

Dynamic Programming

DP → Enhanced Recursion
Parent of DP is Recursion

Code DP first write Recursive solution

Identify DP → ① choice Option → Using Recursion
② asking optional (minimum/maximun)
again calling & function
↓
DP can be use

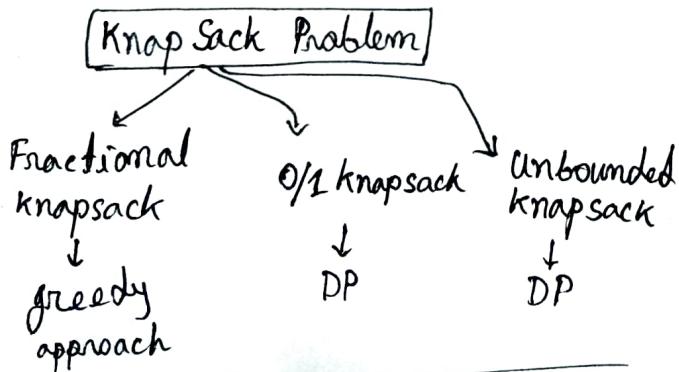
DP = Recursion + Storage
= Memoization

Tabular / Matrix
Bottom-up DP

Variation of DP Questions:

- 1) 0-1 knapsack (6 questions) ↗ Subset Sum
↗ Equal sum Partition
↗ Count of subset
 - 2) Unbounded knapsack (5)
 - 3) Fibonacci (7)
 - (4) LCS (15)
 - 5) LIS (10)
 - 6) Kadane's Algorithm (6)
 - (7) DP on Trees (4)
 - (8) DP on Grid (14)
 - 9) Matrix chain Multiplication (7)
 - 10) Others (5)
- 0-1 knapsack Problem

 - ⑥ Min changes
 - 1) Subset sum
 - 2) Equal sum Partition
 - 3) Count of subset ~~Sum~~
 - 4) Minimum Subset sum ~~Diff~~
 - 5) Target Sum
 - 6) Number of subset sum with given Diff.



Memorization ⇒ Top-Down DP
Matrix DP ⇒ Bottom Up DP ⇒ Tabular

Input:
 $I_1 \ I_2 \ I_3 \ I_4$
 weight [] = 1 3 4 5 → kg
 $V_1 \ V_2 \ V_3 \ V_4$
 value [] = 1 4 5 7 → ₹
 $W = 10 \text{ kg}$
 \downarrow
 max capacity
 \downarrow
 max Profit ? bag



Identify → choice → include or not → max profit ↗

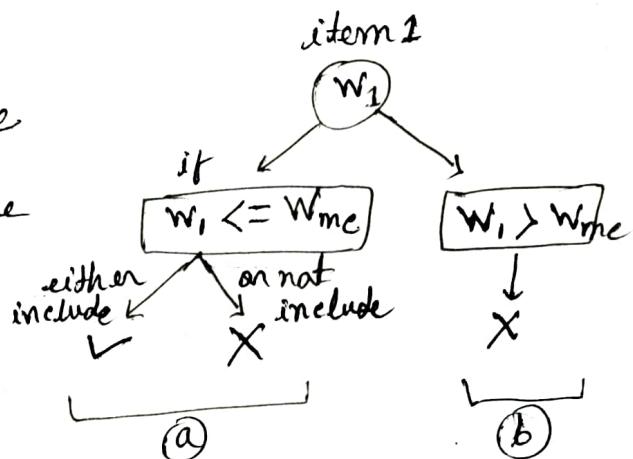
0/1 knapsack Recursion

Input: Weight : [1, 3, 4, 5] kg output \Rightarrow max profit
 Values: [1, 4, 5, 7] ₹
 $W_{\text{max}}^{\text{capacity}} = 7 \text{ kg} \rightarrow W_{\text{mc}}$

(Step 3) Choice Diagram \rightarrow Recursive Code

```
def knapsack{
    Base condition
    choice diagram
}
```

Basecase \rightarrow think of the smallest valid input



n = len(weight)

Recursive Code

```
def knapsack (weight, values, Wmc, n):
    if (n == 0 or Wmc == 0): return 0
```

if weight[n-1] <= Wmc :

value if include
 $\max(\text{values}[n-1] + \text{knapsack}(\text{weight}, \text{values}, \text{Wmc} - \text{weight}[n-1], n-1),$
 $\text{knapsack}(\text{weight}, \text{values}, \text{Wmc}, n-1))$

value if not include

elif weight[n-1] > Wmc :

return knapsack (weight, values, Wmc, n-1)

\downarrow Memorisation

n, w
constraints
 $n \leq 100$
 $w_{\text{mc}} \leq 1000$

		w _{mc}				
		-1	-1	-1	-1	-1
n	-1	-1	-1	-1	-1	-1
	-1	-1	-1	-1	-1	-1

~~DP[100][100]~~ \Rightarrow make matrix (each element -1)

```
def knapsack(wt, val, Wmc, n):
```

if n == 0 or Wmc == 0: return 0

if dp[n][Wmc] != -1: return dp[n][Wmc]

if wt[n-1] <= Wmc :

$\max(\text{val}[n-1] + \text{knapsack}(\text{wt}, \text{val}, \text{Wmc} - \text{wt}[n-1], n-1),$
 $\text{knapsack}(\text{wt}, \text{val}, \text{Wmc}, n-1))$

if wt[n-1] > Wmc :

return dp[n][Wmc] = knapsack (wt, val, Wmc, n-1)

only add these to the recursive code
and memorize.

