Wavelet transforms in power systems

Part 1 General introduction to the wavelet transforms

This tutorial gives an introduction to the field of the wavelet transform. It is the first of two tutorials which are intended for engineers applying or considering to apply WTs to power systems. They show how the WT — a powerful new mathematical tool — can be employed as a fast and very effective means of analysing power system transient waveforms, as an alternative to the traditional Fourier transform. The focus of the tutorials is to present concepts of wavelet analysis and to demonstrate the application of the WT to a variety of transient signals encountered in electrical power systems.

by Chul Hwan Kim and Raj Aggarwal

We shall begin by describing why, in power systems, wavelet transforms (WTs) are better suited for the analysis of certain types of transient waveforms than the Fourier transform (FT) approach. We will look in detail at the fundamental concepts of the WT, and describe the commonly employed digital form of the continuous wavelet transform. A 'wavelet' is described as a 'little' wave, little in the sense of being of short duration with finite energy which integrates to zero, and hence its suitability for transients.

Motivation to use wavelet transforms

Power system transients, which often have an adverse effect on the normal operation of a system, are quite common: lightning transients, transformer inrush currents, motor starting currents, capacitor and line-switching transients are just a few of the typical electromagnetic power system transients that occur in practice. Analysis and understanding of transients associated with such abnormal conditions have always helped to explain and then rectify the cause of the condition. Real-time identification of transients, fast and accurate processing of measured voltage/current signals (employing simulation techniques), online control in transmission and distribution applications and data compression/storage of transients are of particular interest to power engineers. Some of the methods employed for analysis of the transient phenomena at present are:

(i) transforming the data into the frequency domain via Fourier, Laplace or Z- transforms
(ii) using power system simulation programmes such as the electromagnetic transients programme (EMTP) or mathematical solutions of differential equations either analytically or numerically.

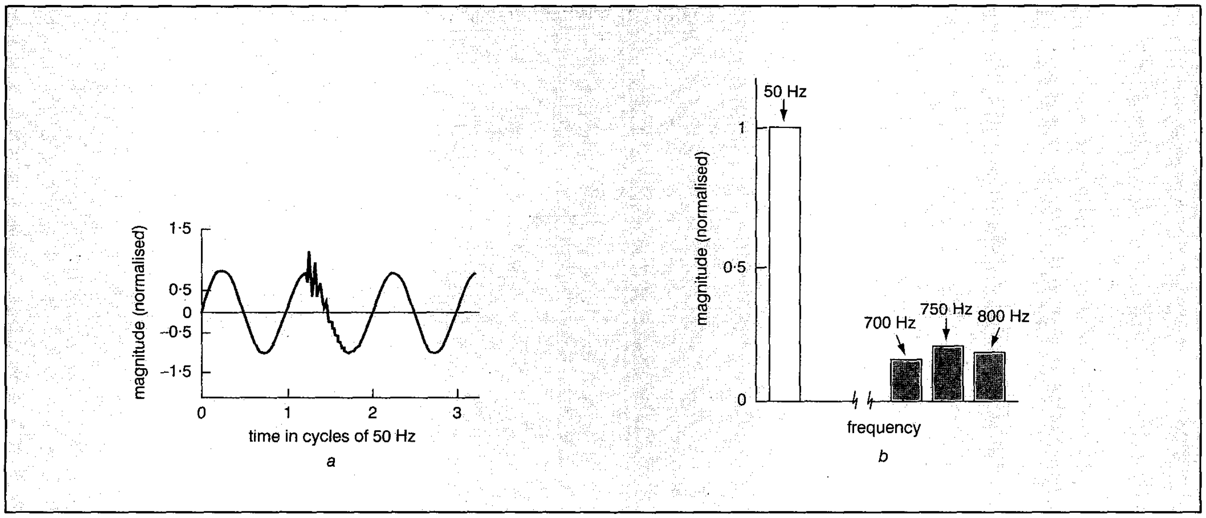
Hitherto, such methods have served the power engineering community well. However, the increasing complexity of power systems, concomitant with a demand to drive the network harder without compromising on the quality of power supply, has meant that power engineers are continuously striving to look for improved alternative methods of transient analysis, for the purposes of designing new equipment to efficiently and expeditiously deal with abnormal transient phenomena. In this respect, the present methods of analysis have limitations. For instance, a Fourier series requires periodicity of all the time functions involved; this effectively means that the basis functions (i.e. sine and cosine waves) used in Fourier analysis are precisely located in frequency but exist for all time. The frequency information of a signal calculated by the classical Fourier transform is an average over the entire time duration of the signal. Thus, if there is a local transient signal over some small interval of time in the lifetime of the signal, the transient will contribute to the FT (albeit in a rather inefficient manner), but its location on the time axis will be lost. Traditional Fourier analysis does not consider frequencies that evolve with time, i.e. non-stationary signals. Finally, certain adverse effects such as the Gibbs phenomenon and aliasing associated with the discrete FT (DFT) exist when analysing certain waveforms and have to be catered for; this is so by virtue of the fact that a Fourier series in DFT averages only over a limited time period.

