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Question # 1

a)

- Canadians are typically not Francophones $\xrightarrow{FOL}$ $\forall x \left((Canadian(x) \wedge \neg Ab_1(x)) \supset \neg Francophone(x) \right)$
- All Quebecois are Canadians $\xrightarrow{FOL}$ $\forall x \left(Quebecios(x) \supset Canadian(x) \right)$
- Quebecois are typically francophones $\xrightarrow{FOL}$ $\forall x \left((Quebecios(x) \wedge Ab_2(x)) \supset Francophone(x) \right)$
- Robert is a Quebecois. $\xrightarrow{FOL}$ $Quebecios(Robert)$

$Ab_1(x)$: abnormal Canadian

$Ab_2(x)$: abnormal Quebecios

Prove Francophone(Robert)

with minimal abnormality

Case 1: $Ab_1(Robert) = False$

$Ab_2(Robert) = False$

$(\underbrace{Canadian(Robert)}_{True} \wedge \underbrace{\neg Ab_1(Robert)}_{True} \supset \neg Francophone(Robert)$ $\longrightarrow$ Robert is not a Francophone

Francophone (Robert) = False

$\underbrace{Qubecios(Robert)}_{True} \wedge \underbrace{\neg Ab_2(Robert)}_{True} \supset Francophone(Robert)$ $\longrightarrow$ Robert is a Francophone

Francophone(Robert) = True

This Model does not entail the KB

Case 2:

$Ab_1(Robert) = True$; Robert is an abnormal Canadian Quebecios (Robert) = True

$Ab_2(Robert) = False$; Robert is normal Quebecions

$\underbrace{Quebecios(Robert)}_{True} \supset Canadian(Robert)$ $\longrightarrow$ means Canadian (Robert) = True

$(Canadian(Robert) \wedge \neg Ab_1(Robert) \supset \neg Francophone(Robert)$ $\longrightarrow$ Robert can be francophone or not

True ... False

$Qubecios(Robert) \wedge \neg Ab_2(Robert) \supset Francophone(Robert)$ $\longrightarrow$ $Francophone(Robert) = True$

True ... True

Case 3:

$Ab_1(Robert) = False$; Robert is an normal Canadian

$Ab_2(Robert) = True$; Robert is abnormal Qucbecions

$Quebecios(Robert) \supset Canadian(Robert)$ $\longrightarrow$ means $Canadian(Robert) = True$

True

$(Canadian(Robert) \wedge \neg Ab_1(Robert) \supset \neg Francophone(Robert)$ $\longrightarrow$ $Francophone(Robert) = False$

True ... True

$Qubecios(Robert) \wedge \neg Ab_2(Robert) \supset Francophone(Robert)$ $\longrightarrow$ Can't be determined

True ... False

Two minimality entailed cases

Case 1:

$Ab_1 = \{Robert\}$ $Ab_2 = \{\}$ saying that Robert is an abnormal Canadian entailes $Francophone(Robert) = True$

Case 2:

$Ab_1 = \{\}$ $Ab_2 = \{Robert\}$ saying that Robert is an abnormal Quebecios entails $Francophone(Robert) = False$

Since both have same number of abnormalities (1) we can't favour one over the other. And since they entail conflicting conclusions, We can say that minimizing abnormality would not be sufficient to determine wheather Robert is a Francophone or not.

b) All Quebecois are abnormal Canadians $\xrightarrow{FOL}$ $\forall x\,(Quebecois(x) \supset Ab_1(x))$

Case 1: $Ab_1 = \{Robert\}$ $Ab_2 = \{\}$ $\underset{True}{Quebecois(Robert)} \supset \underset{True}{Ab_1(Robert)}$ this holds

based on proof shown in part a this case proves that Francophone(Robert) = True

Case 2: $Ab_1 = \{\}$ $Ab_2 = \{Robert\}$ $\underset{True}{Quebecois(Robert)} \supset \underset{False}{Ab_1(Robert)}$ this does not hold

This means that with this addition to the KB case 2 can no longer be a model that entails the KB.