Base condition of Recursive Solution \Rightarrow

Initialisation of Top-Down approach (1st row, 1st column)

Recursive Code

if $n = 0$ or $w = 0$: return 0

if $wt[n-1] \leq w$:

return $\max(val[n-1] + knapsack(wt, val, w - wt[n-1], n-1), knapsack(wt, val, w, n-1))$

elif $wt[n-1] > w$:

return knapsack(wt, val, w, n-1)

$dp = [[0] * (n+1) \text{ for } i \text{ in range}(n+1)]$

for i in range(n+1):

for j in range(w+1):

if $i=0$ or $j=0$: $dp[i][j] = 0$

for i in range(1, n+1):

for j in range(1, w+1):

if $wt[j-1] \leq w$:

$dp[i][j] = \max(val[i-1] + dp[i-1][j - wt[i-1]], dp[i-1][j])$

else:

$dp[i][j] = dp[i-1][j]$

return $dp[n][w]$

Top-down Code

for i in range(n+1):

for j in range(w+1):

if $i=0$ or $j=0$:
 $dp[i][j] = 0$

if $wt[n-1] \leq w$:

$dp[n][w] = \max(val[n-1] + dp[n-1][w - wt[n-1]], dp[n-1][w])$

elif $wt[n-1] > w$:

$dp[n][w] = dp[n-1][w]$

n	0	0	0	0
w	0	0	0	0
	0	0	0	0
	0	0	0	0

$\leftarrow w(j) \rightarrow$	0	0	0	0
i	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Subset Sum Problem: check if we can find the target sum from the subset of the array.

$arr = [3, 7, 8, 12]$

targetsum = 11

ans = True

$arr = []$

targetsum = 0

True

$arr = [3, 7, 8, 12]$

targetsum = 0

True (Empty Subarray)

	0	1	2	3	4	5	6	7	8	9	10	11
0	T	F	F	F	F	F	F	F	F	F	F	F
1	T											
2	T											
3	T											

$arr = []$
targetsum = 11

False
Possible
False

Symmetry of subset sum with 0/1 Knapsack

0/1 Knapsack

wt[]

w

val[]

if $\text{wt}[i-1] \leq j$:

$$\text{dp}[i][j] = \max(\text{val}[i-1] + \text{dp}[i-1][j - \text{wt}[i-1]], \text{dp}[i-1][j])$$

else: $\text{dp}[i][j] = \text{dp}[i-1][j]$

Subset Sum

$\text{wt}[] \Rightarrow \text{arr}[]$

$w \Rightarrow \text{Target Sum}$

$\text{val}[] \Rightarrow \text{Ignore}$

if $\text{arr}[i-1] \leq j$:

$$\text{dp}[i][j] = \text{dp}[i-1][j - \text{arr}[i-1]]$$

or

$$\text{dp}[i-1][j]$$

else: $\text{dp}[i][j] = \text{dp}[i-1][j]$

Equal Sum Partition

$\text{arr} = [1, 3, 2, 2, 5, 7] \Rightarrow$ if $\text{sum}(\text{arr}) \% 2 != 0$ return False
else: target = $\text{sum}(\text{arr}) // 2$
apply subset sum on target.

Count of subset sum with a given sum :

$\text{arr} = [2, 3, 5, 6, 8, 10] \quad \text{sum} = 10$

ans = 3 $\Rightarrow \{2, 3, 5\}, \{2, 8\}, \{10\}$

just change the "or" of subset sum $\Rightarrow '+'$

if $\text{arr}[i-1] \leq j$:

$$\text{dp}[i][j] = \underbrace{\text{dp}[i-1][j - \text{arr}[i-1]]}_{\text{include}} + \underbrace{\text{dp}[i-1][j]}_{\text{do not include}}$$

else: $\text{dp}[i][j] = \text{dp}[i-1][j]$

sum										target
1	0	0	0	0	0	0	0	0	0	0
1										
1										
1										
1										
0										

size of arr

(for sum=0 any subset i.e. empty subset is possible.)

Minimum Subset Sum Difference

$\text{arr} = [1, 2, 7]$

possible subsets									
1	2	3	4	5	6	7	8	9	10
T	T	F	F	F	F	F	F	F	T
T	F	F	F	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F	F

S_1 subsets S_2

Find $\min(S_2 - S_1)$

↓ smaller

$$S_1 + S_2 = \text{total sum}$$

$$\Rightarrow S_2 = \text{total sum} - S_1$$

$$\therefore \min(\text{total sum} - 2 \cdot S_1)$$

In subset sum problem

size of array	0	possible sums									
		0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F	F
1	T	T	F	F	F	F	F	F	F	F	F
2	T	F	F	F	F	F	F	F	F	F	F
3	T	T	(T)	F	F	F	F	(T)	T	T	T

2 can be a subset sum for size 3 arr

7 can be a subset sum

→ This row denotes → if size of arr is 3 then what are the possible subset sums are possible

∴ From 3rd row we get,
possible subset sum for arr = [1, 2, 7] $\Rightarrow S_1$
subset sums = [0, 1, 2, 7, 8, 9, 10] $\Rightarrow S_1$

$$mn = \min_{S1} \text{subsetsumdiff} = \text{float}("inf")$$

for $S1$ in subset sums:

$$mn = \min(mn, \text{totalsum} - 2 \cdot S1)$$

return (mn)

Count the number of subsets with a given difference:

$$\text{arr} = [1, 1, 2, 3] \quad \text{diff} = 1$$

$$S_1 - S_2 = \text{diff} \quad \text{--- (1)}$$

$$S_1 + S_2 = \text{totalsum} \quad \text{--- (2)} \Rightarrow S_1 = \frac{\text{totalsum} + \text{diff}}{2}$$

∴ we have to find now count of subsets whose sum is $\frac{\text{totalsum} + \text{diff}}{2}$

∴ count of subset sum =

Target Sum: $\text{arr} = [1, 1, 2, 3] \quad \text{target} = 1$ assign '+' and '-' before elements of arr such that $\text{sum}(\text{arr}) == \text{target}$



① \rightarrow ②

[basically we are again dividing arr into two subset such that difference b/w subsetsums equals target.

