Name:

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1. Consider the difference equation: $y_t = 1+0.7y_{t-1}-0.12y_{t-2}+u_t$, where (u_t) is a white noise and $y_{100} = 1$, $y_{99} = -1$.

   a) Find 95% prediction interval for y_{102}.

   b) Approximately find the best point forecast for y_{10000}.

2. Consider the difference equation: $y_t = 1 + 0.7y_{t-1} - 0.12y_{t-2} + u_t$, where (u_t) is a white noise.

   a) Find first two values of autocorrelation function.

   b) Find α_{2023} in $MA(\infty)$ representation

$$y_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3} + \dots$$

3. Consider $ETS(AAdN)$ model

$$\begin{cases} u_t \sim \mathcal{N}(0; 20) \\ b_t = 0.9b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + 0.9b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + 0.9b_{t-1} + u_t \end{cases}$$

   with $\ell_{100} = 20$ and $b_{100} = 1$.

   a) Find 95% prediction interval for y_{102}.

   b) Approximately find the best point forecast for y_{10000}.

4. Stationary process (y_t) is described by equation $(y_t - \mu) = 0.5(y_{t-2} - \mu) + u_t$, where (u_t) is a normal white noise with known variance $\sigma_u^2 = 1$.

   Winnie-the-Pooh and Rabbit have a lot of observations to create 95% confidence interval for μ. Rabbit estimates μ in the true model but Winnie-the-Pooh wrongly believes that equation is $(y_t - \mu) = \beta(y_{t-1} - \mu) + u_t$.

   Approximately find the ratio of confidence interval widths.

5. Consider the sum of two independent stationary processes, $y_t = 0.5y_{t-1} + u_t$ and $x_t = v_t + 0.7v_{t-1}$, where (u_t) and (v_t) are independent white noise processes with unit variance.

   a) Classify the sum $s_t = y_t + x_t$ as $ARMA(p, q)$ process with finite p and q.

   b) Explicitely find the polynomials $A(L)$ and $B(L)$ in the corresponding $ARMA(p, q)$ equation $A(L)s_t = B(L)w_t$, where (w_t) is a white noise.