There for since case 1 is the only minimal model in this case it can be concluded that Francophone (Robert) = True

Quebecois are typically abnormal Canadians $\xrightarrow{FOL}$ $\forall x\,(Quebecois(x) \wedge \neg Ab_2(x) \supset Ab_1(x))$

Case 1: $Ab_1 = \{Robert\}$ $Ab_2 = \{\}$

$(\underset{True}{Quebecois(Robert)} \wedge \underset{True}{\neg Ab_2(Robert)}) \supset \underset{True}{Ab_1(Robert)}$ This holds as the model entails the KB

Therefore Francophone (Robert) = True

Case 2: $Ab_1 = \{\}$ $Ab_2 = \{Robert\}$

$(\underset{True}{Quebecois(Robert)} \wedge \underset{False}{\neg Ab_2(Robert)}) \supset \underset{False}{Ab_1(Robert)}$ This holds aswell meaning Francophone (Robert) = False

Similar to part a, as both minimal models entail the KB, and have conflicting conclusion this assertion is not sufficient to conlude Francophone (Robert) = True c)

q : Robert is Quebecios

c : Robert is Canadian

f : Robert is Francophone

Fact 1: Robert is a Quebecios $q = \text{True}$

Fact 2: All Quebecios are Canadians $q \supset c \rightarrow c = \text{True}$

Default 1: Canadians are typically not Francophones $\frac{c : \neg f}{\neg f}$

Default 2: Quebecois are typically francophones $\frac{q : f}{f}$

$F = \langle q, c \rangle \qquad D = \langle c \Rightarrow \neg f,\ q \Rightarrow f \rangle$

two extensions $\Delta_1 = \{\neg f\}$ and $\Delta_2 = \{f\}$

Quebecios are typically Francophones. $\frac{q : \text{True}}{f}$ $\leftarrow$ Non-normal default

$F = \langle q, c \rangle \qquad D = \langle c \Rightarrow \neg f,\ \frac{q : \text{True}}{f} \rangle$

Non-Normal default $\left(\frac{q : \text{True}}{f}\right)$ has extension $\{f\}$

Normal default $(c \Rightarrow \neg f)$ has extension $\{\neg f\}$

like previouse Case there is a contradiction. However, the non-normal default takes priority. Hence f holds, Eliminating the ambiguity.

d)

- Canadians are typically not francophones $\longrightarrow$ $(c \wedge \neg Bf) \supset \neg f$
- All Quebecois are Canadians $\longrightarrow$ $q \supset c$
- Quebecois are typically francophones $\longrightarrow$ $(q \wedge \neg B \neg f) \supset f$
- Robert is a Quebecois. $\longrightarrow$ q

$$KB = \{ (c \wedge \neg Bf) \supset \neg f,\ (q \wedge \neg B \neg f) \supset f,\ q \supset c,\ q \}$$

Case 1: $Bf = True$ $\quad B \neg f = False$

$$(c \cancel{\wedge \underbrace{\neg Bf}_{False}}^{False}) \supset \neg f \equiv True \qquad (q \wedge \underbrace{\neg B \neg f}_{True}) \supset f \equiv q \supset f \qquad A_1^\circ = \{ c, q, f \}$$

$A_1^\circ \models f \longrightarrow$ Therefore the expansion is stable

Case 2: $Bf = False$ $\quad B \neg f = True$

$$(c \wedge \underbrace{\neg Bf}_{True}) \supset \neg f \equiv c \supset \neg f \qquad (q \cancel{\wedge \underbrace{\neg B \neg f}_{False}}^{False}) \supset f \equiv True \qquad A_2^\circ = \{ c, q, \neg f \}$$

$A_2^\circ \models \neg f \longrightarrow$ therefore the expansion is stable

Bf and $B \neg f$ cannot be both True or False at the same time as it causes inconsistency leading to an unstable Expansions

Question #2

$KB = \{ Bird(a), Bird(b), (Bird(c) \vee Bird(d)), \neg Flies(b) \}$ $\qquad a \neq b \neq c \neq d$

a)

Strong default $\forall x \left(Bird(x) \wedge \neg B \neg Flies(x) \supset Flies(x) \right)$

Propositionalized:

$\left(Bird(a) \wedge \neg B \neg Flies(a) \supset Flies(a) \right)$, $\left(Bird(b) \wedge \neg B \neg Flies(b) \supset Flies(b) \right)$

$\left(Bird(c) \wedge \neg B \neg Flies(c) \supset Flies(c) \right)$, $\left(Bird(d) \wedge \neg B \neg Flies(d) \supset Flies(d) \right)$

Case 1:

$B \neg Flies(a) = False \qquad B \neg Flies(b) = False \qquad B \neg Flies(c) = False \qquad B \neg Flies(d) = False$