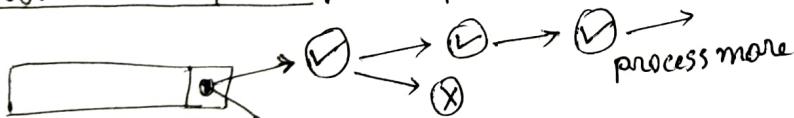
$$\therefore S_1 - S_2 = \text{target}$$

$$\Rightarrow 2S_1 = \text{target} + \text{sum}(\text{arr})$$

$$\Rightarrow S_1 = \frac{\text{target} + \text{sum}(\text{arr})}{2}$$

Find number of possible $S_1 \Rightarrow \text{answer}$

Unbounded Knapsack: multiple occurrence of an element.



once this choice not taken

it is done processing.

then call for remaining arr.
(n-1)

							weight
							max capacity
0	1	2	3	4	5	6	
0	0	0	0	0	0	0	
0							
0							
0							
0							
0							

0-1 knapsack

if $\text{wt}[i-1] \leq j$:

$$\text{dp}[i][j] = \max(\text{val}[i-1] + \text{dp}[i-1][j - \text{wt}[i-1]], \text{dp}[i-1][j])$$

else:

$$\text{dp}[i][j] = \text{dp}[i-1][j]$$

Unbounded knapsack

if $\text{wt}[i-1] \leq j$:

$$\text{dp}[i][j] = \max(\text{val}[i-1] + \text{dp}[i][j - \text{wt}[i-1]], \text{dp}[i-1][j])$$

Taken call the same for further processing

else:

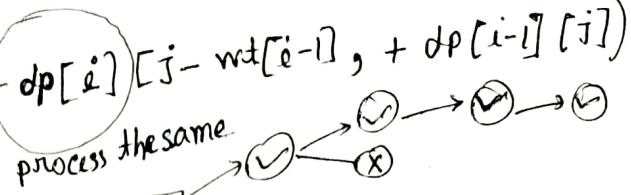
$$\text{dp}[i][j] = \text{dp}[i-1][j]$$

not taken go call for remaining

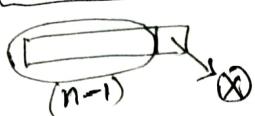
Unbounded knapsack \Rightarrow ① Rod Cutting, ② Coin Change I, ③
coin change II, ④ Maximum ribbon cut

if $wt[i-1] \leq j$:

$$dp[i][j] = \max(\text{val}[i-1] + dp[i][j], dp[i-1][j])$$



else: $dp[i][j] = dp[i-1][j]$



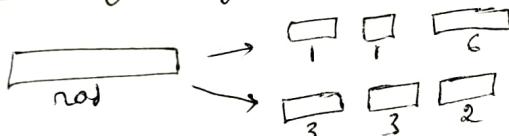
size of arr is 3
but for empty
array or not taking
any element profit = 0

capacity zero

0	0	0	0	0
1	0			
2	0			/\
3	0			

in every
place
placing
profit

Rod cutting length = 8 prices = [1, 5, 8, 9, 10, 17, 17, 20] \rightarrow as for length ≥ 0 profit



0	0	0	0	0	0	0	0
1	0						
2	0						
3	0						
4	0						
5	0						
6	0						
7	0						
8	0						

val \Rightarrow prices
weight \Rightarrow length = [(i+1) for i in range(n)]

if $length[i-1] \leq j$:

$$dp[i][j] = \max(\text{prices}[i-1] + dp[i][j - length[i-1]], dp[i-1][j])$$

else: $dp[i][j] = dp[i-1][j]$

possible length in which we can cut the rod.

\rightarrow weight of Unbounded knapsack is length array here.

Coinchange: any number of coins can be used \Rightarrow Unbounded knapsack

0	0	0	0
1			
1	X	X	
1			

Maximum combination of coins to make target.

\Rightarrow as size = 0; so no coin can be taken \Rightarrow combination 0

total no. of combination

From size 1 empty or no coins target 0 is possible.
 \rightarrow add as total combination is needed

if $coin[i-1] \leq j$: $dp[i][j] = dp[i][j - coin[i-1]] + dp[i-1][j]$

as multiple time one coin can be taken
as No val array is present in this problem
so not taking val

else: $dp[i][j] = dp[i-1][j]$

Coin Change II \rightarrow Minimum number of coins required to make the amount

coin = [1, 2, 5], amount = 11

	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	0					
2	0					
3	0					
4	0					
5	0					

Unbounded Knapsack
as any no. of coins can be taken

\Rightarrow For array of size = n to make amount ∞ numbers of coins required.

