Name: **Mark Daychman**

Emergent Spatial Competition: Extended Hotelling's Model

Abstract

Harold Hotelling was one of the first scientists to develop a comprehensive spatial competition model. In this paper, we extend his original model by simulating a dynamic, multi-seller environment, where sellers can adjust both their locations and prices to optimize revenue. Through simulations, we observe emergent behaviors such as territory division, price clustering, and adaptive movement. Our results show that Hotelling's hypothesis that all agents tend to cluster together fails in the general case. We also uncover some interesting behavior that emerges with more sellers. This study aims to provide insight into the strategic positions of businesses in competitive markets and to highlight the role of spacial factors on market stability.

1 Introduction

Understanding how businesses compete in price and location is one of key parts of market analysis. Hotelling's model of spatial competition, introduced in 1929, was a foundational framework of such behavior. The model showed that sellers would not organize in a socially optimal arrangement, i.e. an arrangement the market would be evenly split between the sellers. Instead, micro-perturbations caused by competition and a potential of higher profits would push a system to a Nash equilibrium where the sellers potentially make less and the buyers have to commute a larger distance.

While insightful, this model has significant limitations: It assumes a one-dimensional space, static pricing for sellers, does not explicitly generalize to environments with more sellers, assumes that the sellers know the most lucrative place where they can be located and that they can instantly move there without any attached cost. Simulations provide a practical way to overcome these limitations. By only allowing sellers to move prices and spatially incrementally, simulations capture more complex interactions.

Our study builds on Hotelling's framework by simulating a two-dimensional grid where sellers adjust their location and price to maximize revenue. Buyers choose sellers based on a cost function that includes both transportation and price. This extension highlights emergent phenomena such as territory division and clustering.

2 Implementation

For this simulation, we use a 50×50 square grid, where at each cell of the grid we have a buyer. At the start of the simulation, we randomly spawn $NSellers$ sellers on the grid. Every seller has its own color in the simulation for ease of visual differentiation. Every seller starts to sell the product at $StartPrice = 10$ and cannot go outside the range $[0, MaxPrice = 20]$. After every day, each seller has a choice to either change its price (decrease or increase by 1) or to move by one cell to any of the four cardinal directions. To do this each seller simulates its potential revenue at a new location or price, assuming that all competitors remain stationary and maintain their current prices. The seller can also choose to not move / change its price. Revenue is calculated as the total number of buyers multiplied by the price of the seller. We assume that demand is perfectly inelastic, i.e. that the buyers would buy the product no matter what, but would still pick the best available deal. We stop the simulation after 100 iterations or if the same state is reached 3 times. We record if a simulation ends in a repetition or not. We run every environment 20 times to get accurate results.