$\left(Bird(a) \wedge \cancelto{True}{\neg B \neg Flies(a)} \supset Flies(a) \right) \longrightarrow \cancelto{True}{Bird(a)} \supset Flies(a) \longrightarrow Flies(a)$

$\left(Bird(b) \wedge \cancelto{True}{\neg B \neg Flies(b)} \supset Flies(b) \right) \longrightarrow \cancelto{True}{Bird(b)} \supset Flies(b) \longrightarrow Flies(b)$

$\left(Bird(c) \wedge \cancelto{True}{\neg B \neg Flies(c)} \supset Flies(c) \right) \longrightarrow Bird(c) \supset Flies(c)$

$\left(Bird(d) \wedge \cancelto{True}{\neg B \neg Flies(d)} \supset Flies(d) \right) \longrightarrow Bird(d) \supset Flies(d)$

- Contradiction $Flies(b)$ as $\neg Flies(b)$ is in the KB. Hence, unstable expansion.
- Any case with $B \neg Flies(b) = False$ would not lead to a stable expansion

Case 2:

$\mathcal{B}\neg Flies(a) = False \qquad \mathcal{B}\neg Flies(b) = True \qquad \mathcal{B}\neg Flies(c) = False \qquad \mathcal{B}\neg Flies(d) = False$

$$\left(Bird(a) \wedge \neg \overset{\text{True}}{\cancel{\mathcal{B}\neg Flies(a)}} \supset Flies(a)\right) \longrightarrow \overset{\text{True}}{\cancel{Bird(a)}} \supset Flies(a) \longrightarrow Flies(a)$$

$$\left(Bird(b) \wedge \neg \overset{\text{False}}{\cancel{\mathcal{B}\neg Flies(b)}} \supset Flies(b)\right) \longrightarrow True$$

$$\left(Bird(c) \wedge \neg \overset{\text{True}}{\cancel{\mathcal{B}\neg Flies(c)}} \supset Flies(c)\right) \longrightarrow Bird(c) \supset Flies(c)$$

$$\left(Bird(d) \wedge \neg \overset{\text{True}}{\cancel{\mathcal{B}\neg Flies(d)}} \supset Flies(d)\right) \longrightarrow Bird(d) \supset Flies(d)$$

$$KB^0 = \{Bird(a), Bird(b), (Bird(c) \vee Bird(d)), \neg Flies(b), Flies(a), Bird(c) \supset Flies(c), Bird(d) \supset Flies(d)\}$$

$KB^0 \not\models \neg Flies(a) \qquad KB^0 \models \neg Flies(b) \qquad KB^0 \not\models \neg Flies(c) \qquad KB^0 \not\models \neg Flies(b)$

stable expansion

Case 3:

$\mathcal{B}\neg Flies(a) = False \qquad \mathcal{B}\neg Flies(b) = True \qquad \mathcal{B}\neg Flies(c) = True \qquad \mathcal{B}\neg Flies(d) = False$

$$\left(Bird(a) \wedge \neg \overset{\text{True}}{\cancel{\mathcal{B}\neg Flies(a)}} \supset Flies(a)\right) \longrightarrow \overset{\text{True}}{\cancel{Bird(a)}} \supset Flies(a) \longrightarrow Flies(a)$$

$$\left(Bird(b) \wedge \neg \overset{\text{False}}{\cancel{\mathcal{B}\neg Flies(b)}} \supset Flies(b)\right) \longrightarrow True$$

$$\left(Bird(c) \wedge \neg \overset{\text{False}}{\cancel{\mathcal{B}\neg Flies(c)}} \supset Flies(c)\right) \longrightarrow True$$

$$\left(Bird(d) \wedge \neg \overset{\text{True}}{\cancel{\mathcal{B}\neg Flies(d)}} \supset Flies(d)\right) \longrightarrow Bird(d) \supset Flies(d)$$

$$KB^{0} = \{Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b),\ Flies(a),\ Bird(d) \supset Flies(d)\}$$

$KB^{0} \not\models \neg Flies(a)$ $\quad$ $KB^{0} \models \neg Flies(b)$ $\quad$ $\underline{KB^{0} \not\models \neg Flies(c)}$ $\quad$ $KB^{0} \not\models \neg Flies(b)$

↳ Not stable

Case 4:

