$$\alpha + q\alpha + q^2\alpha$$

$$\frac{\alpha(q^n - 1)}{q - 1} = 1 \qquad \alpha(q^n - 1) = q - 1$$

$$\alpha q^n - \alpha = q - 1$$

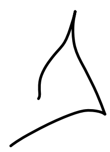
$$\alpha q^n - q = \alpha - 1$$

$$y_t = (1+L)^t x_t$$

$$y_t = (x_t + x_{t-1})(1+L)^{t-1} = x_t(1+L)^{t-1} + y_{t-1}$$

$$y_{t-1} = (1+L)^{t-1} x_{t-1}$$

$$\Rightarrow \quad y_t(1-L) = \ \ldots$$



$$F_n = F_{n-1} + F_{n-2} = L(F_n + LF_n) =$$

$$= L(1+L)F_n$$

$$F_n = L^k(1+L)^k F_n$$

$$F_{n+k} = \frac{F_n}{L^k} = (1+L)^k F_n$$

$1 + C_5^1$

$F_{111} = F_{100+11} = (1+L)^{11} F_{100}$

$(1+L) + C_5^1 (1+L)^2 + C_5^2 (1+L)^3$

2.1

2.2

2.11

2.5 ?

2.7 ?

2.9

2.12

2.13

$z_t = x_t^1 (1 - x_{t-1}^0) y_t^{\neq 0}$

$z_{t-1} = x_{t-1}^0 (1 - x_{t-2}) y_{t-1}$

$z_{t+1} = x_{t+1} (1 - x_t^1) y_{t+1}$

$|\alpha| < 1$

$$\frac{1}{(1-\alpha L)} = 1 + \alpha L + \alpha^2 L^2 + \dots$$

$|\alpha| > 1$

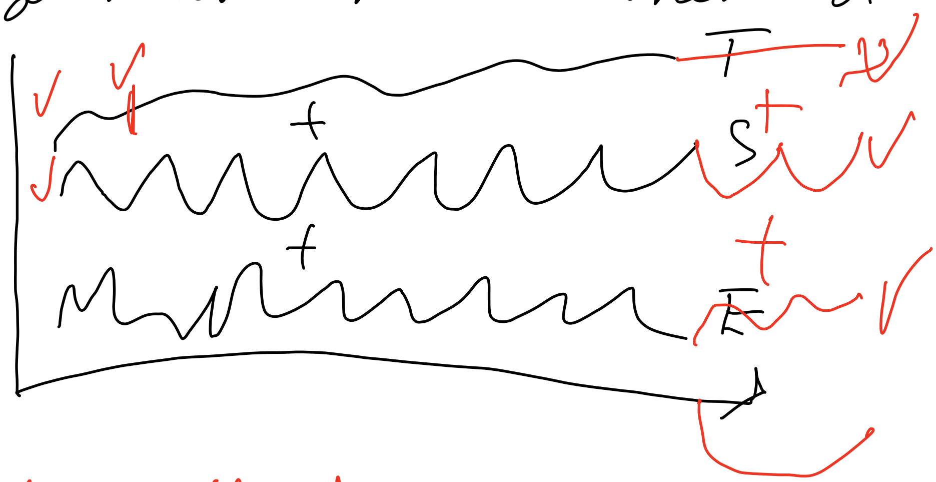
$$\frac{1}{1-\alpha L} = -(\alpha L)^{-1} - (\alpha L)^{-2} - \dots$$