One technique that overcomes the foregoing problems to a certain extent is the windowed FT (also known as the short-time FT or STFT). However, the drawback is that the STFT has the limitation of a fixed window width which needs to be fixed *a priori;* this effectively means that it does not provide the requisite good resolution in both time and frequency, which is an important characteristic for analysing transient signals comprising both high and low-frequency components. A wide window, for example, gives good frequency resolution but poor time resolution, whereas a narrow window gives good time resolution but poor frequency resolution. It should be mentioned that a windowed-FT approach can be applied with a sequence of windows of different widths to get more detail on transient location. However, the latter option is unwieldy and time consuming. Before continuing, it should be emphasised that the purpose of this tutorial is not to 'attack' the Fourier techniques used in power system analysis but to introduce a new complementary method — the wavelet analysis — that performs better in certain instances where interpretation of data through traditional Fourier methods causes ambiguity.

Wavelet analysis overcomes the limitations of the Fourier methods by employing analysing functions that are local both in time and frequency. The WT is well suited to wideband signals that are not periodic and may contain both sinusoidal and impulse components as is typical of fast power system transients. In particular, the ability of wavelets to focus on short-time intervals for high-frequency components and long-time intervals for low-frequency components improves the analysis of signals with localised impulses and oscillations, particularly in the presence of a fundamental and low-order harmonics. In a sense, wavelets have a window that automatically adapts to give the appropriate resolution.

Wavelet and Fourier transform methods

It is desirable (and very often necessary) to apply a frequency-based analysis method in an attempt to isolate the transient components of a signal; this in turn can help, *inter alia*, to identify a particular phenomenon producing



1 Capacitor switching transient signal and its DFT: (*a*) The transient signal; (*b*) The discrete Fourier transform

the transients in the first place. In this respect, it should be mentioned that the waveforms associated with fast electromagnetic transients are typically non-periodic in nature and contain both high-frequency oscillations and very short-duration impulses superimposed on the much lower power frequency signals. These characteristics present a problem for traditional Fourier analysis by virtue of the fact that its use assumes a periodic signal and, as a consequence, a wideband signal requires a very high sampling rate. This inevitably means that a much longer time period is necessary to maintain good resolution in the low-frequency range. Short-term Fourier analysis overcomes this problem to a certain extent and wavelet analysis is very effective in providing the requisite frequency data for non-periodic waveforms.

To explore the advantages and attributes of the WT (particularly in the analysis of transient signals), we compare the properties of the latter with Fourier analysis through a mathematical approach.

The mathematical relationship described herein is only with respect to the very widely employed discrete time signal (as opposed to continuous time) suitable for implementation in digital hardware.

