EECS 388

# Introduction to Computer Security

**Lecture 3:**

## Randomness and Pseudorandomness

September 3, 2024

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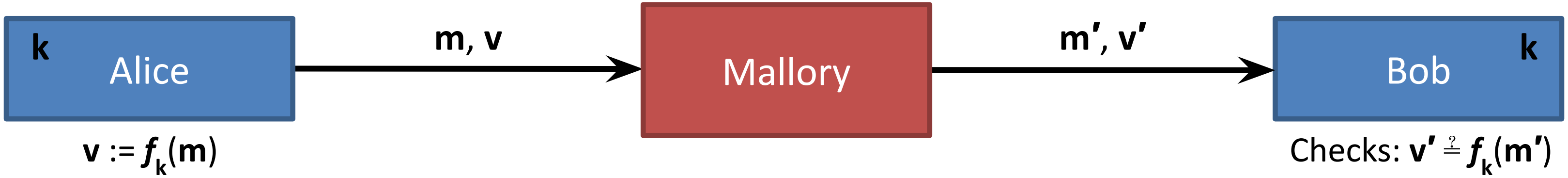
# Review: Message Integrity

**Problem:** Integrity of message from Alice to Bob over an ***untrusted channel***

**Approach:** Alice must append bits to message that only Alice (or Bob) can make

**Ideal solution:** **Random functions**

**Practical solution:** **Pseudorandom functions (PRFs)**




$f_k$() is a PRF if it's practically indistinguishable from a random function (unless you know **k**)

**Today's lecture:** What are some actual functions that (*we hope*) behave as PRFs?
Where do these random keys **k** come from?
What else are PRFs useful for?

# Review: Cryptographic Hashes

*First step towards a practical solution…*

## Cryptographic Hash Function

Fixed function ***H***(). ***No key!***

**Input:** arbitrary length data

**Output:** fixed size *digest* (**n** bits)

Properties of strong hash functions:

### Preimage resistance

Given output **h**, hard to find any input **m** s.t. **h** = ***H***(**m**)

### Collision resistance

Hard to find *any* pair of inputs $\mathbf{m_1}$,$\mathbf{m_2}$ s.t. $H(\mathbf{m_1})=H(\mathbf{m_2})$

Note: Collisions exist [why?], but should be hard to find

### Second-preimage resistance

Given $\mathbf{m_1}$, hard to find different $\mathbf{m_2}$ s.t. $H(\mathbf{m_1}) = H(\mathbf{m_2})$

Computing hashes with **OpenSSL:**

```
$ echo "hello world 1" | openssl dgst -sha256
07bafbe63a0e7c57c572aedf1c228022b537e28785013d7b
e017fc78731a8cc5
$ echo "hello world 0" | openssl dgst -sha256
8bfa9a398e97152beaaf385847808ad2d828c1c7251f1a45
bc7697723827e7e7
```

Observe how changing even a single bit of the input produces output that appears completely unrelated.

Annoying question:

**Are existing hashes *actually strong*?**

Annoying answer:

**We don't know.**

**Candidates:** ~~MD5~~, ~~SHA-1~~, **SHA-256**, SHA-512, SHA-3

# Review: Constructing SHA-256

SHA-256 is a widely used hash function that is currently thought to be strong

**Input:** arbitrary length data
**Output:** 256-bit digest

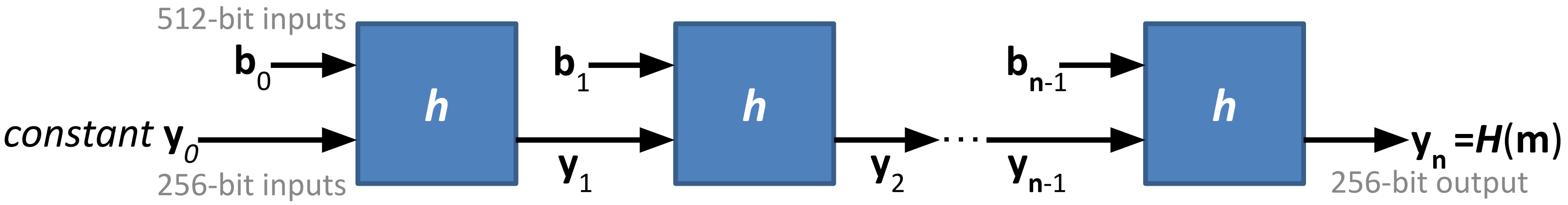
Built from a **compression function *h***:

