Assignment #1 CSC 2502 Parsa Yousse fpour 1004749659

Question #1

a)

- One of Henri or Pierre is truth teller and one is not.

$$(\text{TruthTeller}(\text{Henri}) \wedge \neg \text{TruthTeller}(\text{Pierre})) \vee (\neg \text{TruthTeller}(\text{Henri}) \wedge \text{TruthTeller}(\text{Pierre}))$$

can be simpilified to

$$\text{TruthTeller}(\text{Henri}) \equiv \neg \text{TruthTeller}(\text{pierre})$$

- An inhabitant will answer yes to a question if and only if he is a truth teller and correct answer is yes, or he is not a truth teller and the correct answer is not yes.

$$\forall_x \forall_q (\text{AnswerYes}(x,q) \equiv ((\text{TruthTeller}(x) \wedge \text{True}(q)) \vee (\neg \text{TruthTeller}(x) \wedge \neg \text{True}(q))))$$

can be simplified to

$$\forall_x \forall_q (\text{AnswerYes}(x,q) \equiv (\text{TruthTeller}(x) \equiv \text{True}(q)))$$

- The correct answer to gauche question is yes if and only if the only proper question is to go is left.

$$\text{True}(\text{gauche}) \equiv \text{GoLeft}$$

- The correct answer to a ditOui (x,q) question is yes if and only if x will answer yes to question q.

$$\forall_x \forall_q (\text{True}(\text{ditOui}(x,q)) \equiv \text{AnswerYes}(x,q))$$

- The correct answer to a ditNon (x,q) question is yes if and only if x will not answer yes to q.

$$\forall_x \forall_q (\text{True}(\text{ditNon}(x,q)) \equiv \neg \text{AnswerYes}(x,q))$$

b) $KB \models (AnswerYes(heri, t) \equiv GoLeft)$

To evaluate $AnswerYes(heri, t)$ use $\forall_x \forall_y (AnswerYes(x, q) \equiv (TruthTeller(x) \equiv True(q)))$ from the KB to get

(1) $AnswerYes(heri, t) \equiv (TruthTeller(heri) \equiv True(t))$

To eval $True(t)$ use $ditNon(pierre, gauche)$ as an example question $(t / ditNon(pierre, gauche)$ substitution$)$

from $\forall_x \forall_q (True(ditNon(x, q)) \equiv \neg AnswerYes(x, q))$ in the KB make subs for $x =$ pierre and $y =$ gauche to get

(2) $True(ditNon(pierre, gauche)) \equiv \neg AnswerYes(pierre, gauche)$

Look at deeper into $AnswerYes(pierre, gauche)$
from $\forall_x \forall_y (AnswerYes(x, q) \equiv (TruthTeller(x) \equiv True(q)))$ in the KB make subs for $x =$ pierre and $y =$ gauche

(3) $AnswerYes(pierre, gauche) \equiv (TruthTeller(pierre) \equiv True(gauche))$

fro $True(gauche) \equiv GoLeft$ in the KB the following can be deduced.
(Replacing $True(gauche)$ with $GoLeft$ as they are Logical equivalences)

(4) $AnswerYes(pierre, gauche) \equiv (TruthTeller(pierre) \equiv GoLeft)$

Looking deeper into $TruthTeller(pierre)$ use $TruthTeller(Henri) \equiv \neg TruthTeller(pierre)$ from the KB

(5) $TruthTeller(pierre) \equiv \neg TruthTeller(Henri)$

Sub 5 into 4:

$$\text{AnswerYes}(\text{pierre}, \text{gauche}) \equiv (\neg \text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft})$$

negate this to and sub into 2.

$$\neg \text{AnswerYes}(\text{pierre}, \text{gauche}) \equiv \neg(\neg \text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft})$$

$$\text{True}(\text{ditNon}(\text{pierre}, \text{gauche})) \equiv \neg(\neg \text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft})$$