$B\neg Flies(a) = False$ $\quad$ $B\neg Flies(b) = True$ $\quad$ $B\neg Flies(c) = False$ $\quad$ $B\neg Flies(d) = True$

$$(Bird(a) \wedge \overset{True}{\cancel{\neg B \neg Flies(a)}} \supset Flies(a)) \longrightarrow \overset{True}{\cancel{Bird(a)}} \supset Flies(a) \longrightarrow Flies(a)$$

$$(Bird(b) \wedge \overset{False}{\cancel{\neg B \neg Flies(b)}} \supset Flies(b)) \longrightarrow True$$

$$(Bird(c) \wedge \overset{False}{\cancel{\neg B \neg Flies(c)}} \supset Flies(c)) \longrightarrow Bird(c) \supset Flies(c)$$

$$(Bird(d) \wedge \overset{True}{\cancel{\neg B \neg Flies(d)}} \supset Flies(d)) \longrightarrow True$$

$$KB^{0} = \{Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b),\ Flies(a),\ Bird(c) \supset Flies(c)\}$$

$KB^{0} \not\models \neg Flies(a)$ $\quad$ $KB^{0} \models \neg Flies(b)$ $\quad$ $KB^{0} \not\models \neg Flies(c)$ $\quad$ $\underline{KB^{0} \not\models \neg Flies(b)}$

↳ Not stable

Case 5:

$B\neg Flies(a) = False$ $\quad$ $B\neg Flies(b) = True$ $\quad$ $B\neg Flies(c) = True$ $\quad$ $B\neg Flies(d) = True$

$$(Bird(a) \wedge \overset{True}{\cancel{\neg B \neg Flies(a)}} \supset Flies(a)) \longrightarrow \overset{True}{\cancel{Bird(a)}} \supset Flies(a) \longrightarrow Flies(a)$$

$$\left(Bird(b) \wedge \overset{\text{False}}{\cancel{\neg\mathcal{B}\neg Flies(b)}} \supset Flies(b)\right) \longrightarrow True$$

$$\left(Bird(c) \wedge \overset{\text{False}}{\cancel{\neg\mathcal{B}\neg Flies(c)}} \supset Flies(c)\right) \longrightarrow True$$

$$\left(Bird(d) \wedge \overset{\text{False}}{\cancel{\neg\mathcal{B}\neg Flies(d)}} \supset Flies(d)\right) \longrightarrow True$$

$$KB^{\circ} = \{Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b),\ Flies(a)\}$$

$KB^{\circ} \not\models \neg Flies(a)$ $\quad$ $KB^{\circ} \models \neg Flies(b)$ $\quad$ $\underbrace{KB^{\circ} \not\models \neg Flies(c) \qquad KB^{\circ} \not\models \neg Flies(b)}_{\hookrightarrow \text{Not stable}}$

Case 6)

$\mathcal{B}\neg Flies(a) = True$ $\quad$ $\mathcal{B}\neg Flies(b) = True$ $\quad$ $\mathcal{B}\neg Flies(c) = True$ $\quad$ $\mathcal{B}\neg Flies(d) = True$

$$\left(Bird(a) \wedge \overset{\text{False}}{\cancel{\neg\mathcal{B}\neg Flies(a)}} \supset Flies(a)\right) \longrightarrow True$$

$$\left(Bird(b) \wedge \overset{\text{False}}{\cancel{\neg\mathcal{B}\neg Flies(b)}} \supset Flies(b)\right) \longrightarrow True$$

$$\left(Bird(c) \wedge \overset{\text{False}}{\cancel{\neg\mathcal{B}\neg Flies(c)}} \supset Flies(c)\right) \longrightarrow True$$

$$\left(Bird(d) \wedge \overset{\text{False}}{\cancel{\neg\mathcal{B}\neg Flies(d)}} \supset Flies(d)\right) \longrightarrow True$$

$$KB^{\circ} = \{Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b)\}$$

$\underbrace{KB^{\circ} \not\models \neg Flies(a)}_{\hookrightarrow \text{Not stable}}$ $\quad$ $KB^{\circ} \models \neg Flies(b)$ $\quad$ $\underbrace{KB^{\circ} \not\models \neg Flies(c) \qquad KB^{\circ} \not\models \neg Flies(d)}_{\hookrightarrow \text{Not stable}}$

Any case with $B \neg Flies(a) = True$ Would be unstable as the KB would not entail $\neg Flies(a)$

