Задачки 4

Упр. $f(y_2, \dots y_T \mid y_1)$

$y_t = y_{t-1} + \alpha + u_t$

$u_t \sim N(0; 1)$ независ.

$\Delta y_t = \alpha + u_t$

модель.

$\alpha \sim N(1, 4)$ априор.

априори

$y_1 = 3 \quad y_2 = 5 \quad y_3 = 6$

данные.

$(\Delta y_t \mid \alpha) \sim N(\alpha; 1)$

$\Delta y_2 = 2$

$\Delta y_3 = 1$

а) $\hat{\alpha}_{ML}$ (не байес)

б) $f(\alpha \mid \text{данные})$ (байес)

в) предикт 94% для y_4.

а) $\hat{\alpha}_{ML} = \frac{2+1}{2} = 1.5$

б) $f(\alpha \mid \Delta y_2, \Delta y_3) \propto f(\alpha, \Delta y_2, \Delta y_3) =$

$$= f(\Delta y_2, \Delta y_3 \mid \alpha) \cdot f(\alpha)$$

$$\ln f(\alpha \mid \Delta y_2, \Delta y_3) = \ln f(\Delta y_2 \mid \alpha) + \ln f(\Delta y_3 \mid \alpha) + \ln f(\alpha) + c =$$

$$\ln f(\Delta y_2 \mid \alpha) = \text{const} - \frac{1}{2}(\Delta y_2 - \alpha)^2$$

$$\ln f(\alpha) = \text{const} - \frac{1}{2 \cdot 4} \cdot \alpha^2$$

$$\ln f(\alpha \mid \text{data}) = \text{const} - \frac{1}{2}\left[\frac{\alpha^2}{4} + (\Delta y_2 - \alpha)^2 + (\Delta y_1 - \alpha)^2\right]$$

$(\alpha \mid \text{data}) \sim N(? ; ?)$

$$\ln f(\alpha | data) = const - \frac{1}{2}\left[\frac{\alpha^2}{4} + (\Delta y_2 - \alpha)^2 + (\Delta y_1 - \alpha)^2\right]$$

$$(\alpha | data) \sim N(? ; ?)$$

$$\frac{\partial \ln f}{\partial \alpha} = -\frac{1}{2}\left[\frac{\alpha}{2} + 2(\alpha - \Delta y_2) + 2(\alpha - \Delta y_1)\right]$$

Верш. параболы:

$$\alpha^* + 4(\alpha^* - \Delta y_2) + 4(\alpha^* - \Delta y_1) = 0.$$

$$\alpha^* = \frac{4\Delta y_2 + 4\Delta y_1}{9}$$

$E(\alpha | data)$ апостер.

$$\frac{\partial^2 \ln f}{\partial \alpha^2} = -\frac{1}{2}\left[\frac{1}{2} + 2 + 2\right] = -\frac{1}{2} \cdot \frac{9}{2}$$

$$Var(\alpha | data) \overset{?}{=} \frac{4}{9}$$

$$\boxed{\text{Check} \quad \ln f = \ldots - \frac{1}{2}\cdot\frac{x^2}{\sigma^2} \qquad \frac{\partial^2 \ln f}{\partial x^2} = -\frac{1}{\sigma^2}}$$

$$(\alpha | data) \sim N\left(\frac{4(y_3 - y_1)}{9} ; \frac{4}{9}\right)$$

$$(\alpha | data) \sim N\left(\frac{4}{3} ; \frac{4}{9}\right)$$

б)

$$y_4 = y_3 + \alpha + u_4$$

(94% PI)

$$\left[6 + \frac{4}{3} + 0 - 1.88 \cdot \sqrt{\frac{13}{9}} ; \ldots + \ldots\right]$$

