

THE NEW HIGHER ORDER SPECTRAL ANALYSIS TECHNOLOGY FOR FAULT DIAGNOSIS

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ABSTRACT

There is a huge demand to always maintain machinery in good working condition and to diagnosis faults associated with them accurately. Signal processing techniques are commonly used by researchers for fault diagnostics. The spectral covariance of order 3 and order 4, the bicoherence and tricoherence technique have been compared and studies in this work for fault diagnosis of stationary impacts. A novel comparison between these methods is carried out and it is discovered by simulation that the tricoherence is more effective to investigate fault features than the tricoherence and also the spectral covariance of order 4 more effective than the spectral covariance of order 3. MATLAB and SIMULINK are experimental platforms used to carry out this study using simulated signals.

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CHAPTER 1 – INTRODUCTION

This work entails a novel comparison of the higher order spectral cross covariance and the higher order spectra for damage detection. The spectral covariance for order 3 and order 4, a novel technique for damage detection, is used for the first time and they are compared to the traditional bicoherence and tricoherence. The spectral covariance for order 3 and 4 showed their supremacy against the traditional bicoherence and tricoherence respectively. This is seen in the result. This novel work was accepted and presented at 32nd International Congress and Exhibition on Condition Monitoring and Diagnostic Engineering Management (COMADEM 2019) hosted by the University of Huddersfield. Authors of the paper include L. Gelman, D. Zhao, & A. Ball.

Fault diagnosis is a very important aspect of engineering as it plays a vital role to prevent reactive maintenance and it minimizes the effect of machine failure. It is a preventive maintenance tool which is very important in avoiding unexpected breakdowns which may result to negative financial implications. In engineering, faults are issues and an unaccepted change of a machine structure from the ordinary functioning state which may prompt the end of a framework capacity to work or a brief interference of a frameworks work. A common factor leading to structure and machine fault is the inability of the rolling element bearings and gearboxes to function efficiently. Rolling element bearings are crucial parts in engineering and manufacturing industries and they can be found in so many rotating systems like the gearbox, wind turbine, etc. The rolling element bearings and gearboxes may fail due to reasons like friction of moving parts (wear and tear), harsh conditions of operations, poor maintenance and condition monitoring.

Complex machines and structures are being used today in industries and many of these machines have a very high performance demand because of the accuracy and good working condition required from them. It is sometimes a very difficult task trying to detect faults in these complex machinery and structures by the engineers. Besides from the negative economic impact damaged machinery and structures may cause, they can also lead to safety issues. The importance of monitoring machines during production/working process cannot be overemphasised due to the fact that the reliability of the machine and structures needs to be improved while their unavailability is reduced. Fault diagnosis consists of four (4) major phases. They are; data acquisition phase, extraction of features phase, analysis of fault phase and decision-making phase. These phases must be followed in the particular order in order to diagnose faults. The data acquisition phase involves the collection of data used to monitor measurements. Vibration, temperature, pressure, acoustic and wear debris are examples monitoring measurements used to diagnose faults. The phase which involves extraction of features is a very critical phase. The acquired data needs to be filtered to reduce and remove unwanted parts of the signal to aid good analysis. The best features that indicate fault in the signal needs to be extracted for adequate diagnosis as well as classification. Wrong results resulting to false alarms will occur if the features are not well chosen and extracted. The time domain, frequency domain, time-frequency domain and the time-scale domain are feature extraction tools. The features must be analysed and classified after they have been extracted. Features can be classified by plotting measurements of acquired data as a function of time and this technique is commonly known as trending. The change of magnitude can also provide useful information about the condition of the machine. I utilised the magnitudes of the spectral techniques used for this project work to monitor damage. Spectral analysis is a

diagnostic tool used to identify what should be classified as a fault before a decision can be made.

Mass unbalance, gear fault, misalignment, bearing failure and crack are descriptions of faults in a rotating machine (Gomma, Eissa, & Khader, 2016). Gomma et al explained that the mass unbalance is one of the most common causes of machine vibration.

