Вариант. 2023

№1.

K - const

$$y_t = \sum_{j=1}^{K} u_j \sin(\lambda_j t) + \nu_j \cos(\lambda_j t)$$

$\lambda_j = \text{const}$

u_j, ν_j независимы

$Var(u_j) = Var(\nu_j) = 1$

1) $\gamma_h = cov(y_0, y_h) = cov(y_t, y_{t+h})$

$$\gamma_h = cov(\sum u_j \underbrace{\sin(\lambda_j 0)}_{=0} + \nu_j \underbrace{\cos(\lambda_j 0)}_{=1}, \sum u_j \sin(\lambda_j h) + \nu_j \cos(\lambda_j h)) =$$

$$= cov(\sum \nu_j, \sum \cancel{u_j \sin(\lambda_j h)} + \nu_j \cos(\lambda_j h)) =$$

незав

$$cov(\sum \nu_j, \sum \nu_j \cos(\lambda_j h)) =$$

$$= cov(\nu_1 + \nu_2 + \ldots + \nu_K, \nu_1 \cos(\lambda_1 h) + \nu_2 \cos(\lambda_2 h) + \ldots + \nu_K \cos(\lambda_K h))$$

$$= \underbrace{Var(\nu_1)}_{=1} \cdot \boxed{\sum \cos(\lambda_j h)}$$ ✓

2) стац?

$cov(y_t, y_{t-s}) =$

$$= \operatorname{cov}\Big(\sum u_j \sin(\lambda_j t) + v_j \cos(\lambda_j t),$$
$$\sum u_j \sin(\lambda_j (t-s)) + v_j \cos(\lambda_j (t-s))\Big) =$$
$$\operatorname{cov}\Big(\sum u_j \sin(\lambda_j t), \sum u_j \sin(\lambda_j (t-s))\Big) +$$
$$\operatorname{cov}\Big(\sum v_j \cos(\lambda_j t), \sum v_j \cos(\lambda_j (t-s))\Big) =$$

$$= \sum \sin(\lambda_j t)\sin(\lambda_j (t-s)) +$$
$$\sum \cos(\lambda_j t)\cos(\lambda_j (t-s)) =$$
$$\sum \cos(\cancel{\lambda_j t} - \cancel{\lambda_j t} + \lambda_j s) = \sum \cos(\lambda_j s)$$
$$\gamma_0 = K$$
$$\frac{\gamma_k}{\gamma_0} = \frac{1}{K}\sum \cos(\lambda_j s)$$

$$E(y_t) = 0 \qquad \boxed{y_t - \text{стац.}}$$

3.

$$\begin{pmatrix} a_t \\ b_t \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} + \begin{pmatrix} -2 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} a_{t-1} \\ b_{t-1} \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} a_{t-2} \\ b_{t-2} \end{pmatrix} + \begin{pmatrix} u_t^a \\ u_t^b \end{pmatrix}$$

$$\det(I - \lambda \Phi_1 - \lambda^2 \Phi_2)$$

$$\left|\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \lambda\begin{pmatrix} -2 & 2 \\ 2 & 1 \end{pmatrix} - \lambda^2\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}\right| = 0$$

$$\begin{vmatrix} 1+2\lambda+\lambda^2 & \underline{-2\lambda-2\lambda^2} \\ \underbrace{-2\lambda-2\lambda^2} & 1-\lambda+4\lambda^2 \end{vmatrix} = 0$$

$$\underbrace{(1+2\lambda+\lambda^2)(1-\lambda+4\lambda^2)} + 2(1+\lambda)\lambda = 0$$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \Rightarrow ?$ $\vee$ 1

(4)

u_t – стац в сильном смысле

$u_t = \nu_t \sigma_t$, $\nu_t \sim N(0,1)$

$\sigma_t^2 = 3 + 0{,}1 u_{t-1}^2$ $\quad$ ν_t независимо от

$u_{t-1}, \nu_{t-1}, u_{t-2}, \nu_{t-2} \dots$

1) $Var(u_t) = Var(\nu_t \sigma_t)$

$\overset{\shortparallel}{E(u_t^2)} - E(u_t)^2 = E(u_t^2)$

$E(u_t) = E(\nu_t \sigma_t) = E(\nu_t) E(\sigma_t) = 0$

$E(u_t^2) = E(\underbrace{\nu_t^2}\underbrace{\sigma_t^2}) = \underbrace{E(\nu_t^2)} E(\sigma_t^2)$

