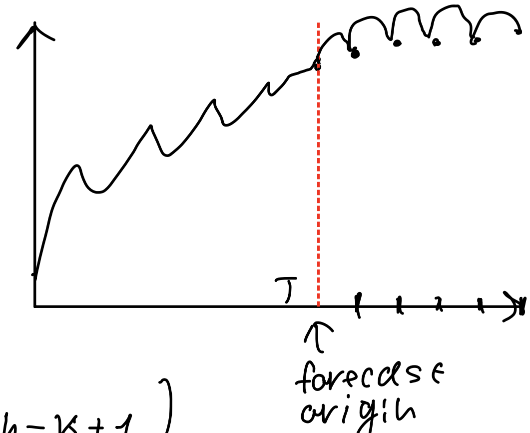
Стратегии

1) Рекурсивная.

forecast horizon = 5



$$\hat{y}_{T+h|T} = \hat{f}(\underbrace{\tilde{y}_{T+h-1}}, \dots, \tilde{y}_{t+h-k+1})$$

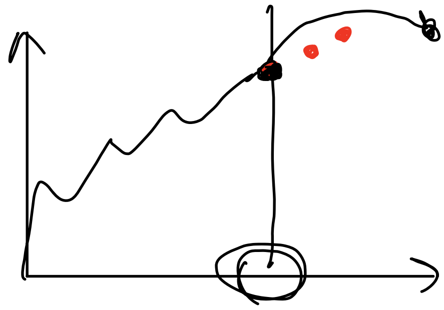
k лагов в регрессорах

$$\tilde{y}_t = \begin{cases} y_t, & t \le T \\ \hat{y}_t, & t > T \end{cases}$$

t	y_t	y_{t-1}	y_{t-2}	x_{t-1}
0	1	—	—	-1
1	2	1	—	-2
2	3	2	1	-3
3	4	3	2	-4
4	5	4	3	-5
5	6,1	5	4	-6
6	6,9	6,1	5	
7		6,9	6,1	

Direct

$$\hat{y}_{T+h} = \hat{f}_h(y_T, \dots, y_{T-K+1})$$



$h = 1$

t	y_t	y_{t-1}	y_{t-2}	x_{t-1}
0	1	—	—	
1	2	1	—	
2	3	2	1	
3	4	3	2	
4	5	4	3	-5

$h = 2$

t	y_t	y_{t-2}	y_{t-3}
0	1	—	—
1	2	—	—
2	3	1	—
3	4	2	1
4	5	3	2

Dirrec

Одношаговая модель такая же

$h = 1$

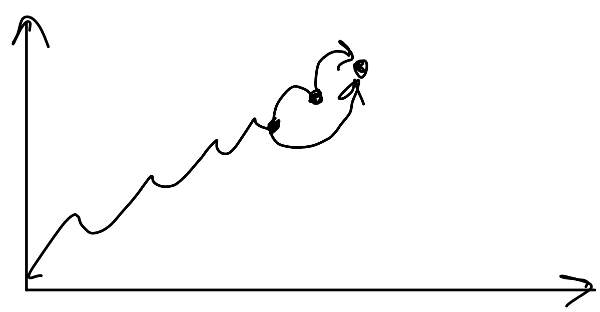
t	y_t	y_{t-1}	y_{t-2}
0	1	–	–
1	2	1	–
2	3	2	1
3	4	3	2
4	5	4	3

x_{t-1} $\rightarrow \hat{y}_t$

-5

$h = 2$

t	y_t	y_{t-2}	y_{t-3}
0	1	–	–
1	2	–	–
2	3	1	–
3	4	2	1
4	5	3	2



4) MIMO

$$\underbrace{[y_{T+h}, \dots, y_{T+1}]} = F(y_T, \dots, y_{T-K+1})$$

5) DirMo

$$h = [\underbrace{1, 2, 3, 4, 5}_{\text{MIMO}} \, \big| \, \underbrace{6, 7, 8, 9, 10}_{\text{MIMO}}]$$

