№2.

$$\begin{array}{c|cccc} & 0 & 1 & 3 & 1 \\ \hline 1 & 1 & 0 & 2 & 0 \\ 2 & 2 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 \end{array}$$

$DTW = 4$

№3.

$f(x) \sim GP(0, K)$

$K = \exp(-(a-b)^2)$

Построить 95%-й интервал для $f(1)$

$f(0) = 0 \quad f(3) = 1$

$\exp(-1) = 0{,}368 \quad \exp(-2) = 0{,}135$

$$(f(0), f(3) \mid f(1)) \sim N\left(0, \begin{array}{c} \begin{array}{ccc} 0 & 3 & 1 \end{array} \\ \left(\begin{array}{cc|c} 1 & e^{-9} & e^{-1} \\ e^{-9} & 1 & e^{-4} \\ \hline e^{-1} & e^{-4} & 1 \end{array}\right) \end{array} \begin{array}{c} 0 \\ 3 \\ 1 \end{array}\right.$$

$e^{-9} = (e^{-1})^9 \quad \frac{1}{e^9} \quad \frac{1}{}$

$$\begin{pmatrix} 1 & 0{,}0001 & 0{,}368 \\ 0{,}0001 & 1 & 0{,}018 \\ 0{,}368 & 0{,}018 & 1 \end{pmatrix}$$

$e^{-4} = e^{-2} e^{-2} =$

$$f(1) = N\left(\underbrace{(0{,}368,\ 0{,}018)\begin{pmatrix}1 & 0{,}0001\\ 0{,}0001 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}_{\mu},\right.$$

$$\left.\underbrace{(1) - (0{,}368,\ 0{,}018)\begin{pmatrix}1 & 0{,}001\\ 0{,}001 & 1\end{pmatrix}^{-1}\begin{pmatrix}0{,}368\\ 0{,}018\end{pmatrix}}_{\sigma^2}\right)$$

$$P(f(1) \in [\underbrace{\mu - 1{,}96\sigma}; \mu + 1{,}96\sigma] = 0{,}95$$

№4.

H_0: стратегии одинаковы

H_A: стратегии различаются

$\{y_t\}_{t=1}^{T}$ - врем. ряд

$\{y_{At}\}_{t=1}^{T}$ - прогнозы A

$\{y_{Bt}\}_{t=1}^{T}$ - прогнозы B

$g(e)$ - ф-я потерь

$\{e_{At}\}_{t=1}^{T}$ $\{e_{Bt}\}$ - остатки обеих мод.

$d_t = g(e_{At}) - g(e_{Bt})$, $t \in \overline{1,T}$

H_0: $E(d) = 0$

$H_1: E(d) \neq 0$

· d_t – слабо стац ряд

$$\sqrt{T}(\bar{d} - \mu) \xrightarrow{d} N(0, \sigma^2)$$

$$S = \frac{\bar{d}}{\sqrt{\bar{\sigma}^2/T}}, \quad \bar{\sigma}^2 = \sum_{t=-\infty}^{t=\infty} \gamma_d(t)$$

$\gamma_d(t)$ – автоков. d порядка t

$$S \overset{H_0}{\sim} N(0,1)$$

AR(1) для d_t:

$$\hat{d}_t = 0{,}3 + 0{,}7 d_{t-1} \qquad \boxed{\hat{\sigma}_u^2 = 1}$$

$\bar{d}$, $\bar{\sigma}^2$

$$d_t = 0{,}3 + 0{,}7 d_{t-1} + u_t \quad | E$$

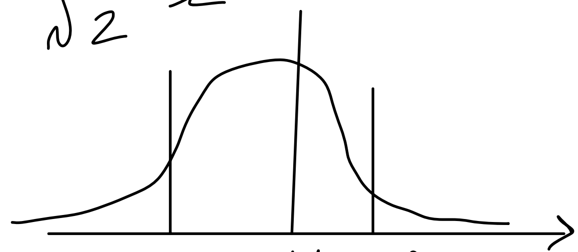
$$\mu = 0{,}3 + 0{,}7\mu + 0 \qquad y = \alpha + \beta X$$

$$\mu = \frac{0{,}3}{1 - 0{,}7} = 1 = \bar{d} \qquad \bar{y}, \bar{X}$$

$$\hat{\sigma}_d^2 = 0{,}49\hat{\sigma}_d^2 + \hat{\sigma}_u^2 = 1$$

$$\hat{\sigma}^2_d = \frac{1}{1-0,49} \approx 2$$

$$\frac{\bar{d} - 0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \approx 0,71$$



H_0 не отв.

№5

Байесовская авторегрессия

$$y_t = \beta y_{t-1} + u_t, \quad u_t \sim N(0,1)$$

$$\beta \sim N(1,1) \ / \ \text{trunc normal}$$

а) Найти апостериорное распр.