**Inputs:** (256 bits, 512 bits), **Output:** 256 bits

reduces 768 bits to 256 bits using a complicated internal function (details out of scope for 388)

Uses the **Merkle–Damgård (MD) construction** (illustrated below) to accept arbitrary-length input by repeatedly applying ***h***():

1. Pad input **m** to the next multiple of 512 bits (adds at least 1 bit, uses fixed algorithm [why?]) and split into 512-bit blocks: $\mathbf{b}_0, \mathbf{b}_1, \ldots \mathbf{b}_{n\text{-}1}$
2. $\mathbf{y}_0$ := *<256-bit constant>*
   $\mathbf{y}_1 := h(\mathbf{y}_0,\mathbf{b}_0)$ … $\mathbf{y}_i := h(\mathbf{y}_{i-1},\mathbf{b}_{i-1})$
3. Return $\mathbf{y}_n$ which is defined to be **SHA-256**(**m**)



# MD Hash Pitfall: Length Extension Attacks

Merkle–Damgård hash functions are susceptible to **length extension attacks**:

Given **y** = ***H***(**x**) for some unknown **x**, attackers can calculate

**z** = ***H***(**x** // *padding* // **s**)

for arbitrary suffix **s**.

concatenation

That is, given:

**y = SHA-256(** ??? )

An attacker can produce:

**z = SHA-256(** ??? *original pad* suffix )

Note that this doesn't violate preimage, second-preimage, or collision resistance.

[But why is it a problem?]

Suppose Alice and Bob use this as a verifier:

**v** := **SHA-256(k** // **m)**


1. Alice sends **m** = "Please go to the bank."
2. Mallory doesn't know **k**, but *can* apply length extension to (**m**, **v**) to calculate **v′** for:

   **m′** = "Please go to the bank.[*original_pad*]
   Then transfer $10,000 to Mallory."

   (*original_pad* is some bytes beyond Mallory's control, but which the recipient might ignore)
3. Since **v′** is the correct verifier for **m′**, Bob will accept the modified message as valid

**You'll explore how this is done in Project 1!**

# Practical Solution: HMAC

## Message Authentication Code (MAC)

Designed to be used as a secure verifier:

**Inputs:** key, arbitrary length data
**Output:** fixed size digest (**n** bits)

**HMAC construction** turns any secure hash function *H*() into a MAC:

$$\mathrm{HMAC}_k(m) = H(k \oplus c_1 \,\|\, H(k \oplus c_2 \,\|\, m))$$

XOR

constant 363636…

constant 5c5c5c…

concatenation

Design protects against length extension!

**Example:** HMAC-SHA-256 is an HMAC constructed using **SHA-256** for *H*()

## For practical purposes, we believe we can treat **HMAC-SHA-256** as a PRF

Can *reduce* PRF security of HMAC-SHA-256 to a (weaker) security property of SHA-256's compression function

```
$ echo "hi" | openssl dgst -sha256 -hmac Secr3t
8074cdfd007e5cfdc71c2c1cd393a5fefa890d7702956a13
66a155d79d1cbe77
```

At last, this gives Alice and Bob a suitable approach for protecting message integrity:

**v := HMAC-SHA-256$_k$(m)**


Computes and sends
**v := HMAC-SHA-256$_k$(m)**

Rejects message if
**v' ≠ HMAC-SHA-256$_k$(m')**

# Randomness and Pseudorandomness

How should we choose a "secret" key **k**?

**Select a uniform random value** [Why?]

*Careful:* People are often sloppy about what is "random"

## True Randomness

Output of a *physical process* that is inherently unpredictable [Examples?]

- ***Inherent*** in all physical systems (Heisenberg)
- ***Absent*** in simplest abstract Turing machine

True randomness in software is scarce.
Getting it requires careful engineering.

