2022.

1) u_t - WN $\quad E(u_t)=0$

$Var(u_t) = \sigma_u^2 \quad cov(u_t, u_s) = 0$

$\forall t \neq s$

$$y_t = \frac{1+3L}{1-0{,}2L}(5+u_t) \;=$$

a) $E(y_t) \quad Var(y_t), \; cov(y_t, y_s)$

$$= \frac{1+3L}{1-0{,}2L}5 + \frac{1+3L}{1-0{,}2L}u_t$$

$$E(y_t) = E\left(\frac{1+3L}{1-0{,}2L}5\right) =$$

$$= E\left(\frac{1+3}{1-0{,}2}5\right) = 25$$

$$Var(y_t) = Var\left(\frac{1+3L}{1-0{,}2L}u_t\right) =$$

$$= Var\left((1+3L)(1+0{,}2L+0{,}2^2L^2+\dots)u_t\right)$$

$$1+3L = (1-0{,}2L)\cdot(-15)+16$$

$$= \frac{(1-0{,}2L)\cdot(-15)+16}{1-0{,}2L} u_t =$$

$$= \left(-15 + \frac{16}{1-0{,}2L}\right) u_t =$$

$$= \left(-15u_t + 16\,u_t(1+0{,}2L+0{,}2^2L^2+\ldots)\right)$$

$$= \underbrace{15^2\sigma_u^2 + \frac{16^2\sigma_u^2}{1-0{,}2^2} - 2\cdot 15\cdot 16\,\sigma_u^2}$$

$\checkmark$

$(y_{t-k}-\mu)\ E$

b) Cmay $A(t-s)$

c) $$y_t = \frac{(1+3L)(5+u_t)}{(1-0{,}2L)}$$

$$\underbrace{y_t(1-0{,}2L)} = (1+3L)(5+u_t)$$

$$y_t = 0{,}2y_{t-1} + 20 + u_t + 3u_{t-1}$$

2) $y_t = 3 + 0,5 y_{t-1} - 0,06 y_{t-2} + u_t - 0,2 u_{t-1}$

$u_t \sim WN$

а) лаговые полиномы + разложить

$(1 - 0,5L + 0,06L^2) y_t = 3 + (1 - 0,2L) u_t$

$1 - 0,5x - 0,06x^2 = 0$

$D = 0,25 - 0,24 = 0,01$

$x_1 = \frac{0,5 - 0,1}{0,12} = \frac{10}{3}$

$x_2 = \frac{0,6}{0,12} = 5$

$\boxed{\left(1 - \frac{10}{3}L\right)(1 - 5L)} y_t = 3 + (1 - 0,2L) u_t$

$y_t = \frac{3}{\left(1 - \frac{10}{3}L\right)(1 - 5L)} + \frac{(1 - 0,2L)}{\left(1 - \frac{10}{3}L\right)(1 - 5L)} u_t$

$corr(y_t, y_{t-s})$ corr

$\mathrm{cov}\big(\mathrm{cons} + (1 - \alpha_1 L - \alpha_2 L^2 - \dots) u_t, \mathrm{const} + (1-\alpha_1 L$

3) ETS(AAN)

$$\begin{cases} y_t = l_{t-1} + b_{t-1} + u_t \\ l_t = l_{t-1} + b_{t-1} + \alpha u_t \\ b_t = b_{t-1} + \beta u_t \\ u_t \sim N(0, \sigma^2) \end{cases}$$

а) Список параметров

$b_0\ l_0, \alpha, \beta, \sigma^2$

логарифм плотности y_2 через парам.