ETS

$(y_t)_{t=1}^{T}$ ~ 1970

$$\hat{y}_{T+1|T} = y_T$$

$$\hat{y}_{T+h|T} = y_T$$

$$\hat{y}_{T+1|T} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

$\alpha \in [0; 1]$

$$\hat{y}_{T+1|T} = \sum_{i=0}^{T-1} \alpha q^i y_{T-i}$$

$$\alpha + q\alpha + q^2\alpha + \dots + q^{T-1}\alpha = 1$$

$$\frac{\alpha(q^T - 1)}{q - 1} = 1$$

$$\frac{\alpha}{1-q} = 1 \Rightarrow \alpha = 1 - q$$
$$q = 1 - \alpha$$

$$\hat{y}_{T+1|T} = \sum_{i=0}^{T-1} \alpha(1-\alpha)^i y_{T-i}$$

$$\hat{y}_{T+h|T} = \sum_{i=0}^{T-1} \alpha(1-\alpha)^i y_{T-i}$$

$$\sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2 \to \min_{\alpha}$$

$$\hat{y}_{T+1|T} = \alpha y_T + (1-\alpha)\hat{y}_{T|T-1} =$$
$$= \alpha(y_T - \hat{y}_{T|T-1}) + \hat{y}_{T|T-1} =$$
$$= \alpha e_T + \hat{y}_{T|T-1}$$

Модель коррекции ошибки

VECM

$$\hat{y}_{T+1|T} = \alpha y_T + \alpha(1-\alpha) y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} \dots$$
$$= \alpha y + (1-\alpha)[\alpha y_{T-1} + \alpha(1-\alpha) y_{T-2}] =$$
$$= \alpha y_T + (1-\alpha)\hat{y}_{T|T-1}$$

$$\shortparallel$$
$$\hat{y}_{T|T-1}$$

$t \in \overline{1,T}$ $\quad \boxed{\hat{y}_{T+1|T} = \alpha y_T + (1-\alpha)\hat{y}_{T|T-1}}$

$$\hat{y}_{2|1} = \alpha y_1 + (1-\alpha) l_0$$

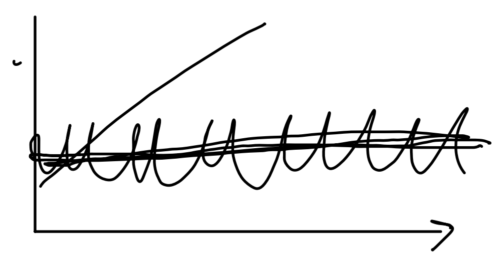
$$\hat{y}_{T+1|T} = \sum_{i=0}^{T-1} \alpha(1-\alpha)^i y_{T-i} + (1-\alpha)^T l_0$$

$$\sum_{t=1}^{T} (y_t - y_{t|t-1})^2 \to \min_{\alpha, l_0}$$

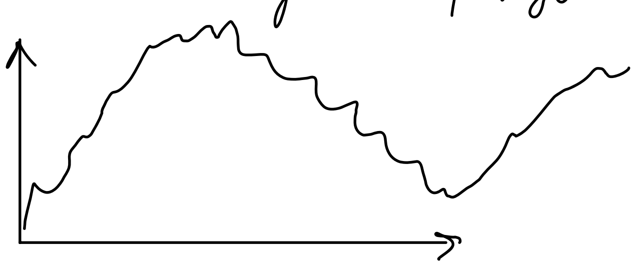
Компонентный вид

Ур-е прогноза $\hat{y}_{t+1|t} = l_t$

Ур-е сглаживания $l_t = \alpha y_t + (1-\alpha) l_{t-1}$

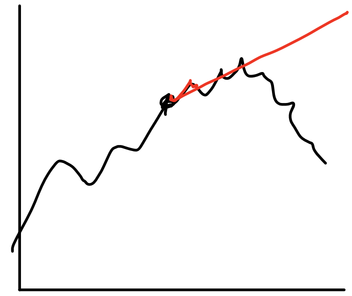


Модели трендов



b_t — локальная скорость роста

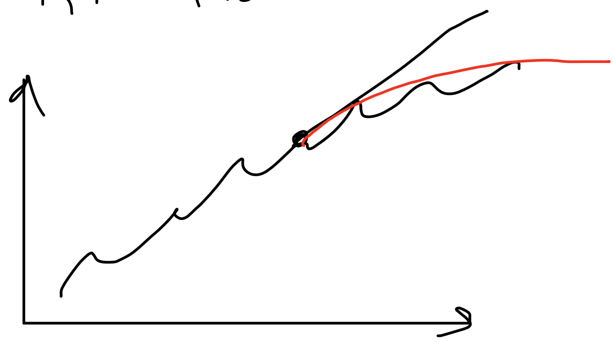
$$\begin{cases} \hat{y}_{t+h} = l_t + h b_t \\ l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1}) \\ b_t = \beta(l_t - l_{t-1}) + (1-\beta)(b_{t-1}) \end{cases}$$



