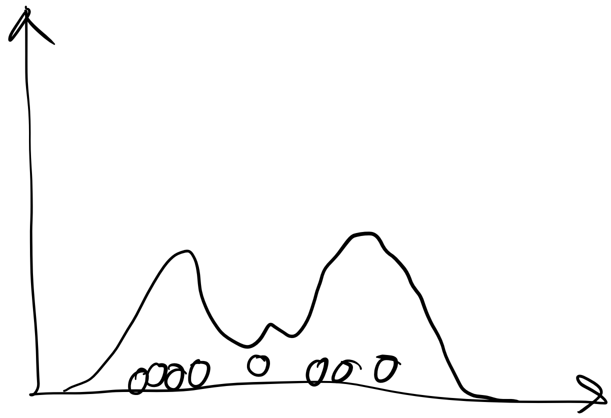
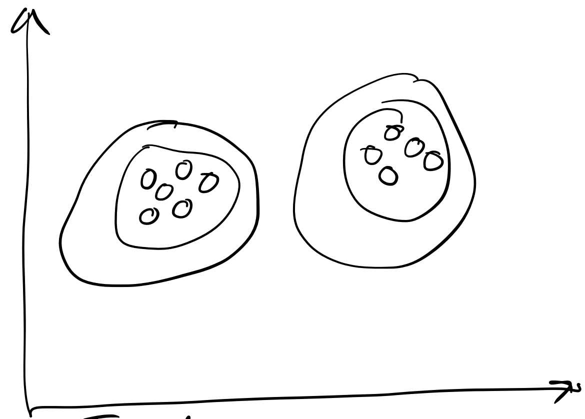
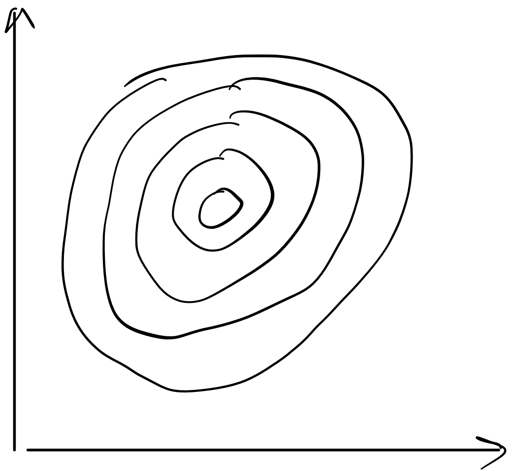
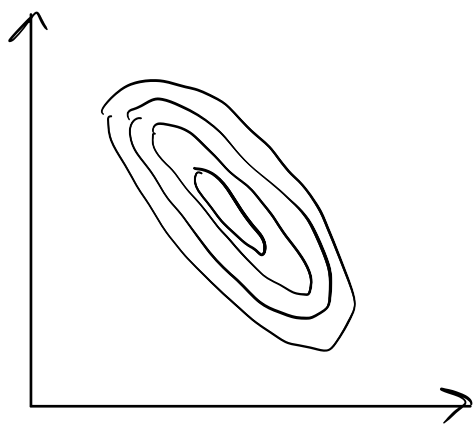
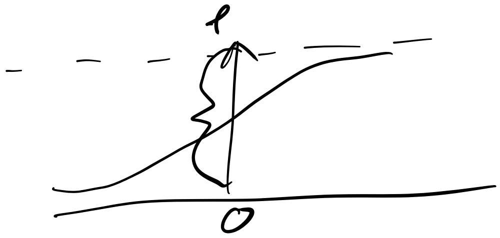




$$N(\mu, \Sigma) = \frac{1}{\sqrt{\det(2\pi\Sigma)}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$$

