Application of Signal Processing and Pattern Recognition By Wavelet Transform

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ABSTRACT

In this paper after reviewing the usage of the wavelet, transform to medical diagnosis, application of wavelet networks as a classifier of vital signals Has been discussed.
The application of this method has been used for preprocessed signals to determine the people who are subject to exacerbation of Ventricular Tachycardia (VT).

KEYWORDS

Wavelet – wavenet - pattern recognition – STFT- signal processing

1 INTRODUCTION

Nowadays, wavelet theory is applied as a powerful tool for analyzing and understanding some phenomena. These phenomena due to their non-stationary nature could not be expressed by Fourier transform.
Widespread usage, the emergence of new methods based on this theory in pre-processing, analysis, compaction as well as extracting important features of various signals are some benefits of this method.

The existence of fast algorithms implementing wavelet transform has provided possibility of real time processes and analysis of events with natural process and consistent with the nature of events. Capabilities and abilities of wavelet transform in cases that the investigated signal has higher complexities are more obvious and its advantage compared with other tools such as Fourier transform is more specified.

Meanwhile, vital signals could be considered the most complex signals due to their time and frequency specifications.
Many developments have been achieved in medical diagnostic methods recently by non-invasive way. So that we could record the vital signals from the body surface reflect the behavior and internal situation of an organ of body or parts of it Finding new methods without invasive measurements seems absolutely necessary to provide and interpret the necessary information from these organs which be used by doctors. But how these signals recorded over time should be interpreted, what basic features they have and how information has been latent in these signals? Answering these questions is the task of the science of pattern recognition. Before description of pattern recognition at first, we

should briefly familiarize with wavelet transform.

2 WAVELET TRANSFORM

In recent years, a new set of time - frequency transforms called wavelet transforms have been presented that the main idea in the design of these transforms is the signal analysis in windows with different lengths [1]. As such, a set of functions $\psi_{a,b}$ as needed windows are defined from a function $\psi(x)$ called wavelet function. Perhaps the most prominent part of this theory is the possibility of making the wavelet bases with limited length in space $L^2(R)$.

Wavelet bases are usually given as a set of transferred or contracted and expanded functions of the mother wavelet as follows:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}}\psi(\frac{x-b}{a}) \quad , \quad (a,b) \in R \quad , a \neq 0 \qquad (1)$$

In this relation a indicates the scale parameter and b shows the transfer parameter [1, 2].

Function $\psi_{a,b}$ based on the expansion or contraction of function $\psi(x)$ is made by the scale parameter a and transfer function with size of b. By changing these two parameters a window with desired length of time and focus, point can be generated. By this method, the problem of nonconformity of window length with non-stationary structures in short-term Fourier transform (**STFT**) can be solved. Wavelet transform based on set of functions (1) is defined as follows:

$$W_x(a,b) = \int_{-\infty}^{+\infty} x(t).\psi^*_{a,b}(t)dt \qquad (2)$$

In following simulation, the mother wavelet with wavelet functions with scale parameter $a = 2$ and the transfer parameter $b = 100, b = 50, b = 0$ respectively, is given.

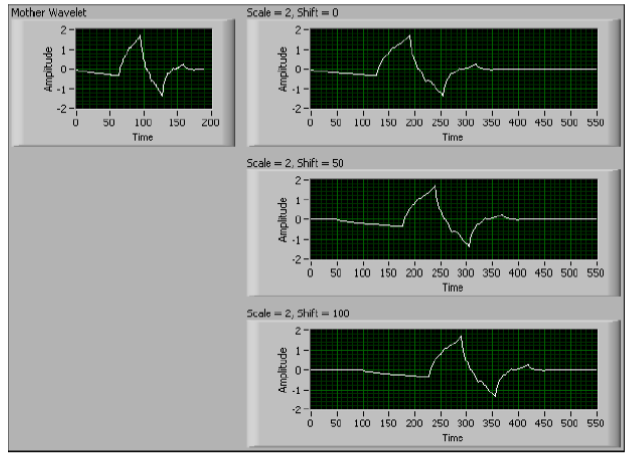


Figure 1 Wavelet Drawing

2.1 Transform and Capability of Locating Time and Frequency Signal

According to the feature of functions used as the analysis window, signal low frequency components with a function with large length and signal high frequency components with a function with short length will be studied and so the time and frequency properties of signals by using wavelet transform can be described with more desirable accuracy than *STFT*.

Since we cannot simultaneously locate the signal properly in both time and frequency domains, we need a compromise so that it would have the most adaptability with the input signal profile. The function that according to uncertainty principle has capability to localize the signal in most desirable way in both time and frequency domains is *Gabor* or *morlet* wavelet, which is in the form of $\Psi(x) = e^{jwx}e^{-x^2/2}$.

Most vital signals of our interest are in form of combination of stroke-like and transient events as well as oscillatory waves form with low period that both can be contained important information. Short term Fourier transforms or other common time - frequency methods are properly compatible with the latter phenomena, but they are not so suitable for analysis of short time pulses. When there are both types of phenomena in data, localizing them in time and frequency is best performed by the wavelet transform. Perhaps this factor is the reason of this transformation success in vital signals processing. Examples such as ECG signal, detection of ventricular late potentials, ECG signal analysis and diagnosis of heartbeat sounds can be presented in this field in relation to the suitability of wavelet transform [3].

