The SKEW Index: Extracting What Has Been Left*

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Abstract

This study disentangles a measure of implied skewness that is related to downward movements in the U.S. equity index from the corresponding implied skewness that is associated with upward movements. A positive SKEW index is constructed from S&P 500 call options, whereas a negative SKEW index is constructed from the S&P 500 put options. We show that the positive SKEW is linked to market sentiment, whereas the negative SKEW is related to existing tail risk measures. The negative SKEW is proposed as a more objective prudent tail risk measure, and it is found to be able to predict recessions, market downturns, and uncertainty indicators up to one year in advance. The predictive power of the negative SKEW is also confirmed when we control for other tail risk measures and also out-of-sample.

Keywords: Market Downturns, Recessions Predictability, Market Sentiment, Tail Risk.

JEL: G13,G15,C12

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1 Introduction

Is an implied skewness index able to enhance the information set revealed by other financial risk and volatility measures? Is there anything *left out* that can be useful to investors from the decomposition of the implied skewness index into directional components? Are the decomposed implied skewness indexes able to predict financial and macroeconomic downturns?

In this paper we show that a more refined directional construction of the implied skewness enriches the information that is extracted from the US equity index option prices. In order to conduct our analysis, we first construct a model-free measure of implied skewness following the methodology of Bakshi et al. (2003), and then we decompose it into its *positive* (SKEW$^+$) and *negative* (SKEW$^-$) components associated with only calls and only puts, respectively. We identify different sets of information associated with the two decomposed implied skewness measures. Our analysis shows that SKEW$^+$ captures market sentiment, whereas SKEW$^-$ is associated with features that are related to tail risk. The latter measure is shown empirically to predict recessions and market downturns well.

The usefulness of financial volatility, in particular of risk measures extracted model-free from options, has been highlighted in the literature (Bakshi and Madan, 2000; Jiang and Tian, 2005; Bakshi and Madan, 2006). Du and Kapadia (2014) pointed out that volatility indexes underestimate real stock market volatility when the period under scrutiny is bearish and jumpy. The 2008 financial crisis has intensified even more the interest in tail events and outliers. Barberis (2013) advocated that investors consistently over-estimate extreme events when they have memories of such events still present in their mind. However, when similar tail events have never taken place before, investors can underestimate the likelihood of a particular type of extreme event. In order to gauge tail risk adequately, implied skewness measures can be used to detect information that is not captured by the implied volatility. While VIX is the market measure that reflects the "likely", the SKEW measure is shown in our paper to be more related to extreme market movements that reflects the "unlikely".

Different measures and methodologies have been proposed to measure tail risk in the equity market (see Bollerslev and Todorov, 2011; Du and Kapadia, 2014; Kelly and Jiang, 2014; Almeida et al., 2017; Gao et al., 2018). Bollerslev and Todorov (2011) proposed a new tail risk measure, called the Investor Fears Index (FI) by decomposing the variance jump risk premium into a positive and a negative component and interpreting the difference between those two as a measure of investor fear. Du and Kapadia (2014) put forward the Jump and Tail Index (JTIX) derived from high moments such as skewness and kurtosis of returns and jumps distribution. Kelly and Jiang (2014) developed a measure of tail risk called TAIL Index under the assumption that the tail distribution is a time varying power law probability distribution. A similar approach in Almeida et al. (2017) considers a tail risk measure based on cross-sectional portfolio returns' expected shortfall (ES). The Chicago Board Options Exchange (CBOE) proposed a SKEW Index computed from the S&P 500 options following the methodology outlined in Bakshi et al. (2003). Gao et al. (2018) developed in a model-free manner a measure of global ex-ante tail risk concern (GRIX) that is constructed for multiple global assets. The tail risk literature and the higher risk-neutral moments literature are now intertwined.[1] The seminal papers by Bakshi and Madan (2000), Bakshi et al. (2003) and Bakshi and Madan (2006) opened a new area of research focused on extracting valuable information about moments of the risk-neutral distribution from option prices.[2]

In general, there is mixed evidence in the literature about the relationship between volatility and skewness, depending on the method of calculation, historical vs. forward-looking measures, and on the sample period. Dennis and Mayhew (2002) found a positive relationship between skewness and volatility by studying individual stock options, Han (2008) detected a negative relationship between risk-neutral skewness and volatility, while Conrad

[1]The literature on skewness is dichotomized by the method of calculation. First, there is the raw approach applied by Dennis and Mayhew (2002), Conrad et al. (2013) and Bali and Murray (2013), using the data without filtering or modification. The second approach works with smoothed data (see Hansis et al., 2010), either by data interpolation between OTM puts with lowest strike and OTM calls with highest strike or by data extrapolation between the highest and lowest strike prices. Another methodology that has been applied to calculate the skewness is the non-parametric approach applied by Xing et al. (2010) and Mixon (2011).

[2]The Bakshi et al. (2003) methodology has been widely used in the financial literature, (e.g. Rehman and Vilkov, 2012; Conrad et al., 2013; DeMiguel et al., 2013; Neumann and Skiadopoulos, 2013; Christoffersen et al., 2018).

et al. (2013) argued that there is no significant relationship between the two. Recently Liu and Faff (2017) presented empirical evidence of a negative relationship between SKEW and VIX, and contributed to the understanding of the behaviour of the SKEW index with respect to asset pricing. By analyzing its relationship with the VIX index and daily market returns, they found that the SKEW index not appearing in line with common asset pricing and skewness literature theory and hypotheses.[3]

Drawing from one of the main conclusions in Liu and Faff (2017), we believe that the total SKEW index contains a mixed set of information, from both the SKEW$^-$ and SKEW$^+$ components, that should be disentangled in order to be better understood with respect to asset pricing implications. Our research goes beyond the relationship between the SKEW index and VIX, and market returns predictability, since we hypothesize that, in spite of the SKEW characteristics and its weak (or negative) relationship with implied market volatility, the SKEW index components contain additional information to the common risk measures extracted from the options market.

To the best of our knowledge, we are the first to investigate the usefulness of the decomposed SKEW index as a market sentiment benchmark, as a tail risk measure, and as a financial stability tool. We study the relationship between the two disentangled components of SKEW and market downturns, recessions and macro indicators predictability, both in-sample and out-of-sample. We find that SKEW$^-$ is a strong predictor of these and therefore very useful from a financial stability point of view, as a tail risk indicator, providing complementary information to VIX.

Policy makers are actively searching for signals of future financial and economic instability, and consequently they are interested in developing policy tools with which to predict or mitigate these outcomes (see Danielsson et al., 2018). While the role of volatility as a

[3]Common theory would suggest that if volatility increases, then return distribution should become more negatively skewed, contributing to higher fear, market downturns and higher negative skewness. Liu and Faff (2017) found the contrary chain of implications for the SKEW index. They also proposed an alternative measure, called SIX, that provides more meaningful information than SKEW in terms of its relationship with VIX and market returns, showing that the theoretical implications are indeed satisfied for SIX.

crisis predictor has been tested in the literature (with mixed results), we demonstrate that SKEW$^-$ can play an important role as a crisis indicator, predicting recessions as well as market downturns up to one year. A decrease in SKEW$^-$ leads to an increase in the probability of a recession, echoing the overoptimism rationale discussed by Brunnermeier and Sannikov (2014), Bhattacharya et al. (2015), and Danielsson et al. (2018).

We find that SKEW$^+$ is possibly playing a dampening role regarding the real level of risk expressed purely by the negative part (SKEW$^-$). Therefore, considering SKEW$^-$ extracted from out-of-the-money (OTM) puts may help investors to better gauge tail risk and avoid under-estimating it. We advocate using SKEW$^-$ as a relevant measure of tail risk, focusing on the left side of the risk-neutral distribution and disregarding investors' beliefs implied from calls that are more attached to market sentiment.

We present empirical evidence to demonstrate that the newly proposed decomposed SKEW indexes contain an additional set of information compared to the total SKEW. The positive implied skewness measure can capture investors' ex-ante *perception* of speculative activity on the calls side, whereas the negative implied skewness measure can capture investors' ex-ante *perception* of tail risk and recessions on the puts side. Considering the latter in isolation from SKEW$^+$ allows us to obtain a more prudent candidate as a tail risk measure that is separated from the more sentiment-driven component.

The remainder of this paper is organized as follows. Section 2 briefly describes the methodology for decomposing the implied skewness and the data used in the study. Section 3 positions the decomposed implied skewness measures within some of the already existing financial risk, volatility and tail risk measures. Section 4 makes a connection between the implied skewness indexes and various sentiment proxies. Section 5 discusses the predictive power of the decomposed SKEW indexes in relation to recessions and market downturns. In section 6 we study the link with uncertainty measures and other macroeconomic variables whilst in section 7 we report more robust empirical confirmation on the usefulness of our measure on an out-of-sample basis. The last section concludes the paper.

2 Extracting the Positive and Negative Skewness

Du and Kapadia (2014) argued that the VIX seems not to take into account stock return jumps, missing relevant information for investors during these situations. They argue that the Bakshi et al. (2003) methodology is more accurate, on average, for computing the second moment of S&P 500 returns, especially, during volatile times. We employ the results from Bakshi and Madan (2000) and Bakshi et al. (2003) for extracting implied volatility and skewness from options prices in a model-free manner. More details of the computation of the SKEW index and its decomposition are discussed in the Technical Appendix.

The total SKEW index is decomposed into two components: a positive SKEW computed only from S&P 500 calls, defined as SKEW^+ and a negative SKEW index computed only from S&P 500 puts, defined as SKEW^-. In practice, in the formulae (14) to (16) in the Technical Appendix, for SKEW^+ we keep only calls when $K_i \geq K_0$, while for SKEW^- we keep only puts when $K_i \leq K_0$. Applying these model-free methodologies, we obtain three daily index series: SKEW, SKEW^+ and SKEW^-.

2.1 Data and Time Series: Preliminary Findings

Daily S&P 500 index options prices and S&P 500 index prices are collected from OptionMetrics for the time period from January 1996 to December 2017. Daily interest rates for the U.S. are obtained from FRED. Options are filtered leaving out options with bid price equal to zero, options with prices below or above two consecutive zero bid prices and options with less than 2 days before expiration. When less than 2 days are left to the expiration date, the maturity is rolled to the 2nd and the 3rd months options. The option maturity selected changes every month in correspondence to the third Friday of the month.

The selected time period includes two recession periods according to the NBER's definition,[4] one in the aftermath of the dot-com bubble in early 2000 and the other in correspon-

[4] "A recession is a significant decline in activity spread across the economy, lasting more than a few months, visible in industrial production, employment, real income, and wholesale-retail sales" See `http:`

dence to the 2007-2009 global financial crisis (GFC). Our time period also includes other volatile events, such as, the Asian and Russian financial crises, the 9/11 terrorist attack, the Eurozone sovereign debt crisis and the UK Brexit vote.

The total implied skewness, S, is a difference between an equity calls portfolio and an equity puts portfolio, as shown in formula (11) in the Technical Appendix. For the S&P 500 this difference appears to be placed entirely in the left skewed area, which drags the total skewness to the left, as illustrated in Figure 1. Bakshi et al. (2003) argue that for return distributions that are left-shifted, all OTM put options will be priced at a premium relative to OTM calls.[5] The positive S^+ bell is representative for calls investors and it is spread across a positive range. On the other hand, the negative S^- bell extracted from puts extends in a wider negative range area and it is the most dispersed among the skewness distributions. In order to clarify, S, S^+ and S^- refer to the implied skewness measures computed through equation (13) in the Technical Appendix and plotted in Figure 1.

It is worth noting here that following the CBOE (2011) methodology we then transform them into SKEW index time series through $SKEW = 100 - 10S$, see equation (12) in the Technical Appendix. Hence, in this paper the implied skewness index, denoted henceforth by SKEW is based on the rescaling and transformation in (12). Hence, SKEW has the opposite sign of S, so if a marginal effect of S on some variable of interest is significant and positive, our analysis will show a significant and negative marginal effect of SKEW on the same variable. The same considerations apply for S^+ and S^- transformed into SKEW$^+$ and SKEW$^-$, respectively.