$y_2 = l_1 + b_1 + u_2$

$l_1 = l_0 + b_0 + \alpha u_1$

$b_1 = b_0 + \beta u_1$

$y_2 = l_0 + b_0 + b_0 + \alpha u_1 + \beta u_1 + u_2 =$

$= \underbrace{l_0 + 2b_0} + (\alpha + \beta) \underset{N(0,\sigma^2)}{u_1} + \underset{N(0,\sigma^2)}{u_2}$

$E(y_2) = l_0 + 2b_0$

$\mathrm{Var}(y_2) = (\alpha+\beta)^2 \sigma^2 + \sigma^2 = \sigma^2(1 + (\alpha+\beta)^2)$

$$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f_{y_2}(x) = \frac{1}{\sqrt{2\pi\sigma^2(1+(\alpha+\beta)^2)}} e^{-\frac{(x-(l_0+2b_0))^2}{2\sigma^2(1+(\alpha+\beta)^2)}}$$

2b) $l_{100} = 30$, $b_{100} = 1$, $\alpha = 0{,}2$, $\beta = 0{,}3$, $\boxed{\sigma^2 = 16}$

$[\hat{y}_F \pm 1{,}96 \sqrt{\hat{y}_{var}}]$

$y_{101} = \underbrace{l_{100} + b_{100}} + \underbrace{u_{101}} = 31 + u_{101} \sim N(31, 16)$

$E(y_{101} | I_{100}) = 31$

$Var(y_{101} | I_{100}) = 16$ $\quad PCI(95\%) = [31 \pm 1{,}96 \cdot 4]$

$y_{102} = \underbrace{l_{101}} + \underbrace{b_{101}} + u_{102} =$

$= l_{100} + b_{100} + \alpha u_{101} + b_{100} + \beta u_{101} + u_{102} =$

$= 30 + 1 + \underbrace{0{,}2 u_{101} + 1 + 0{,}3 u_{101}}_{0{,}5 u_{101}} + u_{102}$

$E(y_{102} | I_{100}) = 32$

$Var(y_{102} | I_{100}) = 0{,}25 \sigma^2 + 16 = 20$

$PCI(95\%) = [32 \pm 1{,}96 \sqrt{20}]$

5) а) стац

б) 1) Можно, он нров

2) Решать через ρ (вычислить α)

4) MA(13)

$y_t = MA(13) - MA(12)$

$$\boxed{y_t = -c + \alpha \varepsilon_{t-13}}$$

2) Если 0 после 13 → MA(13)

Если 0 всё до 13, → ненулевой только 13 коэф

2021) №2

MA(2) - ур $\quad y_t = 3 + u_t + 0{,}5u_{t-1} + 0{,}2u_{t-2}$

$\boxed{u_t - WN}$

$\hat{y}_t = \hat{\beta}_1 + \hat{\beta}_2 y_{t-1}$

а) $\underbrace{E(y_t)}_{=3}, \underbrace{var(y_t)}, \underbrace{cov(y_t, y_s)}_{\frac{\gamma_1}{\gamma_0} = ACF(1)}$

б) Какие $\hat\beta_1$, $\hat\beta_2$ получим вилкой, если у нас много набл.?

y $\quad \hat\beta = (X^T X) X^T y \quad PACF(1)$

$$\underbrace{y_t = \alpha_0 + \alpha_1 y_{t-1} +}\ \nu_t \qquad PACF$$

$$\begin{cases} E(\nu_t) = 0 \\ cov(\nu_t, y_{t-1}) = 0 \end{cases} \qquad \hat\beta_2 = \widehat{PACF(1)} = \widehat{ACF(1)}$$

$$\beta_1 \approx \alpha_0$$

$$E(\nu_t) = E(y_t - \alpha_0 - \alpha_1 y_{t-1}) \approx$$

$$\approx E(y_t - \beta_1 - \beta_2 y_{t-1}) = 0$$

$$E(y_t) = 3 \quad E(y_{t-1}) = 3$$

$$3 - \beta_1 - \beta_2 3 = 0$$

$$\beta_1 = 3 - \beta_2 3$$

$$\underbrace{\underbrace{\beta_1, \boxed{\beta_2}}\ = ACF(1) = PACF(1)}$$

$$y = \underbrace{\underbrace{\hat\alpha_0} + \underbrace{2} y_{t-1} + \underbrace{\hat\alpha_2} y_{t-1} +}\ \cancel{\nu_t}$$

$E(y_t \mid I_{t-1})$