Although $Gabor-Mallat$ wavelet cannot make orthogonal bases from $L^2(R)$, semi-orthogonal wavelet bases can be made that their time - frequency specification would be near enough to the boundary determined by the uncertainty principle. The advantage of these wavelets is that they can be implemented properly by filter banks standard algorithms [2].

2.2 Discrete Wavelet Transform

In signal analysis, parameters a and b can be selected in different ways that one of the most widely used form of selection of these parameters is in the form of $a = a_0^m$ and $b = nb_0 a_0{}^m$ providing that $a_0 \succ 1$ and $b \neq 0$.

Based on such a choice, the family of intended wavelets will include:

$$\Psi_{mn}(x) = a_0^{-m/2}\Psi(a_0^{-m}x - nb_0) \qquad (3)$$

The resulted wavelet transform is called discrete wavelet transform that is an invertible linear transformation and is written as follows:

$$(DWT)_{mn} = <\Psi_{mn}, f> = a_0^{-m/2}\int f(x)\Psi^*(a_0^{-m}x - nb_0)dx \qquad (4)$$

2.3 Wavelet Transform as a Filter Bank

By assuming that the scale parameter a is constant, the wavelet transform that become transfer parameter function b can be considered as a convolution equation that signifies signal crossing from a filter. Frequency response of this filter is equal to:

$$\sqrt{a}\Psi^*(aw) \qquad (5)$$

Ψ^* is complex conjugate of Fourier transform Ψ, so if we consider the above relation for different values of scale (e.g. as powers of 2 i.e. 2^j) we will reach to a filter bank with fixed values Q (factor Q is defined as the result of central frequency division on band width).

This way of analysis breaks down the signal to some sub bands that their bandwidth linearly increases with frequency. In dyadic mode transform each spectral band has approximately one octave width [4].

In figure 3 a relative bandwidth of filters in wavelet transform and its comparison with *STFT* is shown. Thus, the wavelet transform can be considered certain kind of spectrum analyzer.

So, the simplest feature that can be achieved in this way is the energy estimation in various bands. Recently such spectral features of vital signals used to differentiate the different physiological states, for example we can refer to the heartbeat sounds analysis to atherosclerosis diagnosis and to identify fetal cardiac activity states.

If wavelet $\Psi(x)$ is obtained from analysis of signal with multi resolution analysis, its corresponding filter bank can be implemented by fast algorithm $Mallat$. This kind of invertible wavelet transform can be the basis of noise reduction algorithm and improvement of image [1].

Mallat and Zhong used such a filter bank to show multi-scale edge of a signal by its wavelet transform maximum. This method can be applied to reduce noise in evoked potentials and *MRI* images [3].

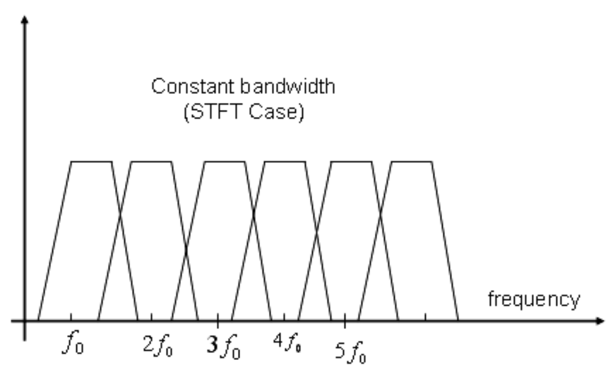


Figure 2 STFT with fixed band width and uniform coverage

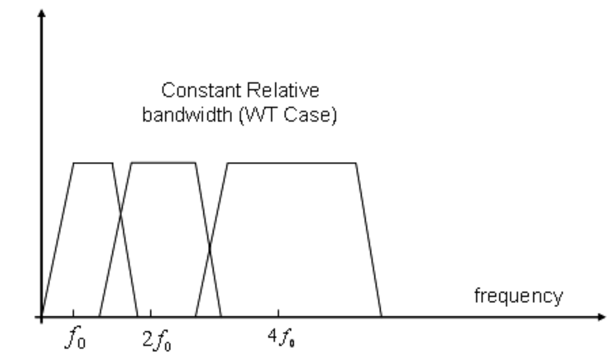


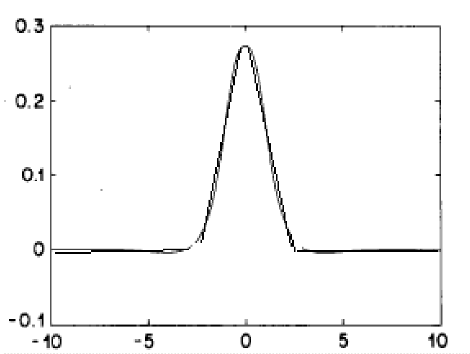
Figure 3 WT with fixed relative band width and logarithmic coverage

2.4 Wavelet Functions

2.4.1. Orthogonal wavelet functions Daublechies with the properties of finite length, high smooth degree and wave asymmetry and the number of high volatility

2.4.2. Orthogonal wavelet functions in bilateral orthogonal scale in time Chui.

These functions are also called *B-Spline* and have properties such as limit length, waveform symmetry, high smooth degree and the number of proper volatility.