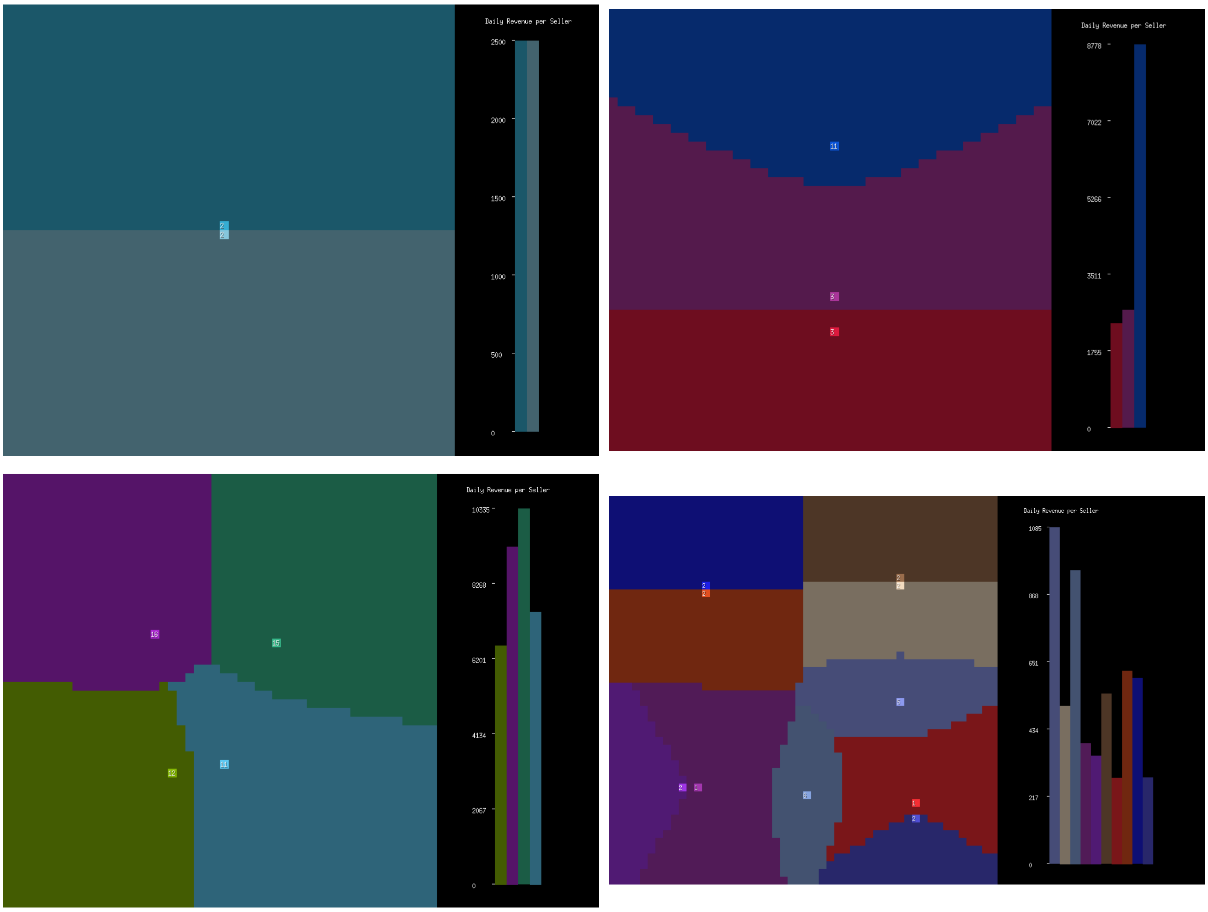

Figure 1: A simulation with 2, 3, 4 and 10 sellers. The seller's location is marked in a lighter color with its price marked inside the square. The area of influence associated to the seller is shaded in a darker color.

The buyer B picks the seller S that minimizes the following cost function:

$$C(B, S) = T_{cost}(B, S) + P_{cost}(B, S), \tag{1}$$

where T_{cost} is the transport cost associated to make the transaction and P_{cost} is the transaction's monetary cost, calculated as follows:

$$T_{cost}(B,S) = T_{coef} \cdot \sqrt{(B_x - S_x)^2 + (B_y - S_y)^2} \tag{2}$$

$$P_{cost}(B,S) = P_{coef} \cdot S_{price} \tag{3}$$

Unless otherwise stated, the P_{coef} in equation 3 is equal to 1. The model described above follows the same principles as the original Hotelling model of spatial competition described in [1]. By tuning T_{coef} we can

The implementation of the code is done in Go and can be found here[1].

3 Results and Analysis

Table 1 shows what percentage of simulations for each configuration ended in a repetition. Note, however, that because the simulation had a repeated state multiple times, does not mean that it reached a stable equillirbrium. Indeed, this is not true and state repetition was used as a simple metric to test the overall stability of the simulation. Notable simulation reflections and observations, as well as the analysis of the stability tests summarized in the table are shown below.

T_coef	**2 Sellers**	**3 Sellers**	**4 Sellers**	**5 Sellers**	**10 Sellers**
0.1	100%	95%	100%	90%	50%
0.5	100%	100%	100%	90%	90%
1.0	100%	100%	90%	85%	85%
1.5	100%	95%	90%	90%	70%
2.0	100%	85%	80%	80%	35%

Table 1: The percentage of simulations for a given number of sellers and T_{coef} that ended in a 3-fold repetition

- **Less repetition for more sellers and higher T_{coef}.** The system terminates in a repetition less often with a higher number of sellers. This can be indicative of the more complex nature of the system that takes longer to stabilize. At a low transport cost, the buyers care less about the distance and more about the price, so the sellers are encouraged to move around more to compete for territory. They quickly reach an equal distribution of land that resembles a Voronoi diagram and converge to a stable state.
- **Higher T_{coef} means higher price of equilibrium.** As expected, with a higher T_{coef}, the transport costs begin to outweigh the price costs. The sellers quickly adapt to this change and begin charging higher prices.
- **No fixed equilibrium.** Although, most of the simulations ended in a repeated state, these states are not fixed. They would often be disturbed by a small perturbation and return back to the equilibrium state after some time.

[1] https://github.com/Pylyr/Collective-Intelligence.git

- **Equilibrium is aperiodic.** The repetition between equilibria states did not happen at regular intervals. In fact, with more sellers and a higher T_{coef}, the longer and more varied the period between repetitions becomes. This is because any positional change at high transport cost can have a big impact on the cost function of the buyer and with more sellers the impact of a single buyer switch is amplified. Both of these factors make the equilibria more fragile.

- **Multiple equilibria.** If one lets the simulation run longer, one can sometimes see the sellers organize themselves in a seemingly stable manner, only to change to a completely different equillibrium later. The agents would cycle between some semi-stable states before moving to the next one. This origin of this phenomenon is unknown, but it was observed previously in [2]. Indeed the authors of this paper claim that a lot of environments under Hotelling's assumptions do not have a unique equilibria and some do not have an equilibria at all.

- **At most pairwise clustering.** Hotelling's original hypothesis was that all sellers would cluster together. In reality, a close (< 3 pixels) clustering only occurs between at most two sellers. This is clearly visible on Figure 1 in a simulation with 10 sellers. What controls the emergence, distribution and quantity of such pair clusters remains unknown.

- **Monopolists have a higher price.** As expected, the "monopolists", i.e. sellers that did not pair with another seller in a cluster, abuse their market power and charge a higher price. On Figure 1 in a simulation with 3 sellers, the two clustered sellers charge a price of 3, whereas the monopolist on top charges 11.

