N 3.18

$y_1 = 0{,}1$, $y_2 = -0{,}2$, $y_3 = 0{,}2$

$\beta < 1$

$y_t \sim AR(1)$ $\quad y_t = \beta y_{t-1} + u_t \quad u_t \sim N(0, \sigma_u^2)$

1) $E(y_1)$ $\quad E(y_2 | y_1)$ $\quad E(y_3 | y_2)$

$E(y_t) = \mu = 0$

$E(y_t) = \beta E(y_{t-1}) + E(u_t)$

$\mu = \beta \mu$

$\mu = 0$

$y_t = \sum_{i=0}^{+\infty} \alpha_i u_{t-i}$ $\quad N(0, \sigma_u^2)$

$E(y_t) = 0$

$E(y_2 | y_1) = \boxed{\beta y_1}$

$E(y_3 | y_2) = \beta y_2$

2) $Var(y_1)$ $\quad Var(y_2 | y_1)$ $\quad Var(y_3 | y_2)$

$\quad \| \sigma^2$

$Var(y_t) = \beta^2 Var(y_{t-1}) + \sigma_u^2$

$$\sigma^2 = \frac{\sigma_u^2}{1 - \beta^2}$$

$Var(y_2 | y_1) = Var(\beta y_1 + u_2 | y_1) = \sigma_u^2$

$Var(y_3 | y_2) = \sigma_u^2$

3) $f(y_1)$, $f(y_2|y_1)$ $f(y_3|y_2)$

$\underbrace{f(y_1)}_{N\left(0, \frac{\sigma_u^2}{1-\beta^2}\right)}$ $\sim N(\beta y_1, \sigma_u^2)$ $\sim N(\beta y_2, \sigma_u^2)$

$y_2 = \beta y_1 + u_2$
$\sim N(0, \sigma_u^2)$

$l(y, \beta, \sigma_u^2)$

$$f(y) = f(y_T | y_{T-1}, \ldots, \cancel{y_1}) \cdot f(y_{T-1} | y_{T-2}, \cancel{\ldots, y_1}) \cdot$$
$$\cdot f(y_2 | y_1) \cdot f(y_1)$$

$$l(y | \sigma_u^2, \beta) = \ln\left(f(y_3|y_2) f(y_2|y_1) \cdot f(y_1)\right)$$

$$\ln\left(\frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(y_3-\beta y_2)^2}{2\sigma_u^2}} \cdot \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(y_2-\beta y_1)^2}{2\sigma_u^2}} \frac{1}{\sqrt{2\pi\frac{\sigma_u^2}{1-\beta^2}}} e^{-\frac{(y_1)^2(1-\beta^2)}{2\sigma_\varepsilon^2}}\right)$$

$$\prod_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(y_t-\beta y_{t-1})^2}{2\sigma_u^2}} \cdot \cancel{\frac{1}{\sqrt{2\pi\frac{\sigma_u^2}{1-\beta^2}}} e^{-\frac{(y_1)^2(1-\beta^2)}{2\sigma_\varepsilon^2}}}$$

Для T набл.

Условное правдоподобие

$$f(y_3, y_2 | \sigma_u^2, \beta, y_1) = \frac{f(y_3, y_2, y_1 | \sigma_u^2, \beta)}{f(y_1 | \sigma_u^2, \beta)}$$

$$\ln\left(\prod_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(y_t-\beta y_{t-1})^2}{2\sigma_u^2}}\right) = -\frac{T-1}{2}\ln 2\pi - \frac{T-1}{2}\ln\sigma_u^2 +$$

$$+\sum_{T=2}^{T} \frac{(\overset{Y_t}{y_t} - \beta \overset{Y_{t-1}}{y_{t-1}})^2}{2\sigma_u^2} \to \max_{\beta,\sigma_u^2}$$

$$\frac{\partial l}{\partial \beta} = \frac{\sum_{t=2}^{T} 2(y_t - \beta y_{t-1})(-y_{t-1})}{2\sigma_u^2} = 0$$

$$\sum_{t=2}^{T}\left(\beta y_{t-1}^2 - y_t y_{t-1}\right) = 0$$

$$\hat\beta = \frac{\sum_{t=2}^{T} y_t y_{t-1}}{\sum_{t=2}^{T} y_{t-1}^2}$$

σ_u^2

N. 3. 20

$$\begin{cases} y_0 = c \\ y_t = \beta + y_{t-1} + u_t \\ u_t \sim N(0, \sigma_u^2), \text{ iid} \end{cases}$$

$E(y_{10})$

$$E(y_t) = E(y_{t-1} + u_t) = E(y_{t-2} + \underbrace{u_{t-1} + u_t}_{0}) = C$$

$$Var(y_t) = \sigma_u^2 t$$

$$E(y_t | y_{t-k}) = y_{t-k}$$

$$Var(y_t | y_{t-k}) = k \sigma_u^2$$

$$y_t | y_{t-k} \sim N(y_{t-k}, k\sigma_u^2)$$

$$y_t | y_{t-1} \sim N(y_{t-1}, \sigma_u^2) \qquad y_1 \sim N(c, \sigma_u^2)$$

$$l(y | c, \sigma_u^2) = \ln(f(y_T | y_{T-1}) \cdot \ldots \cdot f(y_2|y_1) f(y_1))$$

$$=$$

$$l = -\frac{T}{2} \ln(2\pi) - \frac{T}{2} \ln(\sigma_u^2) - \sum_{t=2}^{T} \frac{(y_t - y_{t-1})^2}{2\sigma_u^2} - \frac{(y_1 - c)^2}{2\sigma_u^2} \to \max_{c, \sigma_u^2}$$

$$\frac{\partial l}{\partial c} = \frac{-2(y_1 - c)}{2\sigma_u^2} = 0$$

$$\hat{c} = y_1$$

$$\frac{\partial l}{\partial \sigma_u^2} \quad -\frac{T}{2\sigma_u^2} + \sum_{t=2}^{T} \frac{(y_t - y_{t-1})^2}{2(\sigma^2)^2} = 0 \quad \Big| \cdot 2(\sigma^2)^2$$

$$\hat{\sigma}^2_u = \frac{\sum_{t=2}^{T} (y_t - y_{t-1})^2}{T}$$

№ 3, 26

$$a_t = a_{t-1} + u_t \qquad u_t \sim WN$$

$$b_t = b_{t-12} + \nu_t \qquad \nu_t \sim WN$$

$$a_0 = 0$$

$$b_{-11} = \ldots = b_{-1} = b_0 = 0$$

1) $\Delta = 1 - L$ на a_t

Сначала ли a_t стац?

$$\Delta a_t = a_t - a_{t-1} = (\underbrace{a_t}_{} - 1 - \underbrace{a_{t-2}}_{}) + (u_t - u_{t-1})$$

$$\mathbb{E}(\Delta a_t) = (0 - 0) + (0 - 0)$$

$$Var(\Delta a_t) = Var(a_t - a_{t-1}) = Var(a_t) + Var(a_{t-1}) - 2 cov(a_t, a_{t-1})$$

$$t\sigma^2_u + (t-1)\sigma^2_u - 2(t-1)\sigma^2_u =$$

$$cov(a_t, a_{t-1}) = cov(a_{t-1} + u_t, a_{t-1}) = Var(a_{t-1})$$

$$= t\sigma^2_u - (t-1)\sigma^2_u = \sigma^2_u$$

6c

$$A_{12} = \ell - L^{12}$$