to make amount = 0; no coins are needed so $dp[i][0] = 0 ; 0 \leq i \leq n$

** In this question we have to initialise 1st row also **

for j in range (1, amount+1):

if $j \% \text{coin}[0] == 0$: \Rightarrow the amount(j) is multiple of 1st element of coin[0]

$dp[1][j] = j // \text{coin}[0] \Rightarrow$ no. of coins needed to make j

for i in range (2, n):

for j in range (1, amount): \Rightarrow Instead of $\text{val}[i-1] = 1$ for taking current coin.

if $\text{coins}[i-1] \leq j$:

$dp[i][j] = \min(1 + dp[i][j - \text{coins}[i-1]], dp[i-1][j])$

else: $dp[i][j] = dp[i-1][j]$

Longest Common Subsequence

x: a b c d g h

y: a b e d f h n

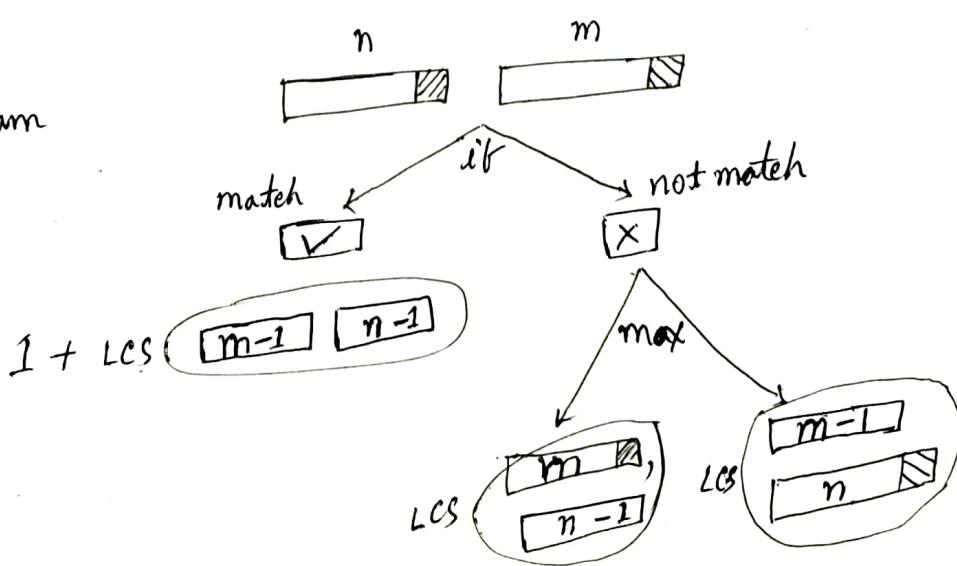
(4)

$m = \text{len}(x)$

$n = \text{len}(y)$

Recursive:

choose diagram

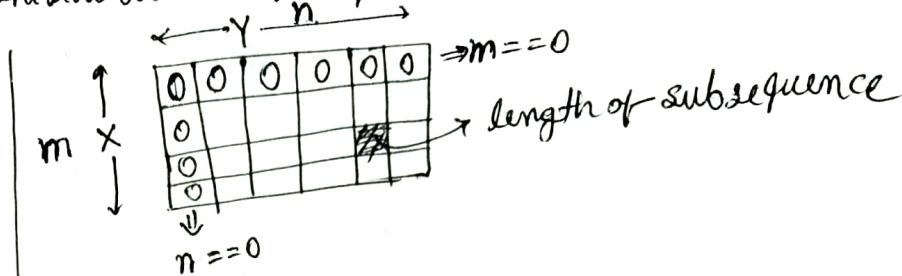


def LCS(x, Y, m, n): \Rightarrow Base condition
 if $m == 0$ or $n == 0$: return 0

from choose diagram {
 if $x[m-1] == Y[n-1]$: return $1 + \text{LCS}(x, Y, m-1, n-1)$
 else: return $\max(\text{LCS}(x, Y, m, n-1), \text{LCS}(x, Y, m-1, n))$
 \downarrow Top down DP.

Base case of recursion \Rightarrow Initialization of Top-down DP

if $m == 0$ or $n == 0$:
 return 0



if $x[m-1] == Y[n-1]$:
 return $1 + \text{LCS}(x, Y, m-1, n-1)$

if $x[m-1] == Y[n-1]$:

$$\text{dp}[m][n] = 1 + \text{dp}[m-1][n-1]$$

else:

return $\max(\text{LCS}(x, Y, m, n-1), \text{LCS}(x, Y, m-1, n))$

else:

$$\text{dp}[m][n] = \max(\text{dp}[m][n-1], \text{dp}[m-1][n])$$

$$\text{dp} = [[0] * (n+1) \text{ for } i \text{ in range}(m+1)]$$

change $m \rightarrow i$; $n \rightarrow j$

for i in range(1, m+1):

for j in range(1, n+1):

$$\text{if } x[i-1] == Y[j-1]: \text{dp}[i][j] = 1 + \text{dp}[i-1][j-1]$$

$$\text{else: } \text{dp}[i][j] = \max(\text{dp}[i][j-1], \text{dp}[i-1][j])$$

return $\text{dp}[m][n]$

Longest Common Substring:

$X = \boxed{a b} @ d @$ \Rightarrow Continuous common substring.

