Computing and Engineering

Department of Engineering

University of HUDDERSFIELD

NME3523

Signal Analysis and Processing

Engineering Control Systems and Instrumentation

Date: May 2016

Time allowed: 3 hours

Instructions to Candidates:

This is an unseen closed book examination.

Candidates should answer four out of six questions. All questions are marked out of 25.

Materials provided: Table of transforms on page 8

Materials allowed: None

A scientific calculator may be used in this exam.

Unannotated paper versions of general bi-lingual dictionaries only may be used by overseas students whose first language is not English. Subject-specific bi-lingual dictionaries are not permitted. Electronic dictionaries may not be used.

Access to any other materials is not permitted.

Wait to be told to Turn Over

Should a candidate have a query about this paper and the Module Leader is not in attendance, please call 01484 472150 or 472450.

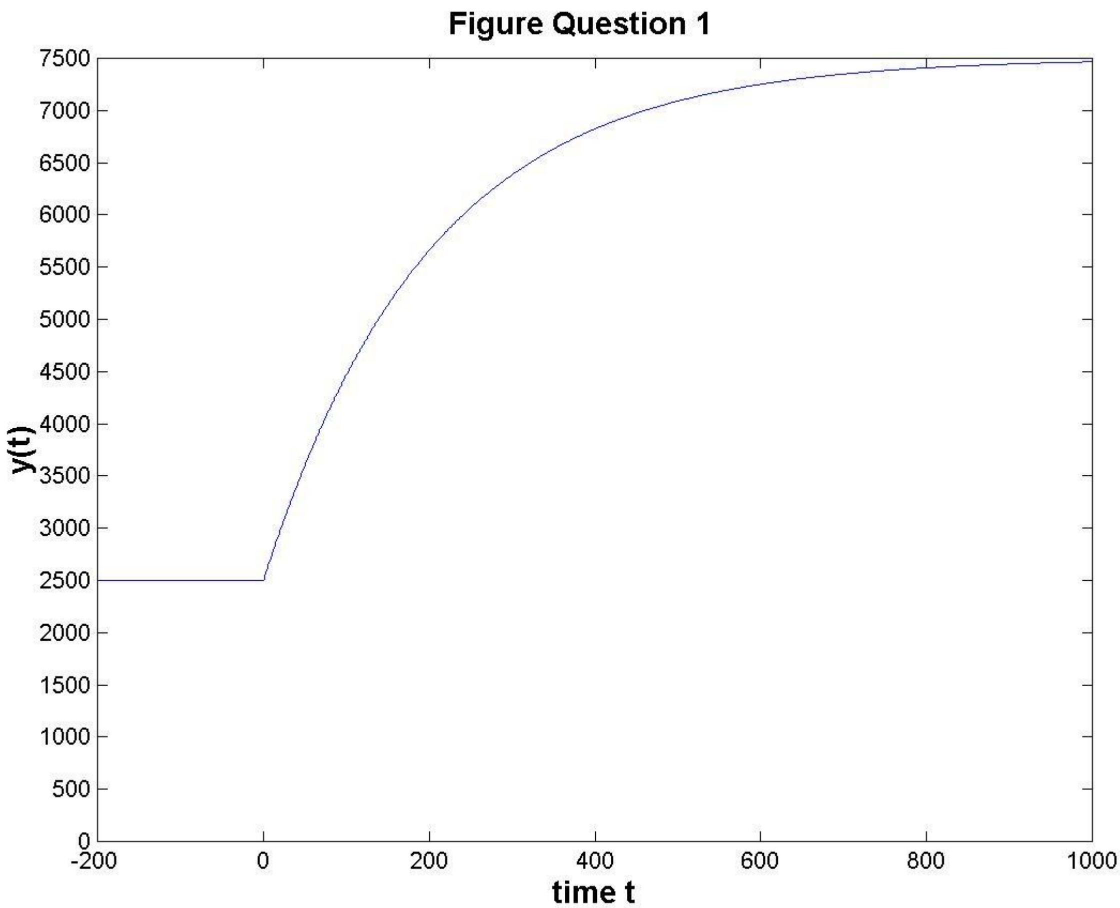
Question 1

(a) A system is defined by the differential equation $\frac{dy(t)}{dt} + (a-b)y(t) = (a+2b)x(t)$ where $y(t)$ is the output and $x(t)$ is the input. For time t < 0 the system is in steady state with the input equal to 5,000. At t = 0, the input undergoes a step change **to** 15,000. The variation of $y(t)$ with time is shown in 'Figure Question 1'.
What are the values of constants a and b ?