Домашка

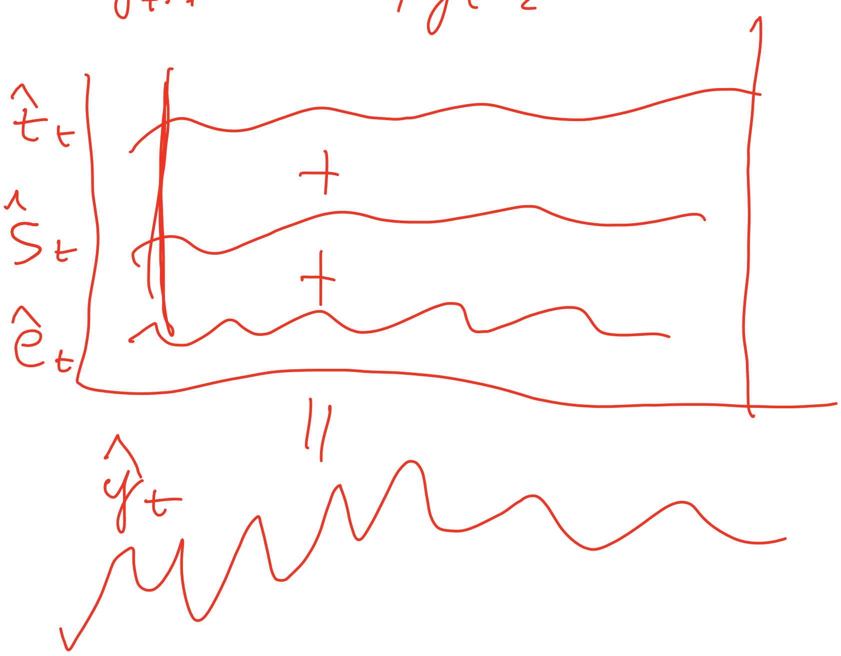
1) STL
2) ETS

1) STL

1) Разложить на компоненты



$\hat{y}_t = \alpha y_{t-1} + \alpha(1-\alpha) y_{t-2}$



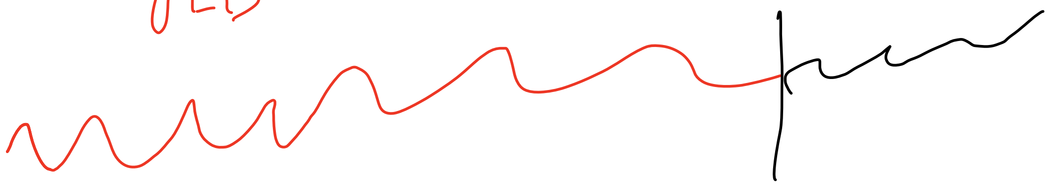
$y_t - \check{y}_t = e^{clean}_t$

ETS $\hat{y}_t = \hat{l}_t + \hat{b}_t + s_t$

$e_{ets} = y_t - \hat{y}_t$

$e_{clean} = e_{ets} - e_{eps}$

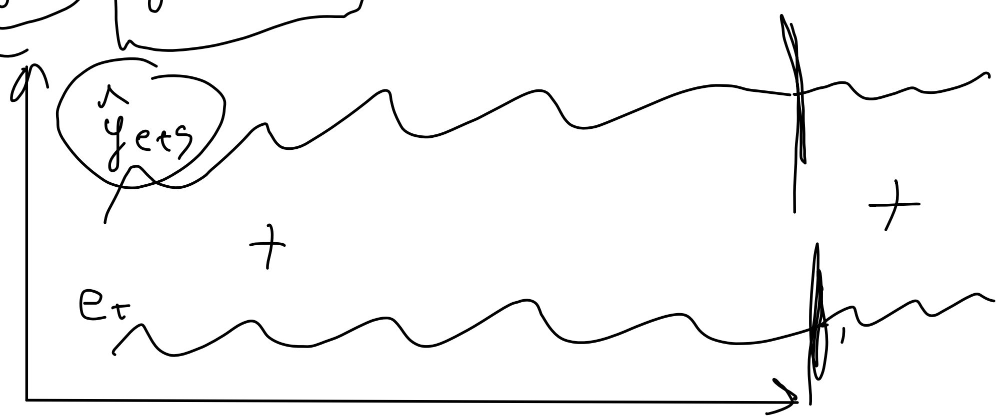
$y_t = \underbrace{ETS}_{\hat{y}^{ETS}} + \varepsilon_t$



ETS

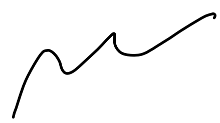
$\hat{y}_t$ $\quad y - \hat{y}_t = e_t \quad \hat{y}^{ETS}_t = y_t - e_t$

$y_t = \hat{y}^{ETS} + \varepsilon$



$\parallel$

y_t



$e^{clean} = y_t - \hat{y}^{ETS} - \hat{e}_t$

№ 2.1

$y_t = 4 + 0{,}4\,y_{t-1} + 0{,}3\,\varepsilon_{t-1} + \varepsilon_t$

$y_t = 4 + 0{,}4\,L y_t + 0{,}3\,L\varepsilon_t + \varepsilon_t$

$$\underbrace{y_t(1-0{,}4L)}_{AR(1)} = \underbrace{4}_{\mu} + \underbrace{(1+0{,}3L)\varepsilon_t}_{MA(1)}$$

$A(L)\,y_t = \beta_0 + B(L)\,\varepsilon_t$ ARMA

2,2.

$y_t = (1+L)^t x_t$ $\qquad L y_t = y_{t-1}$

x_t через y_t $\qquad L^{-1} y_t = y_{t+1}$

$x_t = f(y_t, L)$

$y_t = (1+L)(1+L)^{t-1} x_t$

$(1+L)^{t-1}(x_t + x_{t-1}) = y_{t-1} + (1+L)^{t-1} x_t$

$y_{t-1} = (1+L)^{t-1} x_{t-1}$

$y_t - y_{t-1}$

$(1-L)\,y_t = (1+L)^{t-1} x_t$

$(1-L)^t y_t = x_t$

2,4.