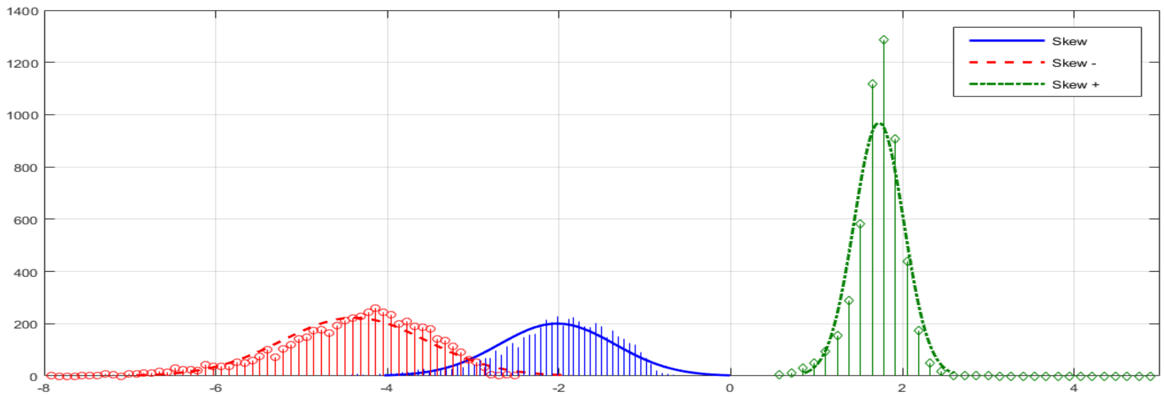
Figure 2 illustrates the historical evolution of the SKEW indexes at daily frequency.[6]

//www.nber.org/cycles/r/recessions.html.

[5] Mitton and Vorkink (2007) advocated that returns of under-diversified portfolios are more positively skewed than those of diversified portfolios. According to them, diversification is a two edged sword: it eliminates undesired volatility but, at the same time, it eliminates also desired skewness. The latter is the right skewness that is reduced when stocks are added, so when diversification is amplified the stocks are contributing in a negative way to the portfolio co-skewness.

[6] In Figure 2 our series are shown at daily frequency to better highlight their trends and events during the time period. Following standard practice in the literature, in the rest of the paper, we rely on monthly frequency, end-of-the month, for ease when compared with other risk measures and macroeconomic variables.

Figure 1: Skew, Skew$^-$ and Skew$^+$ Relationship

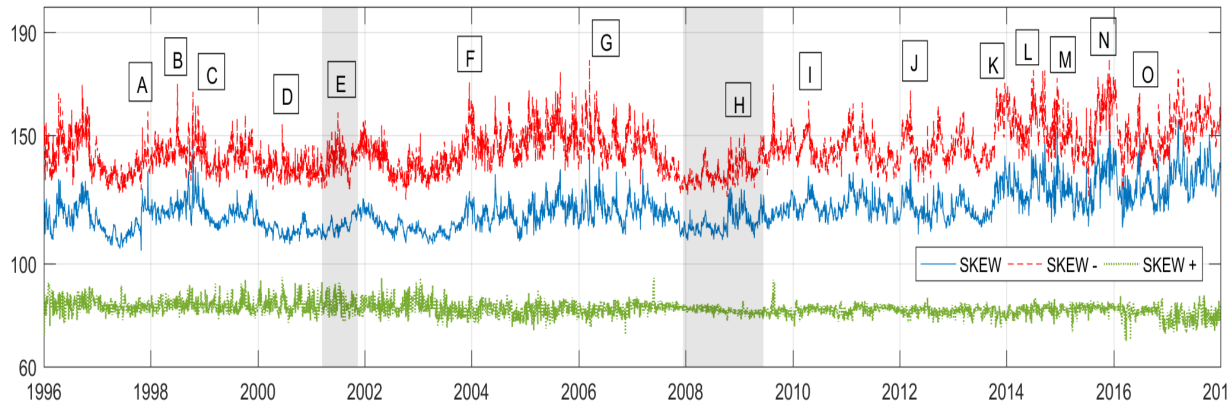


Notes: The figure illustrates the decomposed implied skewness distributions. The total implied skewness is computed using equation 13 in the Technical Appendix. The distribution of Skew$^-$ is computed from the same equation by inputting puts only, while the distribution of Skew$^+$ by inputting calls only. The selected period is from 04-01-1996 to 29-12-2017, at daily frequency.

SKEW$^-$ plays a major role in driving the evolution of the SKEW series, and it always lays above the total SKEW, while the role of SKEW$^+$ is usually more marginal, becoming important only occasionally. SKEW and SKEW$^-$ appear to react in a similar way to events such as the Asian financial crisis, the LTCM collapse, the Iraq invasion, the U.S. housing market bubble in 2006, Lehman Brothers failure in September 2008, during the two stages of the Eurozone sovereign debt crisis, the Chinese Yuan crisis and UK Brexit vote. Even though SKEW and SKEW$^-$ show a similar trend, the latter seems more sensitive to unpredictable events, as reflected by the event outliers marking the increased investors' tail risk perception such as the Russian crisis in August 1998, the dot-com bubble burst in March 2000, the 9/11 terrorist attack, the aftermath of the 2008-2009 recession period, and the Eurozone sovereign debt crisis inception.

The SKEW and SKEW$^-$ levels were low during the 2007-2009 GFC, but higher and more volatile before. SKEW$^-$ reached one of its highest levels in March 2006, underlying a period of increasing concern due to the U.S. housing market bubble and its possible bursting preceding the sub-prime crisis. This behaviour highlights the forward-looking character of

Figure 2: **Decomposed SKEW Indexes**



Notes: This figure shows the total and the decomposed SKEW indexes, namely, SKEW, SKEW$^-$ and SKEW$^+$, computed through equation (12) in the Technical Appendix: $SKEW = 100 - 10S$. NBER recession periods are highlighted in grey. Some of the main events along the selected time period are also highlighted with letters: [A] Asian Financial Crisis [B] Russian Financial Crisis [C] LTCM Collapse [D] Dot-com Bubble Burst [E] 9/11 Terrorist Attack [F] Iraq Invasion [G] U.S. Housing Market Bubble [H] Lehman Brother Failure [I] [J] First and Second Stage Eurozone Sovereign Debt Crisis [K] Syria War Escalation [L] Ukraine-Russia Conflict [M] ISIS Escalation [N] Chinese Yuan Crisis [O] UK Brexit Vote. The selected period is from 04-01-1996 to 29-12-2017, at daily frequency.

our measure computed from put options prices, hence tracking more closely investors' fear of future extreme negative outcomes. The absence of an increase in tail risk during the recent financial crisis, although surprising, is consistent with the idea that the GFC was characterized by a soaring volatility which was, however, predictable over short horizons by standard volatility forecasting models (Brownlees et al., 2011; Kelly and Jiang, 2014).

SKEW appears to be also influenced by the SKEW$^+$ index which shows its effect mainly in the pre financial crisis period and during the optimistic time of the dot-com bubble possibly contributing in dragging down the total SKEW index.[7] Therefore, in order to measure tail risk we suggest using the other component, SKEW$^-$, as a promising candidate that is cleared from the information coming from the calls side representing the optimistic views. This new measure of tail risk can be considered a prudent forward-looking measure of extreme market

[7]While there is evidence in the literature concerning the key role of the equity index OTM puts as a shelter against equity market drops (see Dennis and Mayhew, 2002; Han, 2008; Bondarenko, 2014), the role of the equity indexes OTM calls is less studied and is more commonly associated with optimistic beliefs (e.g. Buraschi and Jiltsov, 2006).

fear, tail risk *perception* and investors hedging willingness. We investigate the role that all three SKEW indexes play in relation to other risk and tail risk measures in section 3, and the relationship between the SKEW indexes and market sentiment in section 4.

3 Positioning the SKEW Indexes in Relation to Other Risk Measures

In this section we position the newly decomposed implied skewness indexes alongside some of the already existing financial risk measures and tail risk measures. We describe in greater depth some of the properties displayed by the decomposed SKEW indexes and compare them with other risk measures that are more common in the financial literature.

3.1 Comparisons with Volatility and Risk Premium Measures

In order to facilitate a more like-for-like comparison we also decompose the VIX into its positive and negative components following (Kilic and Shaliastovich, 2019; Bevilacqua et al., 2019). We apply the same OTM options selection and filter rules as in section 2. See also Section A.3 in the Technical Appendix for more details.

VIX changes are driven more by the negative volatility component (S&P 500 puts) than by the positive volatility component (S&P 500 calls). VIX^- has a prevalent role in the total VIX, in line with the rationale in Bollen and Whaley (2004) and Ang and Bekaert (2006), suggesting that investors weigh downside losses differently from upside gains. VIX^- drives the dynamics of VIX, especially at times when put options are more in need for hedging strategies, and it can be considered a proxy of downside risk. During those negative times, the S&P 500 puts are more expensive than the S&P 500 calls (Bondarenko, 2014) and they are more in demand. VIX^+ reflects news that impacts underlying assets in a positive way (Segal et al., 2015) and which can even be interpreted as “euphoria” (Bollerslev et al., 2015). The decomposed SKEW indexes are also compared with the decomposed good and bad variance risk premia by Kilic and Shaliastovich (2019). The good variance risk premium predicts future asset returns with a positive sign, whereas the bad variance risk premium predicts future asset returns with a negative sign.[8]

Table 1 shows the correlation analysis among the three decomposed SKEW indexes, the three decomposed VIX indexes, and the three decomposed VRP^{+}. Our correlation analysis confirms the conclusions reached by Han (2008), Conrad et al. (2013) and Liu and Faff (2017). There is a negative relationship between VIX and SKEW (-23%) and also between VIX and SKEW^{-} (-49%). When market volatility is high, SKEW^{-} is reduced. There is a positive relationship between VIX and SKEW^{+}. When VIX spikes, SKEW^{+} does the same. The high correlation coefficients among the three decomposed VIX indexes, as well as among the three variance risk premia are in line with the findings of Kilic and Shaliastovich (2019). A positive correlation between the implied volatility indexes and risk premia is also detected. Therefore, it appears that the SKEW indexes contain additional information that is not enclosed in the other financial risk measures. Furthermore, the various decomposed SKEW indexes appear to contain different information from each other.[9]

Liu and Faff (2017) argued that the total SKEW index does not show empirically valuable information linked to VIX, and this contradicts general asset pricing theory which would suggest an increase in financial market volatility being associated with a more negative skewness (higher SKEW index). Hence, motivated by this important finding, we further

[8] They defined the total variance risk premium as total vp, the good variance risk premium as vp good and the negative variance risk premium as vp bad. Their series are available at `https://sites.google.com/view/metekilic/`. We slightly change the notation for consistency with the other decomposed measures, referring to the vp total as VRP, to the vp good as VRP^{+} and to the vp bad as VRP^{-}.

[9] We also compare the SKEW indexes with the measures of implied and realized correlation, and the correlation risk premium by Driessen et al. (2009) computed from options with a 1-month maturity. We thank Grigory Vilkov for kindly sharing the correlation measures at: `http://www.vilkov.net/index.html`. The implied correlation is extracted from the index options price that corresponds to the price of individual stock options. An increase in the price of index options leads to an increase in the implied index variance, in turn, raising the implied correlation. Conversely, an increase in the prices of individual stock options leads to an increase in the implied stock variances, leading to a lower implied correlation (see Buss et al., 2018). The correlation risk premium at time t is defined as the difference between the risk-neutral measure and physical correlation measure: $CRP_t = IC_t - RC_t$. The correlation between the decomposed VIX series and IC is also positive, findings that are in line with the correlation analysis conducted by Buss et al. (2018). Low or even negative correlation is instead found between the decomposed SKEW indexes and the correlation measures, therefore sharing a different set of information.

analyze the decomposed SKEW index in order to investigate first, the different components' information content and, second, the usage of SKEW$^-$ as tail risk indicator rather than as an alternative to the VIX index.

