$$y_t = \mu + \Phi_1 y_{t-1} + \dots + \Phi_p y_{t-p} + \vartheta_t$$

$$\vartheta_t \sim (0, \Sigma)$$

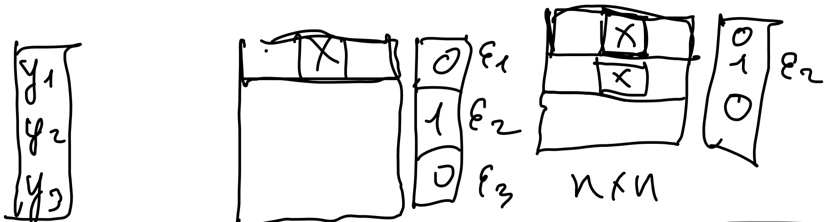
1) $B_0 y_t = \lambda + B_1 y_{t-1} + \dots + B_p y_{t-p} + \varepsilon_t \sim (0, I)$

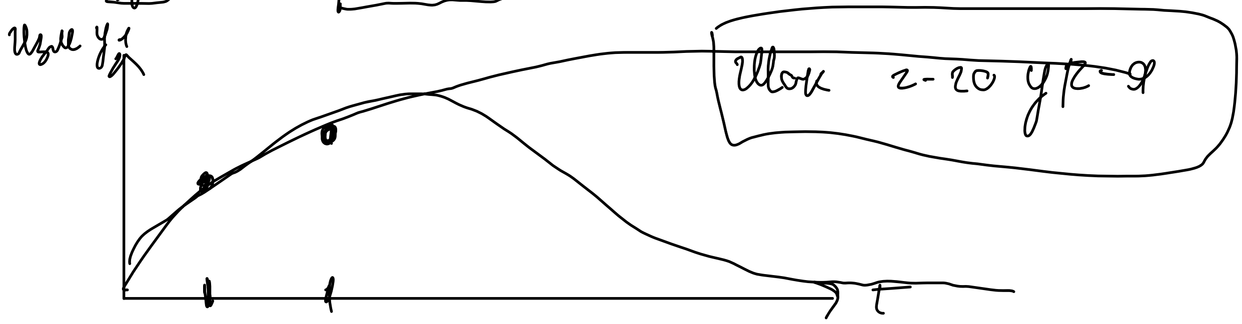
2) $y_t = \underbrace{B_0^{-1}\lambda}_{\mu} + \underbrace{B_0^{-1}B_1}_{\Phi_1} y_{t-1} + \dots + B_0^{-1}B_p y_{t-p} + \underbrace{B_0^{-1}\varepsilon_t}_{\vartheta_t \sim (0, \Omega)}$

3) VAR(p) $\to$ VMA(∞)

$$y_t = \tilde{\mu} + C_0 \vartheta_t + C_1 \vartheta_{t-1} + \dots$$

$$y_t = \tilde{\mu} + \underbrace{C_0 B_0^{-1}} \varepsilon_t + \underbrace{C_1 B_0^{-1}} \varepsilon_{t-1} + C_2 B_0^{-1} \varepsilon_{t-2} + \dots$$





~~t~~ 2

$$y_t = \tilde{\mu} + C_0 B_0^{-1} \varepsilon_t + C_1 B_0^{-1} \varepsilon_{t-1} + C_2 B_0^{-1} \varepsilon_{t-2} +$$

$$y_t = \tilde{\mu} + \Psi_0 \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \Psi_2 \varepsilon_{t-2} + \dots$$

$$\Psi = \Psi_0 + \Psi_1 + \Psi_2 + \dots$$

log of output : x_t

rate r_t

log prices p_t

log money m_t

$$y_t = [\ln x_t, r_t, \ln p_t, \ln m_t]^T$$

$$\ln\left(\frac{p_t}{p_{t-1}}\right) = b_1(\ln x_t - \varepsilon_{as,t}) \quad AS$$

$$\ln x_t = -b_2\left(r_t - \ln\left(\frac{p_t}{p_{t-1}}\right) - \varepsilon_{is,t}\right) \quad IS$$

$$\ln m_t - \ln p_t = b_3 \ln x_t - b_4 r_t - \varepsilon_{md,t}$$ Money demand

$$\ln m_t = \varepsilon_{ms,t}$$ Money supply

$$B_0 \begin{pmatrix} 1 & 0 & -b_1^{-1} & 0 \\ b_2^{-1} & 1 & -1 & 0 \\ b_3 & -b_4 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad B_1 \begin{pmatrix} 0 & 0 & -b_1^{-1} & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\nu_t = B_0^{-1} \varepsilon_t$$

$$E(\nu_t \nu_t^T) = B_0^{-1} E(\varepsilon_t \varepsilon_t^T)(B_0^{-1})^T$$

$$\Omega = B_0^{-1} \underbrace{\Sigma}_{= I} (B_0^{-1})^T$$

$$A y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + \lambda + \boxed{B} \varepsilon_t$$