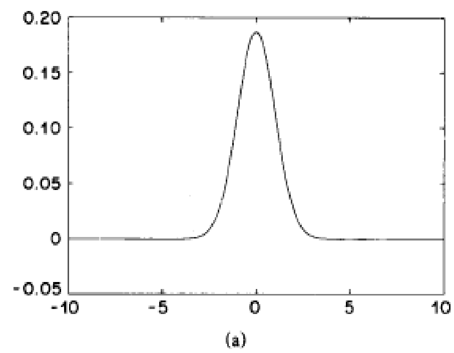


Figure 4 $B-Spline$ Functions

2.4.3. Orthogonal wavelet functions $Spline$ with almost limited length properties, capability to localize high frequency and waveform symmetry.

The third order wavelet function $Spline$ from this family has the property to localize proper frequency [5].

3. Fourier Transforms

Among various transforms, Fourier transform can be considered the most common and familiar transformation.

Fourier by selecting bases $\varpi \in R$, $e^{j\varpi x}$ was the founder of an orthogonal transformation and actually signal analysis dates back to 1826 by using appropriate basis functions, these transformations are expressed as follows:

$$X_f(\omega) = < e^{j\varpi x}, x(t) > \qquad (6)$$

That in above relation the inner multiplication is defined as follows:

$$< f, g > = \int_{-\infty}^{+\infty} f(t)g(t) \qquad (7)$$

As it is observed, Fourier transform coefficients are calculated from the inner multiplication of signal with sinusoidal basis functions in an indefinite time interval and this transformation give us range of signal frequency components regardless the place of time's occurrence of these components. As a result, Fourier analysis has proper operation when it is combination of several stationary components (like sinusoidal component).

But every sudden change over time in a non-stationary signal $x(t)$ will be expanded along frequency axis. Therefore, to analyze non-stationary signals a tool beyond the Fourier transform is needed [6,10].

With a deeper look at Fourier transform it can be found that the main factor of broadcast time information in frequency domain is bases' infinite length (sinusoidal and cosine). One solution is that in Fourier analysis, by introduction a local frequency, we define a kind of time dependency. Thus, the local Fourier transform look at the signal through a window that along this window the signal is almost stationary. Another method which is equivalent to the first method is that we change the sinusoidal basis functions that are used in Fourier transform to basic functions that are more focused on the time axis.

3.1 Fourier Transforms Simulation

In this part a schema of original program or a subprogram that calculate the real discrete Fourier transform is given. It is necessary to say that to obtain the real discrete Fourier transform at first we get unilateral real Fourier and then by using it we calculate the real discrete Fourier transform that the program schema is given below.

In figures 5, 6 local, Fourier transform shows a sinusoidal signal with frequency 3495.43 Hz that is added with a white noise with amplitude 0.001. As it was expressed, this signal is a non-stationary signal because its spectrum is changed over time. It is necessary to mention that in Fourier transform the Hamming window is used.

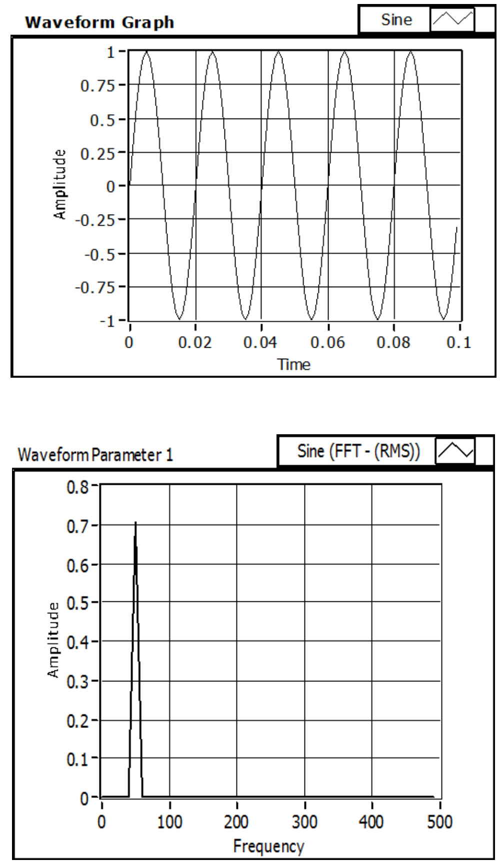
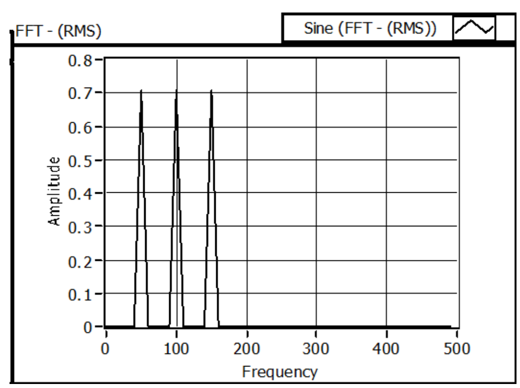