Can't just look at the *output* of a process and determine that it was random

Requires assessing the ***generation procedure***

```
int getRandomNumber()
{
    return 4;  // chosen by fair dice roll.
               // guaranteed to be random.
}
```

Example: **Extract an unbiased random bit from a biased coin** (Von Neumann's method)

Since true randomness is expensive, often want to take a *small amount of it* and create a longer sequence that's "as good as random"

## Pseudorandom Generator (PRG)

# Definition of a PRG

*Review:* **Pseudorandom Function (PRF)**

$f_k : \{0,1\}_m \rightarrow \{0,1\}_n$

**m**-bit input and **n**-bit output

Cannot practically distinguish $f_k(\mathbf{x})$
from random function without knowing **k**

## Pseudorandom Generator (PRG)

$g_k : \perp \rightarrow \{0,1\}_n$ for **n**=poly(|**k**|)

**k** is a truly random seed
No other inputs
Output is much larger than the input
(Can think of output as *stream of bits*)

Cannot practically distinguish $g_k()$ from a
random stream of bits without knowing **k**

**Security definition:** (Similar to PRF definition)

1. Let **k** be a secret seed
2. Toss a coin (in secret) to get bit **b**
3. If ***b***=0, ***s*** := a truly random stream
   If **b**=1, **s** := $\mathbf{g}_k()$
4. Output **s** to Mallory
5. Mallory guesses **b** *in polynomial time**

We say ***g***() is a **secure PRG** if Mallory can't do meaningfully better than random guessing

*** Usual complexity-theoretic caveats…** no brute-forcing

Annoying question again:

**Do PRGs *actually exist*?**

Same annoying answer:

**We don't know.** (Would imply P≠NP.)

# Building a PRG from a PRF

**Given a secure PRF *f*()**, we can construct a PRG ***g***() and prove that it's secure.*

Construction:

For some random key **k** and PRF ***f***(), define $\boldsymbol{g}_k() := \boldsymbol{f}_k(0) \;//\; \boldsymbol{f}_k(1) \;//\; \boldsymbol{f}_k(2) \;//\; \ldots$

Theorem (informal):

If ***f***() is a secure PRF and ***g***() is built from ***f***() by this construction, then ***g***() is a secure PRG.

You're not responsible for the proof, but here it is (*at right*), if you're interested:

(*Hard exercise: Build a PRF from a PRG)

**Proof by contradiction:** If ***g***() as constructed is *not* a secure PRG, ***f***() *can't be* a secure PRF:

1. If ***g***() is not a secure PRG, there exists an algorithm **M*** that can distinguish its output from random. (This is from the definition of a secure PRG.)
2. We can apply **M*** to construct an algorithm **M** to distinguish ***f***() from a random function ***r***():
   a. Query ***h***() with inputs 0, 1, 2, …
   b. Let **s** := ***h***(0) // ***h***(1) // ***h***(2) // …
   c. Apply **M*** to **s** and return the result.
3. If ***h***() is ***r***(), **s** is a random stream, so **M** outputs 0.
   If ***h***() is ***f***(), **s** is ***g***() (by construction), so **M** outputs 1.

   Thus, **M** wins the PRF game for ***f***().

This contradicts our assumption that ***f***() is a secure PRF. Therefore, ***g***() must be a secure PRG.

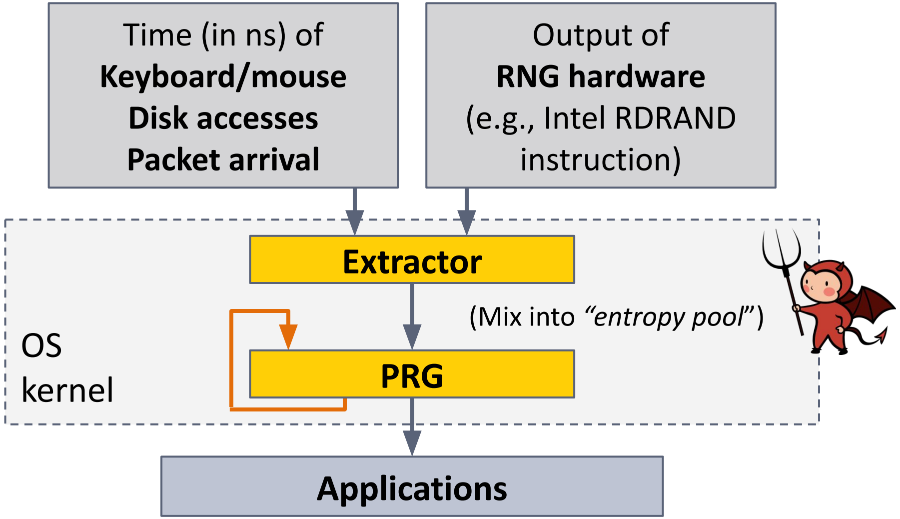
# Getting Randomness

Randomness is **an input** to your program

Typically provided by the OS, via special APIs

OS continuously gathers inputs from physical sources that are hard for adversary to predict, "extracts" uniform bits from them.