Table 1: **Decomposed SKEW and Other Risk Measures: Correlation Analysis**

	SKEW	SKEW$^+$	SKEW$^-$	VIX	VIX$^+$	VIX$^-$	VRP	VRP$^+$	VRP$^-$
SKEW	1								
SKEW$^+$	-0.20***	1							
SKEW$^-$	0.78***	0.03	1						
VIX	-0.23***	0.05	-0.49***	1					
VIX$^+$	-0.37***	0.14**	-0.52***	0.97***	1				
VIX$^-$	-0.14**	-0.02	-0.46***	0.98***	0.93***	1			
VRP	0.01	-0.07*	-0.07*	0.28***	0.23***	0.31***	1		
VRP$^+$	0.02	-0.08*	-0.14**	0.45***	0.39***	0.48***	0.96***	1	
VRP$^-$	0.01	-0.05	-0.02	0.08*	0.05	0.09*	0.96***	0.86***	1

Notes: This table presents the correlation analysis between the total and decomposed skewness indexes (SKEW, SKEW$^+$ and SKEW$^-$), implied volatility indexes (VIX, VIX$^+$ and VIX$^-$), and variance risk premia (VRP, VRP$^+$ and VRP$^-$). The comparison is shown for the common time period among the selected measures, namely from 01-1996 to 08-2014, at monthly frequency. Significance levels: $*\quad p < 0.1$, $**\quad p < 0.05$, $***\quad p < 0.01$.

3.2 Comparison with Other Tail Risk Measures

In this section, we compare the SKEW measures with some of the tail and financial risk measures proposed in the literature, namely, the Kelly and Jiang (2014) tail risk measure, the Almeida et al. (2017) nonparametric tail risk, the Allen et al. (2012) aggregate systemic risk measure (CATFIN) and the CRASH index.

The Kelly and Jiang (2014) measure is computed from a cross-section of equity returns assuming that the lower tail distribution of asset return i at time t follows:

$$P_t(R_{i,t+1} < r \mid R_{i,t+1} < u_t) = \left(\frac{r}{u_t}\right)^{\frac{-a_i}{\lambda_t}} \tag{1}$$

where $P_t(\cdot)$ is the conditional probability on the information set F_t at time t, $r < u_t < 0$, λ_t is the time-varying tail exponent estimated by the Hill estimator: $\lambda_t^{Hill} = \frac{1}{K_t}\sum_{k=1}^{K_t} \ln\left(\frac{R_{k,t}}{u_t}\right)$, $R_{k,t}$ is the k-th return below an extreme value threshold u_t in month t, and K_t measures how many times such exceedance occurs in month t across firms conditioned to F_t.

The Hellinger nonparametric tail risk (Almeida et al., 2017) is also based on a cross-sectional approach on assets' returns. Specifically, it is based on the risk-neutral excess ES of portfolios returns being the aggregate tail risk the average of the single portfolio excess ES defined for every asset i as follows:

$$TR_{i,t} = E^{Q}[R_{i,\tau} - VaR_{\alpha}(R_{i,\tau})|R_{i,\tau} \leq VaR_{\alpha}(R_{i,\tau})], \tag{2}$$

where t is the selected month for the tail risk calculation, τ is the possible state of nature, Q is the risk-neutral density (RND) over the return space R, and α is the VaR threshold. The Hellinger tail risk is computed with five principal components from the 25 Fama and French Size and Book-To-Market portfolios, employing a Hellinger RND given by $\gamma = -0.5$.

The macro index of systemic risk, CATFIN, by Allen et al. (2012) is constructed using value-at-risk (VaR) and ES methods estimated using both nonparametric and parametric approaches. The parametric distributions to estimate the 1% VaR and the 1% ES are the generalized Pareto distribution (GPD) and the skewed generalized error distribution (SGED). The nonparametric methods are measured as a cut off point of the left tail lower one percentile of the monthly excess returns for the VaR and as an average of the extreme financial firms returns beyond the 1% nonparametric VaR. CATFIN is then constructed as an average of the three VaR and ES measures. The CRASH index measures the probability of another catastrophic stock market crash in the U.S. in the next 6 months with regards to institutional investors' expectations of a crash.[10] Table 2 shows the correlation analysis among the selected measures.

Our results indicate that the well-known Kelly and Jiang (2014) TAIL index is linked the most, among the SKEW indexes, with SKEW^{-}, sharing a positive correlation (18%). As in Kelly and Jiang (2014) our results indicate that when tail risk increases, the risk-neutral

[10] The Crash index is available at: `https://som.yale.edu/faculty-research/our-centers-initiatives/international-center-finance/data/stock-market-confidence`. The questions that this index answers is the following: *What do you think is the probability of a catastrophic stock market crash in the U. S., like that of October 28, 1929 or October 19, 1987, in the next six months, including the case that a crash occurred in the other countries and spreads to the U. S.?*

Table 2: **SKEW Indexes and Other Tail Measures: Correlation Analysis**

	SKEW	$SKEW^+$	$SKEW^-$	TAIL	HELLINGER	CATFIN	CRASH
SKEW	1						
$SKEW^+$	-0.19***	1					
$SKEW^-$	0.75***	0.04	1				
TAIL	0.02	0.01	0.18***	1			
HELLINGER	0.01	-0.03	-0.20***	-0.46***	1		
CATFIN	-0.06	-0.02	-0.33***	-0.55***	0.56***	1	
CRASH	0.01	-0.02	0.22***	0.13**	-0.27***	-0.50***	1

Notes: This table presents the correlation analysis between SKEW, $SKEW^-$ and $SKEW^+$ and other tail risk and financial risk indexes. TAIL is the Kelly and Jiang (2014) TAIL index (01-1996 to 07-2014), HELLINGER is the Almeida et al. (2017) Hellinger tail risk (01-1996 to 04-2014), CATFIN is the Allen et al. (2012) catastrophic financial systemic risk (01-1996 to 09-2016). CRASH index is from the Yale School of Management. We conduct the comparison for the common period among the selected measures, namely, between 01-1996 and 04-2014, at monthly frequency. Significance levels: $*\quad p < 0.1$, $**\quad p < 0.05$, $***\quad p < 0.01$.

market return distribution becomes more negatively skewed (thus $SKEW^-$ increases). We adopt the TAIL as a benchmark for sign, and we build on this finding to compare also the correlation analysis between the SKEW measures and other tail risk indicators. $SKEW^-$ is negatively correlated with the Hellinger tail risk and Allen et al. (2012) CATFIN. This can be justified following the interpretation in Almeida et al. (2017) explaining the negative correlation between the Hellinger tail risk and the TAIL (see also Table 2). Thus, in the construction of TAIL, Kelly and Jiang (2014) use the whole individual raw returns below the 5% threshold, while in the Hellinger tail risk 25 portfolios sorted by Size and Book-to-Market are used. Since the Hellinger tail risk is based on the excess ES, this gives values which are, most of the time, negative, therefore explaining the negative correlation we find with $SKEW^-$. If taken in absolute value this would suggest a strong connection between the two. The same rationale holds for CATFIN since this measure is also computed as an average of VaR and ES measures. In fact, the high level of correlation between Hellinger tail risk and CATFIN can be explained on the fact that both measures are based on VaR and ES methodologies. The same $SKEW^-$ appears the most correlated with the CRASH index (22%), thus sharing some similar information on catastrophic stock market risk. On the other hand, we find evidence of lack of correlation between our series, SKEW and $SKEW^+$, and all the tail risk measures adopted here.

Overall, despite the correct direction in correlation, $SKEW^{-}$ and the other tail risk measures do not appear to share high correlation coefficients. One possible explanation may be that the forward-looking measures of tail risk extracted from options are distinct from measures of tail risk built from historical data. The forward-looking measures of tail risk reflect investors' ex-ante subjective beliefs about tail risk (Gao et al., 2018). However, the results in Table 2 confirm that $SKEW^{-}$ is the closest to other tail risk measures, showing the exact correlation sign with respect to the tail risk measures selected, and hence proposed as a tail risk measure in this paper. $SKEW^{-}$ may also be considered a measure of forward-looking tail risk in the financial markets, complementing historical measures (e.g. Kelly and Jiang, 2014) associated with the tail risk from the ex-post realizations of tail events.

For the sake of completeness, we also report the correlation between the decomposed VIX and VRP measures, and the tail risk measures in Table A.1 in the Online Appendix. We find an overall higher correlation between the risk measures and the tail risk measures, but this being in the opposite direction compared to the tail risk benchmark we adopt, namely TAIL. For instance, an increase in the VIX is associated with a decrease in the TAIL or in CRASH. In line with Allen et al. (2012), we find a positive correlation between CATFIN and the volatility measures, due to the fact that CATFIN can be considered as a systemic risk measure, and a positive correlation also between HELLINGER and the volatility measures.

The comparative exercise in this section shows that $SKEW^{-}$ appears to be the SKEW component more connected to tail risk, sharing a positive sign and correct direction with the other tail risk measures proposed. In addition, the relative low correlations between $SKEW^{-}$ and other tail risk measures suggest that $SKEW^{-}$ contains a different set of information compared to other tail risk measures in the literature. As a forward-looking measure $SKEW^{-}$ is likely to capture investors' ex-ante belief about tail risk. Hence, we study its predictive ability more specifically in sections 5 and 6.

4 SKEW Indexes and Market Sentiment

Drawing upon the literature that links investor sentiment to stock prices (e.g., Baker and Wurgler, 2006; Stambaugh et al., 2012) and risk-neutral skewness to market sentiment (e.g., Buraschi and Jiltsov, 2006; Han, 2008; Garleanu et al., 2009; Friesen et al., 2012; Lemmon and Ni, 2014), the hypothesis of this first empirical section is that market sentiment does play a role in affecting the price of equity options, thus changing the implied skewness. In order to test this market sentiment hypothesis, we study firstly how the SKEW indexes are related to investor sentiment and, secondly, whether or not the SKEW indexes are able to predict changes in investor sentiment. Index OTM puts are traded as insurance assets against equity market drops (see Bollen and Whaley, 2004; Han, 2008; Bondarenko, 2014) and their trading is driven mainly by hedging demand from institutional investors (Lakonishok et al., 2007). In our view, this opens up a potential market sentiment impact on the other side of the equity options market, the calls side, more commonly associated with speculation and optimistic beliefs (e.g. Buraschi and Jiltsov, 2006).

When investor sentiment is bearish the index option volatility smile is steeper and the risk neutral skewness is more negative, while when investor sentiment is bullish, the volatility smile is flatter and the skewness less negative. The relationship between the skewness and market sentiment was also tested by Dennis and Mayhew (2002), in the form of a link between the skewness and the Put-Call ratio, finding a positive, but not significant relationship between the two. The sentiment proxies we select are;the investor sentiment index by Huang et al. (2015) (ZHOU), the Baker and Wurgler (2006) Sentiment Index (BW), the Put-Call Ratio from CBOE (PCRatio), the AAII American Association of Individual Investors' survey data (AAII), the Confidence Index (CONF) and the Sentiment Index from University of Michigan Consumer Sentiment (SENT).[11]

[11]ZHOU is the investor sentiment index by Huang et al. (2015) available at http://apps.olin.wustl.edu/faculty/zhou/useful_links, BW is the Baker and Wurgler (2006) sentiment index available at http://people.stern.nyu.edu/jwurgler/. The latter applied a principal component analysis to six underlying sentiment proxies, while Huang et al. (2015) improved this measure extracting the most relevant information for the stock returns from the same six sentiment proxies, but discard-

We first test the impact of market sentiment on the SKEW indexes in a contemporaneous framework to grasp a general insight of such relationships. The following regression is run, where the dependent and independent variables are matched for data availability:

$$SKEW_n^i = \alpha + \beta_1 MarkSent_j + \epsilon \quad with \quad i = Tot, +, -. \tag{3}$$

where $MarkSent_j$, is one of ZHOU, BW, PCRatio, AAII, CONF, SENT.

Table 3: **SKEW Indexes on Investor Sentiment**

	ZHOU		BW		PCRatio	
Predictor	Coef	$R^2(\%)$	Coef	$R^2(\%)$	Coef	$R^2(\%)$
SKEW	-0.029**	1.3	-0.019***	4.5	-0.092*	1.3
$SKEW^+$	0.005***	2.6	0.006***	2.7	0.032*	1.5
$SKEW^-$	-0.028**	2.1	-0.015*	1.8	-0.023**	6.2
	AAII		CONF		SENT	
Predictor	Coef	$R^2(\%)$	Coef	$R^2(\%)$	Coef	$R^2(\%)$
SKEW	-0.013***	3.7	-0.012	0.1	-0.051*	0.7
$SKEW^+$	-0.031	0.1	0.020	0.3	0.024***	1.4
$SKEW^-$	0.041*	0.2	-0.084	0.7	-0.019	0.1

Notes: This table presents the results of the OLS contemporaneous regression estimated trough equation 3 between our dependent variables, SKEW, $SKEW^+$ and $SKEW^-$ and the market sentiment indicators as independent variables. The selected sentiment indicators and their available time period are the following: the sentiment index by Huang et al. (2015) (ZHOU) (from Jan 1996 to Dec 2016), the sentiment index by Baker and Wurgler (2006) (BW) (from Jan 1996 to Sep 2015), the Put-Call ratio (PCRatio) (from Oct 2003 to Dec 2017) and the American Association of Individual Investors' survey index (AAII), the Confidence Index (CONF) and the University of Michigan Consumer Sentiment (SENT) (from Jan 1996 to Dec 2017). The data frequency is monthly. Regressions intercepts are not reported to save on space. The regression coefficients and R^2 in percentage are reported. Significance levels: $* \quad p < 0.1$, $** \quad p < 0.05$, $*** \quad p < 0.01$.

