N 3.19

$y_1 = 0{,}1$, $y_2 = -0{,}2$, $y_3 = 0{,}2$

$y_t \sim AR(1)$, $\boxed{y_t = \beta y_{t-1} + u_t}$

$|\beta| < 1$, $u_t \sim N(0, \sigma_u^2)$

a) $E(y_1), E(y_2 | y_1), E(y_3 | y_2)$

$E(y_t) = \mu$

$\mu = \beta\mu \Rightarrow \mu = 0$

$E(y_1) = 0$

$E(y_2 | y_1) = E(\beta y_1 + \varepsilon_2 | y_1) = \beta y_1$

$E(y_3 | y_2) = E(\beta y_2 + \varepsilon_3 | y_2) = \beta y_2$

2) $Var(y_1)$, $Var(y_2 | y_1)$, $Var(y_3 | y_2)$

$Var(y_t) = \sigma^2$

$\sigma^2 = \beta^2\sigma^2 + \sigma_u^2 \Rightarrow \sigma^2 = \dfrac{\sigma_u^2}{1-\beta^2}$

$Var(y_2 | y_1) = Var(\beta y_1 + \varepsilon_2 | y_1 = \sigma_u^2$

$Var(y_3 | y_2) = \sigma_u^2$

3) $f(y_1),\ f(y_2|y_1),\ f(y_3|y_2)$

$\mathbb{E}(y_1) = 0$

$Var(y_1) = \dfrac{\sigma_u^2}{1-\beta^2}$

$f(y_t) \sim N\left(0, \dfrac{\sigma_u^2}{1-\beta}\right) \sim y_1$

$y_2 = \beta y_1 + \varepsilon_2 \sim N$

$y_2|y_1 \sim N(\beta y_1, \sigma_u^2)$

$y_3|y_2 \sim N(\beta y_2, \sigma_u^2)$

4) $\ln f(y_1, y_2, y_3 | \beta, \sigma_u^2)$

$f(y_1, \dots y_T) = f(y_T|y_1, \dots, y_{T-1}) \cdot$

$\cdot f(y_{T-1}|y_1, \dots, y_{T-2}) \cdot \dots \cdot f(y_2|y_1) \cdot f(y_1)$

Для $AR(1)$:

$f(y_1, \dots, y_T) = f(y_T|y_{T-1}) f(y_{T-1}|y_{T-2}) \cdot \dots$

$\cdot f(y_2|y_1) f(y_1)$

$f(y_t) \sim N\left(0, \dfrac{\sigma_u^2}{1-\beta^2}\right) \sim y_1$

$y_2|y_1 \sim N(\beta y_1, \sigma_u^2)$

$y_3|y_2 \sim N(\beta y_2, \sigma_u^2)$

$$L = \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(y_3-\beta y_2)^2}{2\sigma_u^2}} \cdot \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(y_2-\beta y_1)^2}{2\sigma_u^2}} \cdot$$

$$\cdot \frac{1}{\sqrt{2\pi \frac{\sigma_u^2}{1-\beta^2}}} e^{-\frac{y_1^2(1-\beta^2)}{\sigma_u^2}} = f(y_1, y_2, y_3, \sigma_u^2, \beta)$$

$$l = \frac{1}{2}\ln\sigma_u^2 \left(\frac{(y_3-\beta y_2)^2}{2\sigma^2} + \frac{(y_2-\beta y_1)^2}{2\sigma_u^2} \right)$$

$$+ \left(\underline{\frac{1}{2}\ln\sigma^2} + \frac{1}{2}\ln(1-\beta^2) \right) \left(\underline{\frac{y_1^2(1-\beta^2)}{\sigma_u^2}} \right)$$

$$\frac{\partial l}{\partial \beta} = \frac{1}{2}\ln\sigma_u^2 \left(-\frac{(y_3-\beta y_2)}{\sigma_u^2} - \frac{(y_2-\beta y_1)}{\sigma_u^2} \right) +$$

