№3.14

y_t - стац. процесс

$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$, ε_t - WN

$Cov(\varepsilon_t, y_{t-k}) = 0 \ \forall k \geq 1$

1) $ACF(k)$

2) $\lim\limits_{k \to \infty} ACF(k)$

3) φ_{kk}

$$E(y_t) = \beta_0 + \beta_1 E(y_{t-1}) + 0$$

$$\mu = \beta_0 + \beta_1 \mu \qquad \mu = \frac{\beta_0}{1-\beta_1}$$

$$\beta_0 = \mu - \beta_1 \mu$$

$$y_t = \mu - \beta_1 \mu + \beta_1 y_{t-1} + \varepsilon_t$$

$$(y_t - \mu) = \beta_1 (y_{t-1} - \mu) + \varepsilon_t$$

$ACF(1)$

$$(y_t - \mu) = \beta_1 (y_{t-1} - \mu) + \varepsilon_t \ \Big| \cdot (y_{t-1} - \mu)$$

$$(y_t - \mu)(y_{t-1} - \mu) = \beta_1 (y_{t-1} - \mu)(y_{t-1} - \mu) + \varepsilon_t (y_{t-1} - \mu) \ \Big| \ E$$

$$\gamma_1 = \beta_1 \gamma_0 + \qquad \rho_k = \frac{\gamma_k}{\gamma_0}$$

$$\rho_1 = \frac{\gamma_1}{\gamma_0} \qquad \frac{\beta_1 \gamma_0}{\gamma_0} = \beta_1$$

ACF(2)

$$(y_t - \mu) = \beta_1 (y_{t-1} - \mu) + \varepsilon_t \quad \Big| \cdot (y_{t-2} - \mu)$$

$$(y_t - \mu)(y_{t-2} - \mu) = \beta_1 (y_{t-1} - \mu)(y_{t-2} - \mu) + \varepsilon_t (y_{t-2} - \mu) \ \Big/ E$$

$$\gamma_2 = \beta_1 \gamma_1$$

$$\rho_2 = \beta_1 \frac{\gamma_1}{\gamma_0} = \beta_1 \rho_1 = \beta_1^2$$

$$\rho_K = \beta_1^K$$

$$\lim_{K \to \infty} \rho_K = 0$$

PACF

φ_{11} $$y_t = \alpha_0 + \alpha_1 y_{t-1} + \vartheta_t$$

$$\operatorname{cov}(\vartheta_t, y_{t-1}) = 0$$

$$\operatorname{cov}(y_t - \alpha_0 - \alpha_1 \underbrace{y_{t-1}, y_{t-1}}) = 0$$

$$\gamma_1 - \alpha_1 \gamma_0 = 0$$

$$\alpha_1 = \frac{\gamma_1}{\gamma_0} = \rho_1$$

φ_{22}: $$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \vartheta_t$$

$$\begin{cases} cov(\vartheta_t, y_{t-1}) = 0 \\ cov(\vartheta_t, y_{t-2}) = 0 \end{cases}$$

$$\begin{cases} cov(y_t - \alpha_0 - \alpha_1 y_{t-1} - \alpha_2 y_{t-2}, y_{t-1}) = 0 \\ cov(y_t - \alpha_0 - \alpha_1 y_{t-1} - \alpha_2 y_{t-2}, y_{t-2}) = 0 \end{cases}$$

$$\begin{cases} \gamma_1 - \alpha_1 \gamma_0 - \alpha_2 \gamma_1 = 0 & | : \gamma_0 \\ \gamma_2 - \alpha_1 \gamma_1 - \alpha_2 \gamma_0 = 0 & | : \gamma_0 \end{cases}$$

$$\begin{cases} \rho_1 - \alpha_1 - \alpha_2 \rho_1 = 0 \\ \rho_2 - \alpha_1 \rho_1 - \alpha_2 = 0 \end{cases} \quad \alpha_1 = \rho_1 - \alpha_2 \rho_1$$

$$\rho_2 - (\rho_1 - \alpha_2 \rho_1)\rho_1 - \alpha_2 = 0$$

$$\rho_2 - \rho_1^2 + \alpha_2 \rho_1^2 - \alpha_2 = 0$$

$$\alpha_2 = \frac{\rho_1^2 - \rho_2}{\rho_1^2 - 1} = 0$$

№ 3.15

y_t - стац. процесс

$$y_t = 10 + \boxed{0{,}69}\, y_{t-1} + \varepsilon_t - \boxed{0{,}71}\, \varepsilon_{t-1}$$