б) 95% ДИ для y_4

$y_1 = 5, \ y_2 = 6, \ y_3 = 7$

а) $f(\beta | y_1, y_2, y_3)$

$$\underbrace{f(\beta | data)}_{\text{posterior}} = \frac{f(\beta, data)}{f(data)} = \frac{\overbrace{f(data|\beta)}^{\text{likelih.}} \overbrace{f(\beta)}^{\text{prior}}}{\int f(data|\beta^*) f(\beta^*) d\beta^*}$$

$\propto f(data \mid \beta) \cdot f(\beta)$ $=$

$$f(\beta) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(\beta-1)^2}{2}} \propto e^{\frac{-(\beta-1)^2}{2}}$$

$$f(data \mid \beta) = f(y_4, y_3, y_2, y_1 \mid \beta) =$$

$$= f(y_4 \mid y_3, y_2, y_1, \beta) \cdot f(y_3 \mid y_2, y_1, \beta) \cdot f(y_2 \mid y_1, \beta)$$

$$f(y_2 \mid y_1) \; ; \; f(y_3 \mid y_2, \cancel{y_1}), f(y_4 \mid y_3)$$

$$f(y_2 \mid y_1, \beta) = \underset{\sim N(1,1)}{\beta} \cdot 5 + \underset{\sim N(0,1)}{u_2} \sim N(5, 1) = \cancel{\frac{1}{\sqrt{2\pi\, 26}}} e^{\frac{-(y_2-5)^2}{2 \cdot}}$$

$$E(\beta \cdot 5 + u_t) = 5$$

$$Var(\beta \cdot 5 + u_t) = 26$$

$$f(y_3 \mid y_2, \beta) = \beta \cdot 6 + u_3 = \cancel{\frac{1}{\sqrt{2\pi\, 37}}} e^{\frac{-(y_3 - 6\beta)^2}{2 \cdot 1}}$$

$$f(y_4 \mid y_3, \beta) = \beta \cdot 7 + u_4 = \cancel{\frac{1}{\sqrt{2\pi\, 50}}} e^{\frac{-(y_4 - 7\beta)^2}{2 \cdot 1}}$$

$$e^{\left[-\frac{(\beta-1)^2}{2} - \frac{(y_2 - 5\beta)^2}{2} - \frac{(y_3 - 6\beta)^2}{2} - \frac{(y_4 - 7\beta)^2}{2}\right]}$$

$$= e^{[-(\beta^2 - 2\beta + 1) - (\cancel{y_2^2} - 10 y_2 \beta + 25\beta^2) - (\cancel{y_3^2} - 12 y_3 \beta + 36\beta^2) - (\cancel{y_4^2} - 14 y_4 \beta + 49\beta^2)]}$$

$$e^{\left[\frac{-111\beta^2+\beta(2+10y_2+12y_3+14y_4)}{2}\right]}$$

$$e^{\left[-\frac{(\beta^2-2\beta\hat{\mu}+\hat{\mu}^2-\cancel{\hat{\mu}^2}}{2\cdot 111}\right]} \propto N(\hat{\mu}, 111)$$

$$\hat{\mu} \quad \frac{(1+5y_2+6y_3+7y_4)}{111}$$

$$p(\beta|data) \sim N(\hat{\mu}, \boxed{111})$$

$P(y_4$

$N1.$

$$u_t = \sigma_t \nu_t, \quad \nu_t \sim N(0,1)$$

$$\sigma_t^2 = 4 + 0{,}5u_{t-1}^2$$

$$u_{101} = \sigma_{101}\,\underbrace{\nu_{101}}, \qquad E(u_{101}|u_{100}) = 0$$

$$\underbrace{\hat{\sigma}^2_{101}} = 4 + 0{,}5\cdot 4 = 4+2 = 6$$

$$0 \pm 1{,}96\sqrt{6}$$

$$\hat{\sigma}^2_{102} = 4 + 0{,}5\cdot 6 = 7 \qquad 0 \pm 1{,}96\sqrt{7}$$

$0{,}5t$

$$\mathrm{cov}(u_t^2, u_{t-1}^2) =$$

$$= \mathrm{cov}(\sigma_t^2 v_t^2, u_{t-1}^2) =$$

$$= \mathrm{cov}((\cancel{1} + 0{,}5\, u_{t-1}^2) v_t^2, u_{t-1}^2) =$$

$$= \mathrm{cov}(0{,}5 u_{t-1}^2 v_t^2, u_{t-1}^2)$$

$$= E(0{,}5 u_{t-1}^2 \underbrace{v_t^2} u_{t-1}^2) - E(0{,}5\, u_{t-1}^2 \underbrace{v_t^2}) E(u_{t-1}^2)$$

$$0{,}5 \underbrace{E(v_t^2)}_{1} \cdot \cancel{\mathrm{Var}(u_{t-1}^2)}$$

$$\mathrm{corr} = \frac{\mathrm{cov}}{\mathrm{var}} = 0{,}5\, t$$