$y_t - y_{t-1}$

$l_t - l_{t-1}$

α, β, b_0, l_0



$$\hat{y}_{t+h|t} = l_t + (\varphi + \varphi^2 \dots + \varphi^h) b_t$$

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + \varphi b_{t-1})$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)\varphi b_{t-1}$$

$$\lim_{h \to \infty} \hat{y}_{t+h|t} = l_t + \frac{\varphi b_t}{1-\varphi}$$

Сезонность.

$K = \lfloor (h-1)/m \rfloor$

$$\begin{cases} \hat{y}_{t+h|t} = l_t + h b_t + s_{t+h-m(K+1)} \\ l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1}) \\ b_t = \beta(l_t - l_{t-1}) + (1-\beta)(b_{t-1}) \end{cases}$$

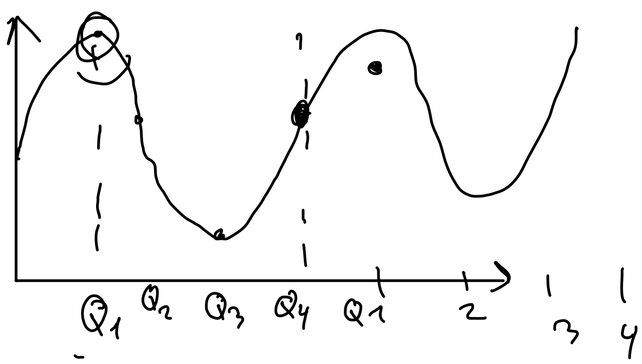
$m = 12$

$$S_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)(S_{t-m})$$

$\alpha, \beta, \gamma,$

l_0, b_0

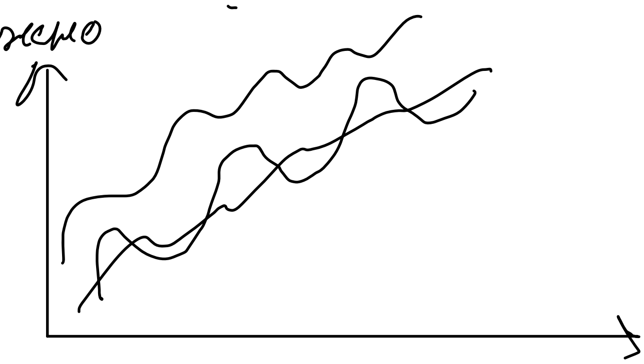
$S_0, \dots, S_{m-1}$



$S_0 + \dots + S_{m-1} = 0$

Из огранич. можно найти один параметр $\Rightarrow$

нужно $m-1$ парам.



$$\begin{cases} \hat{y}_{t+1|t} = l_t \\ l_t = \alpha y_t + (1-\alpha) l_{t-1} \end{cases}$$

$$l_t = l_{t-1} + \alpha(y_t - l_{t-1}) =$$

$$= l_{t-1} + \alpha e_t$$

$e_t = y_t - l_{t-1}$

$y_t = l_{t-1} + e_t$

$$\begin{cases} y_t = l_{t-1} + \varepsilon_t \sim N(0, \sigma^2) \\ l_t = l_{t-1} + \alpha \varepsilon_t \end{cases}$$

ETS(AAN)

$$\begin{cases} y_t = l_{t-1} + b_{t-1} + \varepsilon_t \sim N(0, \sigma^2) \\ l_t = l_{t-1} + b_{t-1} + \alpha \varepsilon_t \\ b_t = b_{t-1} + \beta \varepsilon_t \end{cases} \qquad \alpha, \beta, l_0, b_0, \sigma^2$$

$$f(y_T, \dots, y_1) \cancel{=} \prod_{t=1}^{T} f(y_t)$$

$$f(y_T, \dots, y_1)$$

Случ. величины

$$f(y_T | y_{T-1}, \dots y_1) = \frac{f(y_T, \dots, y_1)}{f(y_{T-1}, \dots, y_1)} \Rightarrow$$

$$(Y_T | Y_{T-1} = y_{T-1} \dots)$$

$$\Rightarrow f(y_T, \dots, y_1) = f(y_T | y_{T-1}, \dots, y_1) \cdot \underbrace{f(y_{T-1}, \dots, y_1)}_{} =$$