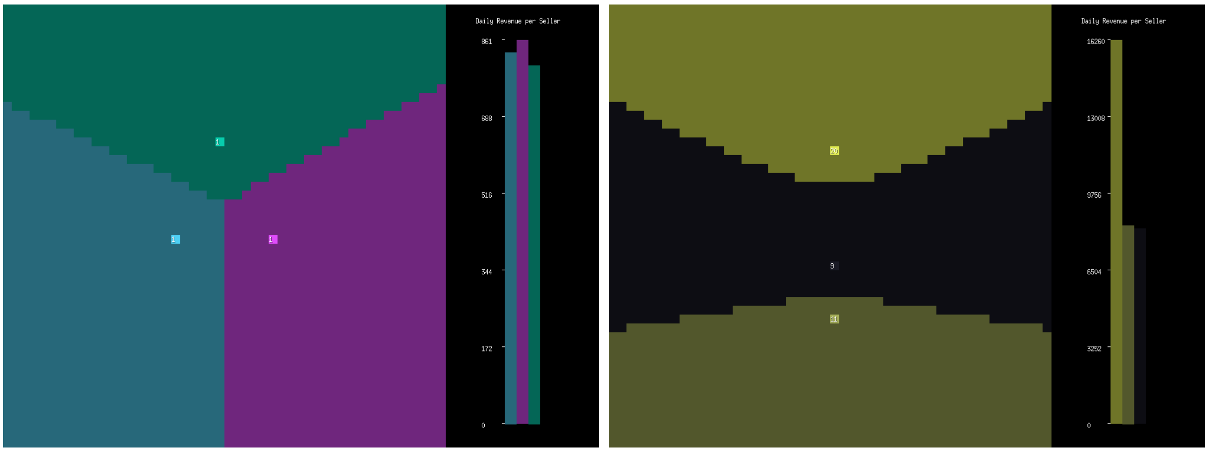

Figure 2: Two possible equilirbia in a 3-seller model. Left: $T_{coef} = 0.1$. Right: $T_{coef} = 2$

- **Sensitive to initial conditions.** Even though the model is fully deterministic, a slight change to the initial conditions can drastically affect the outcome of the model. This was manually tested with some initial configurations. The results would also deviate fairly quickly (within the first 20 iterations)

- **Different equilibrium shapes at different T_{coef}.** Even for simulations with 3 sellers, the shape of the equilibrium is different based on price sensistivity. In Figure 2, on the left a triangle-type equilirbium is shown, which is typical of a low T_{coef}. The sellers cannot differentiate themselves by providing a service that is more conviniently placed to the buyer – the buyers do not care about the distance all that much. Since the sellers are essentially identical, they arrange themselves in a stable equilibrium and get an equal share of the overall revenue. On the right, a similar simulation is shown, only the T_{coef} is higher. The top seller found a convenient place where he can sell to the top third of the grid and charge a much higher price. The other two sellers are clustered together and are forced to lower their prices due to competition. The monopolist on top reaps the highest revenue.

- **Distribution of revenue is not equal.** As evidenced in both Figures 1 and 2, the sellers can have very different revenues. There is no inherit force to the system that tries to equalize the revenue.

4 Conclusion and Further Work

In this paper, we have extended Hotelling's model to a multi-seller environment where sellers adapt their location and prices to maximize revenue. Our simulations showed various emergent behaviors, including the emergence of multiple semi-stable equilibria. The results contradict Hotelling's original hypothesis that sellers always converge to a big single cluster, demonstrating instead a more complex behavior with pairwise clustering and monopolistic pricing in some cases.

Despite the obtained insights, there is still much to explore. Future research could investigate whether social optimum or buyer satisfaction is achieved and how these metrics evolve over time. Incorporating buyer welfare into the model would provide a more comprehensive understanding of the trade-offs between seller strategies and market efficiency.

Additionally, [2] proposes to explore alternative cost functions, such as quadratic transport costs instead of linear, to better reflect real-world buyer behaviors. Introducing regulatory mechanisms, such as price ceilings or penalties for excessive price fluctuations, could shed light on how market interventions affect stability and seller behavior.

Another promising direction is to introduce movement costs for sellers, which would more realistically simulate the inertia and friction businesses face during relocation. Similarly, experimenting with varying levels of seller aggressiveness in price adjustments could show how different competitive strategies impact market dynamics.

Lastly, further work is needed to develop better methods to study equilibrium states. Relying solely on repeated states as a sign for equilibrium may overlook more nuanced patterns of convergence.

By addressing these potential extensions, future research can build on the foundations laid by this study, providing deeper insights into spatial competition and its implications for real-world markets.

References

[1] Harold Hotelling. *Stability in Competition on JSTOR — jstor.org*. `https://www.jstor.org/stable/2224214`. [Accessed 05-01-2025]. 1929.

[2] Ester Iglesias Mas´o. *Spacial Competition Models*. `https://diposit.ub.edu/dspace/bitstream/2445/177186/2/177186.pdf`. [Accessed 14-01-2025]. 2020.