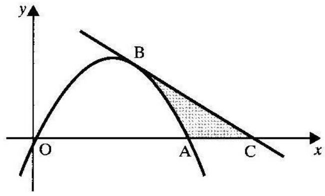
PROBLEMS USING CALCULUS

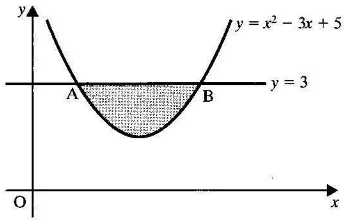
1 The diagram shows the curve $y = 3x - x^2$. The curve meets the x-axis at the origin O and at the point A. The tangent to the curve at the point B(2, 2) intersects the x-axis at C.



a Find the equation of the tangent to the curve at B.

b Find the shaded area.

2



The graph shows sketches of the line $y = 3$ and the curve $y = x^2 - 3x + 5$ (not drawn to scale); they intersect at the points A and B. The shaded region is bounded by the arc AB and the chord AB.

a Find the coordinates of A and B.

b Find the area of the shaded region.

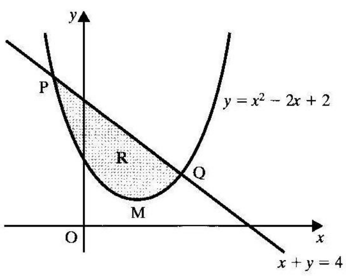
c Show that the equation of the tangent to the curve at A is

$$y + x - 4 = 0$$

and find the equation of the tangent to the curve at B.

d The tangents to the curve at A and B meet at the point C. Show that the coordinates of C are $\left(\frac{3}{2}, \frac{5}{2}\right)$.

3 The diagram below shows sketches of the line with equation $x + y = 4$ and the curve with equation $y = x^2 - 2x + 2$ intersecting at points P and Q. The minimum point of the curve is M. The shaded region R is bounded by the line and the curve.

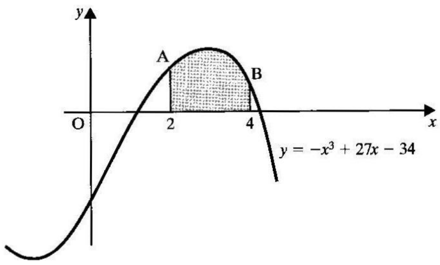


a Show that the coordinates of M are $(1, 1)$

b Find the coordinates of the points P and Q.

c Find the area of the region R

4



The figure shows a sketch of part of the curve with equation $y = f(x)$ where $f(x) = -x^3 + 27x - 34$.

a Find $\int f(x)\,dx$.

The lines $x = 2$ and $x = 4$ meet the curve at points A and B as shown.

b Find the area of the finite region bounded by the curve and the lines $x = 2$, $x = 4$ and $y = 0$.

c Find the area of the finite region bounded by the curve and the straight line AB.

Answers

1. **a** $x + y - 4 = 0$ **b** $\frac{5}{6}$

2. **a** $(1,3), (2,3)$ **b** $\frac{1}{6}$ **c** $x - y + 1 = 0$

3. **b** $(-1,5), (2,2)$ **c** $5\frac{1}{6}$

4. **a** $-\frac{1}{4}x^4 + \frac{27}{2}x^2 - 34x + c,$ **b** 34, **c** 12