AR(2) - процесс: $y_t = \sum \varepsilon_t$

$y_t = \cancel{5} + 0{,}3\, y_{t-1} + \varepsilon_t$ — стационарный

PACF обнуляется после лага 2

ACF:

ε_{t-1}

$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$

$E(y_t) = \mu$

ε_t - iid

$E(\varepsilon_t) = 0 \quad Var(\varepsilon_t) = \sigma_\varepsilon^2$

$E(y_t) = \beta_0 + \beta_1 E(y_{t-1})$

$\mu = \beta_0 + \beta_1 \mu \Rightarrow \beta_0 = \mu - \beta_1 \mu$

$y_t = \mu - \beta_1 \mu + \beta_1 y_{t-1} + \varepsilon_t$

$y_t - \mu = \beta_1 (y_{t-1} - \mu) + \varepsilon_t \;|$

$$ACF(k) = \frac{E((y_t - \mu)(y_{t-k} - \mu))}{\underbrace{\sqrt{Var(y_t)}}_{\sqrt{\gamma_0}} \underbrace{\sqrt{Var(y_{t-k})}}_{\sqrt{\gamma_0}}} = \frac{\gamma_k}{\gamma_0}$$

$k = 1$

$y_t - \mu = \beta_1 (y_{t-1} - \mu) + \varepsilon_t \;| \cdot (y_{t-1} - \mu)$

$(y_t - \mu)(y_{t-1} - \mu) = \beta_1 \underbrace{(y_{t-1} - \mu)^2}_{Var(y_{t-1})} + \varepsilon_t (y_{t-1} - \mu) \;| E$

$$\gamma_1 = \beta_1 \gamma_0 + 0$$

$$E(\varepsilon_t (y_{t-1} - \mu)) = E(\overset{0}{\overset{\shortparallel}{\varepsilon_t y_{t-1}}} - \overset{0}{\overset{\shortparallel}{\varepsilon_t \mu}})$$

$=$

$$\gamma_1 = \beta_1 \gamma_0 \quad | \, / \gamma_0 \qquad \gamma_0 = Var(y_t) =$$

$$ACF(1) = \rho_1 = \beta_1$$

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t \quad | \, Var$$

$$\gamma_0 = \beta_1^2 \gamma_0 + \sigma_\varepsilon^2$$

$$\gamma_0 = \frac{\sigma_\varepsilon^2}{1 - \beta_1^2}$$

$K = 2$

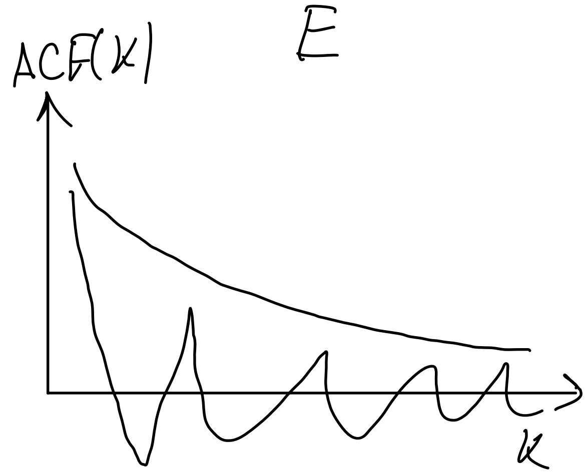
$$y_t - \mu = \beta_1 (y_{t-1} - \mu) + \varepsilon_t \quad | \cdot (y_{t-2} - \mu)$$

E

$$\gamma_2 = \beta_1 \gamma_1 \quad | \, / \gamma_0$$

$$\rho_2 = \beta_1 \rho_1 = \beta_1^2$$

$$\rho_K = \beta_1^K$$



PACF

$K = 1 \qquad y_t = \alpha_0 + \alpha_1 y_{t-1} + v_t$

Yule-Walker $\qquad y_t = \alpha_0 + \cancel{\alpha_1 y_{t-1}} + \alpha_2 y_{t-2}$

$$\begin{cases} E(\vartheta_t) = 0 \quad X \\ cov(\vartheta_t, y_{t-1}) = 0 \end{cases}$$