Figure 5 Fourier transform of a stationary sinusoidal signal with 50Hz frequency



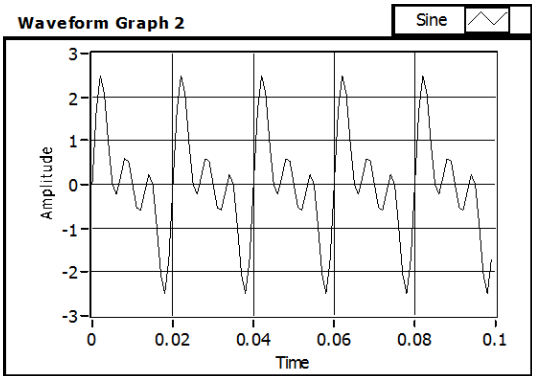


Figure 6 Fourier transform of a total of three stationary sinusoidal signals with 50Hz, 100Hz and 150Hz frequency

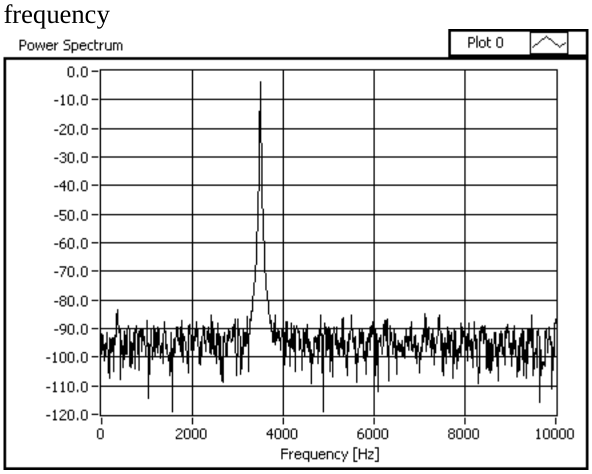


Figure 7 Frequency Spectrum

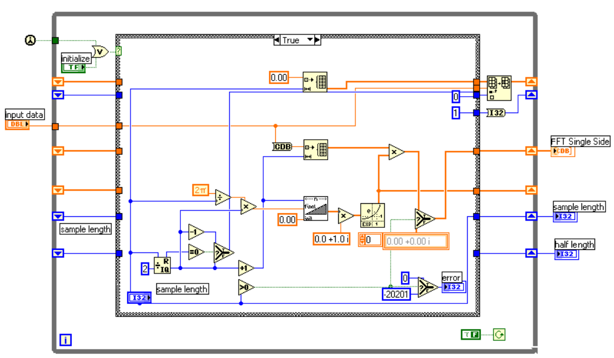


Figure 8 Single Sides Real FFT

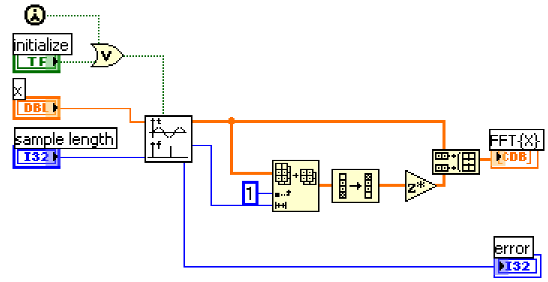


Figure 9 Real FFT

3.2 Time Fourier Transform (STFT): analysis of signal with fixed resolution degree

Instantaneous frequency has been often noticed and used as a way to express time dependence of the frequency. Even if the needed signal is not narrow-band, instantaneous frequency could indicate spectrum different components mean at a special time. But for more accuracy in the time domain, we need to express two-dimensional time and frequency such as $S(t, f)$ from signal $x(t)$ that signifies signal frequency characteristic along time axis. This mode of illustration can be considered similar to music score in which implementation of different frequencies has been expressed over time. Fourier transform at first was revised by Gabor to achieve the above goal. In practice,

Often this is possible that we behave with non-stationary signals like stationary signals by division them into pseudo stationary small pieces i.e. pieces that their statistical properties do not change along the piece.

Consider signal $x(t)$ and suppose that this signal is stationary along time window $g(t)$ with limit length that its centre in time is τ. Fourier transform of signal along this window is called short time Fourier transform (*STFT*) that is expressed as follows:

$$STFT(\tau, f) = \int x(t) g^*(t-\tau) e^{-j2\pi ft} dt \qquad (8)$$

That is a mapping from signal to a two-dimensional function on a time – frequency plane (τ, f).

Parameter f in (8) is like frequency parameter in Fourier transform and many properties of Fourier transform is transferred exactly to *STFT*. But it should be noted here that signal analysis has high dependency on selection of time window g(t).