The discrete Fourier transform (DFT)

The DFT $X(k)$ of a sampled signal is defined as:

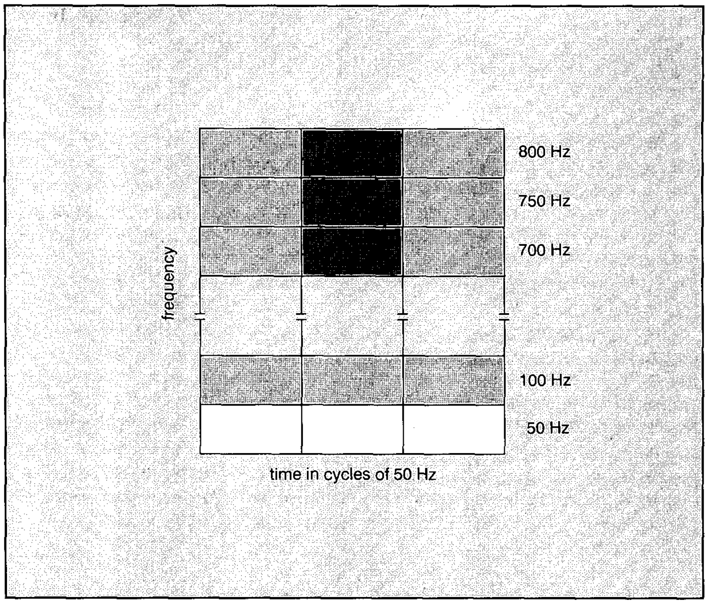
$$X(k) = \sum_{n=0}^{N-1} x(n)\, exp\left(\frac{-j2\pi kn}{N}\right) \qquad (1)$$

where $x(n)$ is a sequence of samples from a continuous time signal $x(t)$ taken every T_S seconds for N samples, i.e.:

$$x(n),\ n=0,1,2,\ \text{------},\ N-1 \qquad (2)$$

The DFT produces a sequence of complex values $X(k)$ whose magnitudes correspond to the discrete frequency components in $x(n)$. As apparent from eqn. 1, the derivation of the DFT strictly requires the input signal $x(n)$ to be periodic, i.e. the signal repeats every N samples. To prevent errors due to aliasing, the sampling rate of the time signal $x(t)$ needs to be at least twice the highest frequency within the signal, i.e. conform to the Nyquist criterion.

The limitations of the DFT of a non-periodic signal can best be illustrated by considering a typical voltage transient waveform associated with capacitor switching in power systems, as shown in Fig. 1*a*.



2 Windowed discrete Fourier transform (WDFT) of a capacitor switching transient signal

The analysis of the transient signal through the DFT (Fig. 1*b*) gives rise to a resonant component at approximately 750Hz and sideband frequencies at approximately 700Hz and 800Hz, respectively, the latter two arising as a direct consequence of the non-periodicity of the input waveform. The presence of significant energy in the sidebands of 750Hz impinges on the latter, and this in turn can prevent precise detection of the resonant frequency of the capacitor-compensated system, which may help find the cause of the transient. The representation of a signal by the DFT is thus best reserved for truly periodic signals.

The aforementioned problems associated with non-periodic signals can be overcome to a certain extent by adapting the DFT to analyse only a small section of the signal at a time — a technique called windowing the signal; this technique is commonly referred to as the short-time (or windowed) Fourier transform. The basic principle of the windowed discrete Fourier transform (WDFT) is outlined below.

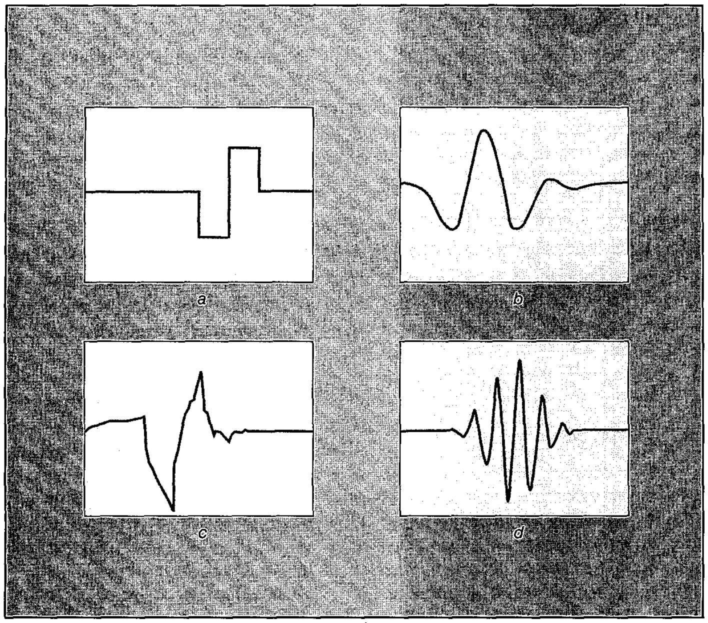
Windowed discrete Fourier transform (WDFT)