Only Case 2 is stable

$KB^{o} = \{ Bird(a), Bird(b), (Bird(c) \vee Bird(d)), \neg Flies(b), Flies(a), Bird(c) \supset Flies(c), Bird(d) \supset Flies(d) \}$

$KB = \{ Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b) \}$

Weak default $\quad \forall x\,(B\,Bird(x) \wedge \neg B \neg Flies(x) \supset Flies(x))$

$(B\,Bird(a) \wedge \neg B \neg Flies(a) \supset Flies(a)),\quad (B\,Bird(b) \wedge \neg B \neg Flies(b) \supset Flies(b)),$

$(B\,Bird(c) \wedge \neg B \neg Flies(c) \supset Flies(c)),\quad (B\,Bird(d) \wedge \neg B \neg Flies(d) \supset Flies(d)),$

The following have to always be True $\quad B\,Bird(a) = True \quad B\,Bird(b) = True$
$B \neg Flies(b) = True$ to prevent conflict.

Case 1: $\quad B\,Bird(a) = True \qquad B\,Bird(b) = True \qquad B\,Bird(c) = True \qquad B\,Bird(d) = True$

$\qquad B \neg Flies(a) = False \qquad B \neg Flies(b) = True \qquad B \neg Flies(c) = False \qquad B \neg Flies(d) = False$

$(\cancelto{True}{B\,Bird(a)} \wedge \cancelto{True}{\neg B \neg Flies(a)} \supset Flies(a)) \longrightarrow Flies(a)$

$(\cancelto{True}{B\,Bird(b)} \wedge \cancelto{False}{\neg B \neg Flies(b)} \supset Flies(b)) \longrightarrow True$

$(\cancelto{True}{B\,Bird(c)} \wedge \cancelto{True}{\neg B \neg Flies(c)} \supset Flies(c)) \longrightarrow Flies(c)$

$(\cancelto{True}{B\,Bird(d)} \wedge \cancelto{True}{\neg B \neg Flies(d)} \supset Flies(d)) \longrightarrow Flies(d)$

$KB^{o} = \{ Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b),\ Flies(a),\ Flies(c),\ Flies(d) \}$

$KB^{o} \models Bird(a) \qquad KB^{o} \models Bird(b) \qquad \underline{KB^{o} \not\models Bird(c)} \qquad \underline{KB^{o} \not\models Bird(d)}$

$KB^{o} \not\models \neg Flies(a) \qquad KB^{o} \models \neg Flies(b) \qquad KB^{o} \not\models \neg Flies(c) \qquad KB^{o} \not\models \neg Flies(d)$

Due to the underlined entailments the expansion is not stable

Case 2: $B\,Bird(a) = True$ $\quad B\,Bird(b) = True$ $\quad B\,Bird(c) = False$ $\quad B\,Bird(d) = False$

$B\neg Flies(a) = False$ $\quad B\neg Flies(b) = True$ $\quad B\neg Flies(c) = False$ $\quad B\neg Flies(d) = False$

$$\left(\overset{\text{True}}{\cancel{B\,Bird(a)}} \wedge \overset{\text{True}}{\cancel{\neg B\neg Flies(a)}} \supset Flies(a)\right) \longrightarrow Flies(a)$$

$$\left(\overset{\text{True}}{\cancel{B\,Bird(b)}} \wedge \overset{\text{False}}{\cancel{\neg B\neg Flies(b)}} \supset Flies(b)\right) \longrightarrow True$$

$$\left(\overset{\text{False}}{\cancel{B\,Bird(c)}} \wedge \overset{\text{True}}{\cancel{\neg B\neg Flies(c)}} \supset Flies(c)\right) \longrightarrow True$$

$$\left(\overset{\text{False}}{\cancel{B\,Bird(d)}} \wedge \overset{\text{True}}{\cancel{\neg B\neg Flies(d)}} \supset Flies(d)\right) \longrightarrow True$$

$$KB^0 = \{Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b),\ Flies(a)\}$$

$KB^0 \models Bird(a)$ $\quad KB^0 \models Bird(b)$ $\quad KB^0 \not\models Bird(c)$ $\quad KB^0 \not\models Bird(d)$

$KB^0 \not\models \neg Flies(a)$ $\quad KB^0 \models \neg Flies(b)$ $\quad KB^0 \not\models \neg Flies(c)$ $\quad KB^0 \not\models \neg Flies(d)$

