BY ATTEMPTING THIS ASSESSMENT YOU ARE CONFIRMING THAT YOU ARE FIT TO DO SO

School of Computing & Engineering

(Department of Engineering and Technology)

University of HUDDERSFIELD

NME3523

Signal Analysis & Processing

Date: May 2015

Time allowed: 3 hours

Instructions to Candidates:

This is an unseen closed book examination.

Candidates should answer four out of six questions. All questions are marked out of 25.

Materials provided: Table of Transforms.

Materials allowed: None
A scientific calculator may be used in this exam.

Unannotated paper versions of general bi-lingual dictionaries only may be used by overseas students whose first language is not English. Subject-specific bi-lingual dictionaries are not permitted. Electronic dictionaries may not be used.

Access to any other materials is not permitted.

Turn Over

Should a candidate have a query about this paper and the Module Leader is not in attendance, please call 01484 472150 or 472450

Question 1

(i) A signal $x(t)$ is sampled N times over a sampling period T to generate a discrete time series x_r. Write down an expression for the Discrete Fourier Transform (DFT) of x_r.

If X_k is the k^{th} term of the DFT what is the frequency associated with X_k?
What is the maximum value of k for which unique values of X_k exist?
To what frequency does this maximum value of k correspond?
[8 marks]

(ii) A cosine wave of frequency 10Hz is heavily contaminated with random noise. Write a MATLAB 'm' file that creates and samples the signal and then performs **ensemble averaging** 10 times to produce an averaged DFT of the signal. (Assume that each time series on which the DFT is performed consists of 512 samples obtained over a sampling period of one second).

Sketch the ensemble averaged DFT and sketch the DFT that would be obtained if ensemble averaging were not used. Highlight the differences between the two sketches.
[17 marks]

Turn Over

Question 2

An underdamped spring-mass-damper system is initially at rest with the force $p(t)$ applied to the mass, the displacement $x(t)$ of the mass and the velocity $x'(t)$ of the mass all being equal to zero. At time $t=0$ the applied force $p(t)$ undergoes a step change from 0 to A. The resultant displacement $x(t)$ of the mass is given by $x(t)=\frac{A}{K}\left\{1-\frac{e^{-\omega_n ct}}{\sqrt{1-c^2}}\left[\sin(t\omega_n\sqrt{1-c^2}+\alpha)\right]\right\}$ where $\alpha=\cos^{-1}c$. The term K is the spring stiffness whilst M represents the mass and f represents the damping constant.

You are given that $\frac{dx(t)}{dt}=0$ when $t=\frac{r\pi}{\omega_n\sqrt{1-c^2}}$ $(r=0,1,2,3,4.....)$.

Show that the damping ratio *c* is given by the expression $c=\sqrt{\frac{y^2}{\pi^2+y^2}}$ where $y=\log_e\left[\frac{Z_1}{Z_2}\right]$ and where Z_1 is the <u>magnitude</u> of the first overshoot and Z_2 is the <u>magnitude</u> of the first undershoot.

[14 marks]

Show that the period T of the decaying oscillations in $x(t)$ can be given by the expression $T=\frac{4\pi M}{\sqrt{4KM-f^2}}$

[11 marks]

Turn Over

Question 3

A digital high pass filter is represented by the expression $y(n) = \dfrac{x(n) - x(n-1)}{2}$.
Derive an expression the gain of this filter as a function of ωT .

Using the expression you derived above, produce a graph of gain versus ωT for the filter and use this graph to explain why any low frequency components to be attenuated and any high frequency components to be passed must all lie below the Nyquist frequency.
[16 marks]

If a sampled cosine wave of with a frequency of one eighth of the sampling frequency is applied to the high pass filter given above what is the phase angle, in degrees, between the output cosine wave and the input cosine wave?

Does the output cosine wave lead or lag the input cosine wave?
[9 marks]

Turn Over

Question 4

(i) The z-transform of a digital filter is given by

$$\frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}} \qquad \text{(equation Q4)}$$

By invoking the time shift theorem, and giving all of your working, develop an expression for the relationship between the n^{th} term $y(n)$ of the output series from the filter and the n^{th} term $x(n)$ of the input series.

What are the necessary conditions for the filter defined by 'equation Q4' to be (a) recursive and (b) non-recursive? What is the order of the filter?

Giving full justification, provide 2 major advantages of digital filters over analogue filters.

[17 marks]

(ii) What is the relationship between the 's-transfer function' of an analogue filter and the 'z-transfer function' of the equivalent digital filter.

How would you select the sampling frequency f_S of the digital equivalent of an analogue low pass filter ?