The WDFT of a discrete input is defined as:

$$\text{WDFT}(k,m) = \sum_n x(n)\, w(n-m)\, exp\left(\frac{-j2\pi kn}{N}\right) \qquad (3)$$

where $w(n-m)$ in its simplest form is a rectangular window function:

$$w(n-m) = \begin{cases} 1 & \text{if } 0 \le (n-m) \le (N-1) \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$



3 Different mother wavelets: (*a*) Harr; (*b*) Symmlet; (*c*) Daubechies; (*d*)Morlet

For each window $w(n-m)$, the WDFT produces a sequence of complex values:

$$\mathrm{WDFT}(k,m),\ k=0,1,\ldots,N-1 \tag{5}$$

As before, their magnitudes are those of the discrete frequencies of the input signal $x(n)$.

The WDFT maps a signal into a two-dimensional function of time and frequency. The WDFT approach represents a sort of compromise between the time and frequency-based views of a signal. It provides some information about ranges of time in the signal when particular frequencies have a relatively large magnitude and at what frequencies a signal event occurs. However, one can only obtain this information with limited precision, controlled by the size of the window.

For example, when applying the WDFT to the capacitor switching transient shown in Fig. 1*a*, and assuming a window size of T = 20ms (this gives a frequency resolution Δf=50Hz), it is apparent from the two-dimensional grid shown in Fig. 2 that there is the same frequency information present at the previous DFT, but associated with a particular part of the 50Hz waveform.

This given window size locates the start time of the transient to only within one 50Hz cycle. As expected, shortening the window by, say four, will locate the short time of the transient with much higher precision, but significantly lower the frequency resolution (Δf will be 200Hz). The energy of the 50Hz fundamental component will start to appear as DC and in the 200Hz component.

The foregoing example shows that both DFT and WDFT techniques have limitations. For a signal comprising a fundamental component ridden with transients, we need higher precision time resolution for short-duration high-frequency signals as well as higher precision frequency resolution for long-duration low-frequency signals. The wavelet transform approach (the focus of this tutorial) lends itself very well to meeting this dilemma. It should be noted that, in power systems, very often signals such as those associated with an autoreclosure sequence in transmission, comprise both short-duration high-frequency transients (prior to fault clearance) and long-duration low frequencies during the 'dead time' prior to circuit breaker reclosure.

Basic concepts of wavelets

Wavelet analysis employs a prototype function called the mother wavelet. This function has a mean of zero and sharply decays in an oscillatory fashion, i.e. it rapidly falls to zero either side of its central path. Mathematically, the continuous wavelet transform (CWT) of a given signal $x(t)$ with respect to a mother wavelet $g(t)$ is generically defined as:

$$\mathrm{CWT}(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)g\left(\frac{t-b}{a}\right)dt \tag{6}$$

where a is the dilation or scale factor and b is the translation factor, and both variables are continuous. It is apparent from eqn. 6 that the original one-dimensional time-domain signal $x(t)$ is mapped to a new two-dimensional function space across scale a and translation b by the wavelet transform (WT). A WT coefficient $\mathrm{CWT}(a,b)$ at a particular scale and translation represents how well the original signal $x(t)$ and scaled and translated mother wavelet match. Thus, the set of all wavelet coefficients $\mathrm{CWT}(a,b)$ associated with a particular signal are the wavelet representation of the original signal $x(t)$ with respect to the mother wavelet $g(t)$.