Sub out ditNon(pierre, gauche) with t and sub result into 1:

$$\text{AnswerYes}(\text{heri}, t) \equiv (\text{TruthTeller}(\text{heri}) \equiv \neg(\neg \text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft}))$$

The following are logical equivalences, by looking at a simple truth table:

$$\neg(\neg \text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft}) \equiv (\text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft})$$

Sub in:

$$\text{AnswerYes}(\text{heri}, t) \equiv (\text{TruthTeller}(\text{heri}) \equiv (\text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft}))$$

using a Truth table we can determine that

$$(\text{TruthTeller}(\text{heri}) \equiv (\text{TruthTeller}(\text{Henri}) \equiv \text{GoLeft})) \equiv \text{GoLeft}$$

T	G	$T \equiv G$	$T \equiv (T \equiv G)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	T	F

Finally we can make the substitution to get the proof.

$$AnswerYes(heri, t) \equiv GoLeft$$

Therefore

$$KB \models AnswerYes(heri, t) \equiv GoLeft$$

c) Goal: Prove GoLeft can be both True and False.

First $GoLeft = True$

if $GoLeft = True$ based on eq $True(gauche) \equiv GoLeft$ in the KB

$$True(gauche) = True$$

Based on $\forall_x \forall_q \left(AnswerYes(x, q) \equiv (TruthTeller(x) \equiv True(q)) \right)$ From the KB

And assuming

$TruthTeller(henri) = True$

$TruthTeller(pierre) = False$

$$AnswerYes(henri, gauche) \equiv (TruthTeller(henri) \equiv True(gauche))$$

$$True \equiv True$$

For the above statement to be True $AnswerYes(henri, gauche)$ needs to be True

$$AnswerYes(henri, gauche) = True$$

$$AnswerYes(pierre, gauche) \equiv (TruthTeller(pierre) \equiv True(gauche))$$

$$False \equiv True$$

For the above statement to be True AnswerYes(pierre, gauche) must be False.

$$AnswerYes(pierre, gauche) = False$$

Now using the $\forall x \forall q \, (True(ditOui(x,q) \equiv AnswerYes(x,q))$ equations from the KB

$$True(ditOui(henri, gauche)) \equiv \underset{True}{AnswerYes(heri, gauche)}$$

Hence $True(ditOui(henri, gauche)) = True$

This means that "heri would say yes when asked if the traveller should go left"
Which is Correct as henri tells the truth and GoLeft is True.

$$True(ditOui(pierre, gauche)) \equiv \underset{False}{AnswerYes(pierre, gauche)}$$

Hence $True(ditOui(pierre, gauche)) = False$

This means that " pierre would say yes if asked the traveller should go left" This is False as pierre lies, GoLeft is true and the expected Answer is no making the derived statement correct.

Now using the $\forall x \forall q \, (True(ditNon(x,q) \equiv \neg AnswerYes(x,q))$ equations from the KB

$$True(ditNon(pierre, gauche)) \equiv \neg AnswerYes(pierre, gauche)$$

$$\neg False$$

Hence $True(ditNon(pierre, gauche)) = True$

This means "would pierre would say Non when asked if the traveller should go left"
Which is correct as pierre lies and GoLeft is True.

$$True(ditNon(henri, gauche)) \equiv \neg AnswerYes(heri, gauche)$$

$$\neg True$$

Hence $True(ditNon(henri, gauche)) = False$

This means "would henri say no when asked if the traveller should go left"
Which is False as heri tells the truth and GoLeft is True making the derived eq correct.

