- Assignments must be completed ***individually***, and you must submit a *single* solution in a PDF file named `a3.pdf`, submitted to MarkUs. Handwritten submissions are acceptable as long as they are written ***neatly*** and ***legibly*** (typed submissions are preferable but not required).
- Please refer to the course information sheet for the ***late submission policy***.
- For each question, please write up detailed answers carefully. Make sure that you use ***notation*** and ***terminology*** correctly, and that you explain and ***justify*** what you are doing. Marks will be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.
- Please read and understand the policy on Academic Integrity given on the course information sheet. Then, to protect yourself, list on the front of your submission ***every*** source of information you used to complete this assignment (other than the lecture and tutorial material). For example, indicate clearly the ***name*** of every student with whom you had discussions, the ***title and sections*** of every textbook you consulted (including the course textbook), the ***source*** of every web document you used, etc.

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**1.** Translate the following DL axioms into first-order logic ($R$ and $S$ are assumed to be atomic roles).

(a) $\exists R.\exists S.\top \sqsubseteq B \sqcup C$

(b) $A \sqcap \neg B \sqsubseteq \forall R.C$

(c) $A \sqcup \exists R.B \sqsubseteq \exists S.\top$

**2.** (a) Build a TBox $\mathcal{T}$ that captures each of the following statements, using only the concept names

$$Vehicle, Boat, Bicycle, Car, Device, Wheel, Engine, Axle, Rotation, Water, Human, Driver, Adult, Child$$

and the role names

$$hasPart, poweredBy, capableOf, travelsOn, controls$$

i. Cars are exactly those vehicles that have at least a wheel and are powered by an engine.
ii. Bicycles are exactly those vehicles that have at least a wheel and are powered by a human.
iii. Boats are exactly those vehicles that travel on water.
iv. Boats have no wheels.
v. Cars and bicycles do not travel on water.
vi. Wheels are exactly those devices that have an axle and are capable of rotation.
vii. Drivers are exactly those humans who control a vehicle.
viii. Drivers of cars are adults.
ix. Humans are not vehicles.
x. Wheels and engines are not humans.
xi. Humans are either adults or children.
xii. Adults are not children.

(b) Extend the TBox $\mathcal{T}$ from Part (a) to a knowledge base $\mathcal{K}$ by adding assertions that capture the following statements, using the individual names *bob* and $qe2$.

i. Bob is a human.
ii. Bob controls QE2.
iii. QE2 is a vehicle that travels on water.

(c) Which of the following statements is true? Formally justify your answer.

i. $Boat \sqcap \exists hasPart.Wheel$ is satisfiable w.r.t. $\mathcal{K}$.
ii. $Car \sqcap Bicycle$ is satisfiable w.r.t. $\mathcal{K}$.
iii. $\exists controls.Car \sqcap Child$ is satisfiable w.r.t. $\mathcal{K}$.
iv. $bob$ is an instance of $(Adult \sqcap Driver)$ w.r.t. $\mathcal{K}$.
v. $qe2$ is an instance of $Boat$ w.r.t. $\mathcal{K}$.
vi. $\exists controls.Car$ is subsumed by $Adult$ w.r.t. $\mathcal{K}$.

## 3. Pots of Water

Consider a world with pots that may contain water. There is a single fluent, $Contains$, where $Contains(p, w, s)$ is intended to say that a pot $p$ contains $w$ liters of water in situation $s$. There are only two possible actions, which can always be executed: $empty(p)$, which discards all the water contained in the pot $p$, and $transfer(p, p')$, which pours as much water as possible without spilling from pot $p$ to $p'$, with no change when $p = p'$. To simplify the formalization, we assume that the following are also available (You may assume that axioms for these have already been provided or built in): a unary function symbol *capacity*, where $capacity(p)$ is intended to represent the maximum capacity of a pot $p$; the usual arithmetic constants, functions, and predicates (e.g., $max(x, y), min(x, y)$, $+$, $-$, $0$).

Imagine that in the initial situation, we have two pots, a 5-liter one filled with water and an empty 2-liter one. Our goal is to obtain 1 liter of water in the 2-liter pot.

**(a)** Write the precondition axioms for the actions.

**(b)** Write the effect axioms for the actions.

**(c)** Suppose we were interested in formalizing the problem using a STRIPS representation. Decide what the operators should be and then write the precondition, add list, and delete list for each operator. You may change the language as necessary.

**(d)** Consider the database corresponding to the initial state of the problem. For each STRIPS operator and each binding of its variables such that the precondition is satisfied, state what the database progressed through this operator would be.

**(e)** Consider the final goal state of the problem. For each STRIPS operator, describe the bindings of its variables for which the operator can be the final action of a plan, and in those cases, what the goal regressed through the operator would be.

**(f)** Provide a **plan** to reach the goal from the initial state. **Justify** your answer by showing that each operator in your plan can be legally performed, and describing the world model after performing each operator.

**(g)** **Optional:** Represent the domain and problem described in this question in PDDL and use a planner to solve the problem.