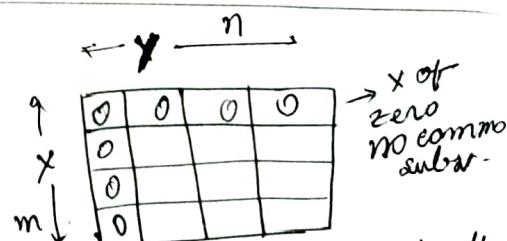
$Y = \boxed{a b} @ c @$ \Rightarrow length = 2,

$\text{res} = 0$ $\boxed{2}$ \downarrow Discontinuous part 0

if $x[i-1] == Y[j-1]$:

$$\text{dp}[i][j] = 1 + \text{dp}[i-1][j-1]; \text{res} = \max(\text{res}, \text{dp}[i][j])$$

else: $\text{dp}[i][j] = 0$



\hookrightarrow for Y of zero length

no common substring

as after dis continuity again assigning $\text{dp}[i][j] = 0$

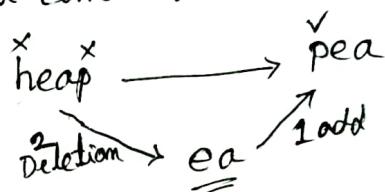
so storing max length in ~~res~~.

Length of Shortest Common Supersequence :

$X: AGCTTAB \quad (m)$ \rightarrow eliminate common subsequence
 $Y: GXTXAYB \quad (n)$ $(X+Y - LCS) \quad LCS = GTAB \Rightarrow \text{length}(LCS) = 4$
 $AGCTTAB + GXTXAYB - GTAB$
 $\Rightarrow m+n-\text{len}(LCS)$
 $\Rightarrow 6+7-4 = 9$

Minimum number of Insertion and deletion to convert string X to string Y :

$X: heap \quad (m)$ $\overset{\checkmark}{\cancel{X ea}} \rightarrow pea$ add: 1
 $Y: pea \quad (n)$ $LCS = ea$ delete: 2
 In X, LCS will remain intact and remaining characters will be removed.
 As LCS is common subsequence so optimal solⁿ can be found using LCS DP
 $X \rightarrow Y \quad LCS: \begin{matrix} \cancel{heap} \\ pea \end{matrix} > ea$



$$\therefore \text{number of insertion} = \text{len}(Y) - \text{len}(LCS)$$

$$\therefore \text{number of Deletion} = \text{len}(X) - \text{len}(LCS)$$

Longest Palindromic Subsequence: input $X = "agbcba"$ \Rightarrow only one string.

How to Find Similarities between LPS and LCS?

Longest Palindromic Subsequence (LPS)	longest common subsequence (LCS)
Input: 1 string problem: Longest (optimal) output: length (int)	Input: 2 strings problem: Longest (optimal) output: length (int)

the other string may be hidden!

$$x = "agbcba", \therefore y = \text{reverse}(x) = "abcbga"$$

$x = \textcolor{blue}{a g b \cancel{C} b a}$ $\Rightarrow LCS = "abcb"$
 $y = \textcolor{blue}{a \cancel{b} c b g a}$ longest palindrome //

$$\therefore LPS(x) \equiv LCS(x, \text{reverse}(x)) //$$

Minimum no. of Deletion / insertion in a string to make it a palindrome:

input = "ag b c b a"

Minimum no. of deletion from input \Rightarrow maximum length of palindrome.
 \Rightarrow no. of deletion = len(input) - len(LPS)

max len palindrome = "a b c b a"

$$= \text{len}(ag\ b\ c\ b\ a) -$$

$$\text{len}(^{\underline{\underline{a\ b\ c\ b\ a}}})$$

$$\therefore \text{length of LPS} \propto \frac{1}{\text{no. of deletion}}$$

$$= 6 - 5 = 1$$

$$\therefore \text{minimum no. of deletion} = \text{len(input)} - \text{LPS}$$

Print Shortest Common Supersequence: input: $x = "ac\ b\ c\ f"$ (m)
 $y = "ab\ c\ d\ a\ f"$ (n)

Supersequence is a string in which both x and y are present elementwise
multiple values that occurs at same time.

$$x = ac\ b\ c\ f \quad > \text{supersequence} = ac\ b\ c\ d\ a\ f$$

$$y = a\ b\ c\ d\ a\ f$$

In $x+y$, LCS exist 2 times, so eliminating 1 LCS we get SCS

Use print LCS technique.

	\emptyset	a	b	c	d	a	f
\emptyset	0 0 0 0 0 0 0						
a	0 1 1 1 1 1 1						
c	0 1 1 2 2 2 2						
b	0 1 2 2 2 2 2						
c	0 1 2 3 3 3 3						
f	0 1 2 3 3 3 3						

$$\text{LCS} = abcf$$

$$\text{LCS} = 4$$

$$i=m; j=n$$

while $i > 0$ and $j > 0$:

$$\text{if } x[i-1] = Y[j-1]:$$

$$\text{res} += x[i-1] \# or Y[j-1]$$

$$j -= 1; i -= 1$$

else:

$$\text{if } dp[i][j-1] > dp[i-1][j]:$$

$$\text{res} += Y[j-1]$$

$$j -= 1$$

$$\text{else: res} += x[i-1]$$

$$i -= 1$$

This path of finding LCS is the supersequence
which contains both x and y.

