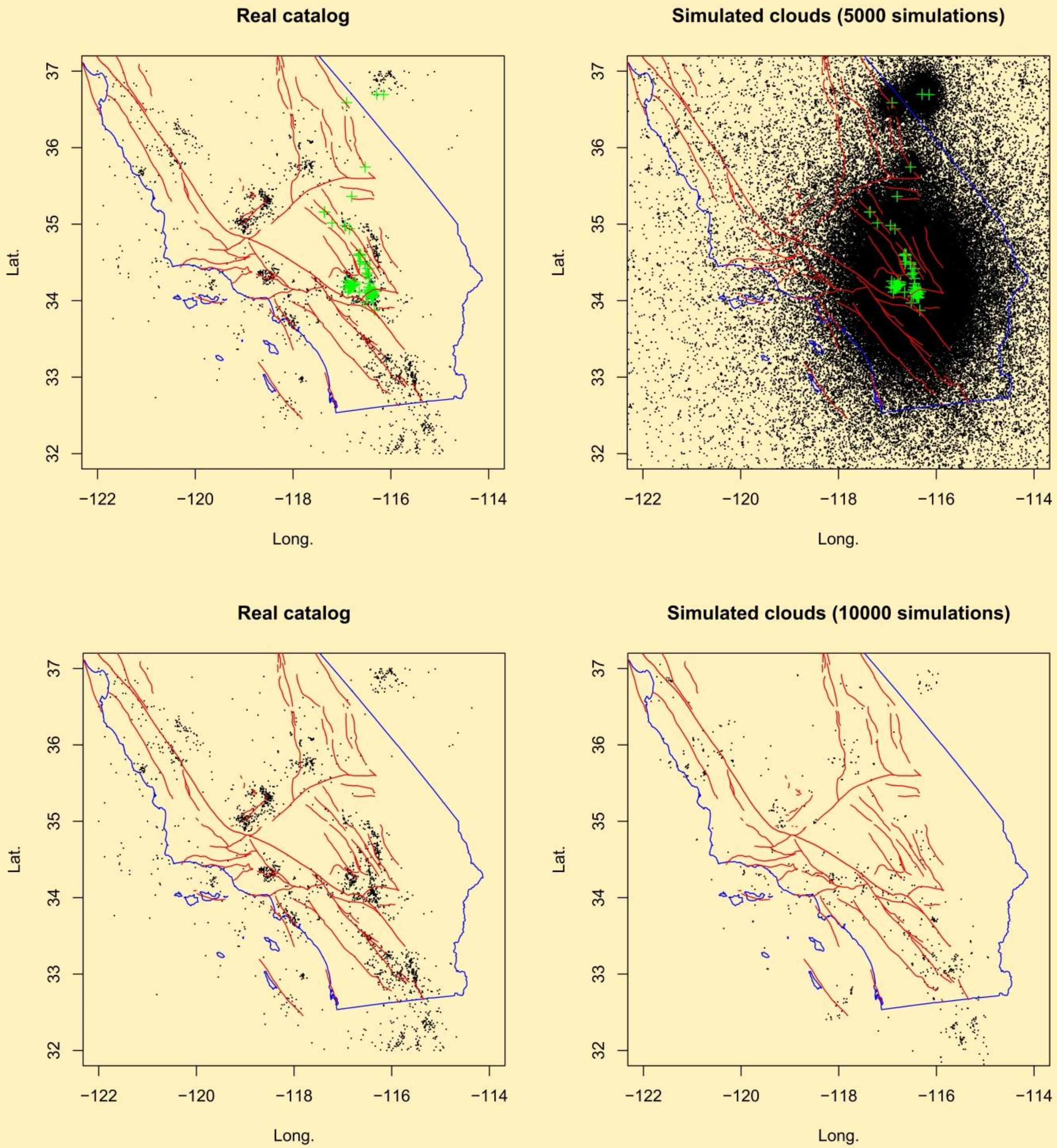
Simulation algorithm

- Generate the background catalog with the estimated background rate, recorded as Generation 0.
- For each event, in the last simulated generation, generate its children, with their occurrence times, locations and magnitude from the pdf.s as assumed in the model, where the number of children is a Poisson random variable with a mean of the productivity function.
- Repeat last step until no more new event is generation. Return with all the events in all generations