We can visualise the mother wavelet as a windowing function. The scale factor a and the size of the windowing function are interdependent, namely smaller scale implies a smaller window. Consequently, we can analyse narrow-band frequency components of the signal with a smaller scale factor and wideband frequency components with a large scale factor, thereby capturing all of the features for a particular signal.

The WT encompasses an infinite set of wavelets due to the need for multi-resolution analysis. For example, a very large family of wavelets (also known as daughter wavelets) can be generated from one mother wavelet by varying the scale and translation factors. The number of coefficients and the level of iterations to generate a daughter wavelet within a family is used to distinguish from other wavelets in the family.

There are many types of mother wavelets that can be employed in practice. To choose the best, the attributes of different mother wavelets need to be considered. Fig. 3 typifies some of the commonly employed mother wavelets, such as Haar, Symmlet, Daubechies, Morlet etc. Of these Haar and Morlet are classed as orthogonal whereas Symmlet and Daubechies are non-orthogonal (see Reference 1 for mathematical forms).

A main feature of wavelets is the oscillating and fast decaying behaviour that comes along with the location in time and frequency. Generally, smooth wavelets (such as the Symmlet wavelet) indicate a better frequency resolution than wavelets with sharp steps such as the Haar wavelet; the opposite applies to time resolution. Another important criterion is a fast computation of the scaled daughter wavelets. In this respect, the orthogonal wavelets calculated recursively have advantages over non-orthogonal wavelets. One of the most widely applied mother wavelets suitable for a wide range of power system applications is the Daubechies wavelet, which is ideally suited for detecting low amplitude, short duration, fast decaying and oscillating type of signals, typical of those encountered in power systems.

Discrete wavelet transform (DWT)

Analogous to the relationship between continuous Fourier transform and discrete Fourier transform, the continuous wavelet transform has a digitally implementable counterpart called the discrete wavelet transform, and is defined as:

$$\mathrm{DWT}(m,k) = \frac{1}{\sqrt{a_o^m}} \sum_n x(n) g\left(\frac{k - nb_o a_o^m}{a_o^m}\right) \quad (7)$$

where g(.) is the mother wavelet and the scaling and translation parameters a and b (as shown in eqn. 6) are functions of an integer parameter m, i.e. $a = a_o^m$ and $b = nb_o a_o^m$, giving rise to a family of dilated mother wavelets, i.e. daughter wavelets. Also in eqn. 7, k is an integer variable that refers to a particular sample number in an input signal. The scaling parameter gives rise to geometric scaling, i.e. $1, \frac{1}{a_o}, \frac{1}{a_o^2}, \cdots$

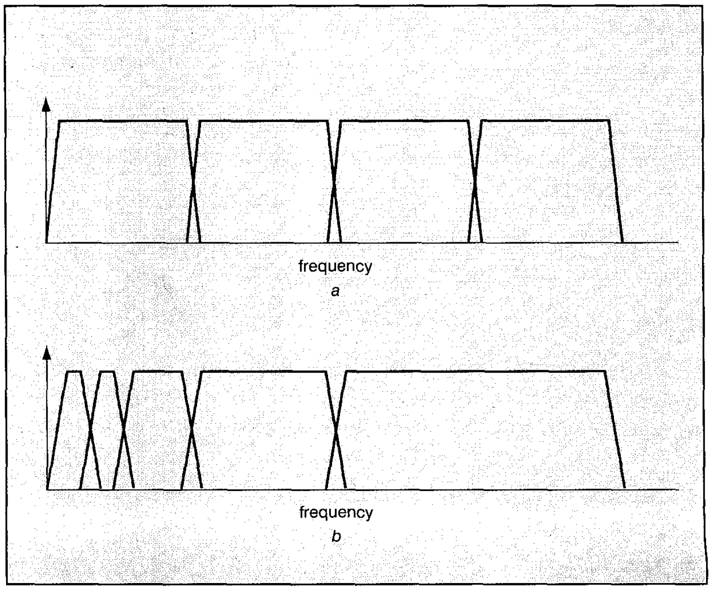
This scaling gives the DWT a logarithmic frequency coverage in contrast to the uniform frequency coverage of, say, the windowed-DFT (i.e. WDFT), as compared in Fig. 4.