while $i > 0$:

$$\text{res} += x[i-1]$$

$$i -= 1$$

while $j > 0$:

$$\text{res} += Y[j-1]$$

$$j -= 1$$

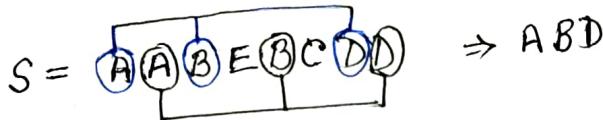
return $\text{res}[: :-1]$

If any of $i=0$ or $j=0$ then

we have to add remaining also

as we have to add all x and y elements

Longest Repeating Subsequence, input $S = "A A B E B C D D"$



$X = A A B E B C D D$ > use LCS with 1 extra condition $i \neq j$

$$Y = A A B E B C D D$$

$$X_A \rightarrow \begin{matrix} 0 \\ 1 \end{matrix}, (0,1)$$

$$Y_B \rightarrow \begin{matrix} 2 \\ 4 \end{matrix}, (2,4)$$

$$X_E \rightarrow \begin{matrix} 3 \\ 7 \end{matrix}, (3,7)$$

$$Y_D \rightarrow \begin{matrix} 6 \\ 7 \end{matrix}, (6,7)$$

$$\text{if } dp = \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & | & & & & & & \\ \hline 0 & & | & & & & & \\ \hline 0 & & & | & & & & \\ \hline 0 & & & & | & & & \\ \hline 0 & & & & & | & & \\ \hline 0 & & & & & & | & \\ \hline \end{array}$$

if $X[i-1] == Y[j-1]$ and $i \neq j$:

$$dp[i][j] = 1 + dp[i-1][j-1]$$

else:

$$dp[i][j] = \max(dp[i][j-1], dp[i-1][j])$$

check in the given 2 string one is subsequence of other or not.

if one string is the subsequence of others then the LCS of these 2 strings equal to $\min(\text{len}(A), \text{len}(B))$.

Alternative if $\text{len}(A) < \text{len}(B)$:

$\Rightarrow A$ and B are list of given input strings

$i=0$
while $i < \text{len}(A)$:
 if $i == \text{len}(B)$: return False
 if $A[i] == B[i]$: $i += 1$
 else $B.pop(i)$
return $A[:] == B[: \text{len}(A)]$

Is subsequence problem

Matrix Chain Multiplication: arr[] = [40, 20, 30, 10, 30]

For array of length n , there will be $n-1$ matrix.

$$\left[\begin{array}{|c|c|} \hline \quad & \quad \\ \hline 2 \times 3 & \end{array} \right] \times \left[\begin{array}{|c|c|} \hline \quad & \quad \\ \hline 3 \times 6 & \end{array} \right] = \left[\begin{array}{|c|c|} \hline \quad & \quad \\ \hline 2 \times 6 & \end{array} \right] \Rightarrow \text{cost on no. of operation} = 2 \times 3 \times 6$$

$$(a \times b) \times (b \times c) = [a \times c]$$

this dimensions must be same for matrix multiplication.
From arr 9 matrices of dimensions $40 \times 20, 20 \times 30, 30 \times 10, 10 \times 30$

most cost efficient way is $(A^* (B^* c))^* D$

cost or no. of operations

11

$$\left(\begin{array}{c} 20 \times 30 \times 10 \\ + \\ 40 \times 20 \times 10 \\ + \\ 40 \times 10 \times 30 \end{array} \right)$$

$$= 26000$$

$$(A^*(B^*C))^*D$$

$$\begin{aligned} & (40 \times 20 * (20 \times 30 * 30 \times 10))^* 10 \times 30 \\ & (40 \times 20 * 20 \times 10)^* 10 \times 30 \\ & (40 \times 10)^* 10 \times 30 \\ & 40 \times 30 \end{aligned}$$

Recursive: Find i, j

def solve(arr, i, j) :

if $i \geq j$: return 0

for K in range(i, j):

temp = solve(arr, i, k) +

solve(arr, k+1, j) +

arr[i-1] * arr[k] * arr[j]

ans = min(ans, temp)

return ans

Memoization

dp[100][100]

ans = IntMax

def solve(arr, i, j):

if $i \geq j$: return 0

if dp[i][j] != -1:

return dp[i][j]

-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1

for K in range(i, j):

temp = solve(arr, i, k) + solve(arr, k+1, j) + ~~arr[i-1] * arr[k] * arr[j]~~

arr[i-1] * arr[k] * arr[j]

ans = min(temp, ans)

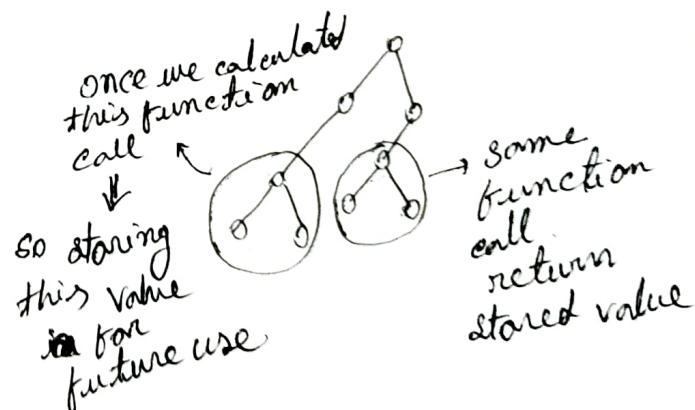
dp[i][j] = ans

return dp[i][j]

arr =

i	K	j	
A	B	C	D

 $\underbrace{\text{solve}(arr, i, k)}_{arr[i-1] * arr[k] * arr[j]} \quad \underbrace{\text{solve}(arr, k+1, j)}_{arr[k]}$

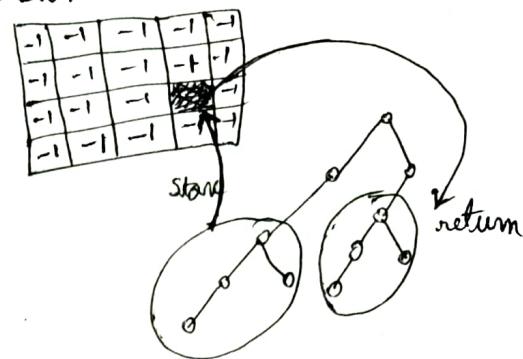


Palindrome Partitioning

$s = "nitik"$ ↓
 No. of partitions
 $n \underbrace{| i t i |}_{\sim} K \Rightarrow 2$ partitions

Minimum no. of partitions to make every partition palindrome.

