
- Assignments must be completed ***individually***, and you must submit a *single* solution in a PDF file named `a1.pdf`, submitted to MarkUs. Handwritten submissions are acceptable as long as they are written ***neatly*** and ***legibly*** (typed submissions are preferable but not required).
- Please refer to the course information sheet for the ***late submission policy***.
- For each question, please write up detailed answers carefully. Make sure that you use ***notation*** and ***terminology*** correctly, and that you explain and ***justify*** what you are doing. Marks will be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.
- Please read and understand the policy on Academic Integrity given on the course information sheet. Then, to protect yourself, list on the front of your submission ***every*** source of information you used to complete this assignment (other than the lecture and tutorial material). For example, indicate clearly the ***name*** of every student with whom you had discussions, the ***title and sections*** of every textbook you consulted (including the course textbook), the ***source*** of every web document you used, etc.

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1. (Chapter 3, Exercise 4 of the textbook) A Canadian variant of an old puzzle:

> *A traveler in remote Quebec comes to a fork in the road and does not know which way to go to get to Chicoutimi. Henri and Pierre are two local inhabitants nearby who do know the way. One of them always tells the truth, and the other one never does, but the traveler does not know which is which. Is there a single question the traveler can ask Henri (in French, of course) that will be sure to tell him which way to go?*

We will formalize this problem in FOL. Assume there are only two sorts of objects in our domain: inhabitants, denoted by the constant symbols *henri* and *pierre*; and French questions, which Henri and Pierre can answer. These questions are denoted by the following function symbols:

- $gauche$, which asks if the traveler should take the left branch of the fork to get to Chicoutimi;
- $ditOui(x, q)$, which asks if inhabitant x would answer yes to the French question q;
- $ditNon(x, q)$, which asks if inhabitant x would answer no to the French question q.

Obviously this is a somewhat impoverished dialect of French, although a philosophically interesting one. For example, the term

$$ditNon(henri, ditOui(pierre, gauche))$$

represents a French question that might be translated as, "Would Henri answer no if I asked him if Pierre would say yes I should go to the left to get to Chicoutimi?"
The predicate symbols of our language are the following:

- $TruthTeller(x)$, which holds when inhabitant x is a truth teller;
- $AnswerYes(x, q)$, which holds when inhabitant x will answer yes to French question q;
- $True(q)$, which holds when the correct answer to the question q is yes;
- $GoLeft$, which holds if the direction to get to Chicoutimi is to go left.

For purposes of this puzzle, these are the ***only*** constant, function, and predicate symbols.

(a) Write FOL sentences for each of the following:

- One of Henri or Pierre is a truth teller, and one is not.
- An inhabitant will answer yes to a question if and only if he is a truth teller and the correct answer is yes, or he is not a truth teller and the correct answer is not yes.
- The correct answer to *gauche* question is yes if and only if the proper direction is to go is left.
- The correct answer to a $ditOui(x, q)$ question is yes if and only if x will answer yes to question q.
- The correct answer to a $ditNon(x, q)$ question is yes if and only if x will not answer yes to q.

(b) Imagine that the facts from Part (a) make up the entire KB of the traveler.
Provide a formal proof showing that there is a ground term t (i.e., a term with no variables) such that

$$KB \models (AnswerYes(henri, t) \equiv GoLeft)$$

In other words, there is a question t that can be asked to Henri that will be answered yes if and only if the proper direction to get to Chicoutimi is to go left.

(c) Imagine that the facts from Part (a) make up the entire KB of the traveler.
Show that this KB does not entail which direction to go. That is, show that there is an interpretation satisfying the KB where $GoLeft$ is true, and another one where it is false.

2. (Chapter 5, Exercise 4 of the textbook) In this question, we will explore the semantic properties of ***propositional*** Horn clauses.
For any set of clauses S, define $\mathcal{I}_S$ to be the interpretation that satisfies an atom p if and only if $S \models p$. (Recall that for propositional formulas, an interpretation is basically a truth assignment)

(a) Show that if S is a set of ***positive Horn clauses***, then $\mathcal{I}_S \models S$.

(b) Give an example of a set of clauses S where $\mathcal{I}_S \not\models S$.

(c) Suppose that S is a set of ***positive Horn clauses*** and that c is a ***negative Horn clause***. Show that if $\mathcal{I}_S \not\models c$ then $S \cup \{c\}$ is unsatisfiable.

(d) Suppose that S is a set of ***positive Horn clauses*** and that N is a set of ***negative ones***. Using part (c), show that if $S \cup \{c\}$ is satisfiable for every $c \in N$, then $S \cup N$ is also satisfiable.

3. (Chapter 5, Exercise 5 of the textbook) In this question, we will formalize a fragment of high school geometry. We will use a single binary predicate symbol, which we write here as $\cong$. The objects in this domain are points, lines, angles, and triangles. We will use constants only to name the points we need, and for the other individuals we will use function symbols that take points as arguments: first, a function that given two points x and y is used to name the line between them, which we write here as $\overline{xy}$; next, a function that given three points x, y, z names the angle between them, which we write here as $\angle xyz$; and finally, a function that given three points x, y, z names the triangle between them, which we write here as $\triangle xyz$. Here are the axioms of interest:

- $\cong$ is an equivalence relation (i.e., it's reflexive, symmetric and transitive).
- For all x, y, $\overline{xy} \cong \overline{yx}$.
- For all x, y, z, $\angle xyz \cong \angle zyx$.
- For all x, y, z, u, v, w, if $\triangle xyz \cong \triangle uvw$, then the corresponding lines and angles are congruent ($\overline{xy} \cong \overline{uv}$, $\angle xyz \cong \angle uvw$, etc.).
- **SAS:** For all x, y, z, u, v, w, if $\overline{xy} \cong \overline{uv}$, $\angle xyz \cong \angle uvw$, and $\overline{yz} \cong \overline{vw}$, then $\triangle xyz \cong \triangle uvw$.

Present an SLD resolution showing that these axioms imply that the base angles of an isosceles triangle must be equal. That is,

$$Axioms \cup \{\overline{ab} \cong \overline{ac}\} \models \angle abc \cong \angle acb$$

where a, b, c are constant symbols.