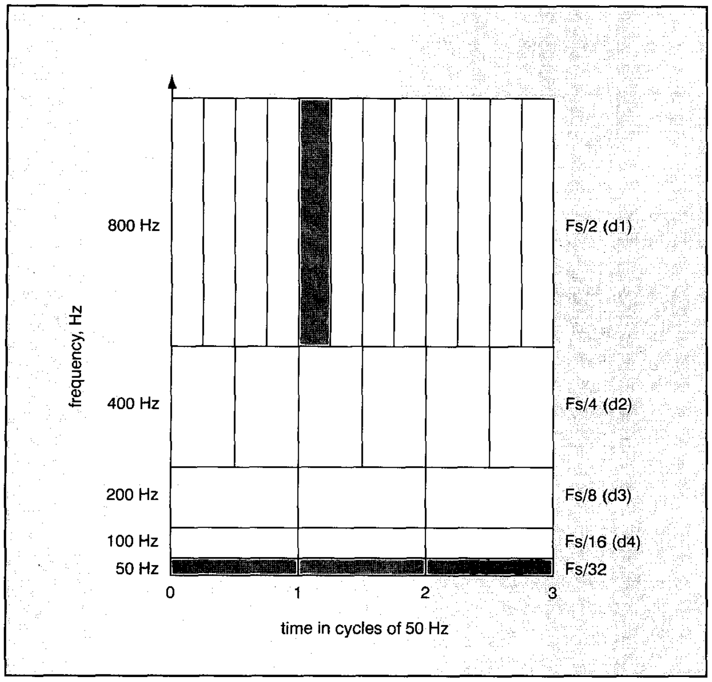
The DWT output can be represented in a two-dimensional grid in a manner similar to the WDFT but with very different divisions in time and frequency, as illustrated by Fig. 5. As can be seen, the DWT analysis produces rectangular time-bandwidths which are narrow at higher frequencies and which progressively get larger as the frequency goes down.

It should be mentioned that, in marked contrast, the frequency coverage associated with the traditional DFT will simply be of a continuous form with no divisions whatsoever.

The aforementioned characteristic feature of the DWT is very different from the WDFT, as seen previously in Fig. 2. The DWT is thus very effective in isolating the highest frequency band at precisely the quarter cycle of its occurrence while the 50 Hz power frequency is fully preserved as a continuous magnitude. This simple example clearly illustrates the suitability of the multi-resolution attributes of the wavelet transform in analysing a non-stationary transient signal comprising both high and low-frequency components.

4 Comparison of the frequency coverage: (*a*) Uniform coverage of the windowed-DFT (WDFT); (*b*) Logarithmic coverage of the discrete wavelet transform (DWT)





5 Discrete wavelet transform of a capacitor switching transient signal

Implementation of the DWT

By simple interchange of the variables n, k and rearrangement of the DWT, eqn. 7 gives:

$$\mathrm{DWT}(m,n)=\frac{1}{\sqrt{a_o{}^m}}\sum_k x(k)g\left(a_o{}^{-m}n-b_ok\right) \quad (8)$$

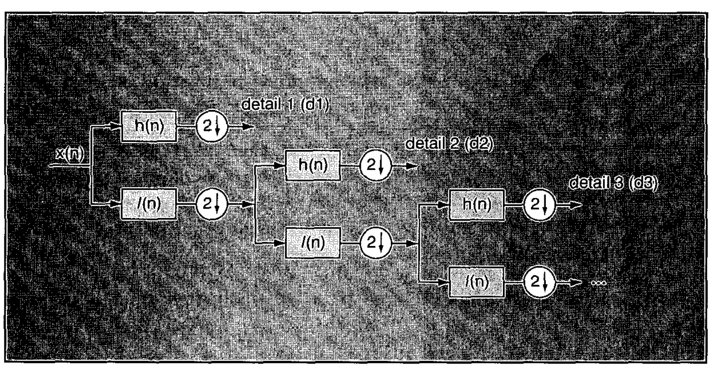
On closer observation of this equation, we notice that there is a remarkable similarity to the convolution equation for the finite impulse response (FIR) digital filters, namely:

$$y(n)=\frac{1}{c}\Sigma\, x(k)h(n-k) \quad (9)$$

where $h\,(n-k)$ is the impulse response of the FIR filter.