This expansion is stable

Case 3: $B\,Bird(a) = \text{True}$ $\quad B\,Bird(b) = \text{True}$ $\quad B\,Bird(c) = \text{False}$ $\quad B\,Bird(d) = \text{True}$

$B\neg Flies(a) = \text{True}$ $\quad B\neg Flies(b) = \text{True}$ $\quad B\neg Flies(c) = \text{False}$ $\quad B\neg Flies(d) = \text{False}$

$$\left(\overset{\text{True}}{\cancel{B\,Bird(a)}} \wedge \overset{\text{False}}{\cancel{\neg B\neg Flies(a)}} \supset Flies(a)\right) \longrightarrow \text{True}$$

$$\left(\overset{\text{True}}{\cancel{B\,Bird(b)}} \wedge \overset{\text{False}}{\cancel{\neg B\neg Flies(b)}} \supset Flies(b)\right) \longrightarrow \text{True}$$

$$\left(\overset{\text{False}}{\cancel{B\,Bird(c)}} \wedge \overset{\text{True}}{\cancel{\neg B\neg Flies(c)}} \supset Flies(c)\right) \longrightarrow \text{True}$$

$$\left(\overset{\text{True}}{\cancel{B\,Bird(d)}} \wedge \overset{\text{True}}{\cancel{\neg B\neg Flies(d)}} \supset Flies(d)\right) \longrightarrow Flies(d)$$

$$KB^{\circ} = \{Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b),\ Flies(d)\}$$

$KB^{\circ} \models Bird(a)$ $\quad KB^{\circ} \models Bird(b)$ $\quad \underline{KB^{\circ} \not\models Bird(c)}$ $\quad \underline{KB^{\circ} \not\models Bird(d)}$

$\underline{KB^{\circ} \not\models \neg Flies(a)}$ $\quad KB^{\circ} \models \neg Flies(b)$ $\quad KB^{\circ} \not\models \neg Flies(c)$ $\quad KB^{\circ} \not\models \neg Flies(d)$

Due to the underlined entailments the expansion is not stable

$\therefore$ $B\neg Flies(a)$ has to be False for a stable expansion

Case 4: $B\,Bird(a) = True$ $\quad B\,Bird(b) = True$ $\quad B\,Bird(c) = False$ $\quad B\,Bird(d) = True$

$B\neg Flies(a) = False$ $\quad B\neg Flies(b) = True$ $\quad B\neg Flies(c) = True$ $\quad B\neg Flies(d) = False$

$$\left(\cancelto{\text{True}}{B\,Bird(a)} \wedge \cancelto{\text{False}}{\neg B\neg Flies(a)} \supset Flies(a)\right) \longrightarrow True$$

$$\left(\cancelto{\text{True}}{B\,Bird(b)} \wedge \cancelto{\text{False}}{\neg B\neg Flies(b)} \supset Flies(b)\right) \longrightarrow True$$

$$\left(\cancelto{\text{False}}{B\,Bird(c)} \wedge \cancelto{\text{False}}{\neg B\neg Flies(c)} \supset Flies(c)\right) \longrightarrow True$$

$$\left(\cancelto{\text{True}}{B\,Bird(d)} \wedge \cancelto{\text{True}}{\neg B\neg Flies(d)} \supset Flies(d)\right) \longrightarrow Flies(d)$$

$KB^{\circ} = \{Bird(a), Bird(b), (Bird(c) \vee Bird(d)), \neg Flies(b), Flies(d)\}$

$KB^{\circ} \models Bird(a)$ $\quad KB^{\circ} \models Bird(b)$ $\quad KB^{\circ} \not\models Bird(c)$ $\quad KB^{\circ} \not\models Bird(d)$

$\underline{KB^{\circ} \not\models \neg Flies(a)}$ $\quad KB^{\circ} \models \neg Flies(b)$ $\quad KB^{\circ} \not\models \neg Flies(c)$ $\quad KB^{\circ} \models \neg Flies(d)$

Due to the underlined entailments the expansion is not stable

All the other combinations of True and False beliefs result in similar extensions as the ones shown.

