1) ММП
2) Тестирование гипотез + Portmanteau tests
3) ДЗ.

ММП.

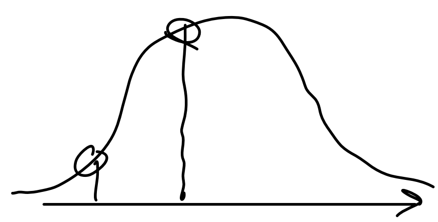
X - выборка

X_i - iid $\qquad \sigma^2 > 0$

$X_i \sim N(\mu, \sigma^2) \qquad f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$L(X)$ = „вероятность" получения нашей выборки $\quad L = \prod_{i=1}^{n} p(X_i = x_i)$

$$L = \underbrace{\prod_{i=1}^{n} f_X(x_i)}_{} \to$$



$$L = f(X_1, \dots, X_n) = \underbrace{\prod_{i=1}^{n} f_X(x_i)}_{} \to \max_{\mu, \sigma^2}$$

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \to \max_{\mu, \sigma^2}$$

$X_1, \dots, X_n$ - с. в.

$x_1, \dots, x_n$ - числа, реализация

$l = \ln(L) =$

$$= \sum_{i=1}^{n} \ln\left(\overset{(2\pi\sigma^2)^{-1/2}}{\overset{\text{''}}{\frac{1}{\sqrt{2\pi\sigma^2}}}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \right) =$$

$$= \sum_{i=1}^{n} \frac{1}{2} \ln(2\pi\sigma^2) - \frac{(x_i-\mu)^2}{2\sigma^2} =$$

$$= \sum_{i=1}^{n} \cancel{\frac{1}{2} \ln(2\pi)} + \frac{1}{2} \ln \sigma^2 - \underbrace{\frac{(x_i-\mu)^2}{2\sigma^2}} \to \max_{\mu, \sigma^2}$$

$$\frac{\partial l}{\partial \mu} = \sum_{i=1}^{n} + \frac{2(x_i-\hat{\mu})}{2\sigma^2} = 0 \Rightarrow \sum \cancel{2}(x_i - \hat{\mu}) = 0$$

$$\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \hat{\mu} = 0$$

$$\sum x_i - n\hat{\mu} = 0$$

$$\hat{\mu} = \frac{\sum x_i}{n} = \bar{x}$$

$$l = \frac{1}{2} \ln p - \frac{(x_i-\mu)^2}{2p}$$

$$\frac{\partial l}{\partial \sigma^2} = \sum_{i=1}^{n} \frac{1}{2} \frac{1}{\sigma^2} - \frac{(x_i-\mu)^2}{2} \cdot \frac{1}{(\sigma^2)^2} = 0$$

$$\sum_{i=1}^{n} \frac{\sigma^2 - (x_i-\mu)^2}{2(\sigma^2)^2} = 0$$

$$(\sigma^2)^{-1} = -(\sigma^2)^{-2}$$

$$\sum_{i=1}^{n} \sigma^2 - (x_i - \bar{x})^2 = 0$$

$$\hat{\sigma}^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$f(X_1, \dots, X_n) = \prod f(X_i)$$

$$Y_t = f(Y_{t-1}, \dots, Y_{t-k})$$

$$f(Y_1, \dots, Y_T) = f(Y_T \mid Y_{T-1}, \dots, Y_1) \cdot f(Y_{T-1} \mid Y_{T-2}, \dots, Y_1) \cdot$$

$$\cdot \; f(Y_2 \mid Y_1) \cdot f(Y_1)$$

ETS(A A N) $\qquad \varepsilon_t \sim N(0, \sigma^2)$

$$\begin{cases} y_t = l_{t-1} + b_{t-1} + \varepsilon_t \\ l_t = l_{t-1} + b_{t-1} + \alpha \varepsilon_t \\ b_t = b_{t-1} + \beta \varepsilon_t \end{cases} \qquad l_0, b_0$$

$$y_1 = \underbrace{l_0 + b_0} + \varepsilon_1^{\sim N(0, \sigma^2)} \qquad y_1 \sim N(l_0 + b_0, \sigma^2)$$

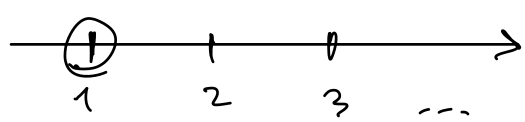
$$E(y_1) = l_0 + b_0 + 0$$

$$Var(y_1) = \sigma^2$$

$$f_{y_1}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x - (l_0 + b_0))^2}{2\sigma^2}}$$

$y_2 \mid y_1$

$= 10$



$$10 = l_0 + b_0 + \varepsilon_1$$

$$y_2 = l_1 + b_1 + \varepsilon_2 = \underbrace{l_0 + b_0 + \alpha\varepsilon_1 + b_0 + \beta\varepsilon_1 + \varepsilon_2}_{y_2 | y_1}$$

$$l_1 = l_0 + b_0 + \alpha\varepsilon_1$$

$$b_1 = b_0 + \beta\varepsilon_1$$

$$E(y_2 | y_1) = l_0 + 2b_0 + (\alpha + \beta)\hat{\varepsilon}_1$$

$$Var(y_2 | y_1) = \sigma^2$$

$$y_2 | y_1 \sim N\left(l_0 + 2b_0 + (\alpha+\beta)(y_1 - (l_0 + b_0)), \sigma^2\right)$$

$$f_{y_2|y_1}(x) \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-(\ldots))^2}{2\sigma^2}}$$

$$Var(Y_T | Y_{T-1}, \ldots, Y_1),\ Var(Y_{T-1} | Y_{T-2}, \ldots, Y_1)$$

$$Var(Y_{T+1} | Y_T, \ldots, Y_1)$$

$$Var(Y_{T+2} | Y_T, \ldots Y_1)$$

$$L = \prod_{i=1}^{n} f(\ldots) \to \max_{\alpha, \beta, \sigma^2, l_0, b_0}$$

$$\begin{cases} \dfrac{\partial L}{\partial \alpha} = 0 \\ \dfrac{\partial L}{\partial \beta} \\ \vdots \\ \dfrac{\partial L}{\partial l_0} = 0 \end{cases}$$