“Unbalance is a condition where the centre of mass does not coincide with the centre of rotation, due to the unequal distribution of the mass about the centre of rotation” (Gomma, Eissa, & Khader, 2016). This fault is a result of unequal distribution of the mass of a rotor and as a result some vibrations are sent to the bearings as well as other machine parts during operation. However, mass of a material may not be evenly distributed as a result of errors relating to design, manufacturing and assembly, and material fault (Gomma, Eissa, & Khader, 2016). Misalignment is another leading cause of faults in a machinery. It leads to other faults in a system. “A misaligned rotor generates bearing forces and excessive vibrations making diagnostic process more difficult” (Gomma, Eissa, & Khader, 2016). Cracks are machine faults that poses a lot of danger especially cracks in a rotor. When such system has a crack, its whole dynamic behaviour changes although research and practice discovered that small cracks will make the system dynamic behaviour change a little, too little that they may not be detectable by this means (Gomma, Eissa, & Khader, 2016). This means that probability of the crack to be detected depends on the crack increasing to a dangerous size. To combat this issue, high resolution filter and frequency are utilised. Signals modelled in this project work were modelled for different crack sizes i.e. 0, 0.1, 0.2. The approach taken in this project work to model the cracks is reducing the stiffness ratio.

Fault diagnosis has become a very popular topic of interest amongst several researchers due to the importance of preventive maintenance which cannot be overemphasised. Different techniques have been created and outlined for fault diagnostics, such as temperature trend analysis, signal processing and analysis, acoustic emission (Wang, Xing, Markert & Liang, 2019). However, the signal processing and analysis technique is commonly used to detect faults. This method provides very crucial information about internal irregularities in machine structure. The mechanical health state of many machine components can be gotten by vibration monitoring. There is a need to have a signal which can be measured physically when monitoring a machinery and structure to diagnose faults using signal analysis and processing methods. The measurable signal may be temperature signal, vibration signal, sound pressure signal although vibration signals are the most commonly used type of signals (Gomma, Eissa, & Khader, 2016). Applying the right signal processing technique to signals ensures that the vibration signals are effective to diagnose faults. Information gotten from signal analysis and processing is studied and used for preventive maintenance planning. Be that as it may, most by far of this research relies upon data gotten from speed and load conditions and accordingly increasing the probabilities of a correct machinery fault diagnosis. In such a situation, the fundamental variable that changes is the presence of fault, which means calculations and signal processing can be exceptionally delicate to such variation (Gelman, Kolbe, Shaw & Vaidhianathasamy, 2017).

Many time domain and frequency domain methods have over the years been proposed and developed to deal with this problem of fault diagnosis and proffer a solution for an accurate and reliable diagnosis method. Techniques like the crest factor, peak value and root mean square value (r.m.s) are time domain techniques that have used for early diagnosis of faults. They have been used in different ways and even combined together but unfortunately

methods based on the r.m.s are not sensitive to early machine faults. Frequency domain techniques also known as spectral techniques deal with the frequency domain characteristics of the signal. The signal cepstral technique is a monitoring technique that is important to detect a signal's insistent pattern which has been utilised for the diagnosis of faults. In a signal cepstrum, vibrations are relatively easy to detect, and this is because the signal spectrum is a Fourier transform of the signal spectrum magnitude (Gomma, Eissa, & Khader, 2016). The spectral and cepstral techniques have a similar problem which is that they do not focus on high frequency unlike the envelope method that deals with high frequencies. "The fundamental idea of high frequency envelope methods is to use the high frequency part of the defect modulated vibration signal and demodulate it to obtain the vibration signature" (Gomma, Eissa, & Khader, 2016).

Non-linearity in a system increases as a result of faults which occur in the system. Non-stationary impacts generated from rolling element bearings and gearboxes are studied extensively to ascertain the level of damage and signal analysis have been a very useful tool to diagnose the faults. Measuring the non-stationary impacts generated from the system gives a clear representation of the variation from the normal working condition of the system to the development of fault in the system. The development of higher order spectra techniques has been very useful for diagnosing faults over the past years. There are other signal analysis and processing techniques which have been proposed and used for fault diagnosis of structures and machinery. Some of these techniques include techniques based on the Wavelet transform, Wigner distribution, Short time Fourier transform (STFT) and the spectral kurtosis. Other non-signal processing techniques like thermal imaging have also been studied and proposed for fault diagnosis. The wavelet transform is a process done similarly

to the short time Fourier transform but unlike the short time Fourier transform, there is a change in the width of the wavelet function with each spectral component of the signal (Gomma, Eissa, & Khader, 2016). The wavelet transform obeys the Heisenberg uncertainty principle hence it is a linear representation of a signal. The wavelet transform unlike the Fourier transform is not associated with spectral leakages. It is regarded as an extension of time-frequency analysis that converts signals into small sinusoidal waves.