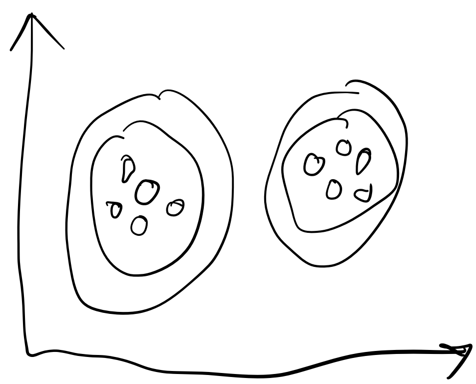


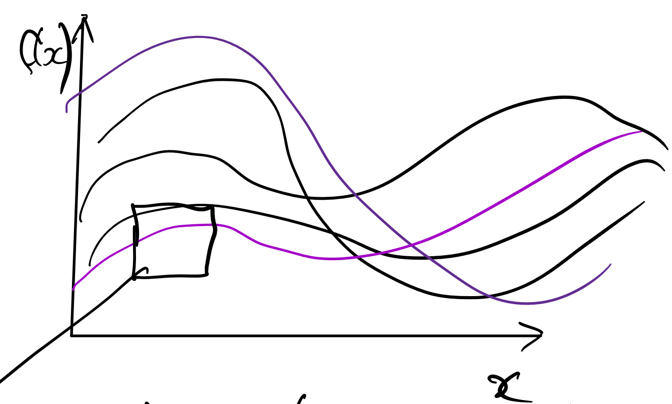


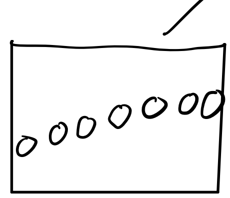


Multivariate dist —
— sample points

Process
— sample functions







$f(x) \sim GP(m(x), k(x,x'))$

$m(x)$ — mean function

$k(x,x')$ — cov. function

Def. Gaussian process

$\forall n \; \forall (x_1, \dots, x_n) \; (f(x_1), \dots, f(x_n)) \sim \mathcal{N}(\mu, \Sigma)$

$\mu = \{m(x_i)\}_{i=1}^{n} \quad \Sigma = \{k(x_i, x_j)\}_{i,j=1}^{n,n}$

Ex 1

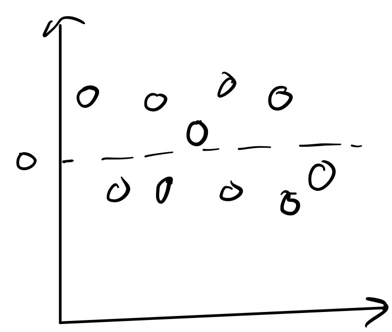
$f(x) \sim GP(m(x), k(x,x'))$

$m(x) = 0$

$k(x,x') = \sigma^2 [x = x']$

$f(x_1), \dots, f(x_n) \sim \mathcal{N}(\mu, \Sigma)$

$\mu = 0 \quad \Sigma = \sigma^2 I$



$$P(f(x_1), \dots, f(x_n)) = \prod_{i=1}^{n} \mathcal{N}(0, \sigma^2)$$

Ex 2

$$f(x) \sim GP(m(x), k(x, x'))$$

$$m(x) = 0$$
$$k(x, x') = C$$

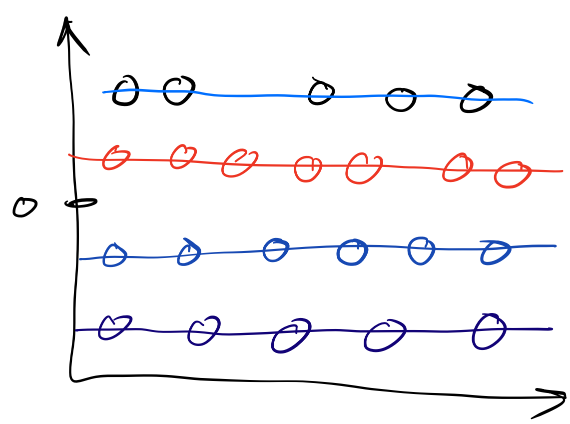
$$f(x_1), \dots, f(x_n) \sim \mathcal{N}(\mu, \Sigma)$$

$$\mu = 0 \qquad \Sigma = \{C\}_{i,j=1}^{n,n} \qquad \text{,, } C$$

$$corr(f(x_i), f(x_j)) = \frac{cov(f(x_i), f(x_j))}{\sqrt{\underbrace{Var(f(x_i))}_{C} \; \underbrace{Var(f(x_j))}_{C}}} = \frac{C}{\sqrt{C^2}} = 1$$

$$Var(f(x_i)) = Var(f(x_j)) = C$$

$$E(f(x_i)) = E(f(x_j)) = 0 \qquad f(x_i) = f(x_j)$$



Ex 3

$f(x) \sim$

$m(x) = 0$

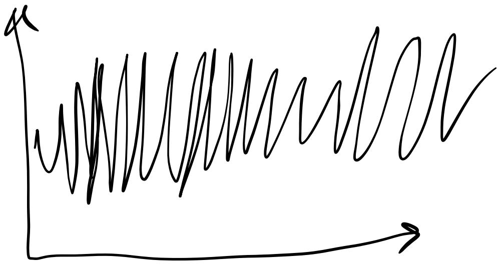
$$K(x, x') = \sigma^2 \exp\left(-\frac{(x-x')^2}{2l^2}\right)$$