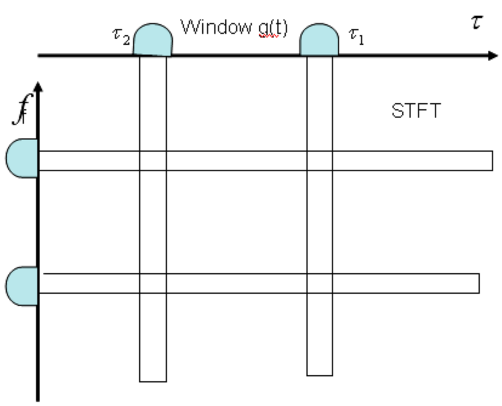


Figure 10 Filter Bank

In Figure 10 vertical strips are seen on time - frequency plane that represent view of applying a window on the signal in *STFT*. Considering a signal that a window around the time t is applied on it, all frequencies existing in *STFT* can be calculated.

This process can be viewed from another angle and it is the filter bank viewpoint. In a special frequency f, relation (8) indicates that signal is filtered along time axis by a band pass filter that filter impulse response is the same applied window function on the signal in which frequency is modulated. This property can be observed in form of horizontal strips. Therefore, *STFT* can be considered as a modulated filter bank.

In above dual attitude mode to signal there is a major weakness that we will explain it in following. Suppose that *STFT* can separate two pure sinusoidal signals. If we indicate window function by $g(t)$ and its Fourier transform by $G(f)$, filter bandwidth, which is shown by Δf, is:

$$\Delta f^2 = \frac{\int f^2 |G(f)|^2 df}{\int |G(f)|^2 df} \qquad (9)$$

That in which the denominator indicates energy $g(t)$. Two sinusoidal signals can be differentiated when they are at least far from each other to extent of Δf. So frequency resolution limit in *STFT* is Δf. Similarly, time resolution capability, which is shown by, Δt is as follows:

$$\Delta t^2 = \frac{\int t^2 |g(t)|^2 dt}{\int |g(t)|^2 dt} \qquad (10)$$

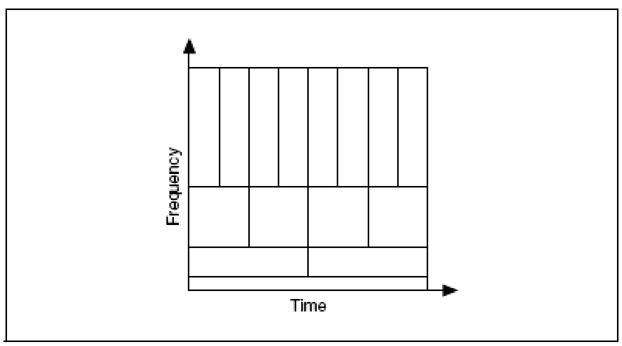
That again the denominator indicates energy $g(t)$. Two pulses in time domain can be differentiated when they are at least far from each other to extent of Δt.

What can be concluded from the above is that whatever Δf and Δt are smaller is better. However, these time and frequency resolution capabilities cannot be small with desirable size and have a lower limit, which is expressed by below relation:

$$\Delta f . \Delta t \geq 1/4\pi \qquad (11)$$

Relation (11) is known as non-equation *Heisenberg* or uncertainty principle. Among various windows, it is only the Gaussian window that satisfies the above condition in equality state and therefore it is more used in practice.

Although this method is useful in some applications it has some major flaws. After choosing a window for *STFT*, frequency and time accuracies Δf and Δt are fixed in the whole time - frequency plane (because in all frequencies the same window is used). For example, if a signal is combination of short-time bursts and long-time pseudo stationary components a simultaneous good resolution limit in both frequency and time is not possible. In other words, if time analysis window is selected small the frequency resolution limit will be reduced. On the other hand, when the window width is high it may make invalid the stationary signal inside the intended window.



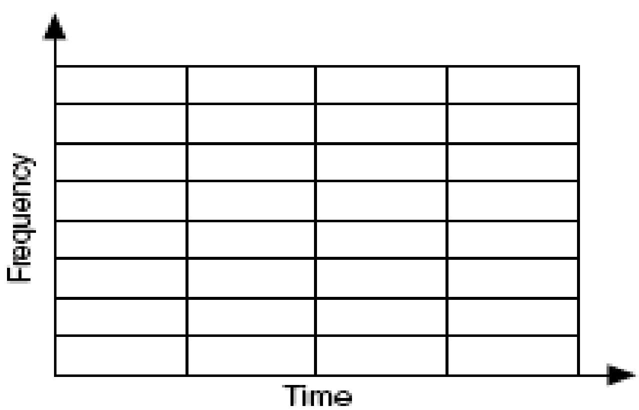


Figure 11 Basis functions and time and frequency resolution limit in short-time Fourier transform and wavelet transform

Mode of time and frequency plane coverage by *STFT* and *WT* is shown respectively in above figure.