Memoization



global

$dp[\text{constraint of } \text{len}(s)] [\text{constraint of } \text{len}(s)] \rightarrow \text{every cell}$

def solve(s, i, j):

if $i >= j$: return 0

if ispalindrome(s, i, j): return 0

if $dp[i][j] != -1$: return $dp[i][j]$

ans = sys.maxsize

for k in range(i, j):

{ temp = 1 + solve(s, i, k) + solve(s, k+1, j)

ans = min(ans, temp)

$dp[i][j] = ans$

return $dp[i][j]$

def ispalindrome(s, i, j): return $s[i:j+1] == s[i:j+1][::-1]$

optimisation:

* if $dp[i][k] != -1$:

left = $dp[i][k]$

else:

left = solve(s, i, k)

$dp[i][k] = left$

if $dp[k+1][j] != -1$:

right = $dp[k+1][j]$

else: right = solve(s, k+1, j)

$dp[k+1][j] = right$

temp = 1 + left + right

this line can be further optimized by checking whether solve(s, i, j) calculated before or not

Boolean Parenthesization total no. of ways to make $S = \text{True}$

[
input $\Rightarrow S = T \wedge F \mid F$
output = 2 $((T \wedge F) \mid F), (T \wedge (F \mid F))$

$S = T \mid F \wedge T \wedge F$ if $(T \mid F \wedge) T (\wedge F)$
 $i \uparrow \quad \uparrow \quad \uparrow \quad j$ $X \quad \uparrow \quad X$

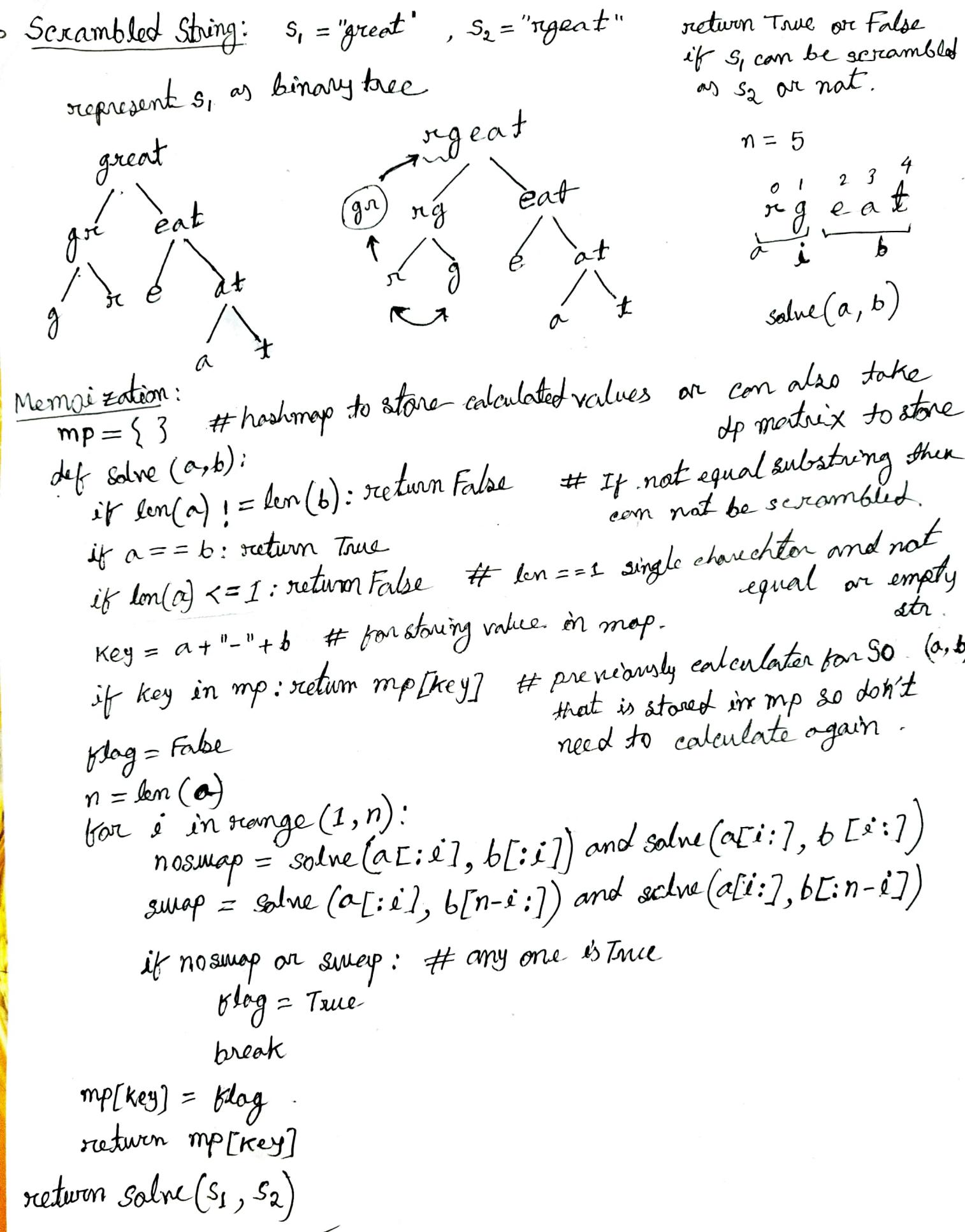
$i = 0; j = \text{len}(S)-1 \quad \therefore K = K+2$

so K can come in
the index of operators
only
and i, j at indices of
 T and F .