[What if attacker can predict *some* of them?]




[What if an attacker learns internal state?]

If compromise is **transient**, can recover by adding more randomness.

How quickly can we recover, though?

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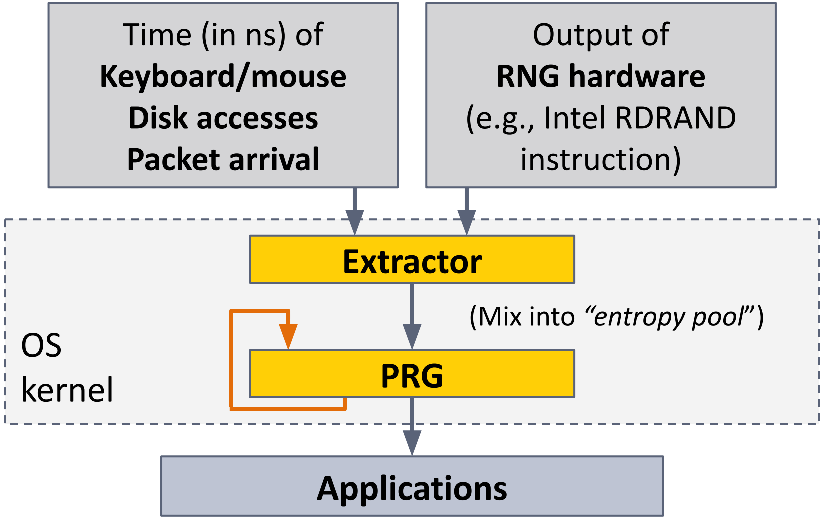


Generating 16 random bytes with **openssl**:

```
$ openssl rand -hex 16
4345b3cccecb66bef87f9289d72b8d2a
$ openssl rand -hex 16
578fbc36ad4d88ab7d98d7fd16e3737d
```

**Caution!** Not all "random" APIs are secure (or even unpredictable!) C's **`rand()`** function is notoriously bad, as are typical math packages.

Use APIs specified as suitable for cryptography:

C (Linux):

```
#include <sys/random.h>
getrandom(buf, size, 0);
```

Python:

```
import secrets
data = secrets.randbits(256);
```

JavaScript:

```
const array = new Uint8Array(32);
self.crypto.getRandomValues(array);
```

# Bad Randomness Example: Elections

DVSorder

DVSorder is a privacy flaw that affects
**Dominion ICP and ICE ballot scanners**

DOMINION VOTING

**To protect privacy, Dominion scanners assign each ballot a random-looking ID**
Linked to ballot scans that many jurisdictions publish online

**My group discovered that these “random” IDs are fully predictable**
Chosen by a *linear congruential generator*, known since 1970s to be unsuitable for security.
Using only public information, <u>anyone</u> can deduce the algorithm and “unshuffle” all ballots

“DVSorder: Ballot Randomization Flaws Threaten Voter Privacy”
Braden Crimmins, Dhanya Narayanan, Drew Springall, and J. A. Halderman, *USENIX Security 2024*