As it was observed above, basis functions *STFT* are obtained from multiplication of Fourier basis functions in a fixed window. To reach appropriate frequency and time resolution limit, some bases with different time width should be used. So that basis functions width in low frequencies is high and in high frequencies it is low. The result of this is that in low frequencies we have good accuracy in frequency because our observance of signal is long, while we have not good time accuracy. In high frequencies that the time width of function is low, we do not have good frequency accuracy because of low observance time but we have high time accuracy and any sudden change can be followed and will show itself in transform coefficients. In other words, to analyze non-stationary signals, we expand them to basic functions that are made from expansion (or contraction) and transfer of a function that is particularly selected for an intended signal. The concept of variable window was the first step in emergence of wavelet transform. This method can be assumed as same as a mathematical microscope that by its adjustment we can evaluate different parts of a signal [5, 6].

3.3 Simulation

1 – Non-stationary signal *STFT* is simulated. In short time Fourier transform we can have frequency – time by a window that is used for signal analysis.

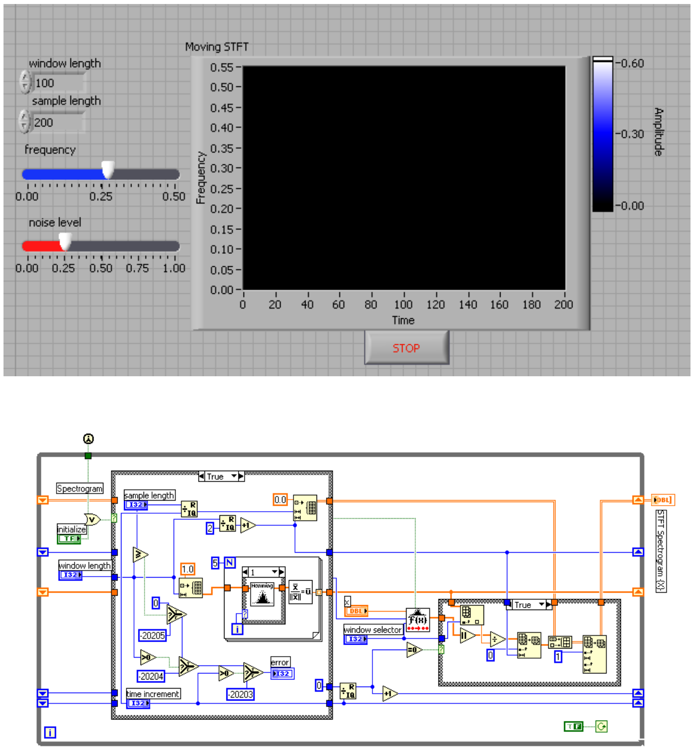


Figure 12 *STFT*

Simulation result:

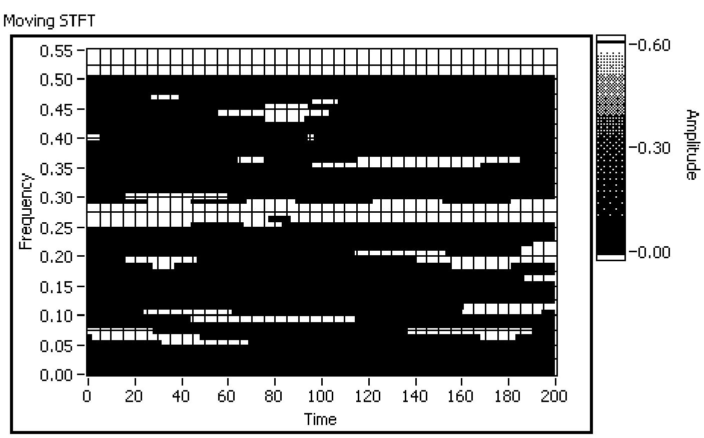


Figure 13 Simulation result

4. Pattern Recognition

Pattern recognition is divided into stages that are begun by features extraction from the samples. Feature extraction is sample transformation to features that all important information is ideally hidden in them.

The next stage is feature selection step. At this stage fewer meaningful features are selected that express the given model in best mode without repetitive content. Finally, classification is done i.e. a particular sample according to features profile selected for it is attributed to a particular class [7,9].

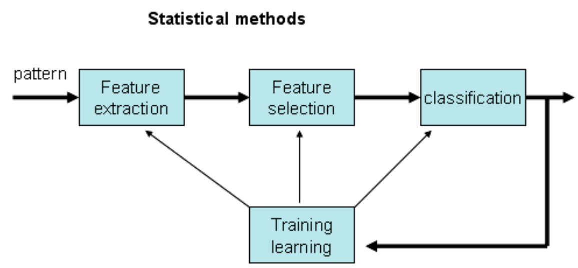


Figure 14 Pattern Recognition Process Block Diagram

Various stages of feature extraction, feature selection and classification are seen in above Figure.

This general and common model can be implemented and practiced by various ways. In order to apply this process in medical diagnosis processes at first the pattern should be introduced for example like disease symptoms, recorded signals, or a collection of images taken from the patient. In addition, obtained classes indicate different possible modes in the diagnosis. According to the patterns, statistical, structural or artificial intelligence methods can be used.