$f(x_1), \ldots, f(x_n) \sim N(\mu, \Sigma)$

$\mu = 0 \quad \Sigma = \{K(x_i, x_j)\}_{i,j=1}^{n,n}$

$$\begin{pmatrix} \sigma^2 & & & \\ & \sigma^2 & & \\ & & \ddots & \\ & & & \sigma^2 \end{pmatrix}$$

$\sigma = 1, \; l = 0{,}05$



$\sigma = 1, \; l = 0{,}5$

$\sigma = 1 \quad l = 3$

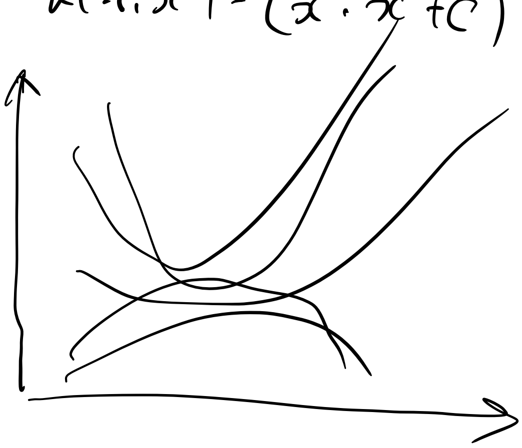
$\sigma = 1, \; l = 16$

l^2 – частоту флуктуаций

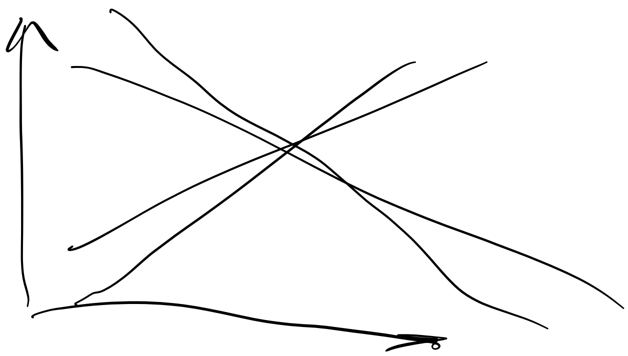
σ^2 – „высота" функций

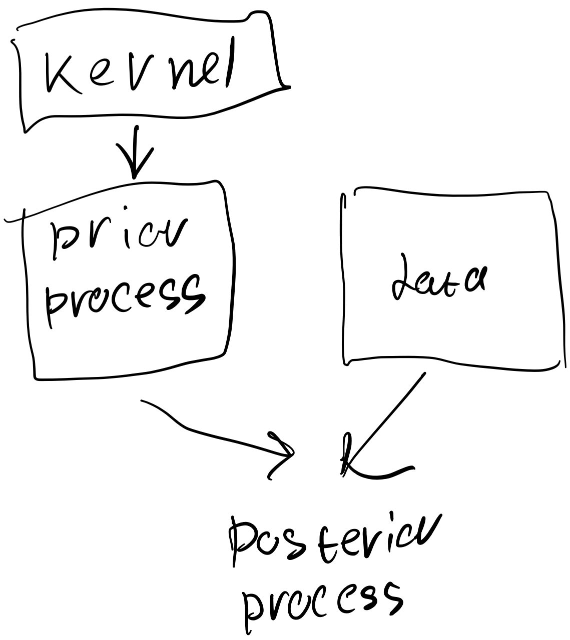
$k(x, x') = \min(x, x')$

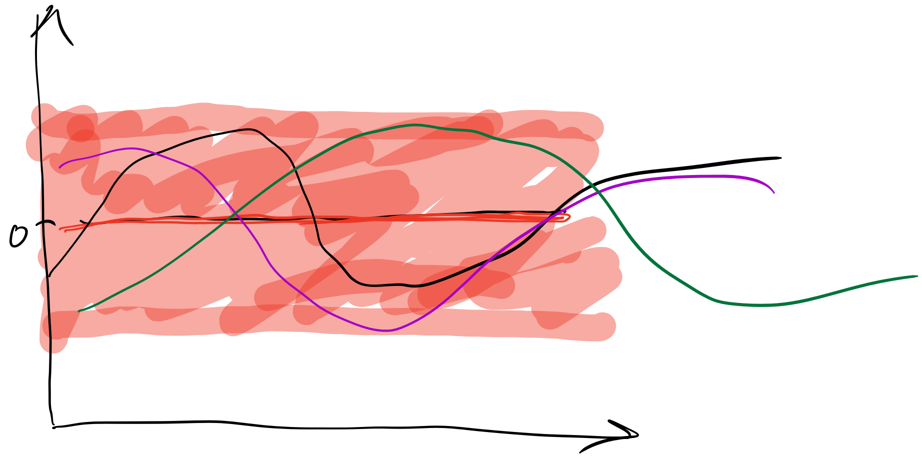
$k(x, x') = (x \cdot x' + c)^2$

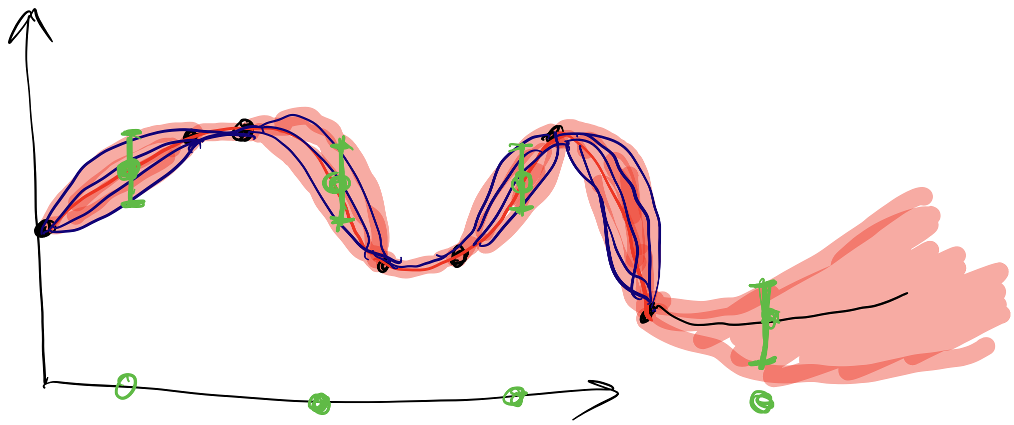


$k(x, x') = x x'$