The results of the estimated contemporaneous OLS regression are presented in Table 3. We observe a positive impact of the market sentiment proxies on $SKEW^+$ (OTM calls

ing their approximation errors. PCRatio is the Put-Call ratio from CBOE, commonly used as a sentiment proxy (see Dennis and Mayhew, 2002); AAII is the sentiment index from the American Association of Individual Investors's survey (`http://www.aaii.com/sentimentsurvey`). The Confidence Index can be downloaded from: `https://som.yale.edu/faculty-research/our-centers-initiatives/international-center-finance/data/stock-market-confidence` and represents the confidence in the financial market, Dow Jones Industrial, in the following months up to 10 years. The question this index answers is the following - *How much of a change in percentage terms do you expect in the following months/years (fill with +/− before the number to indicate expected increase or decrease?* - This is taken with regards to institutional investors. SENT is the University of Michigan Consumer Sentiment retrieved from FRED: `https://fred.stlouisfed.org/series/UMCSENT`.

trade), with AAII being the only exception. On the other hand, we observe that the total SKEW and SKEW$^-$ are negatively associated with the measures of market sentiment in all the cases and in five cases out of six, respectively, with AAII being the only exception.[12]

Overall, we find that two of the most common sentiment proxies in the financial literature, namely Baker and Wurgler (2006) (BW) and Consumer Sentiment (SENT), show a weak or not significant relationship with SKEW$^-$ associated with a negative sign, while a stronger relationship with a positive coefficient sign is found in the case of SKEW$^+$. This is also emphasized by the greater R^2 in the case of SKEW$^+$. We find evidence that the Put-Call ratio (considered as a proxy of market sentiment and trading pressure) is significant in explaining all three SKEW indexes.

This first glimpse of the relationship between market sentiment and SKEW indexes shows that the regression coefficients of SKEW$^-$ and SKEW$^+$ indexes react in an opposite direction to market sentiment indicators, this reflecting more pessimistic and optimistic views, respectively. We further test this relationship, checking whether the SKEW indexes have some predictive power in relation to the selected sentiment indicator by running the following predictive equation:

$$MarkSent_{t+1,j} = \alpha + \beta_{1,i} SKEW^i + \epsilon_{t+1,j} \quad with \quad i = Tot, +, -. \tag{4}$$

where $MarkSent_{t+1,j}$ is, now, the next month level of one of the sentiment indicators, namely, ZHOU, BW, PCRatio, AAII, CONF, SENT. The results are reported in Table 4.

Our empirical evidence suggests that SKEW$^+$ is the decomposed index with the greatest predictive power in relation to market sentiment. SKEW$^+$ is able to predict market sentiment in four cases out of six, namely ZHOU, BW, AAII, and SENT, showing a higher R^2 compared to SKEW$^-$, the only exception being the put-call ratio. Hence, we can con-

[12]The difference in coefficients' signs for AAII is due to the fact that it is a sentiment proxy from a survey about investors' market opinions (bullish, bearish, neutral) and it is measured as the difference between bearish opinion and bullish opinion. It is also considered as a proxy of institutional investors sentiment due to the panel of interviewed investors.

Table 4: **Market Sentiment and SKEW Indexes: Predictive Regression**

	ZHOU		BW		PCRatio	
Predictor	Coef	$R^2(\%)$	Coef	$R^2(\%)$	Coef	$R^2(\%)$
SKEW	-0.169*	0.7	-0.023***	4.4	-0.002***	2.9
$SKEW^+$	0.691***	1.9	0.040***	2.6	0.005	0.0
$SKEW^-$	-0.039	0.0	-0.010	1.3	-0.002***	6.2
	AAII		CONF		SENT	
Predictor	Coef	$R^2(\%)$	Coef	$R^2(\%)$	Coef	$R^2(\%)$
SKEW	-0.003***	5.9	-0.025	0.1	-0.027	0.0
$SKEW^+$	-0.006***	3.3	0.175	0.4	0.582**	1.3
$SKEW^-$	0.001**	1.3	-0.049	0.3	-0.910	0.3

Notes: This table presents the results of the OLS predictive regression estimated through equation 4 between the independent variables, SKEW , $SKEW^+$ and $SKEW^-$ and the next month level of the selected market sentiment indicators, this time as dependent variables. The selected sentiment indicators and their available time period are the following: the sentiment index by Huang et al. (2015) (ZHOU) (from Jan 1996 to Dec 2016), the sentiment index by Baker and Wurgler (2006) (BW) (from Jan 1996 to Sep 2015), the Put-Call ratio (PCRatio) (from Oct 2003 to Dec 2017) and the American Association of Individual Investors' survey index (AAII), the Confidence Index (CONF) and the University of Michigan Consumer Sentiment (SENT) (from Jan 1996 to Dec 2017). The data frequency is monthly. Regressions intercepts are not reported to save on space. The regression coefficients and R^2 in percentage are reported. Significance levels: $*\quad p < 0.1$, $**\quad p < 0.05$, $***\quad p < 0.01$.

clude that $SKEW^+$ is the component that is influenced by market sentiment the most, and which in turn contains useful information for predicting the future levels of some of the most tracked market sentiment indicators, namely ZHOU, BW and SENT. Conversely, $SKEW^-$ is unable to predict the above market sentiment indicators, showing predictive ability only with respect to put-call ratio and AAII.

This market sentiment asymmetric predictive ability between $SKEW^+$ and $SKEW^-$ may be due to the fact that in high sentiment markets, when the information provided by the tail risk index should be more reliable, OTM index puts are expensive (see Bondarenko, 2014; Han, 2008). For this reason, investors with a bullish expectation in equity markets can go long in OTM index calls. Investors who are "end-users" have a net long position in S&P 500 options (e.g. Garleanu et al., 2009). The correlation signs in Table 1 further highlight the positive relationship between $SKEW^+$ and VIX , and also VIX^+. The former is sometimes used as a sentiment proxy, whereas the latter is a proxy for market exuberance (Segal et al., 2015; Bollerslev et al., 2015). Thus, market sentiment can have an undesired effect on the total SKEW dragging it towards the "bright" side thus creating an *optimistic illusion* carried from investors' positive expectations. We aim to provide further evidence that SKEW$^-$ might be a more prudent and valid tail risk indicator. We check this hypothesis and the predictive power of SKEW$^-$in the next sections.

5 Negative SKEW as an Early Warning Indicator

Previous findings in the paper have presented evidence of SKEW$^-$ as the implied skewness component more connected with tail risk measures in the literature and more reactive to extreme events (see Figure 2). It also appears that SKEW$^-$ is roughly speaking orthogonal to the market sentiment indicators (e.g., Brown and Cliff, 2005; Buraschi and Jiltsov, 2006; Lemmon and Ni, 2014); see the results in section 4. In this section we test whether or not SKEW$^-$ is the SKEW index component that is better able to predict recessions and market downturns, hence being a more suitable early warning indicator compared to the total.

This section contributes to the strand of literature covering the role of options-based measures in predicting macroeconomic downturns or increases in uncertainty (Bakshi et al., 2011; Allen et al., 2012; Bollerslev et al., 2015). We also follow Danielsson et al. (2018), whose main idea was that low volatility periods can indicate a banking crisis, and that during low volatility periods, excessive risk-taking can be associated with investor *overoptimism*. Hence, we study here the predictive power of the SKEW indexes as ex-ante indicators of recessions and market downturns. Since we believe that SKEW$^-$ reflects pessimistic investors' beliefs contained in puts, this measure may be useful for monitoring and anticipating financial crises and market downturns. The testing of SKEW and SKEW$^+$ in terms of predicting these indicators is also reported for comparison.

Economic recessions are marked by NBER, while market downturns are modelled with a dummy variable, indicating 1 for a market downturn equal to or less than 5% and 0 otherwise, similarly to Hameed et al. (2010). The following logistic regression is applied with regard to recession predictability:

$$logit(D_t) = \beta_0 + \beta_1 D_{t-h} + \beta_2 SKEW_{t-h} + \epsilon_t, \tag{5}$$

where D is a generic dummy which represents either the $NBER_{i,t}$ recession indicator[13] or the $MktDown_{i,t}$ binary market drop (5%) variable. SKEW is one of the indexes, namely SKEW, SKEW$^+$ or SKEW$^-$, and we also control for the lags of the endogenous variable (e.g., Almeida et al., 2017), with $h = 1, 3, 6, 12$ months ahead.

As a robustness check we also verify whether or not the decomposed SKEW indexes, with special interest on SKEW$^-$, are still informative in predicting recession and market downturns after controlling for the aforementioned tail risk measures by running the following logistic regression:

$$logit(D_t) = \beta_0 + \beta_1 D_{t-h} + \beta_2 SKEW_{t-h} + \beta_3 TR_{t-h} + \epsilon_t, \tag{6}$$

where everything is as in equation (5) with the addition of tail risk (TR) as control variable, which reflects the information content of the previous h lags of the tail risk measures, namely TAIL, HELLINGER, CATFIN and CRASH, jointly.

The results for recession predictability are reported in Table 5 for $h = 1, 3, 6, 12$ months ahead. The first three columns show the results corresponding to the predictive power of the decomposed SKEW indexes (taken separately) and control only for the h lag of the endogenous variable. The regressions (4) to (6) cover the results when the tail risk (TR) controls are added, while the last column (7) shows the predictive power of the multivariate regression when the three SKEW measures are tested jointly, after testing for multicollinearity issues.

The SKEW and SKEW$^-$ indexes exhibit significant predictive power with respect to recessions at all the horizons up to one year, when considered in separate regressions. SKEW$^+$ shows no predictive power in relation to recessions, the only exception being the

[13]The NBER recession period indicator for the U.S. is a dummy variable taking 1 in recession and 0 in expansion periods according to NBER and it available at: `https://fred.stlouisfed.org/series/USREC`.

12-month horizon. When we control for the information content included in the tail risk measures, the predictive ability of the SKEW measures is again significant. This finding confirms the fact that the decomposed SKEW indexes contain a different set of information compared to tail risk measures proposed in the literature. In addition, the evidence of a strong role played by SKEW^- in predicting recessions is emblematic when the three SKEW indexes are collated. SKEW^- shows a stronger significant predictive ability in relation to recessions; this is emphasized even more by the greater pseudo R^2 found in models including SKEW^- compared to models with SKEW or SKEW^+.

The negative relationship between SKEW^- and SKEW on the one hand, and the NBER recession indicator on the other, deserve more commentary. While the NBER dummy variable is activated when the recession has already started, SKEW^- tracks in a forward-looking way the tail risk perception associated with the likelihood of the recession occurrence. We also estimate the marginal effects (ME) for the logistic regression for every predictive horizon.[14] The estimated marginal effects confirm our hypothesis, namely that SKEW^- plays a stronger role in terms of the probability of a future recession, compared to SKEW and SKEW^+. It is important to notice that since the SKEW indexes are calculated based on formula $SKEW = 100 - 10S$, it can be observed that SKEW indexes go in the opposite direction to the computed implied skewness as depicted in Figure 1.

The negative logistic regression coefficient indicates that as the computed skewness increases, the probability of a recession crisis also increases. We find that a 1% decrease in SKEW^- translates into a 0.49%, 1.02%, 1.67% and 0.92% rise in the probability of a recession in the next 1-, 3-, 6-, and 12-month horizon, respectively.[15] The total SKEW shows a similar trend in the marginal effects, with the same sign detected, but it accounts for only half of SKEW^- marginal effects. Small marginal effects are found for SKEW^+, with the highest level being 0.6% at the 6-month horizon.