GoLeft = True satisfies all the equations in the KB

Next GoLeft = False

if GoLeft = True based on eq $True(gauche) \equiv GoLeft$ in the KB

$$True(gauche) = False$$

Based on $\forall_x \forall_q (AnswerYes(x, q) \equiv (TruthTeller(x) \equiv True(q)))$ from the KB
And assuming

$TruthTeller(henri) = True$

$TruthTeller(pierre) = False$

$$AnswerYes(henri, gauche) \equiv (TruthTeller(henri) \equiv True(gauche))$$

$$True \equiv False$$

For the above statement to be True $AnswerYes(henri, gauche)$ needs to be false

$$AnswerYes(henri, gauche) = False$$

$$AnswerYes(pierre, gauche) \equiv (TruthTeller(pierre) \equiv True(gauche))$$

$$False \equiv False$$

For the above statement to be True $AnswerYes(pierre, gauche)$ must be True

$$AnswerYes(pierre, gauche) = True$$

Now using the $\forall x \forall q \, (True(ditOui(x,q)) \equiv AnswerYes(x,q))$ equations from the KB

$$True(ditOui(henri, gauche)) \equiv AnswerYes(heri, gauche)$$

$$False$$

Hence $True(ditOui(henri, gauche)) = False$

This means that "heri would say yes when asked if the traveller should go left"
Which is False as henri tells the truth and GoLeft is False, making the derived eq correct.

$$True(ditOui(pierre, gauche)) \equiv AnswerYes(pierre, gauche)$$

$$True$$

Hence $True(ditOui(pierre, gauche)) = True$

This means that "pierre would say yes if asked the traveller should go left" This is True as pierre lies, GoLeft is False and the expected Answer is yes making the derived statement correct.

Now using the $\forall_x \forall_q \left(True(ditNon(x,q)) \equiv \neg AnswerYes(x,q)\right)$ equations from the KB

$$True(ditNon(pierre, gauche)) \equiv \neg \underbrace{AnswerYes(pierre, gauche)}_{\neg True}$$

Hence $True(ditNon(pierre, gauche)) = False$

This means "would pierre would say No when asked if the traveller should go left"

Which is incorrect as pierre lies and GoLeft is False, making the derived equation correct.

$$True(ditNon(henri, gauche)) \equiv \neg \underbrace{AnswerYes(heri, gauche)}_{\neg False}$$

Hence $True(ditNon(henri, gauche)) = True$

This means "would henri say no when asked if the traveller should go left"

Which is True as heri tells the truth and GoLeft is False making the derived eq correct.

GoLeft = False satisfies all the equations in the KB

This shows that both GoLeft = True and GoLeft = False satisfy the KB

Question #2

a)

Proof by Contradiction

- Suppose ~~there is~~ there exists one clause in S that is not satisfied by I_S. Hence $I_S \not\models S$.

  Let this clause be $p_1 \wedge p_2 \wedge \cdots p_n \supset q$

- For this clause not to satisfy I_S The antedecent must all be True in I_S and the Consequent must be False in I_S

  This means $S \models (p_1, p_2, \cdots p_n)$ and $S \not\models q$.

  However, Since S Contains $p_1 \wedge p_2 \wedge p_3 \wedge \cdots \wedge p_n \subset q$ and $S \models (p_1, p_2, \cdots p_n)$

  in the Consequent must also be entailed by S.

  If $(p_1 \wedge p_2 \wedge \cdots p_n \supset q)$ is True and $p_1 \wedge p_2 \cdots \wedge p_n$ is True, q must be True

  Hence $\underline{\underline{S \models q}}$.

  This is a direct contradiction with the initial assumption $I_S \not\models S$. As $S \not\models q$ and $S \models q$ Cannot both be true. Therefore, $I_S \models S$ must be True.

b) A simple example can be $S = \{p \vee q\}$

$S \not\models p$ or $S \not\models q$ as if either $I(p)$ or $I(q)$ is True and the other False S is satisfied

Hence, $I_S(p)$ and $I_S(q)$ are both false.