$\underbrace{(T \text{ or } F \text{ and } T)}_{(i \text{ to } K-1)} \xrightarrow{K} \text{xor} \underbrace{(F)}_{(K+1 \text{ to } j)}$

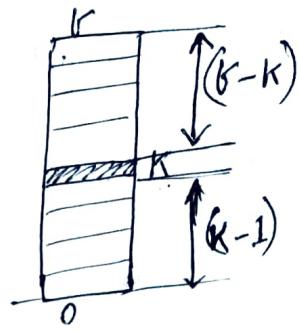
XOR

$T \wedge T \rightarrow F$
 $F \wedge F \rightarrow F$
 $T \wedge F \rightarrow T$
 $F \wedge T \rightarrow T$



Egg Dropping Problem:

$e, k \rightarrow$ no. of floor.
no. of egg



solve(e, k)

if egg Break
solve($e-1, k-1$)

if break at k then
may break for lower
floors. so check for $k-1$

not Break

solve($e, k-k$)

if not break at k then will not break for
lower floors. so attempt for upper floors
at k

Base Case: (i) If no. of floor is 0 then need 0 attempts.
(ii) if 1 floor need 1 attempt, (iii) If 1 egg, in worst case need to
check for all f floors so f attempts.

$$dp = [-1] * (100) \text{ for } i \text{ in range}(100)$$

def solve(e, k):

if $k == 0$ or $k == 1$: return k

if $e == 1$: return k

if $dp[e][k] != -1$: return $dp[e][k]$

ans = sys.maxsize

for K in range(1, $k+1$):

temp = $1 + \max($

solve($e-1, k-1$),

solve($e, k-k$)

)

ans = min(ans, temp)

$dp[e][k] = ans$ # storing ans

for future use.

return solve(e, k)

Time Complexity = $O(n^2 \times k)$

Space Complexity = $O(k \times n)$

previously calculated
for solve(e, k) at $dp[e, k]$

try from 1 to k floor, drop every
floor and find worst case (max) for
that floor; Find min of all 1 to k
floors.

egg break, now we have $e-1$
remaining eggs and $k-1$ floors,
because above k floor all eggs
will break.

egg not break, now we
remain e eggs and $k-k$ floor
because before k (included)
all eggs will remain

DP on Tree: General Structure of code

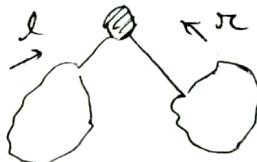
Base Case

```
def solve(node, res):
```

```
    if node == None: return 0
```

Hypothesis

```
l = solve(node.left, res)
r = solve(node.right, res)
```



Induction

```
temp = calculate temp ans.
ans = max(temp, relation)
res = max(res, ans)
return temp
```

$1 + \max(l+r)$

$1 + l+r$

if have to return something
else res then return temp

Diameter of Binary Tree:

Self.res = 0

```
def solve(root):
```

B [if not root: return 0

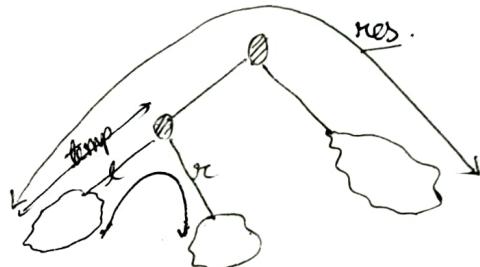
H [l = solve(root.left)
 r = solve(root.right)

I [temp = 1 + max(l, r)

ans = max(temp, 1 + l + r) → for including the current root and left, right as longest path.

self.ans = max(self.ans, ans)

return temp → if longest path extends further



solve(root)

return self.res.

Binary Tree max Pathsum:

Self.res = root.val

```
def solve(root):
```

B [if not root: return 0

H [l = solve(root.left)

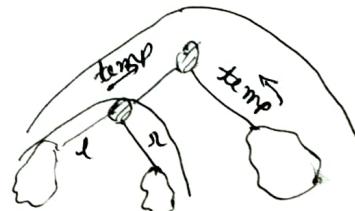
r = solve(root.right)

I [temp = max(root.val + max(l, r), root.val)

ans = max(temp, root.val + l + r)

self.res = max(self.res, ans)

return temp



max single straight path containing current root and any of max sum subtree or only root if l and r are -ve.

either temp or total subtree

solve(root)
 return self.res

Maximum Path Sum of Binary tree from leaf node to leaf node:

self.res = root.val

def solve(root):

B [if not root: return 0

l = solve(root.left)

H [r = solve(root.right)

I [temp = max(l, r) + root.val

ans = max(temp, l+r+root.val)

self.res = max(ans, self.res)

I] return temp

solve(root)

return self.res