The only stable expansion via the strong default is:

$KB^{\circ} = \{Bird(a), Bird(b), (Bird(c) \vee Bird(d)), \neg Flies(b), Flies(a), Bird(c) \supset Flies(c), Bird(d) \supset Flies(d)\}$

The only stable expansion via the weak default is:

$$KB^{o} = \{ Bird(a),\ Bird(b),\ (Bird(c) \vee Bird(d)),\ \neg Flies(b),\ Flies(a) \}$$

The weak default stable expansion implies only $Flies(a)$. However, the strong default stable expansion not only implies $Flies(a)$, but also $Bird(c) \supset Flies(c)$, $Bird(d) \supset Flies(d)$, which means that, if $Bird(c)$ or $Bird(d)$ is True then $Flies(c)$ and $Flies(d)$ is also True respectively.

b)

$$\forall x \left(B\,Bird(x) \wedge \neg B \neg Flies(x) \supset B\,Flies(x) \right)$$

Via the implecation rule $A \supset B \equiv \neg A \vee B$ the default can be rewritten

$$\neg(B\,Bird(x) \wedge \neg B \neg Flies(x)) \vee B\,Flies(x) \equiv \neg B\,Bird(x) \vee B \neg Flies(x) \vee B\,Flies(x)$$

$$\rightarrow \neg B\,Bird(x) \vee \underbrace{B \neg Flies(x) \vee B\,Flies(x)}_{\hookrightarrow \text{ this will always be True}}$$

Therefore, this default does not lead to any reasonable conclusion.

Based on the KB the following have to be True

$Bird(a) = True$ $\quad$ $Bird(b) = True$ $\quad$ $B \neg Flies(b) = True$

Case 1: $B \neg Flies(a) = False$

$$\left(\overset{\text{True}}{\cancel{B\,Bird(a)}} \wedge \neg \overset{\text{True}}{\cancel{B \neg Flies(a)}} \supset \overset{\text{True}}{\cancel{B\,Flies(a)}} \right) \longrightarrow True$$

$$\left(\overset{\text{True}}{\cancel{B\,Bird(b)}} \wedge \neg \overset{\text{False}}{\cancel{B \neg Flies(b)}} \supset B\,Flies(b) \right) \longrightarrow True$$

$(B\, \cancel{Bird(c)}^{\text{False}} \wedge \neg B \neg \cancel{Flies(c)}^{\text{False}} \supset B\, Flies(c)) \longrightarrow True$

$(B\, \cancel{Bird(d)}^{\text{True}} \wedge \neg B \neg \cancel{Flies(d)}^{\text{True}} \supset B\, Flies(d)) \longrightarrow True$

Since all are belief statement no expansion can be derived therefore the default would not lead to a reasonable conclusion.

C)

Default Logic "Birds fly" $KB = \{Bird(a), Bird(b), (Bird(c) \vee Bird(d)), \neg Flies(b)\}$

For a $\dfrac{Bird(a) : Flies(a)}{Flies(a)} \longrightarrow Flies(a) = True$

for b $\dfrac{Bird(b) : Flies(b)}{Flies(b)}$ ← contradiction $\longrightarrow \neg Flies(b) = False$

Only derived expansion is Flies(a) making this similar to the weak default

Circumpscription

$KB = \{\forall x(Bird(x) \wedge \neg Ab(x) \supset Flies(x)),\ Bird(a), Bird(b), (Bird(c) \vee Bird(d)), \neg Flies(b)\}$

M1

$Ab = \{a, b, c, d\}$ $M_1 \models KB$

M_2

$Ab = \{b\}$

By assuming both Bird(c) and Bird(d) are True we can minimize the abnormality as much as possible.

This means Flies(a), Flies(c), Flies(d)

This is similar to Strong default where Flies(a) is derived and Flies(c) and Flies(d) are concluded given that Bird(c) and Bird(d) are True.

Question #3

literals are atoms and their negation

Weak GCWA(Φ) = $\Phi \cup \{\neg\alpha \mid \alpha$ is an atomic ground formula in the vocabulary of Φ s.t. $\Phi \not\models \alpha$ and for all collections of all collections of literals $\beta_1, \ldots, \beta_n$ if $\Phi \models (\alpha \vee \beta_1 \vee \ldots \vee \beta_n)$ then $\Phi \models (\beta_1 \vee \ldots \vee \beta_n)\}$

- Does weak GCWA preserve Consistency? Explain Why.