Now $I_S(p \vee q) = I_S(p) \vee I_S(q) = \text{False} \vee \text{False} = \text{False} \longrightarrow I_S \not\models p \vee q$ or $I_S \not\models S$

c) Proof by Contradiction

Assume $S \cup \{c\}$ is satisfiable $\xrightarrow{\text{this means}}$ There is an interpretation (mini model) I where

$I \models S$ and $I \models c$

Since S is a set of positive horn clauses and $I \models S$, $I(p_i) = \text{True}$

Since I_S is a mini model of S if $\underbrace{I_S(p_i) = \text{True}, \text{ then } I(p_i) = \text{True}}$

c a negative horn Clause

$\neg p_1 \vee \neg p_2 \vee \cdots \vee \neg p_n$

For $I \models c$ one of $p_1, \ldots p_n$ must be False under I_S.

Since $I_S \not\models c$ all $\neg p_i$ are false under I_S $\longrightarrow$ $I_S(p_i) = \text{True}$ $\longrightarrow$ $I(p_i) = \text{True}$

However, since all p_i is True, $\neg p_i$ is all False under I

making c false unde I $\longrightarrow$ $\underline{\underline{I \not\models c}}$

This contradicts the intial assumption of $I \models c$. Showing that if $I_S \not\models c$ then $S \cup \{c\}$ is unsatisfiable.

d) If $S \cup \{c\}$ is satified then $I_S \models c$ based on part c.

Since $I_S \models c$ for every $c \in N$ $\longrightarrow$ $I_S \models N$
We also know that $I_S \models S$

$\left.\begin{matrix} I_S \models N \\ I_S \models S \end{matrix}\right\}$ $I_S \models (N \cup S)$ $\longrightarrow$ $N \cup S$ is satisfiable.

Question #3

- reflexive $\forall_x \forall_y (xy \cong xy)$ $\forall_x \forall_y \forall_z (\angle xyz \cong \angle xyz)$ $\forall_x \forall_y \forall_z (\Delta xyz \cong \Delta xyz)$

- Symmetric $\forall_x \forall_y (\neg(\overline{xy} \cong \overline{uv}) \vee (\overline{xy} \cong \overline{uv})$

- $\forall_x \forall_y (\overline{xy} \cong \overline{yx})$

- $\forall_x \forall_y \forall_z (\angle xyz \cong \angle zyx)$

- $\forall_x \forall_y \forall_z \forall_u \forall_v \forall_w ((\Delta xyz \cong \Delta uvw) \supset ((\overline{xy} \cong \overline{uv}) \wedge (\overline{xz} \cong \overline{uw}) \wedge (\overline{yz} \cong \overline{vw}) \wedge (\angle xyz \cong \angle uvw) \wedge (\angle xzy \cong \angle uwv) \wedge (\angle yzx \cong \angle vwu)))$

$\longrightarrow$ $\neg(\Delta xyz \cong \Delta uvw) \vee ((\overline{xy} \cong \overline{uv}) \wedge (\overline{xz} \cong \overline{uw}) \wedge (\overline{yz} \cong \overline{vw}) \wedge (\angle xyz \cong \angle uvw) \wedge (\angle xzy \cong \angle uwv) \wedge (\angle yzx \cong \angle vwu))$

- SAS: $\forall_x \forall_y \forall_z \forall_u \forall_v \forall_w (((\overline{xy} \cong \overline{uv}) \wedge (\angle xyz \cong \angle uvw) \wedge (\overline{yz} \cong \overline{vw})) \supset (\Delta xyz \cong \Delta uvw))$

$\longrightarrow$ $\neg((\overline{xy} \cong \overline{uv}) \wedge (\angle xyz \cong \angle uvw) \wedge (\overline{yz} \cong \overline{vw})) \vee (\Delta xyz \cong \Delta uvw)$

$\longrightarrow$ $\neg(\overline{xy} \cong \overline{uv}) \vee \neg(\angle xyz \cong \angle uvw) \vee \neg(\overline{yz} \cong \overline{vw}) \vee (\Delta xyz \cong \Delta uvw)$