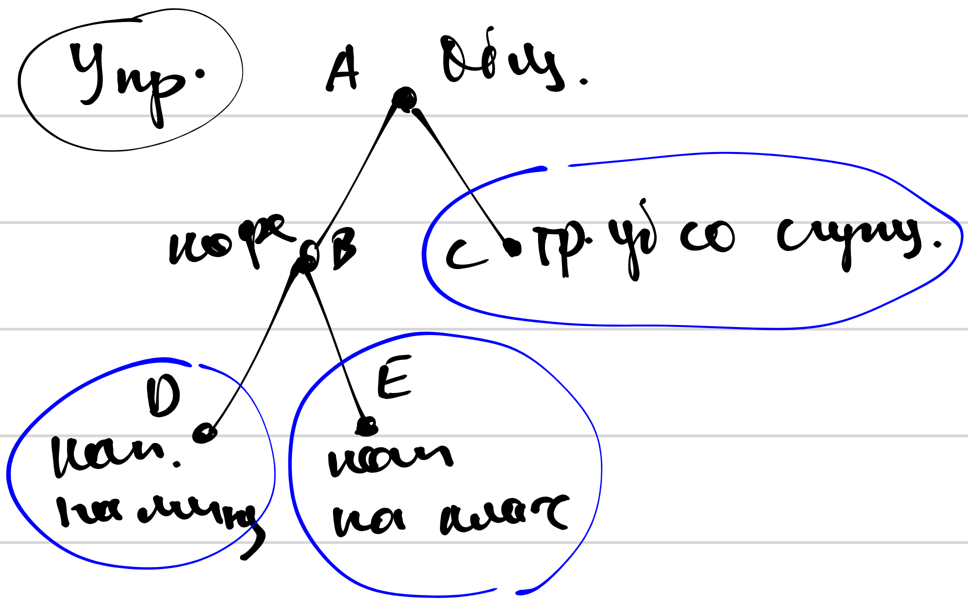
$$(y_4 | data) \sim N\left(y_3 + \frac{4}{3} + 0 ; \underbrace{0 + \frac{4}{9} + 1}_{13/9}\right)$$

data: $y_1, \Delta y_2, \Delta y_3$ ($\Delta y_2 \leftarrow u_2$, $\Delta y_3 \leftarrow u_3$)

$y_3 \quad \alpha \quad u_4 \qquad y_3 \quad \alpha \quad u_4$

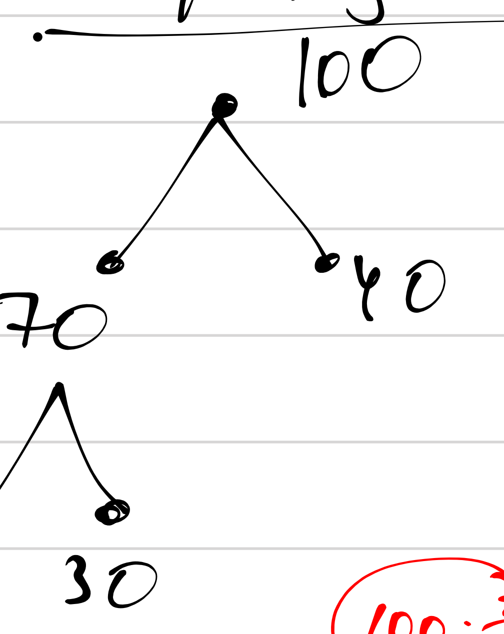
$$E(y_4 | \Delta y_2, \Delta y_3, y_1) = E(y_3 + \alpha + u_4 | \ldots) =$$

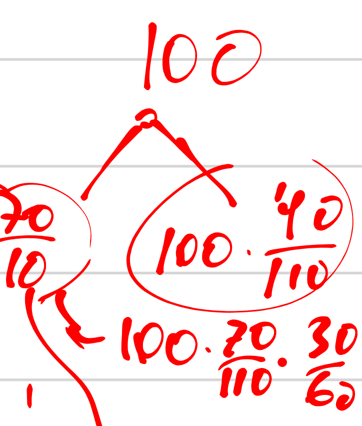
$$= E(y_3 | \ldots) + E(\alpha | \ldots) + E(u_4 | \ldots)$$



на 1 шаг вперед:

прогноз на 1 шаг





$$\begin{pmatrix} y_A \\ y_B \\ y_C \\ y_D \\ y_E \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_C \\ y_D \\ y_E \end{pmatrix}$$

а) [устно] согл. сверху-вниз.