Tripsectral and bispectral analysis do not deal with the trispectrum and bispectrum respectively. Rather, they deal with a normalized form of trispectrum and bispectrum. The bicoherence and tricoherence are normalised higher order spectra techniques used to detect damage relation between amplitudes and phase, of any given signal's frequency component. It should be noted that the bicoherence and tricoherence are just a normalization of bispectrum and the trispectrum respectively and they should not be mistaken with second and third order coherence function respectively (Gomma, Eissa, & Khader, 2016). The dependencies between spectral components can be measured by the bispectrum and trispectrum. They also have the ability to detect nonlinear interactions between spectral components and this is one property that makes them very unique (Gomma, Eissa, & Khader, 2016). However, compared to the bicoherence and tricoherence, the spectral covariance of order 3 and order 4 has shown to be a more powerful tool for fault diagnosis as other methods show limitation.

MATLAB, an experimental platform, used together with SIMULINK are used for this project. SIMULINK is MATLAB-based design and simulation software which is used to simulated signals used for this project. MATLAB has over the years been an important tool for signal processing and analysis for engineers. The ability of MATLAB to integrate data programming,

visualization and numeric computation in a user-friendly environment while providing accurate result made it to be utilised for this project.

1.1 AIMS AND OBJECTIVES

1.1.1 AIMS

The main aim of this project is to research, develop, validate and compare new higher order spectral analysis technology to diagnose faults

1.1.2 OBJECTIVES

1. Compare the bicoherence and the 3rd order spectral covariance by the simulated signals.
2. Compare the tricoherence technique with the 4th order spectral covariance by the simulated signals.

1.2 Theory

Popular signal analysis and processing tools like the power spectrum, variance, autocorrelation etc., have been broadly utilized by researchers for data analysis and processing of the first order and second order spectral analysis. In order to recover the true phase character of a signal and extract information from the signal as a result of the variations from Gaussianity and presence of nonlinearities, techniques based on the power spectrum cannot be used. The higher order spectra (HOS) are utilized for these kinds of cases. Petropulu explained that “HOS consist of higher-order moment spectra, which are defined for deterministic signals, and cumulant spectra, which are defined for random processes” (Petropulu, 1999). The HOS has a wide range of applications in a lot of areas as it is widely used in many areas. The HOS analysis “can be used to identify wave-wave interactions in time-series of water surface elevation” (Ewans, Chistou, Llic & Jonathan, 2019). Some of the areas

where the HOS is applied include “plasma physics for wave interaction and nonlinear phenomena” (Petropulu, 1999), aerospace, biomedical signals, speech analysis and processing, communications system, etc. The power spectrum is also regarded as a member of the higher order spectra. Rosenblatt in 1985 stated that the power spectrum is a second order spectra. However, the power spectrum cannot detect the phase relationship between spectral components. The second order spectral analysis have proven to be good enough to only describe stationary processes because of cases of deviations from stationary to display nonstationary behaviours. In order to adequately study the nonstationary processes, the higher order spectral analysis is used. The high order spectra (HOS) can be used to describe and detect nonlinearities present in the time series. For the nonlinearities to be detected in a signal, the signal must be transformed to the frequency domain from the time domain as this will expose the periodicities of the signal. This is because in the time domain, not all the contents of the signal can easily be extracted.

The bispectrum and trispectrum have been employed for phase coupling between spectral frequency components to provide better diagnosis of damages as well as spectral analysis. Howard related the bispectrum to the power spectrum in 1997. He explained that the Fourier series coefficient, $X(f)$ of a random time signal, $x(t)$ is expressed by

$$X(f) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-i2\pi ft} dt \quad (1)$$

And it can be expressed using amplitude and phase quantities to give

$$X(f) = |X(f)| \exp[i\theta(f)] \quad (2)$$

Where $|X(f)|$ represents the magnitude, and the phase angle is given by and $\theta(f)$.

The power spectrum of the signal $x(t)$ is gotten from the expression below.

$$P(f) = E[X(f)X^*(f)] \quad (3)$$

Where $*$ represents complex conjugate and $E []$ represents the expectation operator.

(Howard, 2019).