[14] The marginal effects of the logistic regression are not shown for the sake of brevity. They are available from the authors upon request.

[15] Note that because of the opposite signs of SKEW vs. *Skew*, our results imply that an increase in *Skew* measures will lead to an increase in the probability of a recession.

Table 5: **Signalling Recessions with SKEW Indexes**

	$h=1$							$h=3$						
$NBER_t$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$SKEW_{t-h}$	-0.322**						-0.119	-0.289**						-0.028
$SKEW^{+}_{t-h}$		0.147					0.510		0.121					0.380
$SKEW^{-}_{t-h}$			-0.415***				-0.588***			-0.302***				-0.347***
$SKEW_{t-h}\|TR$				-0.452*							-0.352**			
$SKEW^{+}_{t-h}\|TR$					0.210							0.149		
$SKEW^{-}_{t-h}\|TR$						-0.602**							-0.293***	
Pseudo R^2 (%)	82.2	77	86.3	84.8	79.9	86.6	87.1	59.0	50.5	62.7	64.6	56.5	64.8	66.1

	$h=6$							$h=12$						
$NBER_t$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$SKEW_{t-h}$	-0.191**						-0.131	-0.133**						-0.086
$SKEW^{+}_{t-h}$		0.022					0.289*		0.214**					0.419*
$SKEW^{-}_{t-h}$			-0.302***				-0.437***			-0.130***				-0.244***
$SKEW_{t-h}\|TR$				-0.233**							-0.167**			
$SKEW^{+}_{t-h}\|TR$					0.033							0.250**		
$SKEW^{-}_{t-h}\|TR$						-0.319***							-0.149***	
Pseudo R^2 (%)	30.4	22.1	42.2	41.2	33.4	49.3	45.1	8.3	7.2	10.3	18.9	18.8	20.5	20.4

Notes: This table presents the results of the logistic predictive regressions estimated through equations 5 and 6 between the SKEW indexes and the NBER dummy variable signalling future recessions. OLS regressions are estimated controlling for the h lags of the dependent variable (columns from (1) to (3)) and also controlling for the all set of tail risk measures (TR), namely TAIL, HELLINGER, CATFIN, and CRASH (columns from (4) to (6)). In the last column (7) we shows the results of the multivariate regression where the three SKEW measures are tested jointly controlling for the lag of the endogenous variable. The results are reported for monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) predictive horizons. Lags of the endogenous variable and SKEW indexes are taken accordingly. The selected period goes from 01-1996 to 12-2017, at monthly frequency. Regressions coefficients and their significance are reported. Regressions intercepts and coefficients of the previous lag of the endogenous are not reported to save on space. The pseudo R^2 (in percent) of every regression is reported in the last row. The significance levels are indicated as follows: * (10%), ** (5%) and *** (1%).

Similar to Danielsson et al. (2018), an *overoptimism* phenomenon may be the cause of our recessions predictability results. This view is consistent with the Brunnermeier and Sannikov (2014) *volatility paradox*: low volatility may actually have an adverse effect on financial stability and it may increase the probability of a systemic event. The volatility in this paradox is the realized volatility reflecting current activities on the market.

When SKEW$^-$ decreases investors take higher risks and enter into riskier positions with the danger of generating a potential economic crisis, thus recession. This is in line with the model put forward by Bhattacharya et al. (2015), in which agents upgrade their optimistic expectations during good times, leading to an increase in risk-taking. Following this intuition, a decrease of 1% in SKEW$^-$ translates into a 1.67% increase in the probability of a recession in the next 6 months.

An example of this is the emblematic drop in SKEW$^-$ that started in 2006/2007, prior to the GFC. We find graphical evidence of SKEW$^-$ shifting from a high to a low value before the GFC and subsequent recession. We apply a Hodrick and Prescott (1997) (HP) filter in order to decompose SKEW$^-$ into trend and deviation from trend. The smoothing parameter (λ) is set at 5000 (Danielsson et al., 2018). We also check the delta of the marginal effects calculated firstly before 2006 and secondly including the GFC, and notice an increase in the ME. Whereas before 2006 the estimated 1-month logistic regression output shows a SKEW$^-$ β coefficient equal to -0.23 with a ME equal to -0.43%, when the downward slope (2006–2008) is included in the logistic regression, the β coefficient becomes equal to -0.42 with a ME equal to -0.85%, thus signaling a double probability of a recession. Similar results are found with the 3-month horizon prediction, and an even higher ME increase is detected with the 6-month horizon from -0.51% to 1.13%. Comparative results for SKEW also show an increase in the ME from 2006 to 2008 for all the horizons, albeit smaller, while SKEW$^+$ ME is even found to decline. This exercise further reveals that SKEW$^-$ tracks the perception that is associated with the likelihood of a recession in a forward-looking way.

Table 6: **Signalling Market Downturns with SKEW Indexes**

	$h=1$							$h=3$						
$MktDown_t$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$SKEW_{t-h}$	-0.062*						-0.021	-0.095*						-0.106
$SKEW^{+}_{t-h}$		0.016					0.008		0.056					0.001
$SKEW^{-}_{t-h}$			-0.084***				-0.104*			-0.056**				-0.002*
$SKEW_{t-h} \vert TR$				-0.055							-0.084*			
$SKEW^{+}_{t-h} \vert TR$					-0.010							0.046		
$SKEW^{-}_{t-h} \vert TR$						-0.063*							-0.017*	
Pseudo R^2 (%)	7.8	6	9.7	11.4	10.6	12.2	9.8	8.2	4.4	8.3	12.6	10.7	11.7	8.3

	$h=6$							$h=12$						
$MktDown_t$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$SKEW_{t-h}$	-0.093**						-0.037	-0.060*						-0.022
$SKEW^{+}_{t-h}$		0.053					0.056		0.108					0.110
$SKEW^{-}_{t-h}$			-0.083**				-0.062*			-0.041*				-0.032
$SKEW_{t-h} \vert TR$				-0.092*							-0.059			
$SKEW^{+}_{t-h} \vert TR$					0.050							0.139		
$SKEW^{-}_{t-h} \vert TR$						-0.065*							-0.030	
Pseudo R^2 (%)	4.6	2.1	4.7	7.8	5.3	7.6	5.4	2.2	1.7	2.1	7.2	8.0	7.5	3.2

Notes: This table presents the results of the logistic predictive regressions estimated through equations 5 and 6 between the SKEW indexes and the market downturns dummy variable which is constructed by indicating with one every market drop equal or lower than 5%, and equal to 0 otherwise. OLS regressions are estimated controlling for the h lags of the dependent variable (columns from (1) to (3)) and also controlling for the all set of tail risk measures (TR), namely TAIL, HELLINGER, CATFIN, and CRASH (columns from (4) to (6)). In the last column (7) we shows the results of the multivariate regression where the three SKEW measures are tested jointly controlling for the lag of the endogenous variable. The results are reported for monthly ($h=1$), quarterly ($h=3$), semi-annual ($h=6$) and annual ($h=12$) predictive horizons. Lags of the endogenous variable and SKEW indexes are taken accordingly. The selected period goes from 01-1996 to 12-2017, at monthly frequency. Regressions coefficients and their significance are reported. Regressions intercepts and coefficients of the previous lag of the endogenous are not reported to save on space. The pseudo R^2 (in percent) of every regression is reported in the last row. The significance levels are indicated as follows: * (10%), ** (5%) and *** (1%).

In Table 6 we report the results with respect to market downturns predictability. The stronger role of SKEW$^-$ in predicting market downturns (compared to SKEW and SKEW$^+$) is again confirmed. At the 1-month horizon, SKEW$^-$ is the only index carrying predictive power after the tail risk information is controlled for.

The predictive power of SKEW$^-$ in relation to market downturns also holds for the 3-month and 6-month horizons after controlling for tail risk, but it vanishes at the 1-year horizon when we control for TR. The estimated marginal effects for SKEW$^-$ are still found to be higher than the ones for SKEW and SKEW$^+$. We find that a 1% decrease in SKEW$^-$ translates into a 0.9%, 0.65%, 0.7%, and 0.29% increase in the probability of a downturn for the 1-, 3-, 6- and 12-month horizons, respectively.[16]

The R^2 carried by the logistic equations that contain SKEW$^-$ are higher than the ones that contain SKEW and SKEW$^+$ up to the 6-month horizon and when no controls are considered. Again, when the three SKEW measures are tested jointly in the same multivariate predictive regression, SKEW$^-$ is the only one showing significant predictive power for future market downturns up to the 6-month horizon.

As an additional robustness check, we also check the role of the decomposed SKEW indexes in predicting recessions after controlling for the lags of the four principal components (PCs) extracted from the macroeconomic and financial predictors by Goyal and Welch (2008).[17] We take the four principal components in order to group the dispersed information that these 14 predictors contain $X_t = (X_{1,t}, ...X_{N,t})'$ is the N-vector ($N = 14$) of the selected predictors and the vector of the first K PCs extracted from X_t is denominated as $\hat{F}_t^P = (F_{1,t}^{GW}, ..., F_{K,t}^{GW})'$. The predictors are standardized before the PCs are computed. We

[16]If S^- had been used in the logistic regression, the beta coefficient would have been positive, and therefore an increase in the $Skew^-$ would have triggered an increase in the probability of a market downturn.

[17]The data set is available from Amit Goyal's webpage at: `http://www.hec.unil.ch/agoyal/`.The 14 predictors are the log dividend price ratio (DP), the log dividend yield (DY), the log earnings-price ratio (EP), the log dividend-payout ratio (DE), the excess stock return volatility (RVOL) computed as a 12-month moving standard deviation estimator (see Mele, 2007; Rapach et al., 2016) and it differs from the measure of stock return volatility used in Goyal and Welch (2008) (sum of squared daily excess stock returns during the month), the book-to-market ratio (BM), the net equity expansion (NTIS), the treasury bill rate (TBL), the long-term yield (LTY), the long-term return (LTR), the term spread (TMS), the default yield spread (DFY), the default return spread (DFR) and the inflation rate (INFL) computed from the CPI.

select only the first four PCs, as selected by the Akaike information criterion, and a reasonable number of components in order to keep the model parsimonious. The results of this robustness check are reported in the Online Appendix with respect to both recessions and market downturns. The total SKEW loses its predictive power, whereas a strong predictive ability (at 1%) is preserved for SKEW^-. The predictive power of SKEW^+ vanishes when we control for the PCs. With respect to the market downturns, we also find similar results and we confirm our hypothesis.

As a final robustness test, we have also replaced TR with each one of the tail risk measures in order to control for the information content of each single tail risk measure separately. The results still hold and the predictive power of SKEW^- is found to be even stronger. The results are placed in the Online Appendix.[18] Our results are found to be in line with the results found in Danielsson et al. (2018) stating that during tranquil periods, when perceived risk is low, economic agents and investors may be misled into taking more risks, possibly leading to market downturns. Therefore we can conclude this section suggesting that the forward-looking information encapsulated in SKEW^- is more valuable in terms of signaling future recessions or market downturns. In the next section we investigate whether SKEW^- is also able to signal future uncertainty and macroeconomic downturns.

6 The Link with Uncertainty and Macroeconomic Indicators

In this section we test whether or not SKEW^- has superior predictive ability, compared to the total and positive SKEW, with respect to macroeconomic and uncertainty indicators.

[18] We also control for decomposed volatility indexes (VIX, VIX^+ and VIX^-), and decomposed variance risk premia (VRP, VRP^+ and VRP^-). We run the same regressions as in 6 with risk measures replacing the tail risk set of controls. We find that SKEW^- preserves its predictive power, whereas the decomposed implied volatility, volatility risk premia do not show predictive power either for recessions or for market downturns. Therefore, the information enclosed in the decomposed implied volatility and volatility risk premia is not useful for predicting a market crisis. On the other hand, the different set of information contained in SKEW^- appears to be useful in this regard.

The SKEW indexes are forward-looking measures, hence they may be useful to monitor and anticipate the uncertainty in the economy or its downturns. In addition, we have shown in the previous section that SKEW$^-$ reacts more in correspondence to recessions and extreme market downturns. Here, we are interested in testing whether or not it may also contain additional information to predict future macroeconomic downturns or increase in uncertainty indicators. The role of SKEW and SKEW$^+$ in predicting these indicators is also analyzed.