Выпишите более простое ур-е, которое могло бы породить похожий процесс

$$(1-0{,}69L)y_t = 10 + (1-0{,}71L)\varepsilon_t$$

$$0{,}69 \approx 0{,}71$$

$$y_t = \underbrace{\frac{10}{1-0{,}69L}}_{\shortparallel \; 10} + \frac{1-0{,}71L}{1-0{,}69L}\varepsilon_t$$

$$(1-0{,}71L)(1+0{,}69L+(0{,}69L)^2+\ldots)\varepsilon_t$$

$$y_t = 10 + \varepsilon_t \qquad 1 \underbrace{-0{,}71L}\underbrace{+0{,}69L} - 0{,}69\cdot 0{,}71L^2 + 0{,}69^2L^2$$

№3.17

y_t – стац.

$$y_t = 2 + 0{,}6\, y_{t-1} - 0{,}08\, y_{t-2} + \varepsilon_t \;, \quad \varepsilon_t \underset{iid}{\sim} N(0,4)$$

1) $E_t(y_{t+1}) = \boxed{E_t}(2 + 0{,}6\, y_t - 0{,}08\, y_{t-1} + \varepsilon_{t+1}) =$

$= 2 + 0{,}6\, y_t - 0{,}08\, y_{t-1}$

$Var_t(y_{t+1}) = Var_t(\cancel{2} + \cancel{0{,}6\, y_t} - \cancel{0{,}08\, y_{t-1}} + \underline{\varepsilon_{t+1}}) = 4$

2) $E_t(y_{t+2}) = E_t(2 + 0{,}6\, y_{t+1} - 0{,}08\, \underline{y_t} + \varepsilon_{t+2}) =$

$= 2 + 0{,}6\, E_t(y_{t+1}) - 0{,}08\, y_t =$

$2 + 0{,}6(2 + 0{,}6\, y_t - 0{,}08\, y_{t-1}) - 0{,}08\, y_t =$

$= (1+0{,}6)\,2 + 0{,}6^2 y_t + 0{,}6\cdot(-0{,}08)\,y_{t-1} - 0{,}08\,y_t$

3) $Var_t(y_{t+2}) = Var(\cancel{2}+0{,}6\,y_{t+1} - \cancel{0{,}08 y_t} + \varepsilon_{t+2}) =$

