

$$C) h = \log_{\frac{10}{7}} n.$$

tree의 같은 level에 있는 node의 합은 $(\frac{7}{10} + \frac{3}{10})^k \cdot n$ 이기 때문에 항상 n 이다.

$$0 \sim \log_{\frac{10}{7}} n \text{ 이기 때문에 } n \cdot (1 + \log_{\frac{10}{7}} n)$$

$$n(1 + \log_{\frac{10}{7}} n) = n(\log_{\frac{10}{7}} \frac{10}{7} n)$$

$$= n \left(\frac{\lg_{\frac{10}{7}} n}{\lg_{\frac{10}{7}} \frac{10}{7}} \right)$$

$$= n \left(\frac{\lg_{\frac{10}{7}} + \lg n}{\lg_{\frac{10}{7}} \frac{10}{7}} \right)$$

$$= \frac{1}{\lg_{\frac{10}{7}} \frac{10}{7}} n \lg n + n$$

$$= \Theta(n \lg n)$$

2.

$$(a) T(n) = 9T(n/3) + n^2$$

$$f(n) = n^2 \quad a=9 \quad b=3$$

$$1. n^2 = O(n^{\log_3 9 - \epsilon}) \quad \epsilon > 0 \quad \text{is false.}$$

$$2. n^2 = \Theta(n^{\log_3 9}) \quad n^{\log_3 9} = n^2 \quad \text{is true.}$$

$$T(n) = \Theta(n^2 \lg n)$$

$$(b) T(n) = 3T(n/4) + n$$

$$f(n) = n \quad a=3 \quad b=4$$

$$1. n = O(n^{\log_4 3 - \epsilon}) \quad \epsilon > 0 \quad \text{일때, } \log_4 3 - \epsilon < 1 \quad \text{이기때문에 false}$$

$$2. n = \Theta(n^{\log_4 3}) \quad \log_4 3 < 1 \quad \text{이기때문에 false.}$$

$$3. n = \Omega(n^{\log_4 3 + \epsilon}) \quad \epsilon > 0 \quad \text{일때, } 0 < \epsilon \leq 6 \quad \text{이면, } \log_4 3 + \epsilon \leq 1 \quad \text{아니}$$

$$af(nb) \leq cf(n), c < 1 \quad \text{작은 상수. } \exists x \frac{n}{4} \leq cn \quad \frac{1}{3} \leq c < 1 \quad \text{이면 상수}$$

위 두가지 만족하기때문에

$$T(n) = \Theta(n)$$