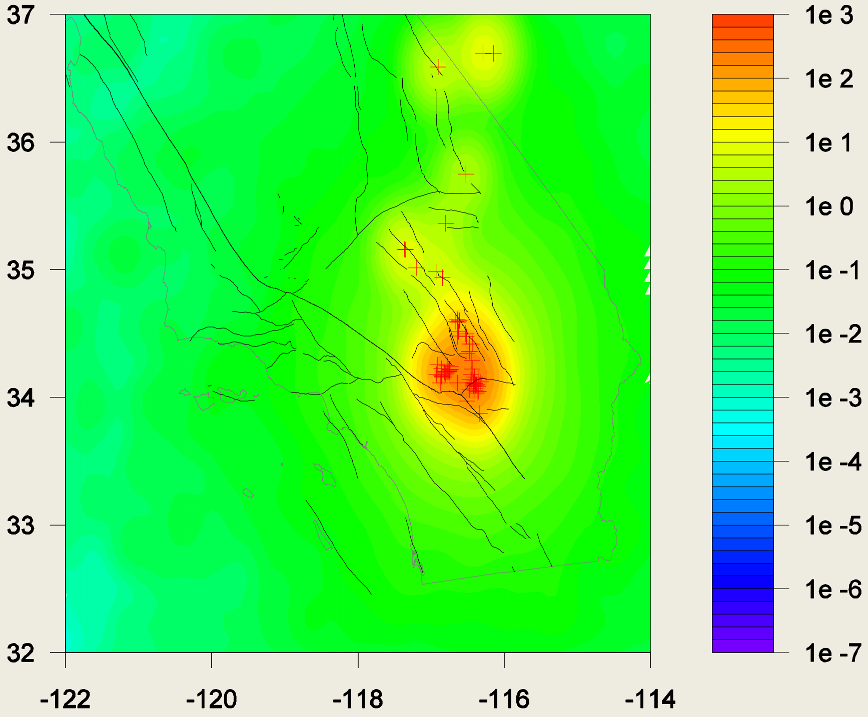
Diagram of SCEC CSEP ETAS



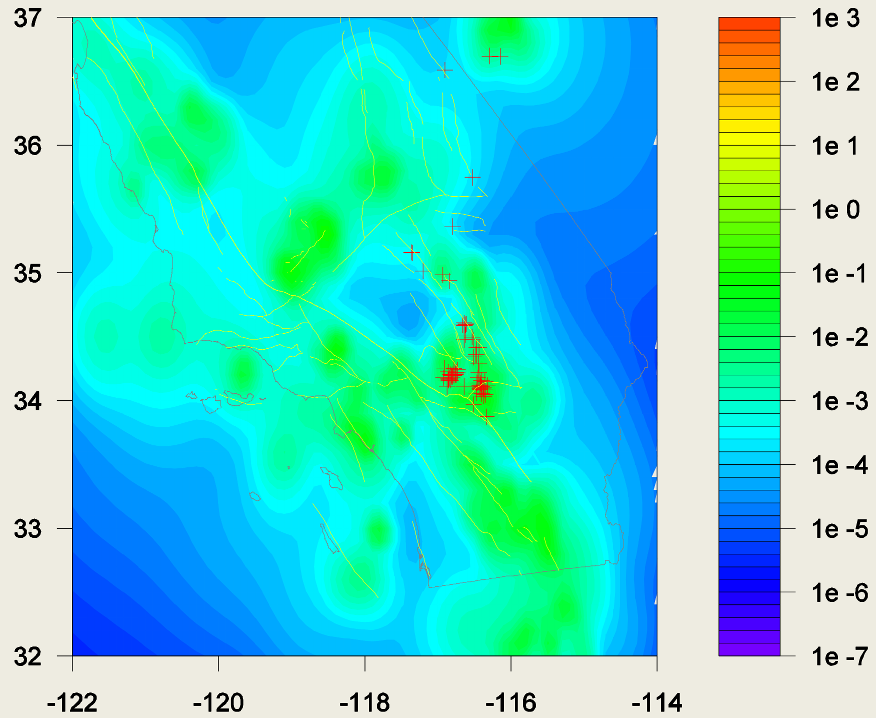
Example: Forecasting experiment after the landers earthquake (1992-6-28 M7.3)



Forecasting Landers aftershocks



ETAS model



Poisson model

Other topics related to the ETAS model

1. Criticality of the model.
   Vere-Jones, Sornetter, Helmstetter, Zhuang

2. Relation to foreshocks.
   Zhuang, Helmstetter, Sornetter, Marzocchi, Christophersen

3. Relation to the Bath law.
   Zhuang, Vere-Jones, Helmstetter, Sornetter, Rhoades

4. HIST-ETAS model.
   Ogata, Kumazawa

5. Finite volume source and hypocenter ETAS model.
   Guo

6. Self-similar ETAS model.
   Vere-Jones

Thank you!