MA(2) – модель

$\mu = 0$

$y_t = \mu + \varepsilon_t + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2}$, $\varepsilon_t \sim iid$

$E(\varepsilon_t) = 0$

$Var(\varepsilon_t) = \sigma_\varepsilon^2$

$\boxed{ACF(L)}$ $PACF(L)$

$E(y_t) = \overset{0}{\mu} + \overset{0}{E(\varepsilon_t)} + \phi_1 \overset{0}{E(\varepsilon_{t-1})} + \phi_2 \overset{0}{E(\varepsilon_{t-2})} = \mu$

$y_t = \varepsilon_t + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} \,|\, y_{t-1}$

$y_t y_{t-1} = \varepsilon_t y_{t-1} + \phi_1 \varepsilon_{t-1} y_{t-1} + \phi_2 \varepsilon_{t-2} y_{t-1} \,/E$

$\gamma_1 = 0 + \phi_1^2 \sigma_\varepsilon^2 + \phi_2^2 \sigma_\varepsilon^2$

$\varepsilon_{t-1} (\varepsilon_{t-1} + \phi_1 \varepsilon_{t-2} + \phi_2 \varepsilon_{t-3})$

$\rho_1 = \frac{\gamma_1}{\gamma_0} = \phi_1^2 + \phi_2^2$

$corr(y_t, y_{t-k}) = \frac{cov(y_t, y_{t-k})}{\sqrt{Var(y_t) Var(y_{t-k})}}$

$= \frac{cov(y_t, y_{t-k})}{Var(y_t)} = \frac{\gamma_k}{\gamma_0}$

$y_t y_{t-2} = \varepsilon_t y_{t-2} + \phi_1 \varepsilon_{t-1} y_{t-2} + \phi_2 \varepsilon_{t-2} y_{t-2}$

$\gamma_2 = 0 \quad 0 + \phi_2 \sigma_\varepsilon^2$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \phi_2$$

$$\rho_k = \begin{cases} \phi_1^2 + \phi_2^2 , & k = 1 \\ \phi_2 , & k = 2 \\ 0 , & k \geq 3 \end{cases}$$

PACF

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \nu_t \qquad PACF(1)$$

$$\begin{cases} E(\nu_t) = 0 \quad \times \\ cov(\nu_t, y_{t-1}) = 0 \end{cases}$$

$$cov(\underbrace{y_t - \cancel{\alpha_0} - \alpha_1 y_{t-1}, y_{t-1}}) = 0$$

$$\gamma_1 - \alpha_1 \gamma_0 = 0$$

$$\alpha_1 = \frac{\gamma_1}{\gamma_0} = \rho_1$$

PACF(2)

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \nu_t$$

$$\begin{cases} E(\nu_t) = 0 \\ cov(\nu_t, y_{t-1}) = 0 \\ cov(\nu_t, y_{t-2}) = 0 \end{cases}$$

$$\begin{cases} cov(y_t - \alpha_0 - \alpha_1 y_{t-1} - \alpha_2 y_{t-2}, y_{t-1}) = 0 \\ cov(y_t - \alpha_0 - \alpha_1 y_{t-1} - \alpha_2 y_{t-2}, y_{t-2}) = 0 \end{cases}$$

$$\begin{cases} \gamma_1 - \alpha_1 \gamma_0 - \alpha_2 \gamma_1 = 0 \\ \gamma_2 - \alpha_1 \gamma_1 - \alpha_2 \gamma_0 = 0 \end{cases} \Big| : \gamma_0$$

$$\begin{cases} \rho_1 - \alpha_1 - \alpha_2 \rho_1 = 0 \\ \rho_2 - \alpha_1 \rho_1 - \alpha_2 = 0 \end{cases}$$

$$\alpha_1 = (1-\alpha_2)\rho_1$$

$$\rho_2 - (1-\alpha_2)\rho_1^2 - \alpha_2 = 0$$

$$\rho_2 - \rho_1^2 + \alpha_2 \rho_1^2 - \alpha_2 = 0$$

$$\alpha_2 = \frac{\rho_1^2 - \rho_2}{\rho_1^2 - 1}$$