Part 4 of the PSD

Indirect Method of PSD Estimation

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Indirect Method of PSD Estimation

- **For a wide-sense stationary (WSS) process, its autocorrelation function is**

$$K(\tau) = E(x^*(t)x(t+\tau))$$

where E **denotes the statistical average, the superscript asterisk denotes complex conjugation**

- **Then, via the Wiener-Khinchine theorem, the power spectral density of the WSS process is the Fourier transform of the autocorrelation function:**

$$S_{xx}(f) = \int_{-\infty}^{\infty} K_{xx}(\tau) e^{-i2\pi f\tau}\, d\tau$$

The Indirect Method of PSD Estimation

- **This approach is called the *indirect* method because it requires two steps**

- **First, the autocorrelation function is computed**

- **Second, the Fourier transform of this function is computed (using the FFT) to obtain the power spectral density**

The Wiener-Khinchine Theorem

The Wiener-Khinchine theorem has two sides:

- **the power spectral density of the wide-sense stationary signal is the Fourier transform of the autocorrelation function:**

$$S_{xx}(f) = \int_{-\infty}^{\infty} K_{xx}(\tau) e^{-i2\pi f\tau} \, d\tau$$

- **the autocorrelation function of the WSS signal is the inverse Fourier transform of the power spectral density**

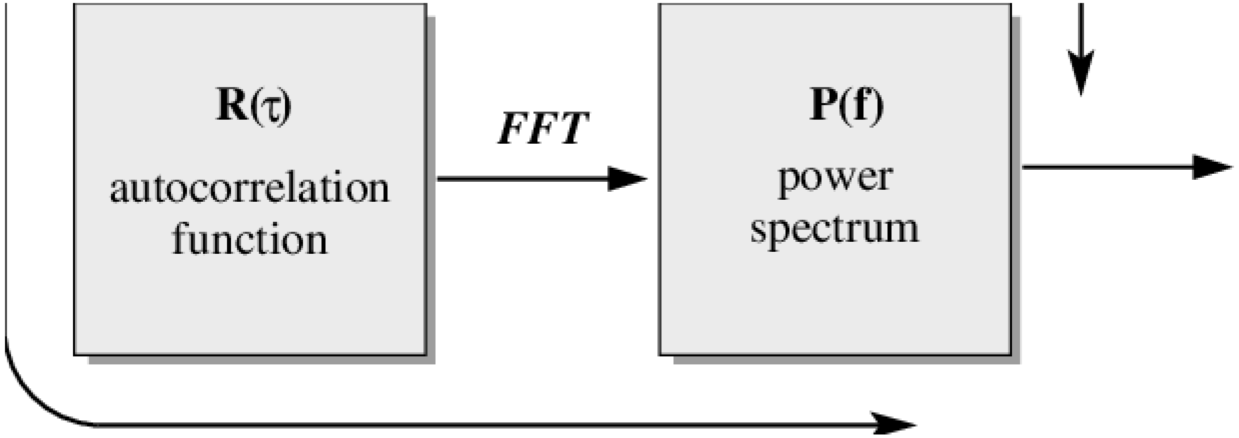
The Wiener-Khinchine Theorem

- **The Wiener-Khinchine theorem is a very important algorithm**

- **It means that the autocorrelation function and the power spectral density contain the *same* information about the signal**

- **Since, neither of these contains any phase information, it is impossible to reconstruct the signal from the autocorrelation function or from the power spectral density**