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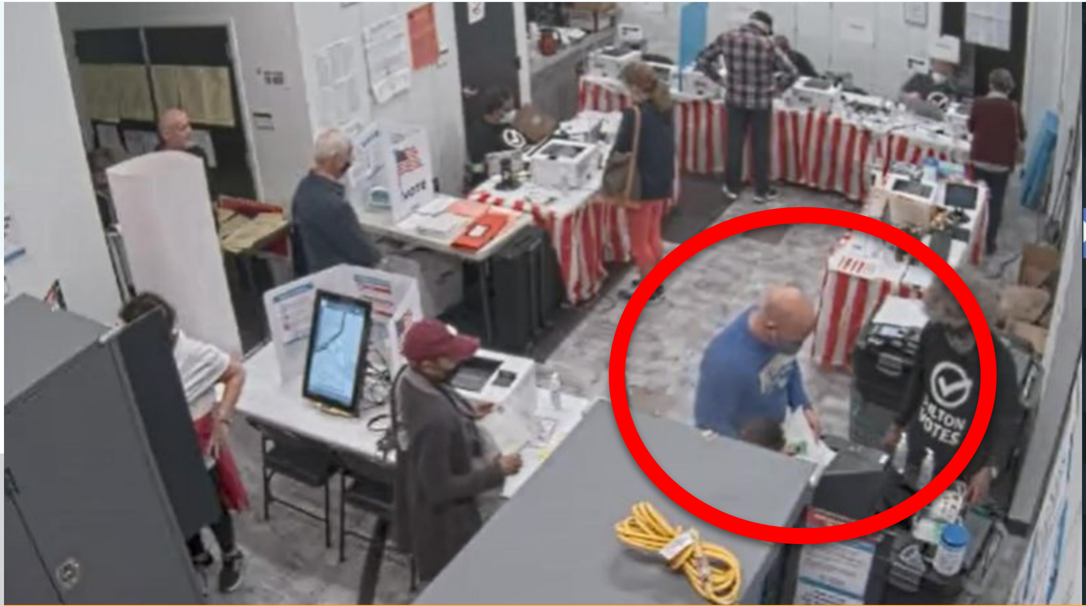


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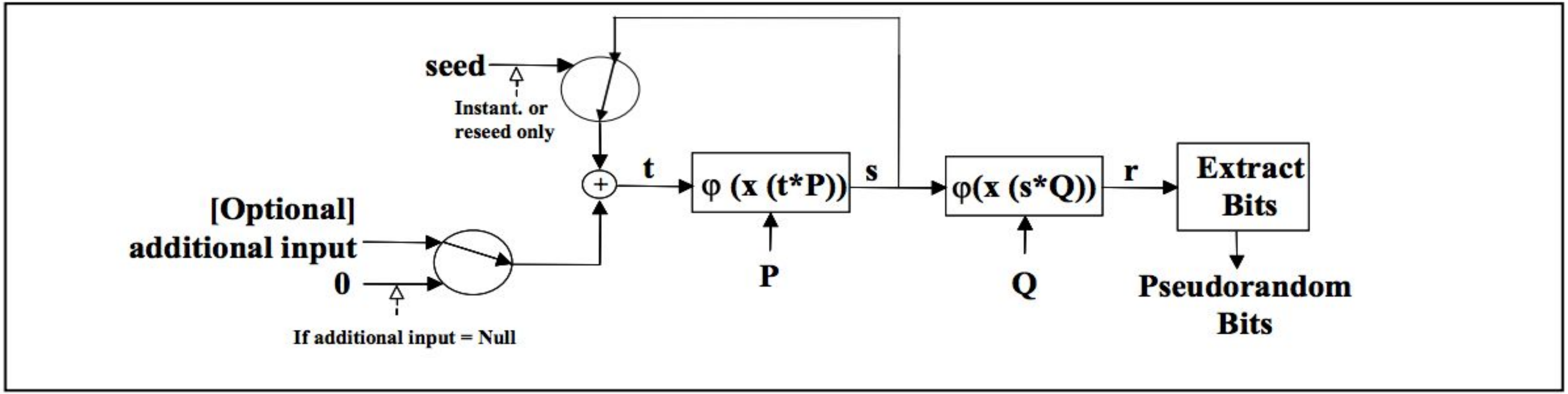
# Randomness as an Attack Target

Good randomness is needed everywhere in cryptography.
RNG is very good attack target!

**Dual-EC DRBG**: 2006 NIST standard that NSA (allegedly) **backdoored**. Evidence in Snowden documents.

Construction allows for the existence of a <u>secret backdoor key</u> that can be used to recover the internal RNG state (and determine future output) given knowledge of small amount of past output.



# Coming Up

Reminders:

**Quiz** on Canvas after every lecture
**Lab Assignment 1** due Thursday at 6 p.m.
**Project 1, Part 1** due Sept. 12 at 6 p.m.

### Thursday

## Confidentiality

Simple ciphers,
AES,
block cipher modes

### Next Week

## Wrap Up Crypto Unit

Combining confidentiality and integrity; Diffie-Hellman,
RSA encryption, digital signatures