[16 marks]

(b) Suppose the input is now varied sinuosoidally with a maximum value of 12,000 and a minimum value of 8,000 at a frequency of 5×10^{-3} radians per second. When all transients have died away what are the maximum and minimum values of the output $y(t)$?

[9 marks]



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Question 2

A signal $f(t)$ is periodic with period 1 second. For $0 \le t \le 1$ seconds, the signal is defined as $f(t) = e^t$.

Express $f(t)$ as an infinite Fourier series.

[Hint: you must calculate a_0 and <u>both</u> the a_k and b_k terms for the correct solution].

[25 marks]

Question 3

The input to an underdamped second order system undergoes a step change from 0 to *A* at time $t = 0$. The Laplace Transform of the resultant system output $X(s)$ is given by

$$X(s) = \frac{A}{Ms\{s^2 + 2c\omega_n s + \omega_n^2\}}$$

where $c < 1$.

Giving all of your working, show that the system output $x(t)$ can be expressed as

$$x(t) = \frac{A}{K}\left\{1 - \frac{e^{-\omega_n ct}\sin(\omega_n t\sqrt{1-c^2} + \alpha)}{\sqrt{1-c^2}}\right\}$$

where $\alpha = \cos^{-1}(c)$ and where $\omega_n = \sqrt{\dfrac{K}{M}}$

[25 marks]

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Question 4

(a) A car has mass of 2000kg, the overall stiffness of the springs K is 30,000kgs^{-2} and the overall damping factor f of the shock absorbers is 3000kgs^{-1}. The distance of the lowest point of the body of the car from the ground when the car is empty is 0.2m. Four people with a combined mass of 360kg get into the car simultaneously and the car drops to a maximum distance below its initial position ('first overshoot'), rebounds ('first undershoot') and then oscillates before stabilising. At the 'first undershoot' it is noted that the car body is 0.0796m below its initial position. [The acceleration of gravity g = 9.81ms^{-2}].

(i) What is the damping ratio for the car (including passengers)?

[3 marks]

(ii) What is the final distance of the car body from the ground when the car has stopped oscillating?

[3 marks]

(iii) What is the closest the car body comes to the ground?

[10 marks]

(b) A system has the transfer function $H(s) = \dfrac{1}{Ms^2 + fs + K}$ where M , f and K are all positive numbers. The system undergoes <u>sinusoidal excitation</u> with angular frequency ω. Show that as $\omega \to \infty$ the system gain G in dB can be approximated by $G = -20\log_{10} M - 40\log_{10} \omega$.

Find an expression for the system gain in dB as $\omega \to 0$.

[9 marks]

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Question 5

(a) A flow meter used for measuring the axial flow velocity of solids particles in a two-component slurry flow consists of an upstream conductance sensor S1 and a downstream conductance sensor S2. The axial separation of S1 and S2 is 50cm. Samples of the output voltages (in mV) from S1 and S2 are taken every 0.2 seconds as shown in 'TABLE QUESTION 5' below. Calculate and plot the cross correlation function for the sampled outputs from S1 and S2. Use this cross correlation function to determine the mean axial flow velocity of the solids particles in the slurry.