Clauses

1. $\overline{xy} \cong \overline{yx}$
2. $\angle xyz \cong \angle zyx$
3. $\neg(\Delta xyz \cong \Delta uvw) \vee (\overline{xy} \cong \overline{uv})$
4. $\neg(\Delta xyz \cong \Delta uvw) \vee (\overline{xz} \cong \overline{uw})$
5. $\neg(\Delta xyz \cong \Delta uvw) \vee (\overline{yz} \cong \overline{vw})$
6. $\neg(\Delta xyz \cong \Delta uvw) \vee (\angle xyz \cong \angle uvw)$
7. $\neg(\Delta xyz \cong \Delta uvw) \vee (\angle xzy \cong \angle uwv))$
8. $\neg(\Delta xyz \cong \Delta uvw) \vee (\angle yzx \cong \angle vwu)$

9. $\overline{ab} \cong \overline{ac}$ (given)

10. $\neg(\overline{xy} \cong \overline{uv}) \vee \neg(\angle xyz \cong \angle uvw) \vee \neg(\overline{yz} \cong \overline{vw}) \vee (\Delta xyz \cong \Delta uvw)$

Prove $\neg(\angle abc \cong \angle acb)$

Substitutions

$x = a \quad y = b \quad z = c$

$u = a \quad v = c \quad w = b$

$\quad c \quad\quad b \quad\quad a$

Attempt #1

$\neg(\angle abc \cong \angle acb)$

(6) $\neg(\Delta abc \cong \Delta acb) \vee (\angle abc \cong \angle acb)$

$\neg(\Delta abc \cong \Delta acb)$

(10) $\neg(\overline{ab} \cong \overline{ac}) \vee \neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb}) \vee (\Delta abc \cong \Delta acb)$

$\neg(\overline{ab} \cong \overline{ac}) \vee \neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb})$

(9) $\overline{ab} \cong \overline{ac}$

$\neg(\Delta abc \cong \Delta acb) \vee (\overline{bc} \cong \overline{cb})$

$\neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb})$

$\neg(\Delta abc \cong \Delta acb) \vee \neg(\angle abc \cong \angle acb)$

Attempt #2

9

$\overline{ab} \cong \overline{ac}$

10

$\neg(\overline{ab} \cong \overline{ac}) \vee \neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb}) \vee (\triangle abc \cong \triangle acb)$

$\neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb}) \vee (\triangle abc \cong \triangle acb)$

$\neg(\triangle abc \cong \triangle acb) \vee (\overline{bc} \cong \overline{cb})$

6

$\neg(\triangle abc \cong \triangle acb) \vee (\angle abc \cong \angle acb)$

$\neg(\angle abc \cong \angle acb)$

$\neg(\triangle abc \cong \triangle acb)$

10

$\neg(\overline{ab} \cong \overline{ac}) \vee \neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb}) \vee (\triangle abc \cong \triangle acb)$

$\neg(\overline{ab} \cong \overline{ac}) \vee \neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb})$

9

$\overline{ab} \cong \overline{ac}$

$\neg(\angle abc \cong \angle acb) \vee \neg(\overline{bc} \cong \overline{cb})$

Attemp #3

6

$\neg(\Delta abc \cong \Delta acb) \lor (\angle abc \cong \angle acb)$

10

$\neg(\overline{ab} \cong \overline{ac}) \lor \neg(\angle abc \cong \angle acb) \lor \neg(\overline{bc} \cong \overline{cb}) \lor (\Delta abc \cong \Delta acb)$

$\neg(\overline{ab} \cong \overline{ac}) \lor \neg(\overline{bc} \cong \overline{cb})$

9

$\overline{ab} \cong \overline{ac}$

1

$\overline{xy} \cong \overline{yx}$

$\neg(\overline{bc} \cong \overline{cb})$

$x/b \quad y/c$

[ ]