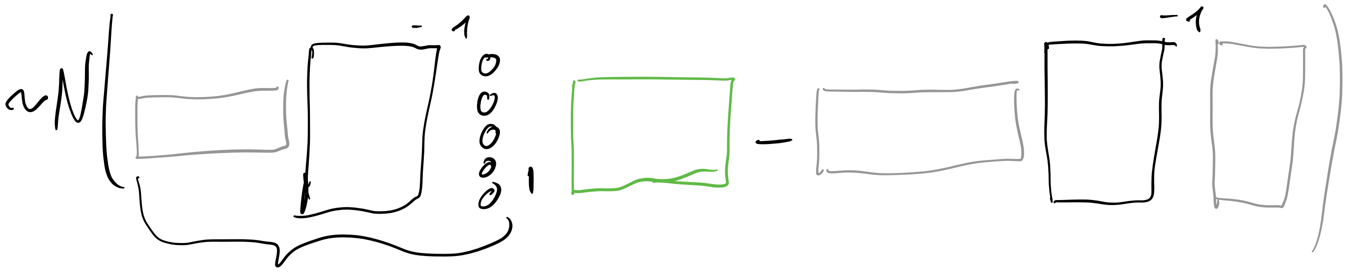
$\forall n \ \forall (x_1 \dots x_n)$

$\mu = \{ m(x) \}_{i=1}^{n} \quad \Sigma = \{ K(x, x') \}_{i,j=1}^{n,n}$

$f(x_1) \dots f(x_n) \sim N(\mu, \Sigma)$

$$(0\,0\,0\,0\,0\,0\,0\,0) \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \right)$$

$(0\,0\,0 \mid 0\,0\,0\,0\,0)$



\+ оценка неопределённости

− slow

$$p(00000) = N\left(\boxed{0},\ \square\right) \to \max_{\sigma^2,\ \ell^2}$$

Sum-kernel

$$k(x,x') = \overbrace{x^T x}^{\text{lin}} + \overbrace{\sigma_1^2 \exp\left(-\frac{\|x-x'\|^2}{2\ell^2}\right)}^{\text{RBF}} +$$

$$+ \underbrace{\sigma_2^2 [x = x']}_{\text{WN}} + \underbrace{\sigma_3^2}_{\text{Constant}}$$