1) Закончить стратегии
2) Правдоподобие ETS
3) Прогнозы.

1) Recursive

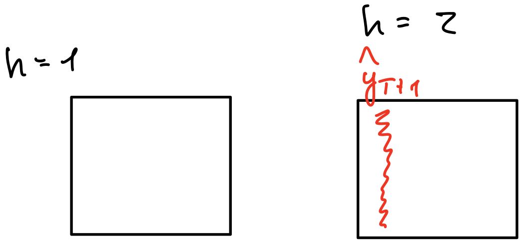
$$\hat{y}_{T+1} = \hat{f}(y_T, \dots, y_{T-k}, x_T)$$
$$\hat{y}_{T+2} = \hat{f}(\hat{y}_{T+1}, y_T, \dots, y_{T-k+1}, \hat{x}_{T+1})$$

2) Direct

h бустингов - неконкур.

$$\hat{y}_{T+h} = f_h(y_T, \dots, y_{T-k}, x_T)$$

+ MLP поверх



3) DirRec

$$\hat{y}_{T+1} = f_1(y_T, \dots, y_{T-k}, x_T)$$
$$\hat{y}_{T+2} \quad f_2(\hat{y}_{T+1}, y_T, \dots, y_{T-k+1}, \hat{x}_{T+1})$$
$$\vdots$$

4) MIMO - Multi-input, multi-output

$$\hat{y}_{T+1}, \hat{y}_{T+2}, \dots, \hat{y}_{T+h} = f(y_T, \dots, y_{T-k}, x_T)$$

1) MLP

2) RNN, Transformer

5) Dirmo

6) Композитное (Rectify)

$ETS(AA^dA)$

Error Trend Season

$$\begin{cases} y_t = l_{t-1} + b_{t-1} + S_{t-12} + u_t \\ l_t = l_{t-1} + b_{t-1} + \alpha u_t \\ b_t = b_{t-1} + \beta u_t \\ S_t = S_{t-12} + \gamma u_t \end{cases} \qquad u_t \sim N(0, \sigma^2)$$

X – iid

$$f(Y) = f(y_T, y_{T-1}, y_{T-2}, \dots, y_1) \neq \prod_{i=1}^{n} f(y_i)$$

$$f(Y) = f(y_T | y_{T-1}, \dots, y_1) \cdot f(y_{T-1} | y_{T-2}, \dots, y_1) \cdot \dots \cdot f(y_2 | y_1) \cdot f(y_1)$$

$$y_1 = l_0 + b_0 + S_{-11} + u_1 \sim N(0, \sigma^2)$$

$$y \sim N(l_0 + b_0 + S_{t-11}, \sigma^2)$$

$$f(y_1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-(l_0+b_0+S_{-11}))^2}{2\sigma^2}}$$

$y_2 | y_1 \sim ?$

$$y_2 = l_1 + b_1 + \underbrace{S_{-10}} + u_2 = l_0 + b_0 + \alpha \hat{u}_1 + b_0 + \beta \hat{u}_1 + S_{-10} + u_2$$

$y_2 | y_1 \Rightarrow$ знаем $l_1, b_1, S_1, \hat{u}_1 = y_1 - l_0 - b_0 - S_{-11}$

$$E(y_2 | y_1) = l_0 + 2b_0 + (\alpha + \beta)\hat{u}_1 + S_{-10}$$

$$E(u_2 | y_1) = 0$$

$$Var(y_2 | y_1) = \sigma^2$$

$$f_{y_2|y_1} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x - (l_0 + 2b_0 + (\alpha+\beta)\hat{u}_1 + S_{-10}))^2}{2\sigma^2}}$$

Прогнозы

1) Точечные
2) Интервальные
3) Прогнозы дисперсии

$(y_{T+1} | \mathcal{F}_T) \sim ?$

$$y_{T+1} = l_T + b_T + S_{T-12} + u_{T+1} \sim N(l_T + b_T + S_{T-12}, \sigma^2)$$

$$E(y_{T+1} | \mathcal{F}_T) = l_T + b_T + S_{T-12}$$

$$Var(y_{T+1} | \mathcal{F}_T) = \sigma^2$$

$$E(y_{T+2} | \mathcal{F}_T) = E(l_{T+1} + b_{T+1} + S_{T-11} + u_{T+2} | \mathcal{F}_T)$$

$= l_0 + 2b_0 + s_{-10}$

$\ln(y_t) \sim ETS(AAN)$

$P(\ln(y_{T+1}) \in [10; 20]) = 0{,}95$

$E\, y_{T+1} = ?$

$E(\ln(y_{T+1})) = \mu \quad Var(\ln(y_{T+1})) = \sigma^2$

$\mu + 1{,}96\sigma = 20$

$\mu - 1{,}96\sigma = 10$

$\mu = 15$

$1{,}96\sigma = 5 \quad \sigma^2 = \left(\frac{5}{1{,}96}\right)^2$

$Med(y_{T+1} | \mathcal{F}_T) = e^{\mu}$

$E(y_{T+1} | \mathcal{F}_T) = e^{\mu} e^{\sigma^2/2} \approx 84$ млн