$$+ \frac{\frac{1}{2}}{1-\beta^2} \cdot (-2\beta) .$$

б) $\ln f(y_2, y_3 \mid \beta, \sigma_u^2\, y_1)$

$$f(y_1, \dots, y_T) = f(y_T \mid y_{T-1}) f(y_{T-1} \mid y_{T-2}) \cdot \dots$$

$$\cdot f(y_2 \mid y_1) \cancel{f(y_1)}$$

$$l = \frac{1}{2}\ln\sigma_u^2 \left(\frac{(y_3-\beta y_2)^2}{2\sigma^2} + \frac{(y_2-\beta y_1)^2}{2\sigma_u^2} \right)$$

$$\ln \left(\prod_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(y_t-\beta y_{t-1})^2}{2\sigma_u^2}} \right) =$$

$$= -\frac{T-1}{2}\ln 2\pi - \frac{T-1}{2}\ln \sigma_u^2 - \sum_{t=2}^{T}\frac{(\overset{Y_t}{y_t} - \beta \overset{Y_{t-1}}{y_{t-1}})^2}{2\sigma_u^2}$$

$$\to \max_{\beta, \sigma_u^2}$$

$$\frac{\partial l}{\partial \beta} = \sum_{t=2}^{T}\frac{2(y_t - \beta y_{t-1})(-y_{t-1})}{2\sigma_u^2} = 0$$

$$\sum_{t=2}^{T}(y_t - \beta y_{t-1})y_{t-1} = 0$$

$$\hat\beta = \frac{\sum_{t=2}^{T} y_t y_{t-1}}{\sum_{t=2}^{T} y_{t-1}^2}$$

$$\frac{\partial l}{\partial \sigma^2} = -\frac{T-1}{2\sigma_u^2} + \frac{\sum_{t=2}^{T}(y_t - \beta y_{t-1})^2}{2(\sigma_u^2)^2} = 0$$

$$(T-1)\sigma_u^2 = \sum_{t=2}^{T}(y_t - \beta y_{t-1})^2$$

$$\hat\sigma_u^2 = \frac{\sum_{t=2}^{T}(y_t - \beta y_{t-1})^2}{T-1}$$

№3.21

$$\begin{cases} y_0 = C \\ y_t = y_{t-1} + u_t \\ u_t \sim N(0, \sigma_u^2) \end{cases}$$