We select indicators commonly used in the literature which track the economic, political and geopolitical uncertainty. Monthly data is collected on the Economic Uncertainty Index (EUI) by Bali et al. (2014), the Economic Policy Uncertainty (EPU) index (Baker et al., 2016), the GeoPolitical Risk (GPR) index (Caldara and Iacoviello, 2018), the CBOE VIX index, the macroeconomic index (Jurado et al., 2015) (MUI) and the CRASH index, already described in Section 3.2 and used here as a proxy for future uncertainty.[19] We also analyze the predictability power of the SKEW indexes with regards to some widely used macroeconomic indicators. We select the Aruoba-Diebold-Scotti macroeconomic conditions index by Aruoba et al. (2009) (ADS), the Chicago FED National Activity Index, CFNAI, and the Industrial Production (IP) indicator.[20]

[19] The Economic Uncertainty Index (EUI) by Bali et al. (2014) is a measure of economic uncertainty based on the time-varying conditional volatility of a large set of macroeconomic and financial variables and it is available at `http://faculty.msb.edu/tgb27/workingpapers.html`. The EPU index by Baker et al. (2016) is computed from news associated with the ten most important American newspapers, reflecting the concerns and uncertainty around specific economic or political events. It is collected from: `http://www.policyuncertainty.com/`. The GeoPolitical Risk index by Caldara and Iacoviello (2018) (GPR) is an index computed in a similar way to EPU from newspapers articles associated with geopolitical risk and events such as wars, terrorist attacks or international conflicts. It is available at: `https://www2.bc.edu/matteo-iacoviello/gpr.htm`. The VIX index from the CBOE is commonly considered a proxy of market fear. The macroeconomic index by Jurado et al. (2015) (MUi) is a refined measure of macroeconomic uncertainty from a macro data set including information in hundreds of macroeconomic and financial indicators available at: `https://www.sydneyludvigson.com/data-and-appendixes/`. For more details on the CRASH Index, see Section 3.2.

[20] The Aruoba-Diebold-Scotti (ADS) index tracks real business conditions at high frequency, it is based on economic indicators and it is collected from: `https://www.philadelphiafed.org/research-and-data/real-time-center/business-conditions-index`, the Chicago FED National Activity Index (CFNAI) is a monthly index which tracks the overall economic activity and the inflationary pressure and it is computed as a weighted average of 85 monthly indicators and it is collected from: `https://www.chicagofed.org/publications/cfnai/index`. The Industrial Production (IP) indicator is commonly used as a proxy of real output and it is collected from `https://fred.stlouisfed.org/`.

We run the following predictive regression:

$$Indicator_t = \beta_0 + \beta_1\ Indicator_{t-h} + \beta_2\ SKEW_{t-h} + \epsilon_t, \tag{7}$$

where $Indicator_t$ is one among either the uncertainty indicators, namely, EUI, EPU, GPR, VIX , MUi and CRASH or one among the macroeconomic indicators, namely ADS, CFNAI and IP. SKEW is one of the SKEW measures, $h \in (1, 3, 6, 12)$ months, and we control for the h lag of the endogenous variable. Similar to the previous section, we also add the tail risk controls with respect to the uncertainty and macroeconomic indicators predictability, therefore running the following regressions:

$$Indicator_t = \beta_0 + \beta_1\ Indicator_{t-h} + \beta_2\ SKEW_{t-h} + \beta_3\ TR_{t-h}\epsilon_t, \tag{8}$$

where everything stays as in equation (7) with the addition of TR, that includes the four tail risk measures we use as controls, namely TAIL, HELLINGER, CATFIN, and CRASH.

The results are reported in Table 7 and Table 8 for the uncertainty indicators, and in Table 9 for the macro indicators.[21] We observe that both SKEW and SKEW$^-$ are able to predict future levels of EUI up to one year in advance, while SKEW$^+$ only from 6 to 12 months in advance. However, when we control for the tail risk measures, only SKEW and SKEW$^-$ keep their predictive ability, with the latter being the only one able to predict the next month EUI. Even if SKEW and SKEW$^-$ show similar predictive ability, greater R^2 is associated with models including SKEW$^-$, further emphasizing the usefulness of SKEW$^-$ to predict economic uncertainty. The EUI by Bali et al. (2014) is found to be higher during bad states of economy, low output growth and low economic activity. The negative coefficients carried by SKEW$^-$ in predicting future levels of EUI might be justified from the negative correlation that the two indexes share (-22%), and from the fact that SKEW$^-$ tends to anticipate the

[21] The same set of empirical has been conducted also controlling for all the 12 lags of the dependent variables as in Allen et al. (2012). The results did not materially change in their significance and they are reported in the Online Appendix.

perception of decreasing business conditions since extracted from options. SKEW$^-$ is the only index able to predict future levels of EPU up to 6-months in advance, however when we control for tail risk information this predictability holds only at the first month horizon. With respect to GPR, SKEW$^-$ is the only index showing significant predictability, this time at the longer horizon, even after controlling for TR.

Hence, the same conclusions drawn for the NBER recession dummy may apply to the negative coefficient sign found between SKEW$^-$ and uncertainty indicators such as EPU, EUI and MUi, being possibly associated with the counter-cyclical relationship between current uncertainty measures and the forward-looking implied skewness. The tail risk proxy reacts in advance compared to the EPU index which reacts when uncertainty is already spreading.

In Table 8, future levels of VIX are predicted well by all the SKEW indexes. The total SKEW is able to predict future VIX at the 3- and 6-month horizons, after controlling for tail risk information. SKEW$^+$ predicts VIX well within one year when no controls are added, with positive relationship. The decomposed SKEW$^-$ and SKEW$^+$ are able to predict VIX at the longer 6- and 12-month horizons, even after controlling for TR.

The negative relationship between SKEW and SKEW$^-$, and VIX is also depicted in Table 1. The negative coefficients found, most of the time, between SKEW and SKEW$^-$, and EPU can be interpreted as a consequence of the negative relationship between SKEW indexes and VIX and the positive between VIX and EPU.[22] On the other hand, the overall positive relationship between SKEW indexes and GPR is consistent with the fact that the geopolitical index shows a negative relationship with VIX (see Caldara and Iacoviello, 2018). This might be interpreted from the nature of the events to which the SKEW indexes and these uncertainty indexes react.[23] GPR as well as SKEW$^-$ do not appear to respond to financial events in the same way as VIX and EPU (see also Figure 2).[24] Thus, the positive coefficients

[22]Baker et al. (2016) shown VIX to have a correlation of 58% with the EPU index. We find the EPU index and VIX to be positively correlated at 42%.

[23]The GPR index captures events such as wars, terrorist attacks and global conflicts and appears to carry an additional source of risk compared to the EPU index. The GPR index and SKEW$^-$ react to events, such as, 9/11, Iraq invasion, Syria War and Ukraine-Russia conflict. See also Figure 2.

[24]EPU index as well as VIX react to most of the economic downturn and financial crisis. On the other

Table 7: SKEW Indexes and Uncertainty Indicators (1)

h		EUI	R^2	EPU	R^2	GPR	R^2
1	$SKEW_{t-h}$	-0.005**	95.9	-0.021	69.0	0.520	60.3
	$SKEW^+_{t-h}$	0.006	95.3	-0.648	69.2	-0.669	60.1
	$SKEW^-_{t-h}$	-0.008**	96.1	-0.228**	69.5	0.211	60.1
	$SKEW_{t-h}\|TR$	-0.005	96.1	-0.148	75.7	0.148	75.7
	$SKEW^+_{t-h}\|TR$	0.006	96.0	-0.534	75.8	-0.534	75.8
	$SKEW^-_{t-h}\|TR$	-0.002*	96.4	-0.066**	76.6	0.066	75.7
3	$SKEW_{t-h}$	-0.020***	84.9	-0.085	44.9	0.455	20.8
	$SKEW^+_{t-h}$	0.009	84.4	-0.730	45.1	1.461	20.9
	$SKEW^-_{t-h}$	-0.030***	85.7	-0.046*	45.5	0.410	20.8
	$SKEW_{t-h}\|TR$	-0.020**	89.0	-0.462	53.0	0.462	53.0
	$SKEW^+_{t-h}\|TR$	0.001	88.8	-0.885	53.2	-0.885	53.0
	$SKEW^-_{t-h}\|TR$	-0.013*	88.9	-0.324	52.9	0.324	52.9
6	$SKEW_{t-h}$	-0.043***	67.0	-0.028	37.3	0.368	12.5
	$SKEW^+_{t-h}$	0.044**	66.1	-0.334	37.4	0.688	12.4
	$SKEW^-_{t-h}$	-0.053***	68.4	-0.145*	37.6	0.402	12.6
	$SKEW_{t-h}\|TR$	-0.051***	74.7	-0.194	45.3	0.194	45.3
	$SKEW^+_{t-h}\|TR$	0.031	73.4	-0.131	45.2	-0.131	45.2
	$SKEW^-_{t-h}\|TR$	-0.032***	74.8	-0.051	45.2	0.051	45.2
12	$SKEW_{t-h}$	-0.084***	32.4	-0.365	29.6	0.581	8.2
	$SKEW^+_{t-h}$	0.088**	27.0	-0.399	29.5	0.624	8.0
	$SKEW^-_{t-h}$	-0.091***	36.1	-0.008	29.1	0.646**	9.5
	$SKEW_{t-h}\|TR$	-0.079***	41.4	-0.438	40.2	0.438	40.2
	$SKEW^+_{t-h}\|TR$	0.069	38.6	-0.002	39.8	-0.002	39.8
	$SKEW^-_{t-h}\|TR$	-0.067***	41.8	-0.197	39.9	0.191*	40.7

Notes: This table presents the results of the predictive regressions estimated through equations 7 and 8 between the SKEW indexes, and the selected uncertainty proxies, namely the Economic Uncertainty Index (EUI) by Bali et al. (2014), the Economic and Policy Uncertainty (EPU) by Baker et al. (2016) and the GeoPolitical Risk Index (GPR) by Caldara and Iacoviello (2018). OLS regressions are estimated controlling for the h lags of the dependent variable and also controlling for the all set of tail risk measures (TR), namely TAIL, HELLINGER, CATFIN, and CRASH. The results are reported for monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) predictive horizons. Lags of the endogenous variable and SKEW indexes are taken accordingly. The selected period goes from 01-1996 to 12-2017, at monthly frequency. Regressions coefficients and their significance are reported. Regressions intercepts and coefficients of the previous lag of the endogenous are not reported to save on space. The R^2 (in percent) of every regression is reported in the last row. The significance levels are indicated as follows: * (10%), ** (5%) and *** (1%).

between SKEW and SKEW$^-$, and GPR may be due to the fact that the events to which the indexes react are of a similar nature.

We also find that the information enclosed in SKEW$^-$ is the only one able to predict future MUi from 6- to 12-month horizons, even when we control for tail risk. The same

hand, GPR is found to show an almost nil correlation with VIX and low correlation with EPU.

SKEW$^-$ shows predictive ability with respect to future levels of CRASH up to one year; however, when controlling for TR, this holds only at the one year horizon. This confirms the results of Table 2 and it is consistent with the fact that both measures carry information about investors' perceptions about catastrophic events.