б) оптим. соглас. мин. дисперсию ошибок прогнозов, если ков. м-ца ошибок прогнозов

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 1 & 0 & 0 & 0 & 0 \\ B & 0 & 1 & 0 & 0 & 0 \\ C & 0 & 0 & 1 & 0 & 0 \\ D & 0 & 0 & 0 & 4 & 0 \\ E & 0 & 0 & 0 & 0 & 4 \end{array} = W \qquad W^{-\frac{1}{2}}$$

оц-ся по прош. ими прогнозам.

$$\begin{pmatrix} \hat{y}_A \\ \hat{y}_B \\ \hat{y}_C \\ \hat{y}_D \\ \hat{y}_E \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \tilde{y}_C \\ \tilde{y}_D \\ \tilde{y}_E \end{pmatrix} + \begin{pmatrix} u_A \\ u_B \\ u_C \\ u_D \\ u_E \end{pmatrix}$$

завис. предик.

$$\begin{pmatrix} 100 \\ 70 \\ 40 \\ 30 \\ 30 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

привели к одн. дисперсии $W^{-\frac{1}{2}}$.

$$W^{-\frac{1}{2}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\text{"}y\text{"} = \begin{pmatrix} 100 \\ 70 \\ 40 \\ 15 \\ 15 \end{pmatrix} \qquad \text{"}X\text{"} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \rightarrow \hat{\beta} = (X^T X)^{-1} X^T y$$

$$X^T y = \begin{pmatrix} 140 \\ 177.5 \\ 177.5 \end{pmatrix}$$

Упр. тест DM,

$$d_t = |e_t^A| - |e_t^B|$$

алгоритм прогн А и алг. прогн Б.

$d_t \sim$ стац. процесс

. 100 набл. в выборке, сравн. алго.

$\alpha = 5\%$

$$H_0: E(d_t) = 0.$$
$$H_A: E(d_t) > 0$$

$d_t \sim MA(2)$ $E(d_t) = \alpha$ $\hat{E}(d_t) = \hat{\alpha}$

$d_t = \alpha + u_t + \beta_1 u_{t-1} + \beta_2 u_{t-2}$

$\hat{\alpha} = 3$ $\hat{\beta}_1 = 2$ $\hat{\beta}_2 = 1$

$\hat{\sigma}_u^2 = 4$

$$DM = \frac{\bar{d} - 0}{se(\bar{d})} = \frac{3-0}{\left(\frac{8}{10}\right)} = \frac{30}{8} \approx 4$$

$$se^2(\bar{d}) = \hat{Var}\left(\frac{d_1 + d_2 + \ldots d_{100}}{100}\right) =$$

$$= \frac{1}{100^2} \cdot \left(100 \cdot \hat{Var}(d_t) + 2 \cdot 99 \cdot \hat{Cov}(d_1, d_2) + 2 \cdot 98 \cdot \hat{Cov}(d_1, d_3)\right) \approx$$

$$\approx \frac{1}{100}(24 + 32 + 8) = \frac{64}{100}$$

$$\hat{Var}(d_t) = 4 + 2^2 \cdot 4 + 1^2 \cdot 4 = 6 \cdot 4$$

$$\hat{Cov}(d_1, d_2) = \hat{Cov}(\alpha + u_1 + \beta_1 u_0 + \beta_2 u_{-1}, \alpha + u_2 + \beta_1 u_1 + \beta_2 u_0)$$

$$= \hat{\beta}_1 \cdot \hat{\sigma}_u^2 + \hat{\beta}_1 \hat{\beta}_2 \cdot \hat{\sigma}_u^2 = 2 \cdot 4 + 2 \cdot 4 = 16$$

$$\hat{Cov}(d_1, d_3) = \hat{\beta}_2 \cdot \hat{\sigma}^2 = 4.$$