By comparing eqns. 8 and 9, it is evident that the impulse response of the filter in the DWT equation is $g(a_o{}^{-m}n-b_ok)$.

6 Implementation of the discrete wavelet transform (DWT)



By selecting a_o=2 or ($a_o{}^{-m}$=1, $^1/_2$, $^1/_4$, $^1/_8$,...) and b_o = 1, the DWT can be implemented by using a multi-stage filter with the mother wavelet as the lowpass filter $l(n)$ and its dual as the highpass filter $h(n)$, as shown in Fig. 6. As evident from the example considered, down-sampling the output of the lowpass filter $l(n)$ by a factor of $2(\downarrow 2)$ effectively scales the wavelet by a factor of 2 for the next stage, thereby simplifying the process of dilation.

It is important to note that the highpass and lowpass filters are not independent of each other, but are related by:

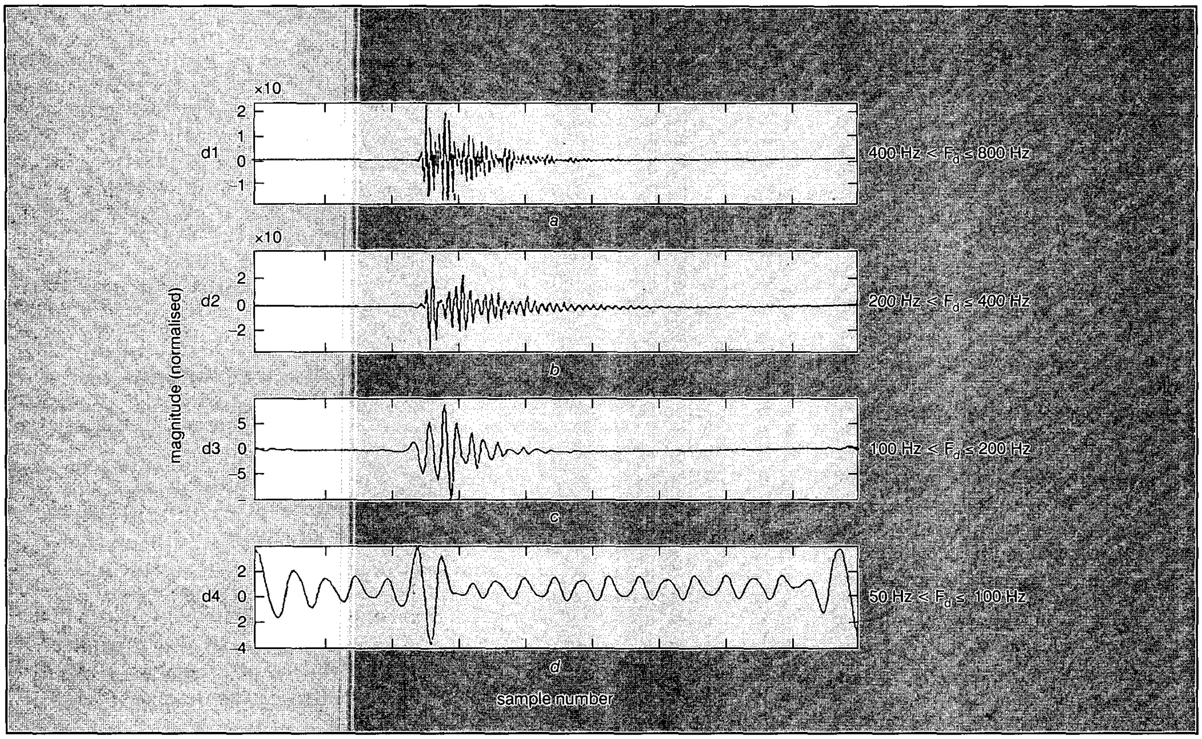
$$h[L-1-n]=(-1)^n\, l(n) \quad (10)$$

where L is the filter length. Note that the two filters are 'odd index' alternated reversed versions of each other, and the lowpass to highpass conversion is provided by the $(-1)^n$ term; filters satisfying this condition are commonly used in signal processing, and they are known as the 'quadrature mirror filters'.

