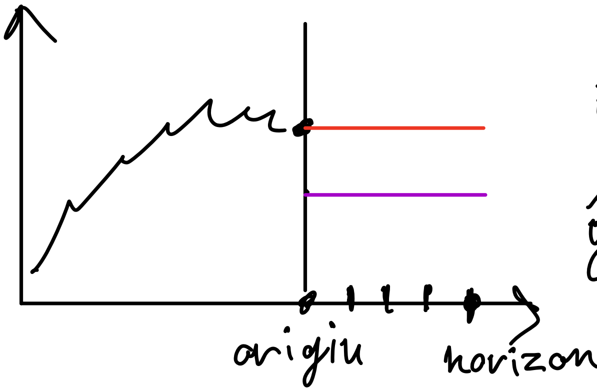


$\hat{y}_{t+h} = y_T$ $\quad y_1, \dots, y_T$

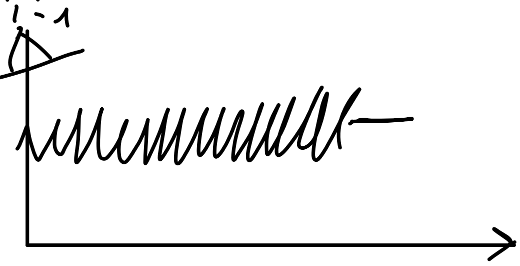
$\hat{y}_{t+h} = \frac{1}{T} \sum_{t=1}^{T} y_t$

$\alpha \in (0;1)$

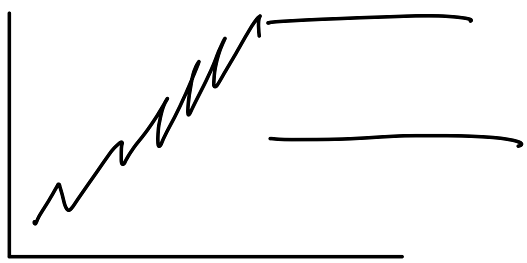
$\hat{y}_{T+1|T} = \alpha y_T + \alpha(1-\alpha) y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} + \dots$

$\hat{y}_{T+1|T} = \alpha y_T + (1-\alpha)\left(\alpha y_{T-1} + \alpha(1-\alpha) y_{T-2} + \dots\right)$

$\hat{y}_{T+1|T} = \alpha y_T + (1-\alpha) \hat{y}_{T|T-1}$



Weighted average form



$\hat{y}_{2|1} = \alpha y_1 + (1-\alpha) \hat{y}_{1|0}$

$\hat{y}_{2|1} = \alpha y_1 + (1-\alpha) l_0$

$\hat{y}_{3|2} = \alpha y_2 + (1-\alpha) \hat{y}_{2|1}$

$\vdots$

$y_{T+1|T} = \underbrace{\alpha y_T + (1-\alpha) \hat{y}_{T|T-1}}$

$\hat{y}_{T+1|T} = \sum_{j=0}^{T-1} \alpha(1-\alpha)^j y_{t-j} + (1-\alpha)^T l$

Component form

Forecast eq $y_{t+h|t} = l_t$

Smoothing eq $l_t = \alpha y_t + (1-\alpha) l_{t-1}$

1) Прогноз – константа
2) а) α, l_0 – подобрать (задачке)
   б) оптимизировать

$$\frac{1}{T}\sum (y_t - \hat{y}_{t|t-1})^2 \to \min_{\alpha, l_0}$$

Holt's linear trend

y_{t+1} y_{t+2}

Forecast $y_{t+h|t} = l_t + h b_t$

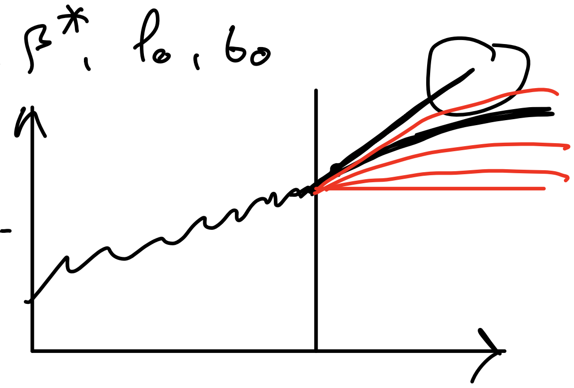
Smoothing $l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$

