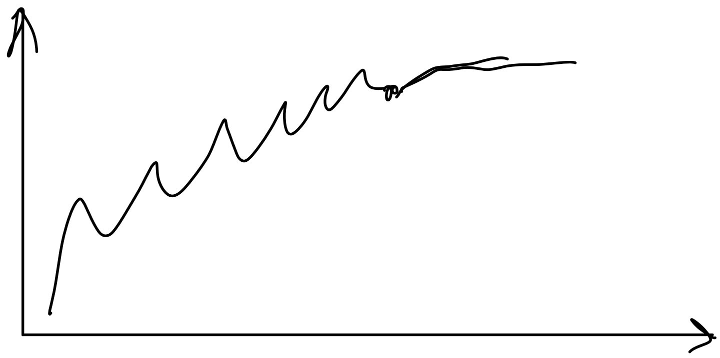
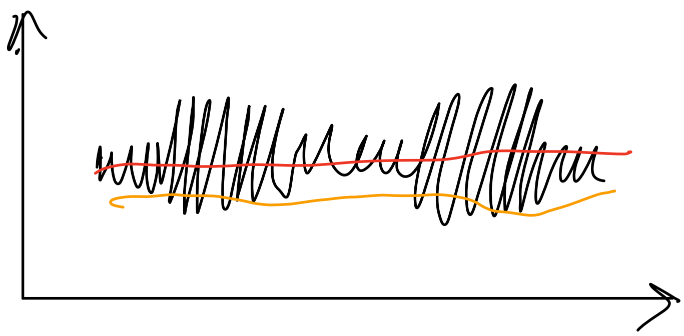
Conditional heteroschedasticity models





Volatility clustering

Ex model: ARMA(p, q)

r_t – log-returns

$$r_t = \mu_t + a_t \quad a_t \sim N(\cancel{0, \sigma_\varepsilon^2}) \qquad \mu_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{i=1}^{q} \theta_{t-i}$$

$$\sigma_t^2 = Var(r_t \mid F_{t-1}) = Var(a_t \mid F_{t-1})$$

CH TSAY

Exact function of σ^2 (GARCH)

stochastic function of σ_ε^2 (stochastic volatility)

$\sigma_t = \sqrt{\sigma^2_t}$

ARCH

1) a_t - serially uncorrelated, but dependent
2) dependence - lagged quatratic

$a_t = \sigma_t \varepsilon_t$ $\quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m}^2$