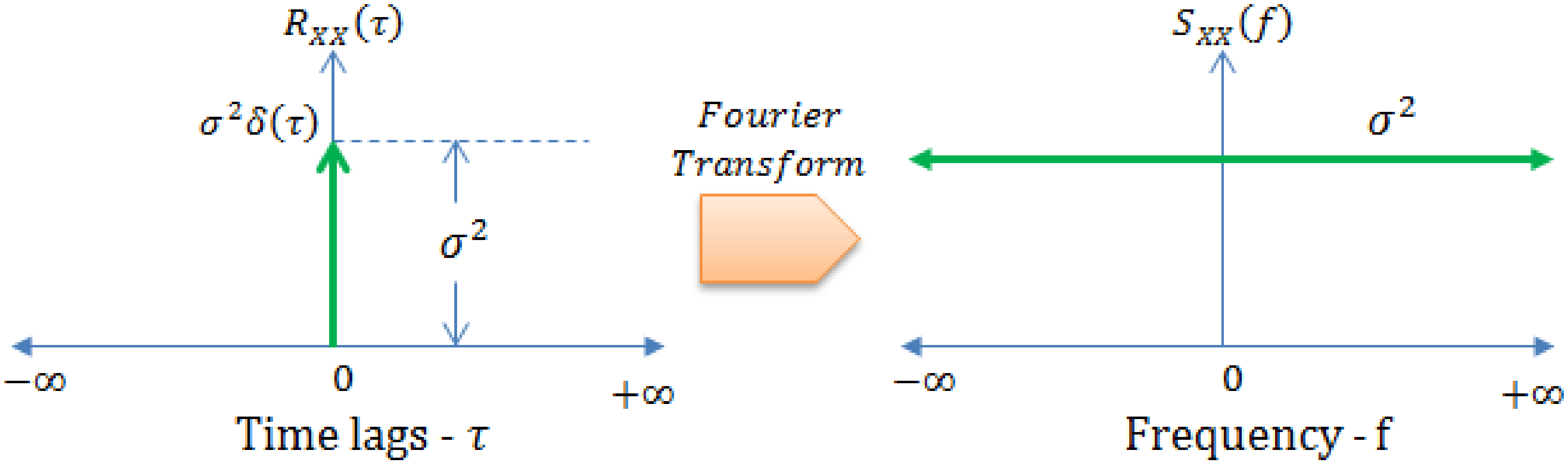
The Indirect Method of PSD Estimation



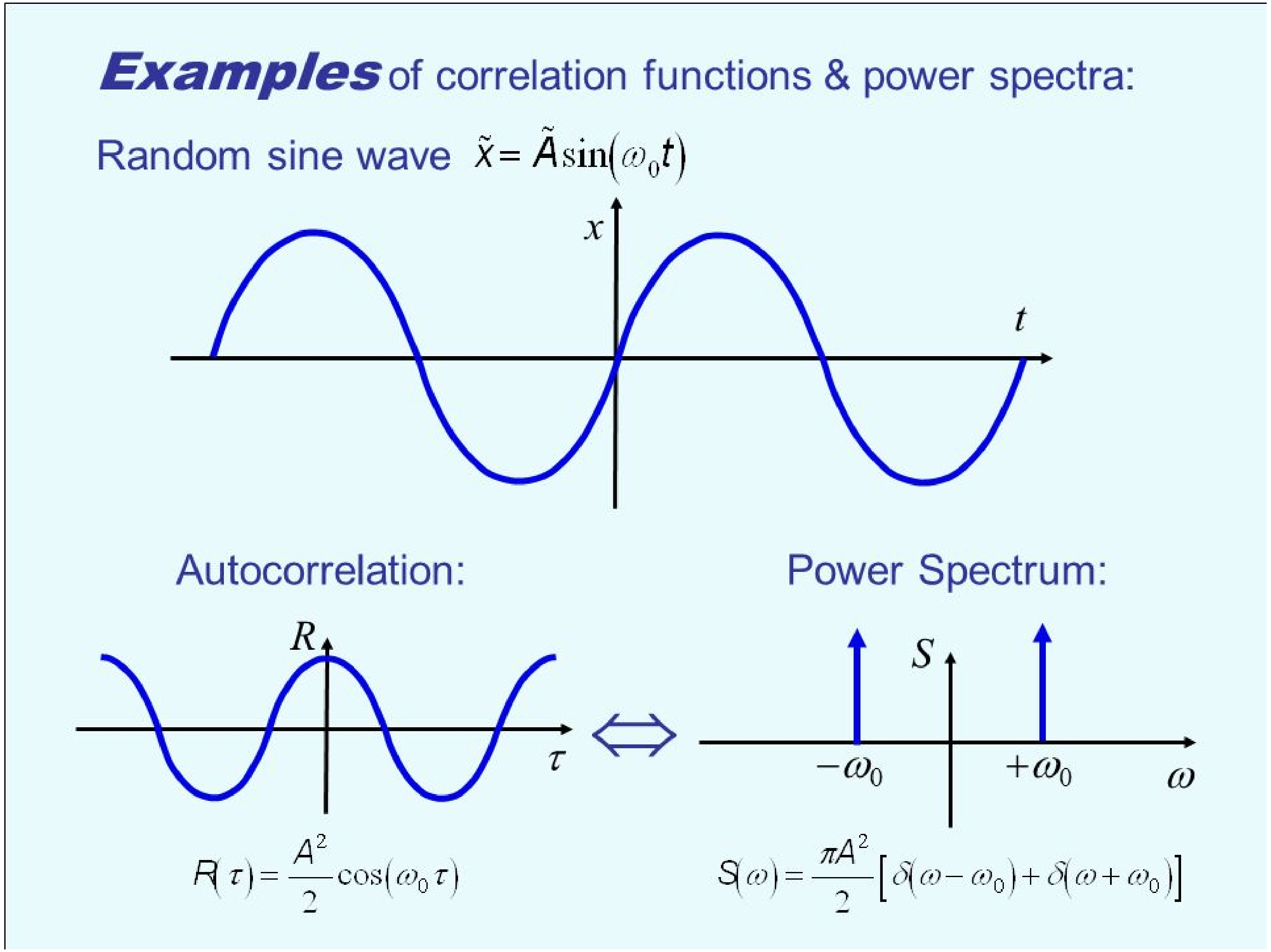
The Autocorrelation Functions and the PSDs

Signal	The autocorrelation function	The power spectral density (one-sided)
Sine wave	$\frac{A^2}{2}\cos 2\pi f_0\tau$	$\frac{A^2}{2}\delta(f - f_0)$
White noise	$\frac{a}{2}\delta(\tau)$	a
Low-pass white noise	$aB\left(\frac{\sin 2\pi B\tau}{2\pi B\tau}\right)$	a if $0 \le f \le B$
Band-pass white noise	$aB\left(\frac{\sin \pi B\tau}{\pi B\tau}\right)\cos 2\pi f_0\tau$	a if $f_0 - (B/2) \le f \le f_0 + (B/2)$