Table 8: SKEW Indexes and Uncertainty Indicators (2)

h		VIX	R^2	MUi	R^2	CRASH	R^2
1	$SKEW_{t-h}$	-0.043	70.2	-0.004	94.1	0.007	90.2
	$SKEW^+_{t-h}$	0.112*	70.2	0.001	94.0	-0.038	90.2
	$SKEW^-_{t-h}$	-0.033	70.1	-0.003	94.1	0.050*	90.3
	$SKEW_{t-h}\|TR$	-0.041	71.5	-0.001	94.1	0.016	91.0
	$SKEW^+_{t-h}\|TR$	0.107	71.6	0.001	94.1	-0.550	91.0
	$SKEW^-_{t-h}\|TR$	-0.039	71.5	-0.003	94.2	0.021	91.0
3	$SKEW_{t-h}$	-0.135**	38.1	-0.001	69.1	0.032	60.2
	$SKEW^+_{t-h}$	0.200**	37.2	0.001	68.8	-0.026	59.9
	$SKEW^-_{t-h}$	-0.058*	37.0	-0.001	69.5	0.090*	60.4
	$SKEW_{t-h}\|TR$	-0.141*	43.8	-0.001	69.7	0.055	65.4
	$SKEW^+_{t-h}\|TR$	0.203	43.4	0.001	69.3	-0.058	65.3
	$SKEW^-_{t-h}\|TR$	-0.047	43.1	-0.001	69.9	0.014	65.3
6	$SKEW_{t-h}$	-0.201***	21.8	-0.028	37.3	0.072	29.2
	$SKEW^+_{t-h}$	0.245*	19.5	0.002	37.0	-0.073	29.0
	$SKEW^-_{t-h}$	-0.168***	21.1	-0.002***	39.3	0.092*	29.4
	$SKEW_{t-h}\|TR$	-0.197**	33.4	-0.001	44.1	0.101	39.0
	$SKEW^+_{t-h}\|TR$	0.273*	32.6	0.001	43.3	-0.024	38.7
	$SKEW^-_{t-h}\|TR$	-0.179**	33.7	-0.002**	45.0	0.002	38.7
12	$SKEW_{t-h}$	-0.212**	14.0	-0.002	8.2	0.070	28.6
	$SKEW^+_{t-h}$	0.0447**	13.8	0.003	7.1	-0.103	28.5
	$SKEW^-_{t-h}$	-0.209***	14.3	-0.003***	14.2	0.207**	30.7
	$SKEW_{t-h}\|TR$	-0.159	14.1	-0.002	14.2	0.095	34.0
	$SKEW^+_{t-h}\|TR$	0.432**	15.1	0.004	13.3	-0.126	33.9
	$SKEW^-_{t-h}\|TR$	-0.227***	16.3	-0.004***	20.6	0.169**	35.2

Notes: This table presents the results of the predictive regressions estimated through equations 7 and 8 between the SKEW indexes, and the selected uncertainty proxies, namely the VIX index, the macroeconomic index by Jurado et al. (2015) (MUi), and CRASH. OLS regressions are estimated controlling for the h lags of the dependent variable and also controlling for the all set of tail risk measures (TR), namely TAIL, HELLINGER, CATFIN, and CRASH. We exclude the latter from TR when CRASH is predicted. The results are reported for monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) predictive horizons. Lags of the endogenous variable and SKEW indexes are taken accordingly. The selected period goes from 01-1996 to 12-2017, at monthly frequency. Regressions coefficients and their significance are reported. Regressions intercepts and coefficients of the previous lag of the endogenous are not reported to save on space. The R^2 (in percent) of every regression is reported in the last row. The significance levels are indicated as follows: * (10%), ** (5%) and *** (1%).

Regarding macroeconomic indicators predictability, we show the results in Table 9.

SKEW and SKEW$^-$ are able to predict future ADS from 3- to 6-month and from 3- to 12-month horizons, respectively, even after controlling for the tail risk variables. SKEW$^-$ shows, once again, higher R^2 compared to both models including SKEW and SKEW$^+$. SKEW$^-$ is the only SKEW index showing predictive ability for CFNAI up to one year, also after controlling for TR. Finally, both SKEW and SKEW$^-$ are strong predictors of future levels of industrial production growth at all horizons, with SKEW$^-$ presenting slightly greater performance in terms of R^2.[25]

Interestingly, positive coefficients are found between SKEW and SKEW$^-$ and macroeconomic indicators, such as ADS and CFNAI. This can be due to trading activity reasons. The better the business and economic conditions, the higher the investors' trading volume which for S&P 500 is mainly generated by puts trading. The positive relationship between macroeconomic indicators and SKEW$^-$ might also come from their correlation being 25% between SKEW$^-$ and CFNAI and 20% between SKEW$^-$ and ADS. When the macroeconomic indicators (correlated at 90%) decrease, we also find our tail risk measure to be lower. In calm periods, which might coincide with periods of better-than-average business conditions, according to the ADS index (Aruoba et al., 2009), SKEW$^-$ is found to be higher as well since investors are concerned about a possible future tail event. A similar interpretation may apply for the positive coefficients found between SKEW$^-$ and CFNAI.

7 Out-of-Sample Analysis

Finally, in order to consolidate our findings, we also check the out-of-sample predictability of the SKEW measures with respect to all the dependent variables considered so far. We run the same equations as in (5) and in (7) where now $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are the OLS estimates of the coefficients from the beginning of the sample until month t.

[25] Also with respect to the macroeconomic and uncertainty indicators, we have checked the performance of the SKEW indexes when controlling for each one of the tail risk variables separately. We do not report the results for the sake of space. They are available from the authors upon request and showing that the predictive power of SKEW$^-$ holds even stronger.

Table 9: SKEW Indexes and Macroeconomic Activity Indicators

h		ADS	R^2	CFNAI	R^2	IP	R^2
1	$SKEW_{t-h}$	0.002	82.3	0.008*	46.4	0.016***	98.2
	$SKEW^+_{t-h}$	0.003	82.3	0.002	45.8	-0.002	98.1
	$SKEW^-_{t-h}$	0.003	82.4	0.012***	47.2	0.017***	98.5
	$SKEW_{t-h}\|TR$	0.005	84.1	0.014	53.2	0.027***	99.0
	$SKEW^+_{t-h}\|TR$	0.006	84.0	0.002	52.4	-0.004	98.8
	$SKEW^-_{t-h}\|TR$	0.004	84.1	0.008*	52.9	0.014***	99.3
3	$SKEW_{t-h}$	0.007*	54.1	0.009*	46.5	0.044***	96.2
	$SKEW^+_{t-h}$	-0.011	53.7	-0.001	45.9	-0.034*	96.1
	$SKEW^-_{t-h}$	0.008**	54.4	0.011**	47.0	0.047***	96.8
	$SKEW_{t-h}\|TR$	0.015*	60.2	0.014	54.2	0.073***	97.5
	$SKEW^+_{t-h}\|TR$	-0.008	59.3	-0.004	53.5	-0.041	97.2
	$SKEW^-_{t-h}\|TR$	0.011**	60.3	0.010*	54.1	0.042***	97.6
6	$SKEW_{t-h}$	0.015***	31.9	0.010**	28.1	0.095***	90.0
	$SKEW^+_{t-h}$	-0.007	29.9	-0.016	27.0	-0.051*	89.1
	$SKEW^-_{t-h}$	0.015***	32.3	0.016***	29.1	0.095***	90.3
	$SKEW_{t-h}\|TR$	0.027***	40.6	0.021**	34.8	0.155**	91.2
	$SKEW^+_{t-h}\|TR$	-0.008	37.3	-0.018	33.5	-0.073	90.3
	$SKEW^-_{t-h}\|TR$	0.023***	41.0	0.018**	35.1	0.094***	91.8
12	$SKEW_{t-h}$	0.009	3.6	0.013*	3.2	0.188***	70.8
	$SKEW^+_{t-h}$	-0.019**	3.8	-0.020*	2.9	-0.206**	67.8
	$SKEW^-_{t-h}$	0.015***	5.5	0.020***	5.8	0.181***	72.1
	$SKEW_{t-h}\|TR$	0.016	16.2	0.015	13.2	0.267**	73.0
	$SKEW^+_{t-h}\|TR$	-0.019*	15.8	-0.025	12.9	-0.241*	69.9
	$SKEW^-_{t-h}\|TR$	0.023***	18.9	0.023***	15.4	0.195***	73.8

Notes: This table presents the results of the predictive regressions estimated through equations 7 and 8 between the SKEW indexes, and the selected macroeconomic proxies, namely the Aruoba-Diebold-Scotti macroeconomic conditions index by Aruoba et al. (2009) (ADS), the Chicago FED National Activity Index, CFNAI, and the Industrial Production (IP) indicator. OLS regressions are estimated controlling for the h lags of the dependent variable and also controlling for the all set of tail risk measures (TR), namely TAIL, HELLINGER, CATFIN, and CRASH. The results are reported for monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) predictive horizons. Lags of the endogenous variable and SKEW indexes are taken accordingly. The selected period goes from 01-1996 to 12-2017, at monthly frequency. Regressions coefficients and their significance are reported. Regressions intercepts and coefficients of the previous lag of the endogenous are not reported to save on space. The R^2 (in percent) of every regression is reported in the last row. The significance levels are indicated as follows: * (10%), ** (5%) and *** (1%).

In order to evaluate the forecast of the SKEW measures we use the Campbell and Thompson (2007) out-of-sample R^2_{OS}, namely, $R^2_{OS} = 1 - (MSFE_i/MSFE_0)$ which measures the reduction in mean squared forecast errors (MSFE) for the competing predictive regressions against the AR benchmark forecast we adopt over the evaluation period (see Rapach et al., 2010). When $R^2_{OS} > 0$ it means that the competing predictive regression forecast is more accurate than the benchmark in terms of forecast errors, meaning that $MSFE_i < MSFE_0$.

Since we are comparing nested models, to test whether or not the predictive regression forecast produces a significant improvement in the MSFE, we also study the Clark and West (2007) MSFE-adjusted statistic that is testing the null hypothesis that the benchmark MSFE is less than or equal to the predictive competing regression MSFE against the one-sided (upper-tail) alternative hypothesis that the benchmark MSFE is greater than the predictive competing regression MSFE which corresponds to $H_0 : R^2_{OS} \leq 0$ against $H_A : R^2_{OS} > 0$.[26] This corresponds to the null hypothesis that information contained in the full models with SKEW measures does not improve forecasts in terms of errors compared to the restricted models without SKEW measures. We show the out-of-sample predictability results in Table 10, for different horizons with $h \in \{1, 3, 6, 12\}$ and with an in-sample period and an out-of-sample evaluation forecast period selected as 1996-2005 and 2006-2017, respectively.

Table 10: **Skew Out-of-Sample Predictive Regression Results**

h	Predictor	NBER	MktDrop	EUI	EPU	GPR	VIX	MUi	CRASH	ADS	CFNAI	IP
1	SKEW	0.29*	1.30*	2.27*	2.88**	0.56***	5.46***	-4.02	0.31**	-2.56	-2.73	2.13*
	$SKEW^+$	-0.22	0.34*	1.91*	4.47***	-1.72	5.53***	-7.10	0.44**	-1.48	-2.72	-0.56
	$SKEW^-$	1.49*	3.18**	4.11**	3.73***	-4.45	5.39***	-3.87	4.41***	-2.01	0.38**	3.94***
3	SKEW	1.47*	1.21*	3.34**	3.46**	-2.26	5.95***	-5.67	0.16**	-2.08	0.88*	2.39*
	$SKEW^+$	-0.05	0.26*	1.32*	3.90**	-3.54	4.90**	-4.76	1.45**	-1.11	1.42	-0.70
	$SKEW^-$	2.38*	3.12***	6.40**	3.13**	0.28***	5.98***	-4.21	1.11***	0.38*	4.15***	4.76**
6	SKEW	0.49*	0.62*	3.57**	1.27**	-4.37	5.02***	-4.86	0.05*	-0.92	1.97**	3.09**
	$SKEW^+$	-1.69	0.10*	1.42*	2.44**	-1.04	4.74**	-3.15	0.76*	-1.35	0.17	-0.95
	$SKEW^-$	2.30*	3.79***	3.70**	2.93***	-5.39	4.83***	-2.71	0.89***	-0.48	3.79***	3.50**
12	SKEW	-0.88	2.27**	2.69*	3.15**	-2.06	6.26***	-5.01	0.28*	-1.44	1.77*	2.79*
	$SKEW^+$	-0.17	1.87*	1.48*	2.29**	-7.19	4.11**	-5.82	2.38**	-0.33	0.48	-0.21
	$SKEW^-$	0.01*	0.55**	5.17***	2.55**	0.37***	4.95***	-4.03	4.91***	0.18*	3.24***	6.32***

Notes: This table reports the out-of-sample results for the SKEW measures predictive ability with respect to all the variables considered in the empirical section of the paper. The table reports the Campbell and Thompson (2007) R^2_{OS} statistic (in percent) comparing forecasts from the competing forecasting models including the SKEW measures to the AR benchmark forecasting model. Statistical significance for the R^2_{OS} statistic is based on the p-value of the Clark and West (2007) out-of-sample adjusted-MSFE statistic that corresponds to a one-sided test of the null hypothesis that the competing forecasting model has equal expected square forecast error relative to the AR benchmark forecasting model against the alternative hypothesis that the competing forecasting model has a lower expected square forecast error than the AR benchmark. The significance levels are indicated as follows: * (10%), ** (5%) and *** (1%).