$\varepsilon_t \sim iid$, $E(\varepsilon_t) = 0$ $Var(\varepsilon_t) = 1$

$\alpha_0 \geq 0$, $\alpha_i \geq 0$ $\forall i > 0$

Weaknesses

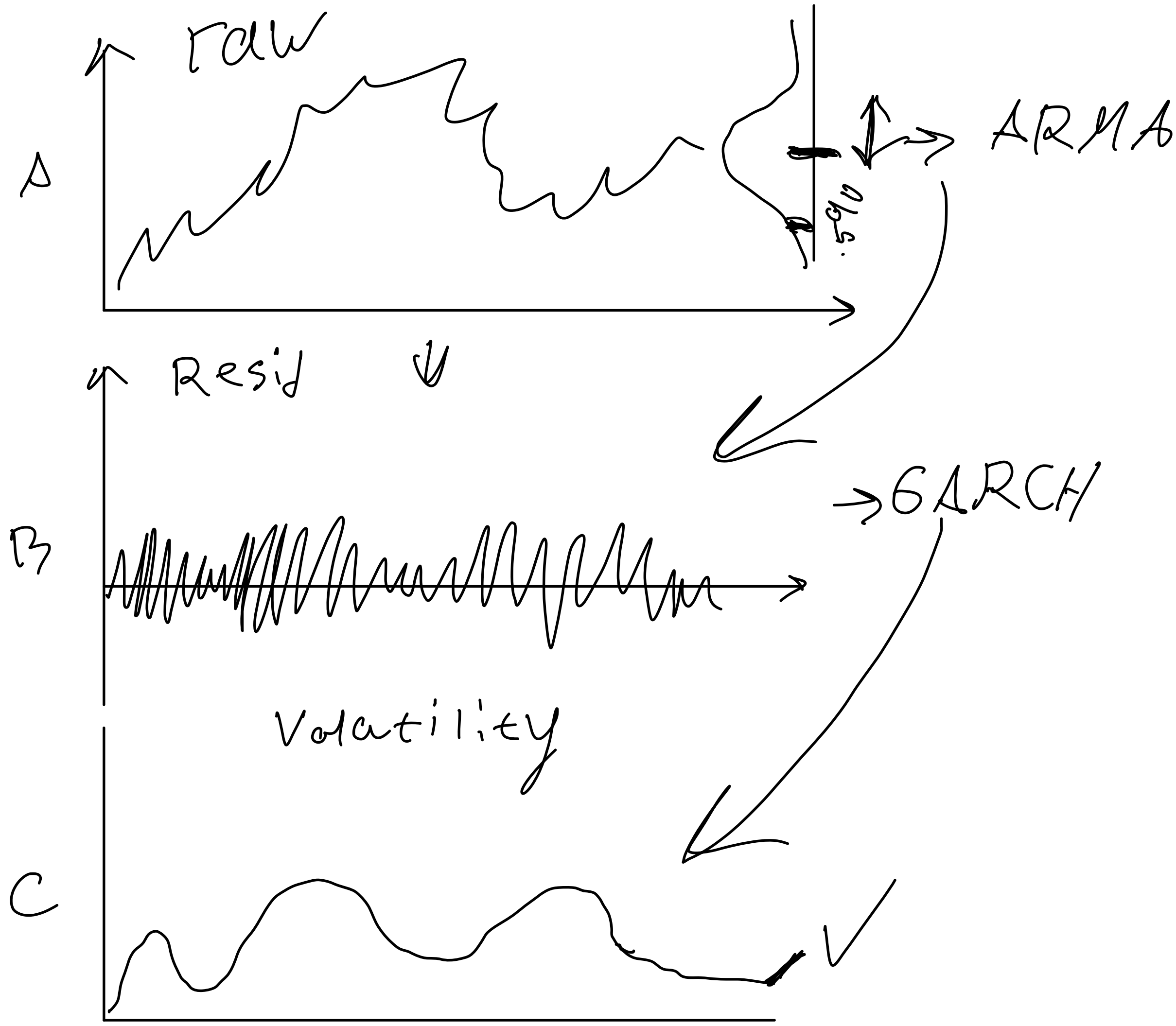
1) Equally responses to pos and neg shocks
2) $\alpha_i^2 \in [0; \frac{1}{3}]$
3) Mechanical way
4) Overpredicts

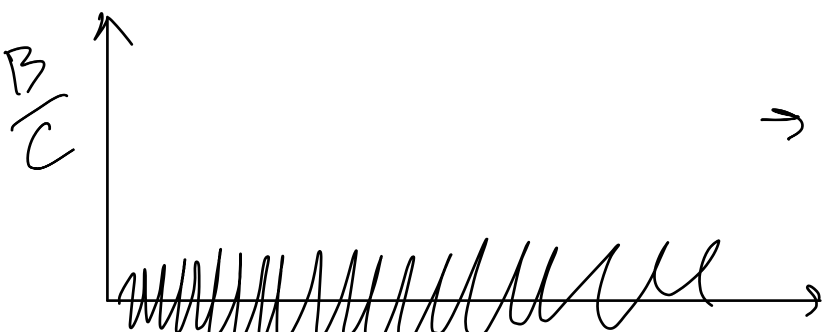
1) Building mean model
2) Getting residuals $y_t - \hat{y}_t$
3) Specify ARCH
4) Checking and re-estimation

F-test, Ljung-box $\tilde{a} = \frac{a_t}{\sigma_t} \sim \varepsilon_t$

$$\sigma^2_{T+1} = \alpha_0 + \alpha_1 a_T^2 + \dots + \alpha_m a_{T-m}^2$$

$$\sigma^2_{T+2} = \alpha_0 + \alpha_1 \sigma^2_{T+1} + \dots$$





→ распределение

1) Гистограмма

2) Плотность GARCH

$$a_t = \sigma_t^2 \varepsilon_t$$

Распределение
(плотность)