- Are there any subset relationship between GCWA and weak GCWA? That is, can we say that for any arbitrary set of Sentences Φ, either of weak GCWA(Φ) $\subseteq$ GCWA(Φ) or GCWA(Φ) $\subseteq$ weak GCWA(Φ) holds? Explain Why

GCWA

example 1 $\Phi = \{p \vee \neg q\}$

$\Phi \not\models p \rightarrow$ since $\Phi \models p \vee \beta_1 \vee \ldots$ is false, the implication is vacuously True

There for the condition holds

$\Phi \not\models q \rightarrow$ Same logic can be applied to q

GCWA(Φ) = $\{(p \vee \neg q), p, \neg q\}$ ⟵ Consistent

Weak GCWA

$\Phi \not\models p$ $\Phi \models (p \vee \neg q)$ however $\Phi \not\models q$ Does not hold

$\Phi \not\models q$ $\Phi \models (p \vee \neg q)$ however $\Phi \models p$ Does not hold

$$\text{Weak GCWA}(\Phi) = \{p \vee \neg q\} \leftarrow \text{Consistent}$$

Example 2

$\Phi = \{(p \vee \neg q), (p \vee \neg q \vee r)\}$

GCWA

$\Phi \not\models p$ $\longrightarrow$ Since the antecedent is false (no collection of atoms make it True) the implication is vacuously True.

$\Phi \not\models q$ $\longrightarrow$ Similar Reasing as above

$\Phi \not\models r$ $\longrightarrow$ Similar Reasing as above

$$GCWA(\Phi) = \{(p \vee \neg q), (p \vee \neg q \vee r), \neg p, \neg q, \neg r\} \leftarrow \text{Consistent}$$

Weak GCWA

$\Phi \not\models p$ $\Phi \models \{p \vee \neg q \vee r\}$ but $\Phi \not\models \{\neg q \vee \neg r\}$ Does not hold

$\Phi \models q$ $\Phi \models \{p \vee p \vee \neg q \vee r\}$ and $\Phi \models \{p \vee \neg q \vee r\}$ holds

$\Phi \models r$ $\Phi \models \{r \vee p \vee \neg q\}$ and $\Phi \models \{p \vee \neg q\}$ holds

$$\text{Weak GCWA}(\Phi) = \{(p \vee \neg q), (p \vee \neg q \vee r), \neg q, \neg r\} \leftarrow \text{Consistent}$$

Example 3

$\Phi = \{(p \vee q), (p \vee \neg q \vee \neg r)\}$

GCWA

$\Phi \not\models p \longrightarrow \Phi \models (p \vee q)$ but $\Phi \not\models q$ Does not hold

$\Phi \not\models q \longrightarrow \Phi \models (p \vee q)$ but $\Phi \not\models p$ Does not hold

$\Phi \not\models r \longrightarrow$ No collection of atom in the antecendent can be True

Therefore Vacuously holds

$GCWA(\Phi) = \{(p \vee q), (p \vee \neg q \vee \neg r), \neg r\} \leftarrow$ Consistent

Weak GCWA

$\Phi \not\models p \longrightarrow$ Due to similar reason to GCWA Doesn't hold

$\Phi \not\models q \longrightarrow$ Due to similar reason to GCWA Doesn't hold

$\Phi \not\models r \longrightarrow \Phi \models (r \vee p \vee \neg q \vee \neg r)$ and $\Phi \models (p \vee \neg q \vee \neg r)$

then it holds

$\text{Weak } GCWA(\Phi) = \{(p \vee q), (p \vee \neg q \vee \neg r), \neg r\} \leftarrow$ Consistent

Conclusion

Consistency

Based on the examples shown it can be seen that the expanded set is always consistent. Since GCWA is awalys consistent, the addition of restrictions in weak GCWA ensures that Weak GCWA is also always consistent.

Relationship

The relationship that can be seen from the examples shown is

Weak GCWA (Φ) $\subseteq$ GCWA (Φ). As mentioned this is due to the additional restrictions added to the weak GCWA compared to GCWA, which causes the extension to be less as shown in examples 1 and 2 or at most equal as shown in example 3. A general pattern noticed was with introduction of negations the $\Phi \models (a \vee \beta_1, \ldots \beta_n)$ antecedent was usually false which led to the implication being implications being vacuously True, hence the negation of the atom was added. However, since in weak GCWA β_i were literal the $\Phi \models (a \vee \beta_1, \ldots \beta_n)$ usually holds True hence not causing the implication to be vacuously True. This added restriction noticed was one of the reasons that

Weak GCWA (Φ) $\subseteq$ GCWA (Φ).