The implementation of the DWT with a filter bank is computationally efficient. The output of the highpass filter in Fig. 6 gives the detailed version of the high-frequency component of the signal. As can be seen, the low-frequency component is further split to get the other details of the input signal. By using this technique, any wavelet can be implemented.

Similar to the WDFT (as shown in Fig. 2), the DWT can be represented in a two-dimensional grid, but with a very different division in time and frequency. For example, if the original signal is being sampled at F_S hertz, then the highest frequency that the sampled signal would faithfully represent is $F_S/2$ (based on the Nyquist theorem). This would be seen as the output of the highpass filter, which is the first detail in Fig. 6, i.e. the first detail would capture the band of frequencies between $F_S/2$ and $F_S/4$. Likewise, the second detail would capture the band of frequencies between $F_S/4$ and $F_S/8$, and so on. The rectangles depicting these divisions are also illustrated in Fig. 5.

Fig. 7 typifies the decomposition of the capacitor switching transient signal, shown in Fig. 1*a* to four levels of detail $d1 \rightarrow d4$ (these are essentially the same as those shown in Figs. 5 and 6) using the Daubechies mother wavelet. Associated with wavelet analysis, both the high and low-frequency characteristic features of varying magnitudes at different levels of detail are clearly evident. These are obtained by



7 Analysis of the capacitor switching transients through the discrete wavelet transform (DWT) d1 → d4 levels of detail

applying the DWT to a number of cycles of the transient signal and, as expected, the highest frequency component present at the sampling frequency of say, F_S = 1600 Hz, *is* F_d = 800Hz; this is reflected in the signal dilation depicted in Fig. 7*a*. Likewise, Fig. 7*b*, 7*c* and 7*d* illustrate the dilations in the lower frequency ranges of 200Hz → 400Hz, 100Hz → 200Hz, and 50Hz → 100Hz, respectively. It is interesting to note that the higher frequency components (certainly those in the range 100< F_d ≤800 Hz are concentrated in a relatively short time window and this can be attributed to the fact that the capacitor switching transients occur only for a short period of time following a switching operation. The dilation in the low-frequency range (50Hz <F_d ≤100Hz) persists for a much longer period (Fig. 7*d*). This also explains why the different dilations show little shifts (or translations), particularly at higher frequencies. Of course, had the transients manifested themselves also onto other parts of the input signal, then the dilations (albeit of smaller magnitudes) would have also appeared in other parts of the decomposed signal components, i.e. the translations would have been apparent.

It is evident from the foregoing that the reconstruction of the original signal is very simple and straightforward; it involves simply up-sampling the signal components at every level by two (↑2), passing them through the appropriate highpass and lowpass filters and then adding them up.

Finally, it should be mentioned that, for readers who are interested in applying the wavelet transform for transient signal analysis, the wavelet tool box is available as part of the very widely employed MATLAB and Mathematica software packages.

Further reading

1 STRANG, G., and NGUYEN, T.: 'Wavelets and filter banks' (Wellesley-Cambridge Press, ISBN 0 96140 887 1, 1996)

2 MISITI, M. MISITI, Y. OPPENHEIM, G. and POGGI, J.: 'Wavelet toolbox manual — user's guide', The Math Works Inc., USA, March 1996 (available only with the MATLAB 'wavelet toolbox')

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Chul Hwan Kim is an Associate Professor in the School of Electrical and Computer Engineering, SungKyunKwan University, South Korea. He is a member of the Korean IEE. Raj Aggarwal is Professor in the Electrical Power and Energy Group, Department of Electronic and Electrical Engineering, University of Bath, Bath BA2 7AY, UK. He is an IEE Fellow.