$t = \dots, -2, -1, 0, 1, 2, \dots \qquad y_t = x_{-t}$

1) $L y_t = L x_{-t} = x_{-t-1}$ X

$L y_3 = y_2 \qquad x_{-3-1}$

2) $L y_t = y_{t-1} = x_{-t+1}$ ✓

3) $\underset{\underset{y_{-t}}{\parallel}}{x_t} L y_t = x_t y_{t-1}$ ✓

4) $\underset{y_{-t-1}}{x_t L y_t} = \underset{y_{-t+1}}{x_{t-1} y_t}$ X

2,5.

y_t - стац. процесс $\qquad E(y_t) = \mu$

$\qquad cov(y_t, y_{t-k}) = \gamma_k$

1) $z_t = 2 y_t$

$E(z_t) = 2\mu$ ✓

$cov(2y_t, 2y_{t-k}) = 4\gamma_k$

3) $z_t = \Delta y_t = y_t - y_{t-1}$

$E(z_t) = \mu - \mu = 0 \quad \checkmark$

$Cov(y_t - y_{t-1}, y_{t-k} - y_{t-k-1}) =$

$= \gamma_k - \gamma_{k-1} - \gamma_{k+1} + \gamma_k = f(k) \quad \checkmark$

2.9.

$\varepsilon_t - WN \qquad y_t = 2 + 0{,}5\, y_{t-1} + \varepsilon_t$

$E(\varepsilon_t) = 0 \qquad y_1 = C$

$Var(\varepsilon_t) = \sigma_\varepsilon^2 \qquad y_3 \quad y_2 \quad y_1$

$cov(\varepsilon_t, \varepsilon_{t-k}) = 0 \;\forall k \neq 0 \qquad E(y_3) = 2 + 0{,}5\, E(y_2) =$

$E(y_t) \qquad Var(y_t) \qquad = 2 + 0{,}5(2 + 0{,}5\, E(y_1))$

$t = 3$

$2 + 0{,}5 \cdot 2 + 0{,}5^2\, 2$

$E(y_t) = 2 + 0{,}5\, E(y_{t-1}) + 0 = 2\left(\sum_{k=1}^{t-1} 0{,}5^{(k-1)}\right) + 0{,}5^{t-1}$

$2 + 0{,}5 \cdot 2 + 0{,}5^2 \cdot 2$

$$= 2\left(\sum_{k=1}^{t-1} 0{,}5^{k-1}\right) + 0{,}5^{(t-1)}\, y_1$$

$Var(y_t) = 0{,}5^2\, Var(y_{t-1}) + \sigma_\varepsilon^2$

$0{,}5^4 + 0{,}5^2\, \sigma_\varepsilon^2 + \sigma_\varepsilon^2$

$$= \sigma_\varepsilon^2 \left(\sum_{k=1}^{t} 0{,}5^{2(k-1)}\right) + 0{,}5^{2(t-1)}\, Var(y_1)$$

$$\frac{b_1(1-0{,}5^{k})}{1-0{,}5}$$

$y_1 = 0$

$y_1 = 4$

$y_1 = 4 + \varepsilon_1$

$y_1 = 4 + \frac{2}{\sqrt{3}}\varepsilon_t \qquad \mu = 2\left(\frac{(1-0{,}5^{t})}{1-0{,}5}\right) + 0{,}5^{(t-1)} E(y_1)$

$$\sigma^2 = \sigma_\varepsilon^2\left(\frac{1(1-0{,}5^{2t})}{1-0{,}5^2}\right) + 0{,}5^{2(t-1)} Var(y_1)$$

$y_1 = 0 \qquad \mu = 2\left(\frac{(1-0{,}5^{t})}{1-0{,}5}\right)$ Ас. стац.

$y_1 = 4$ ✗

$y_1 = 4 + \varepsilon_1 \qquad \mu = \frac{2 - 2\cdot 0{,}5^{(t-1)}}{0{,}5} + 0{,}5^{(t-1)} y_1 =$

$= 4 - 4\,0{,}5^{(t-1)} + 0{,}5^{(t-1)}(4+0)$

$=$ ✗

$y_1 = 4 + \frac{2}{\sqrt{3}}\varepsilon_1 \qquad 4 - 4\cdot 0{,}5^{(t-1)} + 0{,}5^{(t-1)}(4+0)$

✓

2,12 , 2.13 , 2.15

2,16 , 2.17 ?