[16 marks]

Time (seconds)	Output from S1 (mV)	Output from S2 (mV)
0	2	5
0.2	-9	4
0.4	-22	-7
0.6	-1	6
0.8	-10	6
1.0	6	-1
1.2	5	9
1.4	17	-11
1.6	6	-19
1.8	-6	3
2.0	4	-5
2.2	-10	1
2.4	0	6
2.6	0	18
2.8	0	1
3.0	-3	-10
3.2	11	9
3.4	-19	-11
3.6	4	2
3.8	9	0

TABLE QUESTION 5

(b) It is required to produce a random signal x containing 512 evenly spaced points in the time interval 0 to 1 seconds. It is also required to produce a second random signal y which is identical to x but delayed by $\frac{20}{512}$ seconds. It is then required to cross correlate the signals x and y using the MATLAB 'xcorr' function. Write an 'm' file which will perform the above functions and which will also enable the signals x, y and the resultant cross correlation function to be plotted.

[9 marks]

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Question 6

(a) A digital high pass filter is represented by the expression $y(n) = \dfrac{x(n) - x(n-1)}{2}$.

Derive an expression for the gain of this filter as a function of ωT.

Using the expression you derived above, produce a graph of gain versus ωT for the filter and use this graph to explain why any low frequency components to be attenuated and any high frequency components to be passed must all lie below the Nyquist frequency.

[16 marks]

(b) If a sampled cosine wave with a frequency of <u>one eighth</u> of the sampling frequency is applied to the high pass filter given above what is the phase angle, in degrees, between the output cosine wave and the input cosine wave?

Does the output cosine wave lead or lag the input cosine wave?

[9 marks]

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BY ATTEMPTING THIS ASSESSMENT YOU ARE CONFIRMING THAT YOU ARE FIT TO DO SO

DATA SHEET

Fourier Series

If $f(t)$ is a periodic waveform with period T then $f(t)$ can be expressed in the form

$$f(t) = a_0 + \sum_{k=1}^{\infty} \left\{ a_k \cos\left(\frac{2\pi kt}{T}\right) + b_k \sin\left(\frac{2\pi kt}{T}\right) \right\}$$

where;

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

and;

$$a_k = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi kt}{T}\right)dt$$

and;

$$b_k = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi kt}{T}\right)dt$$

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DATA SHEET

	$f(t)$	$F(s)$	$f(k)$	$F(z)$
1	Unit impulse	1	$\delta(k)$	1
2	Unit step	$\frac{1}{s}$	$u(k)$	$\frac{z}{z-1}$
3	Unit ramp t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
4	t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2 z(z+1)}{(z-1)^3}$
5	t^3	$\frac{6}{s^4}$	$(kT)^3$	$\frac{T^3 z(z^2+4z+1)}{(z-1)^4}$
6	e^{-at}	$\frac{1}{s+a}$	$(e^{-aT})^k$	$\frac{z}{z-e^{-aT}}$
7	$1-e^{-at}$	$\frac{a}{s(s+a)}$	$1-(e^{-aT})^k$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
8	$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$kT(e^{-aT})^k$	$\frac{Tz\,e^{-aT}}{(z-e^{-aT})^2}$
9	$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$(1-akT)(e^{-aT})^k$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$
10	$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$	$(e^{-aT})^k-(e^{-bT})^k$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
11	Item 6 with $e^{-aT}=c$		c^k	$\frac{z}{z-c}$
12	Item 8 with $e^{-aT}=c$		kTc^k	$\frac{kTz}{(z-c)^2}$
13	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	$\sin k\omega T$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$
14	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	$\cos k\omega T$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$
15	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$(e^{-aT})^k\sin k\omega T$	$\frac{z\,e^{-aT}\sin\omega T}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2aT}}$
16	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$(e^{-aT})^k\sin k\omega T$	$\frac{z(z-e^{-aT}\cos\omega T)}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2aT}}$
17	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$	$\sinh k\omega T$	$\frac{z\sinh\omega T}{z^2-2z\cosh\omega T+1}$
18	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$	$\cosh k\omega T$	$\frac{z(z-\cosh\omega T)}{z^2-2z\cosh\omega T+1}$

Note: T is the sampling period.

End of Exam Paper