Statistical pattern recognition uses measurements and transform applied to patterns structure as feature vector. Feature selection often is performed by a series of automatic sequential operations or sometimes by an expert and based on his experience. Also classifiers generally are dependent on probability distribution of features. Bayesian linear classifiers assuming a normal distribution for features is one of the ways that is used generally and especially it is used in medical applications. However, other nonparametric classification techniques such as K-nearest neighbor or polynomial classifier should also be noticed because in many times assuming normal distribution for features is not correct.

Structural and syntactic pattern recognition techniques classify themselves based on descriptions and existing graphs in model structure. Straight lines, arches and corners that are named *primitive* are some features that are set following each other and consequently and threaded. Each class is expressed with a distinct grammar. The process of classification, in accordance with graphic specifications seem the signals recorded, but use of them in clinical problems have been seldom reported that time-consuming of these methods can be referred as one of the main reasons [8,11].

The third technique, which is used for pattern recognition, is artificial intelligence. Beside the symbolic artificial intelligence that is often referred, our emphasis is on connectionist systems that for example can be realized as artificial neural networks and their geometry profile is highly dependent on the problem that should be resolved.

After initial selecting of the network type, connection among network elements (neurons) must be adjusted through training. Artificial neural networks store information about a specific problem-solving strategy that is designed for it in these connections. Therefore, in practical problems, teaching and learning strategies have particular importance.

Although the methods presented above have significant differences in implementation and realization mode, all of them in medical diagnosis are applied for one purpose, i.e. decision making about medical problems. Therefore, knowledge and experience that experts need to do this should be used in different methods. In other words, structures, models or molds used should be affected by a learning process and it can be done according to the method selected by using various ways such as estimating probabilities or covariance matrix or by creating special grammars or adjustment of weights coefficients in connections between network nodes. Knowledge required is expressed correspondingly in form of the shapes, coefficients or structures. Another difference that exists between these methods is in mutual time with the user in order to optimize the process of classification. Statistical methods for feature selection, appropriate number of features and model parameters should be selected and evaluated and finally should be carefully set manually. Artificial neural networks can do this manual adjustment by applying learning strategies as self-organizing and optimal [5,7].

The considered process to combine the advantages of artificial neural networks with proper methods of feature extraction includes two parts: a) few features among input patterns with high dimensions are calculated. This is done by time - frequency transform and among sampled spots of the signal as the input samples. B) Calculated features are applied as inputs of a simple artificial neural network that this network serves as a classifier.

The quality of this system in addition to network weights is very dependent on the frequency – time transform parameters. Therefore, we also generalize learning phase of neural network from weights coefficients to time – frequency parameters adjustment in input nodes. This process allows us automatic and optimal adjustment of input parameters in order to perform correct classification. Due to main components used in this method we call it wavelet network.

The idea of using wavelet network for classification is trying to combine wavelet transform capabilities in extraction and selection feature with decision making power of neural networks. In learning stage, wavelet network not only learns appropriate decision functions and complex regions of decision making which determined by the weights coefficients but also determines some parts of the parameter space that are appropriate for reliable classification of input signals. A wavelet network can be considered a generalized perceptron neural network by wavelet nodes as pre-processing units for feature extraction.

A wavelet network for classification consists of two parts including feature extraction and classification.

Wavelet nodes which are set during the learning phase are modified versions of basis wavelet $h(t)$ as $h((t-\tau_k)/a_k)$. Nodes are described by a transmission parameter τ_k and a scale parameter a_k which has an inverse relation with node frequency. These parameters are like wavelet transform variables.

Node output that is displayed by Φ_{ik} is as node inner multiplication h_k and signal S_i that is the wavelet network input and can be expressed (i represents signal number: $i=1,2,...N$).

$$\Phi_{ik} = \langle h_k, S_i \rangle = \int h^*(\frac{t-\tau}{a_k}) S_i(t) dt \qquad (12)$$

Thus, the output of a wavelet node can be considered as a result of a continuous wavelet transform $W(a_k, \tau_k)$ that express the correlation between wavelet h_k and signal S_i. Based on the attitude of bandpass filters, this quantity can be expressed as a specific amount from bandpass filter $S_k(\tau, a_k)$ in time τ_k.

Upper part shows classifier that its decision is based on wavelet nodes output. Information that is obtained by observing output nodes activity level (which have been shown here by y_{ij}) is that signal $S_i(t)$ probably belongs to which class $(j=1,2,....M)$ j. In training stage, least squares error means:

$$E = \sum_{i=1}^{N}\sum_{j=1}^{N}(d_{ij} - y_{ij})^2 = \min \qquad (13)$$

Between the network desired output vector i.e. d_i and real output i.e. y_i a method such as gradient method should be used that through it at least weights partial derivatives in part of classifier based on the general rule Delta are calculated.