$= \boxed{0{,}6^2}\,Var_t(y_{t+1}) + 4 = 0{,}06^2\cdot 4 + 4$

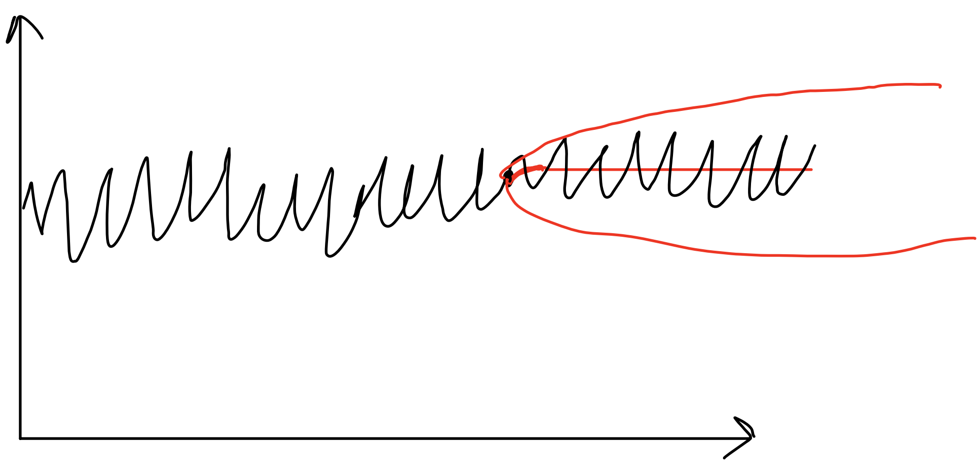
$0{,}36\cdot 4 + 4 = 5{,}44$

3) CI, 95% für y_{102} $\quad y_{99}=5,\ y_{100}=5{,}1$

$3{,}2 + 0{,}6^2\cdot 5{,}1 + 0{,}6\cdot(-0{,}08)\cdot 5 - 0{,}08\cdot 5{,}1 =$

$= \hat{y}_{t+2}$

$[\hat{y}_{t+2} - 1{,}96\sqrt{5{,}44}\ ;\ \hat{y}_{t+2} + 1{,}96\sqrt{5{,}44}]$



$E(y_t)$, $Var(y_t)$

$y_t = 2 + 0{,}6\,y_{t-1} - 0{,}08\,y_{t-2} + \varepsilon_t$

$E(y_t) = \mu \quad \forall t$ $\qquad 2 = \mu - 0{,}6\mu + 0{,}08\mu$

$\mu = 2 + 0{,}6\mu - 0{,}08\mu$

$\mu = \frac{2}{0{,}48} = 4\frac{1}{6}$

$$\mu(1-0{,}6+0{,}08)=2$$

$$y_t = 2+0{,}6\,y_{t-1} - 0{,}08\,y_{t-2} + \varepsilon_t \qquad Var(y_t) = \sigma_y^2 \ \forall t$$

$$Var(y_t) = Var(2+0{,}6\,y_{t-1} - 0{,}08\,y_{t-2} + \varepsilon_t)$$

$$\sigma_y^2 = 0{,}36\,\sigma_y^2 + 0{,}0064\,\sigma_y^2 + \sigma_\varepsilon^2 - 0{,}6\cdot 0{,}08\,\gamma_1$$

$$= \sigma_y^2 = \frac{\sigma_\varepsilon^2 - 0{,}6\cdot 0{,}08\,\gamma_1}{1-0{,}36-0{,}0064} = \gamma_0$$

$$y_t = 2+0{,}6\,y_{t-1} - 0{,}08\,y_{t-2} + \varepsilon_t$$

$$2 = \mu - 0{,}6\mu + 0{,}08\mu$$

$$(y_t-\mu) = 0{,}6(y_{t-1}-\mu) - 0{,}08(y_{t-2}-\mu) + \varepsilon_t \,/\, (y_{t-1}-\mu)$$

$$\gamma_1 = 0{,}6\,\gamma_0 - 0{,}08\,\gamma_1$$

$$1{,}08\,\gamma_1 = 0{,}6\,\gamma_0$$

$$\gamma_1 = \frac{0{,}6}{1{,}08}\gamma_0$$

$$\frac{4 - 0{,}6\cdot 0{,}08\,\gamma_1}{1-0{,}36-0{,}0064} = \gamma_0$$

$$\frac{4 - \frac{0{,}6^2\cdot 0{,}08}{1.08}\gamma_0}{0{,}6336} = \gamma_0$$

$$\gamma_0 = \frac{4 - 0{,}026\,\gamma_0}{0{,}6336} = \gamma_0$$

$$\frac{4}{0{,}6336} = r_0\left(1 + \frac{0{,}026}{0{,}6336}\right)$$

$$r_0 = \frac{4}{0{,}6336} \cdot \frac{1}{\left(1 + \frac{0{,}026}{0{,}6336}\right)}$$

3,16 , 3,25 , 3,13