a) $E(y_{10})$, $Var(y_{10})$, $y_{10} \sim ?$

$y_1 = y_0 + u_1$

$y_2 = y_0 + u_1 + u_2$

$y_t = y_0 + \overset{\sim N}{\sum_{k=1}^{t} u_k}$

$y_t \sim N(c, t\sigma_u^2)$

$E(y_t) = c$

$Var(y_t) = t\sigma_u^2$

б) $E(y_{10} | y_7)$ $Var(y_{10} | y_7)$, $y_{10} | y_7 \sim ?$

$E($

$y_3 = c + u_1 + u_2 + u_3$

$y_3 | y_2 = c + u_3 + y_2$

$y_{10} | y_2 = u_{10} + \dots + u_3 + y_2$

$E(y_t | y_s) \overset{t>s}{=} y_s$

$Var(y_t | y_s) = \sigma_u^2 (t-s)$, $t \neq s$

$y_t | y_s \sim N(y_s, \sigma_u^2 (t-s))$

$y_{101} | y_{100} \sim N(y_{100}, \sigma_u^2)$

$y_{102} | y_{100} \sim N(y_{100}, 2\sigma_u^2)$

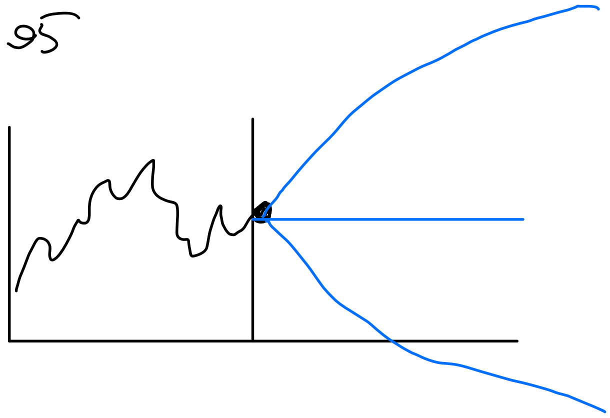
$c = 4,\ \sigma_u^2 = 9,\ y_{100} = 20$

$P(y_{101} \in [\qquad]) = 0{,}95$

$20 \pm 1{,}96 \cdot 3$

y_{102}

$20 \pm 1{,}96\ 3\sqrt{2}$



c, σ^2 – ММП

$y_{t|s} \sim N(y_s \mid \sigma_u^2(t-s))$

$y_t \sim N(c, t\sigma_u^2)$

$f(y_1, \dots y_T) = f(y_T \mid y_1, \dots, y_{T-1}) \cdot$

$\cdot f(y_{T-1} \mid y_1, \dots, y_{T-2}) \cdot \dots \cdot f(y_2 \mid y_1) \cdot f(y_1)$

$y_t = y_{t-1} + u_t$

$f(y_1, \dots, y_T) = f(y_T \mid y_{T-1}) f(y_{T-1} \mid y_{T-2}) \cdot \dots$

$\cdot f(y_2 \mid y_1) f(y_1)$

$y_t \mid y_{t-1} \sim N(y_{t-1}, \sigma_u^2)$

$y_1 \sim N(y_0'' c, \sigma_u^2)$

$$\ell = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln \sigma_u^2 - \sum_{t=2}^{T}\frac{(y_t - y_{t-1})^2}{2\sigma_u^2} - \frac{(y_1 - c)^2}{2\sigma_u^2}$$

$$\frac{\partial \ell}{\partial c} = \frac{2(y_1 - c)}{2\sigma_u^2} = 0 \Rightarrow \hat{c} = y_1$$

$$\frac{\partial \ell}{\partial \sigma^2} = -\frac{T}{2\sigma_u^2} + \sum_{t=2}^{T}\frac{(y_t - y_{t-1})^2}{2(\sigma_u^2)^2} + \cancel{\frac{(y_1 - c)^2}{2(\sigma_u^2)^2}}$$

$$\hat{\sigma}_u^2 = \frac{\sum_{t=2}^{T}(y_t - y_{t-1})}{T}$$

N 3.27

a_t — RW $\quad a_t = a_{t-1} + u_t \sim WN$

b_t — SRW $\quad b_t = b_{t-12} + \nu_t \sim WN$

u_t и ν_t — незав.

1) $\Delta = 1 - L$, $\Delta^d a_t$ стала стац.

$$\Delta a_t = a_t - a_{t-1} = a_{t-1} + u_t - a_{t-1} = u_t \to \text{стац.}$$

2) $\Delta b_t = b_t - b_{t-1} = b_{t-12} + u_t - b_{t-1} =$

$$= \underbrace{b_{t-12}}_{b_{t-24}} - \underbrace{b_{t-1}}_{} + u_t \quad \text{нет}$$

b) $\Delta_{12} = 1 - L^{12}$

$\Delta_{12} a_t = a_t - a_{t-12} =$

$= a_{t-1} + u_t - a_{t-12} = \quad \dots \quad =$

$= \cancel{a_{t-12}} + \boxed{u_{t-11} + \dots + u_t} - \cancel{a_{t-12}} =$

$=$

$E(\Delta_{12} a_t) = 0$

$Var(\Delta_{12} a_t) = 12 \sigma_u^2$

$\Delta_{12} b_t = b_t - b_{t-12} = b_{t-12} + u_t - b_{t-12}$

$= u_t$