The results suggest that the SKEW measures' predictability echoes the one found in

[26] The Clark and West (2007) can be considered as an adjusted version of the Diebold and Mariano (1995) statistic. In fact the latter can be undersized when comparing forecasts from nested linear models, leasing to tests with very low power (see also Rapach et al., 2010).

the in-sample analysis.[27] Noticeably, SKEW$^-$ achieves a good out-of-sample forecast performance with respect to the recession dummy at all horizons, with the R^2_{OS} relative to SKEW$^-$ found to be higher than the one relative to the total SKEW. All SKEW measures show out-of-sample performance with respect to the market downturns dummy variables. The R^2_{OS} is, again, found to be higher for SKEW$^-$, the only exception being the longer horizon in which the opposite is true.

Regarding the uncertainty indicators, we find a good out-of-sample performance for all the SKEW measures with respect to EUI, EPU, VIX and CRASH associated with a positive R^2_{OS}, implying that the predictive regression forecast including the SKEW measures outperforms the benchmark in terms of MSFE. The R^2_{OS} is found to be higher for SKEW$^-$ with respect to out-of-sample EUI and CRASH predictability, while mixed results in terms of the R^2_{OS} are found with respect to EPU and VIX. The total SKEW out-performs the benchmark at the 1-month horizon with respect to GPR predictability, while SKEW$^-$ does the same at the 3- and 12-month horizons. With respect to the macroeconomic indicators, we find weak out-of-sample predictability for ADS, this being all associated with SKEW$^-$. Finally, SKEW$^-$ appears to show a greater out-of-sample predictability with respect to CFNAI and industrial production, compared to the total SKEW, both in terms of significance of the MSFE-adjusted statistic and also in terms of size of the R^2_{OS}.

Therefore, we can conclude that overall the decomposed SKEW$^-$ measure is found to out-perform the total SKEW and the positive SKEW$^+$ also out-of-sample, showing a greater size of the positive R^2_{OS} and the Clark and West (2007) adjusted MSFE significantly less than the historical average MSFE in the majority of the cases. We also find a similar picture when we compare the out-of-sample performance of SKEW$^-$ with the tail risk controls as in the previous section.[28] Hence, we show that the out-of-sample exercise further corroborates the main message of the paper, namely that information contained in the SKEW indexes,

[27]The out-of-sample results are robust under various specification of the forecasting in-sample estimation window. When a different benchmark is chosen (e.g. previous lag of the dependent) the results still hold.

[28]The all set of results is available from the authors upon request and it confirms the main key message of Table 10.

especially in SKEW$^-$, carries predictive ability about future recessions and market downturns as well as several indicators of uncertainty and macroeconomy, highlighting the importance of the left tail – SKEW$^-$ as a tail risk forward-looking measure.

8 Conclusion

We extract further information enclosed in the implied skewness recovered model-free from the U.S. equity index options market, by decomposing it into its positive and negative components, namely, SKEW$^+$ and SKEW$^-$. The decomposed indexes reflect investors' belief characteristics and an additional information content that are not captured by the total SKEW.

The answers to our very first questions in the opening of the paper are positive. We have empirically demonstrated that disentangling the information contained in the two S&P 500 options portfolios, calls and puts, respectively, provides new insights into investor perceptions, speculative activity and the associated sentiment that are reflected in the call options, as well as tail risk information and market downturns, that are reflected in put options.

By relating the SKEW indexes with market sentiment proxies, we find that, most of the time, the total SKEW index can be influenced by the positive information flows coming from the calls side, leading to a possible over-optimistic bias in the measurement of the market tail risk. On the other hand, we find that the negative SKEW component, namely SKEW$^-$, is the measure more connected to existing tail risk measures. Therefore, we suggest that the left tail risk should be separated from the market view on skewness on the more speculative right tail. We propose employing SKEW$^-$ as a more reliable and prudent tail risk index, since it can predict future market downturns and signal potential recession periods in advance. There is something *left* to discover about the option-based implied skewness, even after accounting for other financial and tail risk measures.

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Appendix A Technical Appendix

A.1 The Bakshi et al. (2003) Methodology

For the calculation of the cubic risk-neutral moment, we employ the following result from (Bakshi and Madan, 2000; Bakshi et al., 2003) that underpins our methodology for extracting implied volatility and skewness from options prices:

Theorem 1: Under all martingale pricing measures, the following contract prices can be recovered from the market prices of OTM European calls and puts: the t-period risk-neutral return skewness, S(t,T) is given by:

$$S(t,T) \equiv \frac{\mathbb{E}_t[(R(t,T)-\mathbb{E}_t[R(t,T)])^3]}{[\mathbb{E}_t(R(t,T)-\mathbb{E}_t[R(t,T)])^2]^{3/2}} = \frac{e^{rt}W(t,T)-3\mu(t,T)e^{rt}V(t,T)+2\mu(t,T)^3}{(e^{rt}V(t,T)-\mu(t,T)^2)^{3/2}} \quad (9)$$

where the price of the volatility and cubic contracts are, respectively:

$$V(t,T) = 2\int_{S_t}^{\infty} \frac{1-ln(\frac{K}{S_t})}{K^2}C(t,T;K)dK + 2\int_{0}^{S_t} \frac{1+ln(\frac{S_t}{K})}{K^2}P(t,T;K)dK \quad (10)$$

$$W(t,T) = \int_{S_t}^{\infty} \frac{6(ln(\frac{K}{S_t}))-3(ln(\frac{K}{S_t}))^2}{K^2}C(t,T;K)dK - \int_{0}^{S_t} \frac{6(ln(\frac{S_t}{K}))+3(ln(\frac{S_t}{K}))^2}{K^2}P(t,T;K)dK \quad (11)$$

A.2 SKEW Index and its Decomposition

The final formula to compute the SKEW index, based on the CBOE (2011) SKEW Index methodology, is the following:

$$SKEW = 100 - 10S \tag{12}$$

where S is the computed implied skewness $S = \mathbb{E}[(\frac{R-\mu}{\sigma})^3]$, R is the S&P 500 30 days log-return, μ is its expected value and σ is its standard deviation. The $SKEW$ index is based on the rescaling in (12). Following Bakshi et al. (2003) and expanding equation 9, S is calculated starting from a portfolio of S&P 500 options with a pay off reflecting the skewness payoff:

$$S = \frac{\mathbb{E}[R^3] - 3\mathbb{E}[R]\mathbb{E}[R^2] + 2\mathbb{E}[R]^3}{(\mathbb{E}[R^2] - \mathbb{E}^2[R])^{3/2}} \tag{13}$$

Simplifying the notation $S = \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^2)^{3/2}}$ and the calculation of P_1, P_2 and P_3 from the options market is as follows:

$$P_1 = \mathbb{E}[R_t] = -e^{rT} \sum_i \frac{Q_K \delta_K}{K_i^2} + \varepsilon_1 \tag{14}$$

$$P_2 = \mathbb{E}[R_t^2] = 2e^{rT} \sum_i \frac{(1 - \ln(\frac{K_i}{F_0}))Q_K \delta_K}{K_i^2} + \varepsilon_2 \tag{15}$$

$$P_3 = \mathbb{E}[R_t^3] = 3e^{rT} \sum_i \frac{\ln\left(\frac{K_i}{F_0}\right) - \ln^2\left(\frac{K_i}{F_0}\right) Q_K \delta_K}{K_i^2} + \varepsilon_3 \tag{16}$$

where T denotes the options expiration date, F_0 is the forward of S&P 500 calculated from the put-call parity as $F_0 = e^{rT}[c(K,T) - p(K,T)] + K$, K_0 is the reference price, the first exercise price less or equal to the forward level $F_0(K \leq F_0)$, K_i is the strike price of i-th out-of-the-money options used in the calculation, r is the risk free rate with expiration T, δ_{K_i} is the sum divided by two of the two nearest prices to the exercise price K, Q_K is a generic price of a European call or put with strike price respectively above or below K_0, and ε_i represents the adjustments for the difference between K_0 and F_0 given by:

$$\varepsilon_1 = -\left(1 + \ln\left(\frac{F_0}{K_0}\right) - \frac{F_0}{K_0}\right). \tag{17}$$

$$\varepsilon_2 = 2\ln\left(\frac{K_0}{F_0}\right)\left(\frac{F_0}{K_0} - 1\right) + \frac{1}{2}\ln^2\left(\frac{K_0}{F_0}\right) \tag{18}$$

$$\varepsilon_3 = 3\ln^2\left(\frac{K_0}{F_0}\right)\left(\frac{1}{3}\ln\left(\frac{K_0}{F_0}\right) - 1 + \left(\frac{F_0}{K_0}\right)\right) \tag{19}$$

These equations are applied both for the near term expiration date and also for the far term expiration date. The target is to interpolate these two expirations around 30-days, as done for the VIX (see next section A.3). The same set of S&P 500 options used for VIX index is also used for SKEW and the same filters' rules discussed in Section 2 are applied. As in Dennis and Mayhew (2002), in formulae (14), (15) and (16) we use a trapezoidal approximation, consisting of a discrete sum of our available options prices instead of the integrals. In order to compute the negative and positive SKEW indexes, the total SKEW index is decomposed into two components: the positive SKEW computed only from S&P 500 calls, defined as SKEW$^+$ and the negative SKEW index computed only from S&P 500 puts, defined as SKEW$^-$. In formulae from (14) to (16), for SKEW$^+$ we keep only calls when $K_i \geq K_0$, while for SKEW$^-$ we keep only puts when $K_i \leq K_0$. Applying these model-free methodologies, we obtain three daily indexes series: SKEW, SKEW$^+$ and SKEW$^-$.

A.3 VIX Decomposition Methodology

The following formula is used in order to calculate the implied variance for both the near and also the far term expiration:

$$\sigma^2_{VIX} = \frac{2}{T}\sum \frac{\Delta(K_i)}{K_i^2} e^{rT} Q_t(K_i) - \frac{1}{T}\left[\frac{F_t}{K_0} - 1\right]^2 \tag{20}$$

where T is the expiration date, F_t is the forward of S&P 500 calculated from the put-call parity as $F_t = e^{rT}[c(K,T) - p(K,T)] + K$, where K_0 is the reference price, the first exercise price less or equal to the forward level $F_t(K_0 \leq F_t)$ and K_t is the strike price of i-OTM used in the calculation. This is a call if $K_i > K_0$, a put if $K_i < K_0$ and both call and put if $K_i = K_0$. $Q_t(K_i)$ is the average bid-ask of OTM options with exercise price equal to K_i. If $K_i = K_0$ it will be equal to the average between ATM call and put price, relative to that strike price. r is the risk free rate and $\Delta(K_i)$ is the sum divided by two of the two nearest prices to the exercise price K_0. The formula (20) is based on the variance swap formula approximation: $\sum_{i=1}^{n} \frac{\Delta(K_i)}{K_i}^2 e^{rT} Q(K_i)$. $\Delta(K_i)$ is equal to $\frac{(K_{i+1} - K_{i-1})}{2}$ for $2 \leq i \leq n-1$. $Q(K_i)$ is a generic price of a European call or put with strike price respectively above or below K_0. When K_i is equal to K_0, $Q(K_i)$ is equal to the average call and put. An adjustment term is needed in order to convert ITM calls in OTM puts: $\frac{1}{T}\left[\frac{F_0}{K_0} - 1\right]^2$. The VIX calculation is given by an interpolation between $\sigma^2_{VIX_1}(T_1)$ and $\sigma^2_{VIX_2}(T_2)$ around 30-days target. Finally, the VIX is computed as:

$$VIX = \sqrt{\frac{365}{30}\left[T_1 \sigma^2_{VIX_1}(T_1)\frac{N_2 - 30}{N_2 - N_1} + T_2 \sigma^2_{VIX_2}(T_2)\frac{30 - N_1}{N_2 - N_1}\right]}\,100 \tag{21}$$

In order to decompose the VIX index the same formula used for the total VIX is applied (see Kilic and Shaliastovich, 2019; Bevilacqua et al., 2019). For VIX$^+$ we keep calls only ($K_i \geq K_0$), while for VIX$^-$ we keep puts only ($K_i \leq K_0$). We obtain three daily indexes series: VIX, VIX$^-$ and VIX$^+$.