№2.1.

$$y_t = 4 + 0{,}4\, y_{t-1} + 0{,}3\, \varepsilon_{t-1} + \varepsilon_t$$

$$y_t = 4 + 0{,}4\, L y_t + 0{,}3\, L \varepsilon_t + \varepsilon_t$$

$$\underbrace{(1 - 0{,}4 L)}_{A(L)}\, y_t = 4 + \underbrace{(1 + 0{,}3 L)}_{B(L)}\, \varepsilon_t$$

№2.2

x_t - c.n. $t = 0, 1, 2 \ldots$

$$\underbrace{y_t = (1+L)^t x_t}$$

$$y_t = (1+L)(1+L)^{t-1} x_t$$

$$y_t = (1+L)^{t-1}(x_t + x_{t-1})$$

$$y_t = (1+L)^{t-1} \underbrace{x_t} + \underbrace{(1+L)^{t-1} x_{t-1}}_{y_{t-1}}$$

$$y_t = (1+L)^{t-1} x_t + y_{t-1}$$

$$y_t - y_{t-1} = (1+L)^{t-1} x_t$$

$$x_t = y_t - \sum_{k=0}^{t-1} y_k$$

N 2.4.

$x_t, \quad t = \ldots, -2, -1, 0, 1, 2, \ldots$

$y = x_{-t} \qquad y_1 = y_{-1}$

1) $Ly_t = Lx_{-t}$ ✗

$y_{t-1} = x_{-t-1}$

$x_{-t+1} \quad x_{-t-1}$

2) $\underbrace{L(y_t) = y_{t-1}} = \underbrace{x_{-t+1}}$ ✓

3) $x_t \underbrace{Ly_t} = x_t y_{t-1}$ ✓

4) $x_t Ly_t = x_{t-1} y_t$ ✗

$x_t y_{t-1} = x_{t-1} y_t$

N 2.5

y_t – стац. процесс

$E(y_t) = \mu$

$cov(y_t, y_{t-k}) = \gamma_k$

1) $z_t = 2y_t$

$E(z_t) = 2\mu$

$cov(z_t, z_{t-k}) = cov(2y_t, 2y_{t-k}) = 4\gamma_k$

3) $z = \Delta y_t = y_t - y_{t-1}$

$$E(z_t) = 0$$

$$cov(z_t, z_{t-k}) = cov(y_t - y_{t-1}, y_{t-k} - y_{t-k-1}) =$$

$$= \gamma_k - \gamma_{k-1} - \gamma_{k+1} + \gamma_k =$$

$$= 2\gamma_k - \gamma_{k-1} - \gamma_{k+1}$$

4) $z_t = 2y_t + 3y_{t-1}$

2.9.

ε_t - WN

$y_t = 2 + 0,5\, y_{t-1} + \varepsilon_t$

$y_1 = 0$

$y_1 = 4$

$y_1 = 4 + \varepsilon_1$

$y_1 = 4 + \frac{2}{\sqrt{3}}\, \varepsilon_1$

$E(y_t)$

$cov(y_t, y_{t-k})$

$$E(y_2) = E(2 + 0,5\, y_1 + \varepsilon_2) = 2 + 0,5\, E(y_1)$$

$$E(y_3) = E(2 + 0,5\, y_2 + \varepsilon_2) = 2 + 0,5\, E(y_2) =$$

$$= 2 + 0,5(2 + 0,5\, E(y_1)) = \underbrace{2 + 0,5 \cdot 2} + 0,5^{(2)} E(y_1)$$

$$E(y_t) = \sum_{k=0}^{t-2} 2 \cdot 0,5^k + 0,5^{t-1} E(y_1)$$

$$Var(y_2) = Var(2 + 0{,}5 y_1 + \varepsilon_2) = 0{,}5^2 Var(y_1) + 6^2$$

$$Var(y_3) = 0{,}5^2 Var(y_2) + 6^2 = 0{,}5^4 Var(y_1) + 0{,}5^2 6^2 + 6^2$$

$$Var(y_t) = \sum_{k=0}^{t-2} 6^2 (0{,}5)^{2k} + (0{,}5)^{2(t-1)} Var(y_1)$$

$$\frac{6_1(1-q^k)}{1-q}$$

$$\sum_{k=0}^{t-2} 2 \cdot 0{,}5^k = \frac{2(1-0{,}5^{(t-1)})}{0{,}5}$$

$$E(y_t) = \sum_{k=0}^{t-2} 2 \cdot 0{,}5^k + 0{,}5^{t-1} E(y_1) =$$

$$= 4(1-0{,}5^{(t-1)}) + 0{,}5^{(t-1)} E(y_1)$$

$$Var(y_t) = \sum_{k=0}^{t-2} 6^2 (0{,}5)^{2k} + (0{,}5)^{2(t-1)} Var(y_1) =$$

$$= \frac{6^2(1-0{,}5^{2(t-1)})}{1-(0{,}5)^2} + (0{,}5)^{2(t-1)} Var(y_1)$$

$y_1 = 4$ $\quad E(y_t) = 4$

$Var(y_t) = f(t)$

$y_1 = 4 + \varepsilon_1$ $\quad E(y_t) = 4$

$$Var(y_t) = \frac{4}{3} 6^2 (1 - 0{,}5^{2(t-1)}) + 0{,}5^{2(t-1)} 6^2$$

$$\boxed{y_1 = 4 + \frac{2}{\sqrt{3}}\varepsilon}$$

$$E(y_t = 4)$$

$$Var(y_t) = \frac{4}{3}\sigma^2(1 - \cancel{0{,}5^{2(t-1)}}) + \cancel{\frac{4}{3}\sigma^2 0{,}5^{2(t-1)}}$$

$$= \boxed{\frac{4}{3}\sigma^2}$$

$$cov(y_t, y_{t-1}) = cov(0{,}5y_{t-1} + \cancel{\varepsilon_t}, 0{,}5y_{t-2} + \underline{\varepsilon_{t-1}}) =$$

$$= 0{,}5\, Var(y_{t-1}) =$$

$$cov(y_t, y_{t-2}) = cov(\cancel{2} + 0{,}5y_{t-1} + \cancel{\varepsilon_t}, y_{t-2}) =$$

$$= 0{,}5\, cov(y_{t-1}, y_{t-2}) =$$

$$= 0{,}5\, cov(2 + 0{,}5y_{t-2} + \varepsilon_{t-1}, y_{t-2}) =$$

$$= 0{,}5 \cdot 0{,}5\, Var(y_{t-2})$$

$$cov(y_t, y_{t-k}) = 0{,}5^k\, Var(y_{t-k})$$

Для $y_1 = \boxed{4 + \frac{2}{\sqrt{3}}\varepsilon_1}$:

$$cov(y_t, y_{t-k}) = 0{,}5^k \frac{4}{3}\sigma^2$$