Trend $b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*) b_{t-1}$

$\alpha, \beta^*, l_0, b_0$

$0 < \phi < 1$

$$y_{t+h|t} = l_t + \phi b_t + \phi^2 b_t + \dots + \phi^h b_t = l_t + (\phi + \phi^2 + \dots + \phi^h) b_t$$



$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + \phi b_{t-1})$$

$$b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)\phi b_{t-1}$$

$$[0.8;\ 0.98]$$

$$y_{t+h|t} \xrightarrow[h\to\infty]{} l_t + \frac{\phi b_t}{1-\phi}$$

Seasonality

Holt-Winters

$$\begin{cases} y_{t+h|t} = l_t + h b_t + s_{t+h-m(k+1)} \\ l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1}) \\ b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*) b_{t-1} \\ s_t = \gamma(\underbrace{y_t - l_{t-1} - b_{t-1}}) + (1-\gamma) s_{t-m} \end{cases}$$

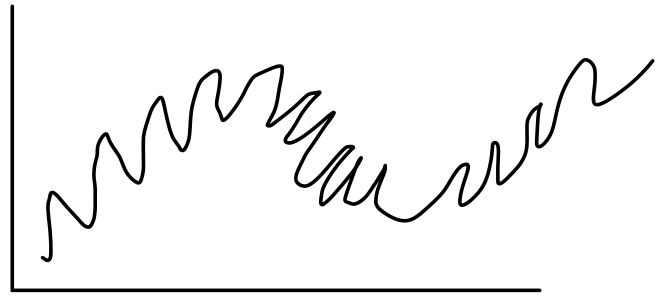
m – период сез.

$$k = \operatorname{int}\left(\frac{h-1}{m}\right)$$

$$s_t = \gamma^*(y_t - l_t) + (1-\gamma^*) s_{t-m}$$

$$\gamma = \gamma^*(1-\alpha)$$

$$\alpha, \beta^*, \gamma, l_0, b_0, s_0^1, s_0^2, \ldots, s_0^m$$



Относительные

$$\begin{cases} y_{t+h|t} = (l_t + h b_t)\, s_{t+h-m(k+1)} \\ l_t = \alpha \dfrac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1}) \\ b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)\, b_{t-1} \\ s_t = \gamma \dfrac{y_t}{l_{t-1} + b_{t-1}} + (1-\gamma)\, s_{t-m} \end{cases}$$

Начнём с начала

Simple exp smoothing ~ ETS(ANN)

$$\begin{cases} \hat{y}_{t+1|t} = l_t \\ l_t = \alpha y_t + (1-\alpha)\, l_{t-1} \end{cases}$$

$$l_t = l_{t-1} + \alpha(y_t - l_{t-1}) = l_{t-1} + \alpha(y_t - \hat{y}_{t|t-1})$$

$$= l_{t-1} + \alpha e_t$$

$$e_t = y_t - l_{t-1} = y_t - \hat{y}_{t|t-1}$$

$$y_t = \hat{y}_{t|t-1} + e_t$$

$$\sim N(0, \sigma^2)$$

$$\begin{cases} y_t = l_{t-1} + e_t & \text{observation (measi} \\ l_t = l_{t-1} + \alpha e_t & \text{state/transition} \end{cases}$$

$$y_{t+1|t} = E(y_{t+1} | t) = E(l_t + e_{t+1} | t) =$$

$$= l_t$$

$$\varepsilon_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}}$$

$$\begin{cases} y_t = l_{t-1}(1 + \varepsilon_t) \\ l_t = l_{t-1}(1 + \alpha \varepsilon_t) \end{cases}$$

ETS(AAN)

$$\begin{cases} y_t = l_{t-1} + b_{t-1} + \varepsilon_t \\ l_t = l_{t-1} + b_{t-1} + \alpha\varepsilon_t \\ b_t = b_{t-1} + \underbrace{\beta}_{= \alpha\beta^*} \varepsilon_t \end{cases}$$

$\alpha, \beta, \gamma, \phi \quad l_0, b_0, s_0^1 \ldots, s_0^m, \sigma^2$

Add $\quad \sum s_i = 0$

Mul $\quad \sum s_i = m$