$$cov(y_t - \alpha_0 - \alpha_1 y_{t-1}, y_{t-1}) = 0$$

$$\gamma_1 - \alpha_1 \gamma_0 = 0 \quad | : \gamma_0$$

$$\rho_1 - \alpha_1 = 0$$

$$\alpha_1 = \rho_1$$

$K = 2 \qquad corr(y_t, y_{t-2} | y_{t-1})$

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \vartheta_t$$

$$\begin{cases} E(\vartheta_t) = 0 \quad X \\ cov(\vartheta_t, y_{t-1}) = 0 \\ cov(\vartheta_t, y_{t-2}) = 0 \end{cases}$$

$$\begin{cases} cov(y_t - \alpha_0 - \alpha_1 y_{t-1} - \alpha_2 y_{t-2}, y_{t-1}) = 0 \\ cov(y_t - \alpha_0 - \alpha_1 y_{t-1} - \alpha_2 y_{t-2}, y_{t-2}) = 0 \end{cases}$$

$$\begin{cases} \gamma_1 - \alpha_1 \gamma_0 - \alpha_2 \gamma_1 = 0 \\ \gamma_2 - \alpha_1 \gamma_1 - \alpha_2 \gamma_0 = 0 \end{cases} \quad | : \gamma_0$$

$$\begin{cases} \rho_1 - \alpha_1 - \alpha_2 \rho_1 = 0 \quad \Rightarrow \alpha_1 = \rho_1 (1 - \alpha_2) \\ \rho_2 - \alpha_1 \rho_1 - \alpha_2 = 0 \end{cases}$$

$$\rho_2 - \rho_1(1-\alpha_2)\rho_1 - \alpha_2 = 0$$

$$\underbrace{\rho_2}_{\beta_1^2} - \underbrace{\rho_1^2}_{\beta_1^2} + \rho_1^2\alpha_2 - \alpha_2 = 0$$

$$\rho_K = \beta_1^K$$

$$\alpha_2(\rho_1^2 - 1) = 0$$

$$\beta_1^2 - 1$$

$$\alpha_2 = 0$$

$$PACF(K) = \begin{cases} \beta_1, & K = 1 \\ 0, & K > 1 \end{cases}$$

3.14.

$$y_t = 10 + \underline{0{,}69}\,y_{t-1} + \varepsilon_t - \underline{0{,}71}\,\varepsilon_{t-1}$$

$$(1 - 0{,}69L)\,y_t = 10 + (1 - 0{,}71L)\,\varepsilon_t$$

$$(1 - 0{,}7L)$$

$$A(L)\,y_t = B(L)\,\varepsilon_t$$

$$y_t = (1 - 0{,}7L)^{-1}\,10 + \varepsilon_t$$

$$y_t = \frac{10}{0{,}3} + \varepsilon_t \qquad \frac{10}{1 - 0{,}7\cancel{L}}$$

$$10\,(1 - 0{,}7L)^{-1}$$

$$= (1 + 0{,}7L + 0{,}7^2L^2 + \ldots)\,10$$

$MA(q)$

$N 3.17.$

$y_t = 7 + u_t + 0{,}2\, u_{t-1} \quad u_t \sim N(0, 4)$

$y_{100} = 7.2 \quad u_{100} = 1.3$

$\sum_{i=0}^{\infty} y_{100-i} (0{,}2)^i = 5.6$

H_t

$E(y_{101} | F_{100}) =$

$= E(7 + u_{101} + 0{,}2\, u_{100} | F_{100}) =$

$= 7 + 0{,}2 \cdot 1.3$

$E(y_{102} | F_{100}) = E(7 + u_{102} + 0{,}2\, u_{101} | F_{100}) =$

$= 7$

$Var(y_{101} | F_{100}) = Var(7 + u_{101} + 0{,}2\, u_{100} | F_{100}) =$

$= 4$

$Var(y_{102} | F_{100}) = 4 + 0{,}04 \cdot 4 = 4{,}16$

$AR(\infty)$

$u_{103} + 0{,}2\, u_{102} \,|\, F_{100}$

$4 \quad 0{,}04 \quad 4$

$AR(p) \to MA(\infty)$

$A(L) y_t = \varepsilon_t$

$Var(y_t) = 4{,}16$

$$y_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \dots$$

$$A(L)\, y_t = B(L)\, \varepsilon_t$$

$$y_t = \frac{B(L)}{A(L)} \varepsilon_t \quad MA(\infty)$$

$$\varepsilon_t = \frac{A(L)}{B(L)} y_t$$

$$y_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2} + \dots$$

$$E(y_{t+1} \mid H_t)$$