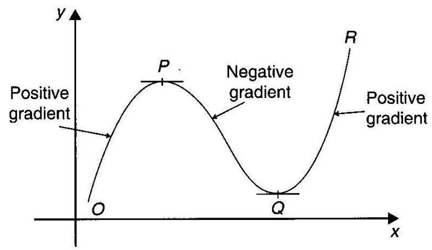
Maximum & Minimum

Maximum & Minimum Points

The graph shown has a maximum point at P and a minimum point at Q.

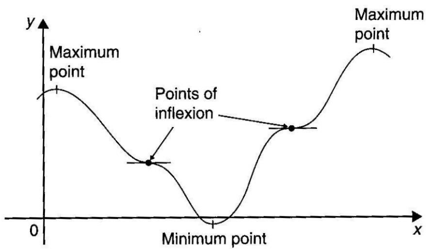


The gradient of the curve at P and Q is 0.

We can use this fact to locate maximum and minimum points (also called turning points).

Points of Inflexion

This graph shows two points of inflexion with zero gradient.



Maximum and minimum points and points of inflexion are given the general term of stationary points.

Finding Turning Points

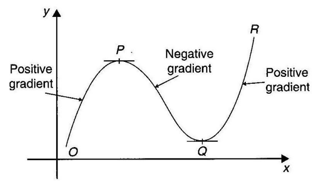
- Turning points (maximum and minimum points) can be found by finding places on the curve where the gradient is zero.
- If $y = f(x)$ solve $f'(x) = 0$.
- Substitute the x values found back into $y = f(x)$ to find the corresponding y values.

Example

- Find the co-ordinates of the turning point on the curve $y = 3x^2 - 6x$

Determining the Nature of a Turning Point

- Method 1 – Examine the gradient on either side of the turning point



As x increases:

- If the gradient changes from positive to negative you have found a maximum point.
- If the gradient changes from negative to positive you have found a minimum point.

Example (continued)

Determine the nature of the turning point on the curve $y = 3x^2 - 6x$ using method 1.

Determining the Nature of a Turning Point

- Method 2 – Calculate the second derivative at the turning point

- If the second derivative is negative you have found a maximum point.
- If the second derivative is positive you have found a minimum point.
- If the second derivative is zero you have found a point of inflexion (probably)

Example (continued)

Determine the nature of the turning point on the curve $y = 3x^2 - 6x$ using method 2.

Example (continued)

- We can use the information we have found to sketch the curve of $y = 3x^2 - 6x$.
- We found a minimum point at (1,-3).

Example 2

(a) Find the maximum and minimum values of the curve $y = x^3 - 3x + 5$

(b) Sketch the curve