[8 marks]

Turn Over

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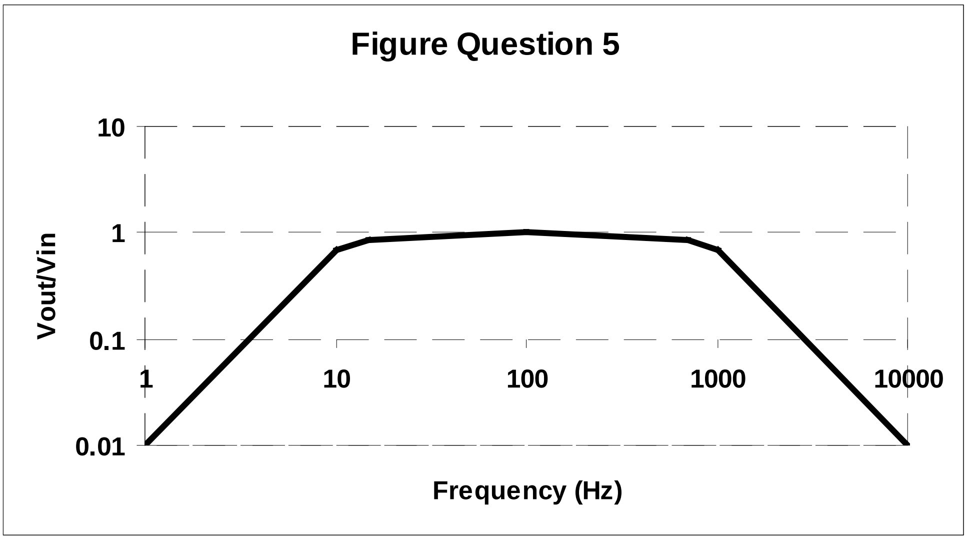
Question 5

(i) Design a circuit comprising a Butterworth ‘2-pole’ low pass filter in series with a Butterworth ‘2-pole’ high pass filter which has a gain versus frequency characteristic approximating that shown in ‘Figure Question 5’ below. Give all capacitance and resistance values in your circuit diagram.



[10 marks]

(ii) A simple passive high pass filter consists of a 100nF capacitor and a $10\,\mathrm{k\Omega}$ resistor. The output from the filter appears across a very high impedance load which draws negligible current. The input signal to the filter consists of three components (a) a 1V dc component; (b) a sine wave of 25Hz and $\pm 2\mathrm{V}$ peak to peak; and (c) a sine wave of 500Hz and $\pm 3\mathrm{V}$ peak to peak.

What are the amplitudes of the three components at the output from the filter?
[8 marks]

What is the attenuation of the 25Hz component in dB?
[3 marks]

What is the phase of the 25Hz component at the output relative to its phase at the input?
[4 marks]

Turn Over

Question 6

(i) With the aid of diagrams, produce a design for a cross correlation flow meter for measuring the velocity of solids particles being conveyed by water in a pipeline. Your design should include sketches for both the mechanical parts and the electrical circuitry of the flow meter. Explain how your flow meter design is intended to function.
[12 marks]

(ii) A cross correlation flow meter for measuring the velocity of slurry in a pipeline consists of two sensors A and B separated by an axial distance of 0.25m (A is upstream of B). Under a given flow condition the sampled outputs from A and B are given in the table below (Table Q6). By calculating and plotting the relevant cross correlation function determine the slurry flow velocity.
[13 marks]

Time (seconds)	Output from A (mV)	Output from B (mV)
0	5	12
0.1	2	6
0.2	-9	4
0.3	-22	-1
0.4	-1	-1
0.5	-10	4
0.6	6	3
0.7	5	-8
0.8	17	-14
0.9	6	-2
1	-6	-7
1.1	4	10
1.2	-10	10
1.3	0	12
1.4	0	7
1.5	0	-5
1.6	-3	-1
1.7	11	-14
1.8	-19	5
1.9	4	-1

TABLE Q6

Turn Over

	$f(t)$	$F(s)$	$f(k)$	$F(z)$
1	Unit impulse	1	$\delta(k)$	1
2	Unit step	$\frac{1}{s}$	$u(k)$	$\frac{z}{z-1}$
3	Unit ramp t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
4	t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2z(z+1)}{(z-1)^3}$
5	t^3	$\frac{6}{s^4}$	$(kT)^3$	$\frac{T^3z(z^2+4z+1)}{(z-1)^4}$
6	e^{-at}	$\frac{1}{s+a}$	$(e^{-aT})^k$	$\frac{z}{z-e^{-aT}}$
7	$1-e^{-at}$	$\frac{a}{s(s+a)}$	$1-(e^{-aT})^k$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
8	$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$kT(e^{-aT})^k$	$\frac{Tz\,e^{-aT}}{(z-e^{-aT})^2}$
9	$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$(1-akT)(e^{-aT})^k$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$
10	$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$	$(e^{-aT})^k-(e^{-bT})^k$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
11	Item 6 with $e^{-aT}=c$		c^k	$\frac{z}{z-c}$
12	Item 8 with $e^{-aT}=c$		kTc^k	$\frac{kTz}{(z-c)^2}$
13	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	$\sin k\omega T$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$
14	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	$\cos k\omega T$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$
15	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$(e^{-aT})^k\sin k\omega T$	$\frac{z\,e^{-aT}\sin\omega T}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2aT}}$
16	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$(e^{-aT})^k\sin k\omega T$	$\frac{z(z-e^{-aT}\cos\omega T)}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2aT}}$
17	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$	$\sinh k\omega T$	$\frac{z\sinh\omega T}{z^2-2z\cosh\omega T+1}$
18	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$	$\cosh k\omega T$	$\frac{z(z-\cosh\omega T)}{z^2-2z\cosh\omega T+1}$

Note: T is the sampling period.

End of Exam Paper