Short-Run restrictions

$$y_t = [q_t, r_t, m_t]^T$$

$$\nu_t^q = -a_{12} \nu_t^r + b_{11} \varepsilon_t^{IS} \qquad IS$$

$$\nu_t^r = -a_{21} \nu_t^q - a_{23} \nu_t^m + b_{22} \varepsilon_t^{LM} \qquad LM\text{-}inv$$

$$\nu_t^m = b_{33} \varepsilon_t^m$$

$$\begin{pmatrix} 1 & a_{12} & 0 \\ a_{21} & 1 & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \nu_t = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix} \varepsilon_t$$

$$A y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + \lambda + B \varepsilon_t$$

$$y_t = A^{-1} A_1 y_{t-1} + \dots A^{-1} A_p y_{t-p} + A^{-1} \lambda + \underbrace{A^{-1} B \varepsilon_t}_{\nu_t'}$$

$$\vartheta_t = A^{-1} B \varepsilon_t$$

$$A \vartheta_t = B \varepsilon_t$$

Long-run restrictions

$$\Psi = \Psi_0 + \Psi_1 + \Psi_2 + \dots =$$

$$= \underbrace{(I - \Phi_1 - \dots - \Phi_p)^{-1}}_{F} A^{-1} B = F A^{-1} B$$

$$\vartheta_t = A^{-1} B \varepsilon_t$$

$$\Omega = A^{-1} B B^T A^{T^{-1}}$$

$$\Psi \Psi^T = F \Omega F^T$$

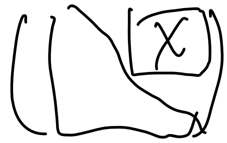
$$\hat{\Psi} - \hat{F} \operatorname{chol}(\Omega) \qquad \hat{A}^{-1} \hat{B} = \hat{F}^{-1} \hat{\Psi}$$

Blanchard, Quah (1989)

ΔQ – growth rat

u_t – unemployment

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \Delta Q_t \\ u_t \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} + A_1 \begin{pmatrix} \Delta Q_{t-1} \\ u_{t-1} \end{pmatrix} + \dots A_p \begin{pmatrix} \Delta Q_{t-p} \\ u_{t-p} \end{pmatrix} +$$



$$+ \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix}$$

$$\Psi(1,2) = 0$$

Sign Restrictions

(1) $B_0 y_t = \lambda + B_1 y_{t-1} + \dots + B_p y_{t-p} + \varepsilon_t \quad \varepsilon_t \sim iid\, N(0, \Sigma)$

(2) $y_t = \mu + \Phi_1 y_{t-1} + \dots + \Phi_p y_{t-p} + \vartheta_t \quad \vartheta_t \sim iid(0, \Omega)$

B_0 - lower-triangular

Σ - identity

$$\vartheta_t = B_0^{-1} \varepsilon_t$$

$$\Omega = B_0^{-1} (B_0^{-1})^T$$

Alternative

$$\tilde{B}_0 y_t = \tilde{\lambda} + \tilde{B}_1 y_{t-1} + \dots + \tilde{B}_p y_{t-p} + \omega_t$$

$$\omega_t \sim iid\, \underline{N(0, I)}$$

$\tilde{B}_i = Q B_i, \quad \tilde{\lambda} = Q\lambda$

Q - orthogonal matrix $\quad Q^T = Q^{-1} \quad QQ^T = Q^T Q = I$

$\cdot Q^T | \quad \underline{Q B_0 y_t = Q\lambda + Q B_1 y_{t-1} + \dots + Q B_p y_{t-p} + \omega_t}$

(1) $B_0 y_t = \lambda + B_1 y_{t-1} + \dots + B_p y_{t-p} + \underbrace{Q^T \omega_t}_{\varepsilon_t}$

(2). $y_t = \mu + \Phi_1 y_{t-1} + \dots + \Phi_p y_{t-p} + u_t$

$$\mu = B_0^{-1}\lambda \quad \Phi_i = B_0^{-1} B_i \quad u_t = (QB_0)^{-1}\omega_t$$

$$E(u_t u_t^T) = (QB_0)^{-1} \underbrace{E(\omega_t \omega_t^T)}_{I} \left((QB_0)^{-1}\right)^T =$$

$$= B_0^{-1} \underbrace{Q^T Q}_{I} \left(B_0^{-1}\right)^T = \left(B_0^{-1}\right)\left(B_0^{-1}\right)^T = \Omega$$

$$\varepsilon_t = Q^T \omega_t$$

$$Q = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} \qquad 0 \le \gamma \le \pi$$

$$B_0 y_t$$

$$\underbrace{\begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} b_{011} & 0 \\ b_{021} & b_{022} \end{pmatrix}}_{QB_0} =$$

$$= \begin{pmatrix} b_{011}\cos\gamma - b_{021}\sin\gamma & -b_{022}\sin\gamma \\ b_{011}\sin\gamma + b_{021}\cos\gamma & b_{022}\cos\gamma \end{pmatrix}$$

$\gamma = 0 \Rightarrow B_0$ is lower triangular

$\gamma = \frac{\pi}{2} \Rightarrow$?

$\gamma \in (0; \pi)$ – ulead

1) Оценим VAR(P), вычислить всё для VMA, B_0^{-1} s.t. $B_0^{-1}(B_0^{-1})^T = \Omega$

2) Сэмплируем γ, вычисляем Q, $\tilde{B}_0$, $\tilde{\Psi}_i$;

3) Вычисляем IRF (для квартальных на 4 периода)

$$y_t = \tilde{\mu} + \tilde{\Psi}_1 \varepsilon_t +$$

4) Если у IRF верные знаки, то это наша модель

Иначе — повторить 2–4