As mentioned earlier, in addition to weights, the wavelet nodes parameters should also be adjusted. Partial derivatives of scale parameters and wavelet node transmission, i.e. a_k and τ_k are dependent on selected basis wavelet and they can be obtained by back propagation error. In each repeated learning cycle, weights and wavelet parameters are changed so that the error E is reduced. This process will be still repeated until network falls into a minimum. After convergence is resulted wavelet nodes show some parts of time - frequency plane that are more reliable for decision making or classification. To optimization parameters, use of algorithm is proposed for its robustness and fast convergence. This method can be also considered as a modified version and quick propagation algorithm.

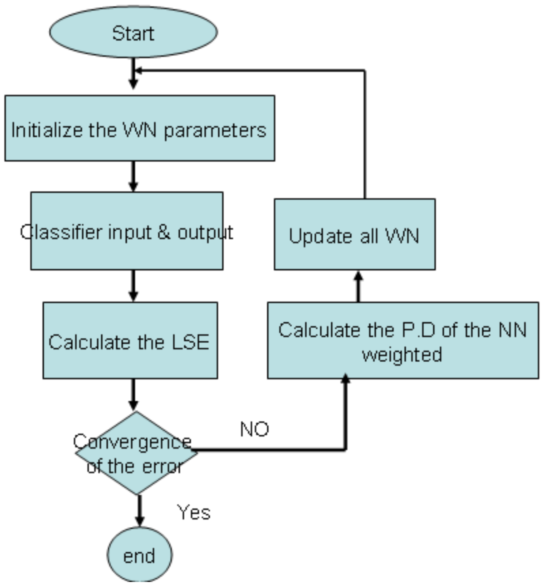


Figure 15 wavelet network learning algorithm view process for classification

The number of wavelet nodes in a wavelet network depends on this intended problem, but generally the number of these nodes should be considered low in order to increase network generalizability. Another problem that should be noted is initialization wavelet network parameters. It is recommended that similar to multilayer perceptron neural network, the wavelet network be taught in several times with different random initial values and then the best answer will be selected. To determine initial values of wavelet parameters the problem information should be used. For example, if signal examining is supposed to do in high frequencies at least one of the wavelet nodes in this area should be initialized from time - frequency plane.

5. CONCLUSION

The classification of electrocardiogram signals (ECG signals) by using wavelet network has been represented.

The analysis of ventricular late potentials has been used for identifying patients with ventricular tachycardia. The wavelet networks method for medical diagnosis applications has been introduced. The reliability and feasibility of this method for a specific clinical situation by considering vital signals as input, and the advantages of this method have been shown.

The proposed wavelet network could be considered an appropriate method for pattern recognition purposes which is often used in problems of medical diagnosis. As the advantages of this method we could refer to the decrease of the necessary communication with the user and existence of features that best match with signal in the network input.

REFERENCES

[1] W.W. Boles and B. Boashash, “A human identification technique using images of the iris and wavelet transform,” IEEE Transactions on Signal Processing, vol. 46 Issue. 4, pp. 1185– 1188, April 1998.

[2] M.User and A. aldroubi, “A review of Wavelets in biomedical applications,” Proceedings of the IEEE Vol. 84 Issue 4, pp. 626–638, Apr 1996.

[3] M. Li, P.Xia and W.C. Lee, “Wavenet: A Wavelet-Based Approach to Monitor Changes on Data Distribution in Networks,” The 28th International Conference on Distributed Computing Systems, pp.395-402, June 2008.

[4] M. Azadi, A. Rahideh, A.A Safavi and O. Mahdiyar, “Wavenet Based Vector Control of a Permanent Magnet Synchronous Motor Drive,” IEEE International Conference on Electric Machines & Drives, pp. 1663 – 1668, May 2007.

[5] G.Y. CHEN, T.D. BUI and A. Krzyżak, “Invariant pattern recognition using radon, dual-tree complex wavelet and Fourier transforms,” Pattern Recognition journal, pp. 2013-2019, Volume 42, Issue 9, September 2009.

[6] M. K. Kiymik, I. Güler, A. Dizibüyük and M. Akin, “Comparison of STFT and wavelet transform methods in determining epileptic seizure activity in EEG signals for real-time application,” journal of Computers in Biology and Medicine Vol. 35, Issue 7, pp. 603-616, October 2005.

[7] B.G. Xu and A.G. Song, , “Pattern recognition of motor imagery EEG using wavelet transform,” Journal of Biomedical Science and Engineering, pp. 64-67, May 2008.

[8] C.C. LIU AND D.Q. DAI, "Face Recognition Using Dual-Tree Complex Wavelet Features," IEEE Transactions on Image Processing, PP. 2593-2599, VOL. 18 ISSUE 11, NOV. 2009.
[9] Dr.Ing. David Cuesta Frau, "Pattern Recognition Techniques Applied to Biomedical Signal Processing", PhD Course- Prague, October 2003, Polytechnic University of Valencia-Spain.
[10] O Rioul, M Vetterli, "Wavelets and signal processing", IEEE signal processing magazine, 1991 - infoscience.epfl.ch.
[11] H. Li, B.S. Manjunath,S.K. Mitra, "Multisensor Image Fusion Using the Wavelet Transform", Graphical Models and Image Processing, Elsevier,Volume 57, Issue 3, May 1995, Pages 235–245.