The Autocorrelation Function and the PSD: White Noise



The Autocorrelation Function and the PSD: Random Sine Wave



The Blackman and Tukey Method for PSD Estimation

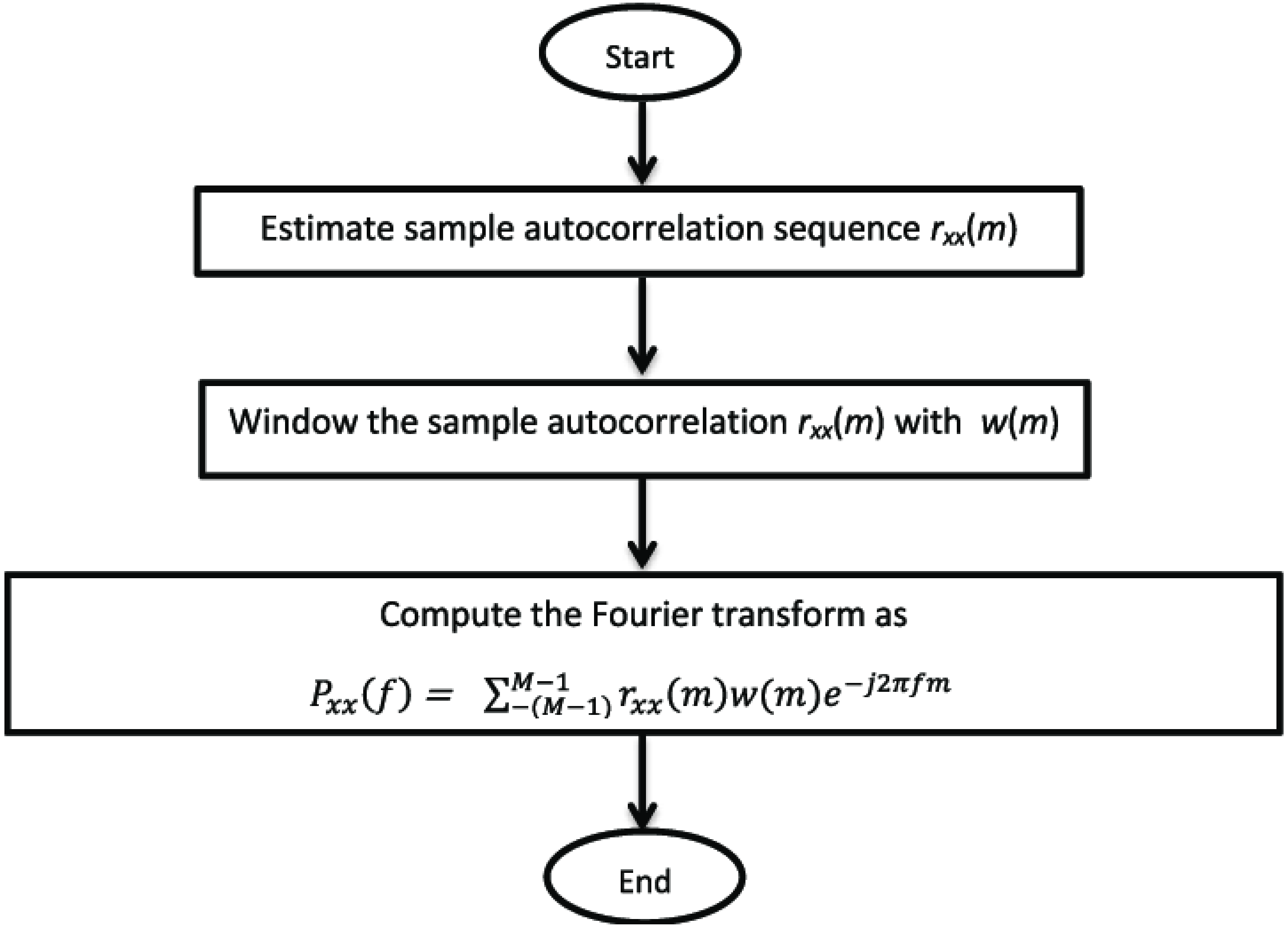
- **Blackman and Tukey proposed the indirect method in which the estimated autocorrelation sequence is, first, windowed and then Fourier transformed to yield an estimate of the power spectral density**

- **Thus, the Blackman-Tukey estimate is**

$$\hat{S}_{xx}^{BT}(f) = \sum_{m=-(L-1)}^{L-1} K_{xx}(m) W(m) e^{-i2\pi fm}$$

where $W(m)$ **the window function**

The Blackman and Tukey Method for Indirect PSD Estimation



The Blackman and Tukey Method for Indirect PSD Estimation

It is clear that the effect of windowing the autocorrelation function is to smooth the periodogram estimate, thus, decreasing the variance of the estimate at the expense of poorer frequency resolution.

Correlation Estimates via the FFT

The following steps are employed to compute the autocorrelation function via the FFT:

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- **Compute the power spectral density of a signal**

- **Compute the autocorrelation function using the inverse FFT**