$$= f(y_T | y_{T-1}, \dots, y_1) \cdot f(y_{T-1} | \underbrace{y_{T-2}, \dots, y_1}_{\mathcal{F}_{T-2}})$$

$$f(y_{T-2}, \dots, y_1) = \dots \qquad \mathcal{F}_{T-2} =$$

$$= \underbrace{\qquad\qquad}_{} \cdot f(y_2 | y_1) \underbrace{f(y_1)}_{}$$

$$= \prod_{t=1}^{T} f(y_t | \mathcal{F}_{t-1})$$

$$\begin{cases} y_t = l_{t-1} + b_{t-1} + \varepsilon_t \sim N(0, \sigma^2) & \alpha, \beta, l_0, b_0, \sigma^2 \\ l_t = l_{t-1} + b_{t-1} + \alpha \varepsilon_t \\ b_t = b_{t-1} + \beta \varepsilon_t \end{cases}$$

$$y_1 = l_0 + b_0 + \varepsilon_1 \sim N(l_0 + b_0, \sigma^2)$$

$$y_2 | y_1 = l_1 + b_1 + \varepsilon_2 =$$

$$= l_0 + b_0 + \alpha \hat{\varepsilon}_1 + b_0 + \beta \hat{\varepsilon}_1 + \varepsilon_2 =$$

$$\hat{y}_1 = l_0 + b_0 + \hat{\varepsilon}_1 \Rightarrow \hat{\varepsilon}_1 = \hat{y}_1 - l_0 - b_0$$

$$= l_0 + 2b_0 + \alpha(\hat{y}_1 - l_0 - b_0) + \beta(\hat{y}_1 - l_0 - b_0) + \varepsilon_2$$

$$= \underbrace{l_0 + 2b_0} + \underbrace{(\alpha + \beta)(\hat{y}_1 - l_0 - b_0)} + \underbrace{\varepsilon_2}$$

$$y_2 | y_1 \sim N\left(l_0 + 2b_0 + (\alpha + b)(y_1 - l_0 - b_0), \sigma_2\right.$$

Задача.

$$\varepsilon_t \sim N(0, 4)$$

$$\begin{cases} y_t = l_{t-1} + b_{t-1} + s_{t-2} + \varepsilon_t \\ l_t = l_{t-1} + b_{t-1} + 0{,}3\, \varepsilon_t \\ b_t = b_{t-1} + 0{,}2\, \varepsilon_t \\ s_t = s_{t-2} + 0{,}1\, \varepsilon_t \end{cases}$$

$s_{100} = 2, \quad s_{99} = -1{,}9 \quad b_{100} = 0{,}5, \quad l_{100} = 4$

y_{102}

$y_{101} = l_{100} + b_{100} + s_{99} + \varepsilon_t$

$(y_{101} | \mathcal{F}_{100}) \sim N(\underbrace{l_{100} + b_{100} + s_{99}}_{2{,}6}, 4)$

$E(y_{t+h} | \mathcal{F}_t)$ – точечный прогноз

$Var(y_{t+h} | \mathcal{F}_t)$ – неопред.

95%

$E(y_{101} | \mathcal{F}_{100}) = 2{,}6$

$Var(y_{101} | \mathcal{F}_{100}) = 4$

$[2{,}6 - 1{,}96 \cdot 2; \ 2{,}6 + 1{,}96 \cdot 2]$

$(y_{102} | \mathcal{F}_{100}) \sim$

$y_{102} = l_{101} + b_{101} + s_{100} + \varepsilon_{102}$

$l_{101} = \underbrace{l_{100} + b_{100}}_{4{,}5} + 0{,}3\,\varepsilon_{101}$

$b_{101} = 0{,}5 + 0{,}2\,\varepsilon_{101}$

$y_{102} | \mathcal{F}_{100} = 7 + 0{,}5\,\varepsilon_{101} + \varepsilon_{102}$

$E(y_{102} | \mathcal{F}_{100}) = 7 \qquad y_{102} | \mathcal{F}_{100} \sim N(7, 5)$

$Var(y_{102} | \mathcal{F}_{100}) = 1 + 4 = 5$

$$\left[7 - 1{,}96\sqrt{5}\ ;\ 7 + 1{,}96\sqrt{5}\right]$$