Solve Number Cross with SMT solver

21500185 PyeongGangKim

1. Constraints

1-1. All matrix's value is either 1(Black) or 0(White).

1-2. Sum of the numbers in black squares of the column is the same as the first column in input file.

1-3. Sum of the numbers in white squares of the row is the same as the last column in each row in input file.

2. Constraint as a logic formula

2-1. We save that sum of rows in each column from the first row of the input file in R(j).(j is column)

2-2. We save that sum of columns in each row from the last column of the input file in C(i).(i is row)

2-3. We put the value of M X N matrix in input file in A(i,j). (i is row, j is column)

2-4. We need two M X N matrix(E(i,j),F(i,j)). One(E(i,j)) stores that the black is 1 and white is 0. The other ones(F(i,j)) store that the white is 1 and black is 0.

Q1. E(i,j) must have either 0 or 1.

$$\bigwedge_{i=0}^{M-1} \bigwedge_{j=0}^{N-1} 0 \leq E(i,j) \leq 1$$

Q2. F(i,j) must have either 0 or 1.

$$\bigwedge_{i=0}^{M-1} \bigwedge_{j=0}^{N-1} 0 \leq F(i,j) \leq 1$$

Q3. The sum of the columns of the black squares multiplied by the columns of the input matrix is same as R(j)(it means that sum of rows in each column)

$$\bigwedge_{j=0}^{N-1} \left(\sum_{i=0}^{M-1} E(i,j) * A(i,j)\right) = R(j)$$

Q4. The sum of the rows of the white squares multiplied by the rows of the input matrix is same as C(i)(it means that sum of columns in each row)

$$\bigwedge_{i=0}^{M-1} \left(\sum_{j=0}^{N-1} F(i,j) * A(i,j)\right) = C(i)$$

Q5. The colors of each square in two matrices must match. So I make two cases(one case subtract one and the other added one. It would make same color.) and compare them.

$$\bigwedge_{i=0}^{M-1} \bigwedge_{j=0}^{N-1} (E(i,j) = (F(i,j) - 1)) \vee (E(i,j) = (F(i,j) + 1))$$

3. Demonstration.

Case 1

Input

```
20 23 30 29 34 6 9 21 19
8 2 1 8 1 3 5 7 6 18
9 1 4 2 5 6 3 1 7 28
3 5 1 4 9 1 3 9 1 8
8 6 6 3 5 1 1 4 1 4
8 6 6 2 6 8 3 3 9 31
8 7 8 8 4 5 2 1 1 18
4 8 3 5 5 2 1 2 8 24
1 2 8 8 8 3 7 2 7 22
8 3 9 5 9 2 1 4 9 35
```

Output

```
1 1 0 1 0 0 1 0 0
0 1 1 0 1 0 0 0 0
1 1 1 0 1 0 0 1 1
1 1 1 1 1 1 1 0 1
0 0 0 1 1 0 0 1 1
0 1 1 1 0 0 1 1 0
0 0 1 0 0 0 1 1 1
1 1 1 1 0 1 0 1 0
0 0 0 0 1 1 0 1 0
```

Case 2

Input

```
20 23 30 29 34 6 9 21 19 20 23 30 29 34 6 9 21 19
8 2 1 8 1 3 5 7 6 8 2 1 8 1 3 5 7 6 36
9 1 4 2 5 6 3 1 7 9 1 4 2 5 6 3 1 7 56
3 5 1 4 9 1 3 9 1 3 5 1 4 9 1 3 9 1 16
8 6 6 3 5 1 1 4 1 8 6 6 3 5 1 1 4 1 8
8 6 6 2 6 8 3 3 9 8 6 6 2 6 8 3 3 9 62
8 7 8 8 4 5 2 1 1 8 7 8 8 4 5 2 1 1 36
4 8 3 5 5 2 1 2 8 4 8 3 5 5 2 1 2 8 48
1 2 8 8 8 3 7 2 7 1 2 8 8 8 3 7 2 7 44
8 3 9 5 9 2 1 4 9 8 3 9 5 9 2 1 4 9 70
```

Output

```
1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 0 0 0
0 0 0 1 1 0 0 1 0 0 0 1 1 1 0 0 1 0
1 0 1 1 1 0 1 1 0 1 0 0 1 1 1 0 1 1
1 1 1 0 1 1 0 1 1 1 1 1 0 1 0 1 1 1
0 1 1 1 1 0 1 0 1 0 1 0 1 0 0 0 0 0
0 1 1 1 0 0 1 1 1 0 1 1 1 0 0 0 1 1
0 0 0 1 0 0 0 0 1 0 0 1 1 1 1 0 0 0
1 1 1 0 1 0 0 1 0 1 1 1 0 0 0 1 1 1
0 0 0 0 0 1 1 1 0 0 0 0 0 1 0 1 1 1
```

4.Discussion

4-1. We need that represented a matrix by black is 1, white is 0. When we print matrix, use the E(i,j) matrix only.

4-2. I suggest new ideas.
When we get the value of sum of columns in each rows, we multiply -1 to sum of columns in each Rows. And get the value of sum of rows in each columns in the same way as above. I can guess rows's colors in each columns in the same way as above. When we get the columns's colors in each rows, we subtract one to represented matrix and multiply input columns in each rows to it. Then we can get a matrix with color represented by either 1 or 0.

This logic spends too much time, when it runs in z3 solver. So I think that this logic is not good program from a time perspective.