$E(u_t^2) = E(\sigma_t^2)$

$\underbrace{Var(\nu_t)}_{=1} + \underbrace{E(\nu_t)^2}_{=0}$

$E(u_t^2) = E(3 + 0{,}1 u_{t-1}^2)$

$E(u_t^2) = X$

$x = 3 + 0{,}1x \qquad 0{,}9x = 3$

$$x = \frac{3 \cdot 10}{9} = \frac{30}{9} = \boxed{3\frac{1}{3}} \checkmark$$

$E(u_t^2) = E(\sigma_t^2)$

$u_t^2 = \sigma_t^2 + \omega_t$ ← ненабл

$\boxed{E(\omega_t) = 0}$

набл / не набл

$\sigma_t^2 = 3 + 0{,}1 u_{t-1}^2$

$u_t^2 - \omega_t = 3 + 0{,}1 u_{t-1}^2$

$$\underbrace{u_t^2 = 3 + 0{,}1 u_{t-1}^2}_{AR(1)} + \omega_t \quad WN(?)$$

$E(\omega_t) = 0 \quad \checkmark$

$Var(\omega_t) = C$

$cov(\omega_t, \omega_k) = 0 \quad \forall k \neq t$

$$cov(\omega_t, \omega_{t-1}) = E(\omega_t \omega_{t-1}) - \underbrace{E(\omega_t)}_{0} E(\omega_{t-1}) =$$

$$= E\left((u_t^2 - \sigma_t^2)(u_{t-1}^2 - \sigma_{t-1}^2)\right) =$$

$$= E\left((\nu_t^2 \sigma_t^2 - \sigma_t^2)(\nu_{t-1}^2 \sigma_{t-1}^2 - \sigma_{t-1}^2)\right) =$$

$$= E\Big(\underbrace{\sigma_t^2}\, \underbrace{\sigma_{t-1}^2}_{t-2} (\nu_t^2 - 1)(\underbrace{\nu_{t-1}^2 - 1}_{t-2})\Big) =$$

$$= \underbrace{E(\nu_t^2 - 1)}_{Var(\nu_t)} E(\sigma_t^2 \sigma_{t-1}^2 (\nu_{t-1}^2 - 1)) = 0 \quad \square$$

$$Var(\nu_t) = 1$$

$$(E(\nu_t^2) - E(1))$$

$$Var(\nu_t) = 1$$

$$Var(w_t) = E(w_t^2) = E((u_t^2 - \sigma_t^2)^2) =$$

$$E((\nu_t^2 \sigma_t^2 - \sigma_t^2)^2 = E(\sigma_t^4 (\nu_t - 1)^2) =$$

$$E((3 + 0{,}1 u_{t-1}^2)^2 (\nu_t - 1)^2) = \boxed{?}$$

$$E((u_{\cancel{t}}^2 - 3 - 0{,}1 u_{\cancel{t-1}}^2)^2)$$

$$\underbrace{E((X^2 - 3 - 0{,}1 X^2))^2}_{\forall t} = C$$

$$y_1 = N(0,1) \quad E(y_1) = 0 \quad E(y_1^2 - 1) = 0$$

$$y_2 = N(0,1) \quad E(y_2) = 0 \quad E(y_2^2 - 1) = 0$$

$$\chi^2$$

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$$y_t = u_t + 0{,}5 u_{t-1} \quad u_t - WN$$

$$\begin{cases} x_t = F x_{t-1} + v_t \\ \underbrace{y_t}_{\text{- скаляр}} = G x_t + w_t \end{cases}$$

x_0, (v_1, w_1), (v_2, w_2) - некорр

F, G - матрицы

$$\overbrace{\begin{pmatrix} u_t \\ u_{t-1} \end{pmatrix}}^{x_t} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \overbrace{\begin{pmatrix} u_{t-1} \\ u_{t-2} \end{pmatrix}}^{x_{t-1}} + \begin{pmatrix} u_t \\ 0 \end{pmatrix}$$

$$y_t \quad (1 \;\; , 0{,}5) \begin{pmatrix} u_t \\ u_{t-1} \end{pmatrix} + \underset{\substack{\shortparallel \\ 0}}{w_t}$$

$$x_0 = \begin{pmatrix} u_0 \\ 0 \end{pmatrix} \quad \underset{v_1}{\begin{pmatrix} u_1 \\ 0 \end{pmatrix}} \quad \underset{v_2}{\begin{pmatrix} u_2 \\ 0 \end{pmatrix}} \quad \underset{w_1}{0} \quad \underset{w_2}{0}$$