Howard explained that the complex conjugate operation shows the squared of the magnitude of the frequency component. The phase information is removed from the signal by this operation of getting the magnitude squared. Howard said

“whereas the phase information is available from the Fourier series integral, as shown in equation 2 above, the actual phase is always arbitrary, being independent on the start time of the time series used in the Fourier integral” (Howard,2019). He further explained that “the only means of detecting phase relationships between different frequency components is thus to determine the statistical dependence between them, using the higher-order techniques.” (Howard,2019).

The kurtosis is also a normalized trispectrum. In 2015, Wang, Xiang, Markert, & Liang reviewed developments on the spectral kurtosis theories and their applications in detecting of faults and the diagnosis of faults. Wang et al. said, “the kurtosis is a measure of peakedness, and hence it is a good indicator of signal impulsiveness in the context of fault detection for rotating components” (Wang, Xiang, Markert, & Liang, 2015). Wang et al. expressed the kurtosis as

$$kurtosis (x) = \frac{E\{(x - \mu)^4\}}{\sigma^4} - 3 \quad (4)$$

Where μ and σ represents the mean and the standard deviation respectively and $E \{ \}$ is the expectation operation (Wang, Xiang, Markert, & Liang, 2015). Wang et al. explained that the function of “minus 3” in the expression is to make the kurtosis of the normal distribution equal to zero (Wang, Xiang, Markert, & Liang, 2015). Wang et al. explained that new methodologies have been proposed based on the kurtosis in the time domain. Techniques like the envelope kurtosis (EK) and scalar indicator kurtosis ratio (KR) were amongst the listed methodologies listed by Wang et al. based on the time domain kurtosis.

In 2007, Gelman and Petrunun proposed a new multidimensional time/multi-frequency transform for higher order spectral analysis (HOS). This transform is proposed for transient signals having nonlinear polynomial deviations of instantaneous frequency (high order chirps). They proposed this transform to compare it with the classical multi-frequency HOS based on the Fourier transform. This classical HOS has been previously studied by various researchers to detect nonlinearity in stationary temporal processes and it has been seen not to be suitable for transient temporal processes with any deviations of the instantaneous frequency because during the whole process, averaging of the HOS leads to errors as a result of the nonstationary signal. Another reason is because of the variations in the stationary kernel of the Fourier transform with the transient processes. Gelman and Ottley previously proposed a higher order chirp-Fourier transform for higher order chirps which is suitable to develop a new HOS. The short time high order chirp spectra (HOCS) proposed by Gelman and Petrunun is good because the non-traditional transient kernel of the higher order chirp Fourier transform is good for higher order chirps and also because transient signals are good with the time-domain technique for windowing (Gelman & Petrunun, 2007). Gelman and Petrunun explained that in order to estimate the short time HOCs, external windows should be used to divide the time domain signal into overlapping blocks and internal windows be

used to split each time block into overlapping segments. This proposed short time HOCS of the 3rd order detects nonlinearity in a system based on the higher order chirp-Fourier transform of the 2nd order as a result of the deviations in the frequency of the input signal is linear. They modelled an input signal which I adopted for the purpose of this project work. The signal the following parameters; input signal of 10 Hz/s, damping of 250, resonance frequencies of 1,460.5 Hz and 1.500 Hz. Signal to noise ratio of 40dB is also added to the output signal to impede early detection of the nonlinearity present in the system. For the estimation of the proposed short time HOCS and the classical HOS, Gelman and Petrunun used a segment size of 0.4s, made the duration of the signal to be 8s, used the Hamming window as the internal window with overlapping of 50% and used the rectangular window as the external window. The signal model used by them is very good and it is adopted by me and it yielded good results. (Gelman & Petrunun, 2007).

In 2017, Gelman, Parrish, Pentrunin and Walters used the chirp Fourier order spectra to detect combustion instability. "Combustion instabilities, known as rumble and screech are the self-excited aerodynamic instabilities in the gas turbine combustor. They cause the premature failures of the gas turbine components, and, consequently, the failure of the gas turbine as a whole" (Gelman, Parrish, Pentrunin & Walters, 2017). The combustion instabilities were characterised as non-linear, complex and non-stationary by Gelman et al hence the signal processing technique is utilized by them (Gelman, Parrish, Pentrunin & Walters, 2017). HOS techniques based on the third order normalized spectra were compared to figure out which is more effective. The Short time higher order chirp Fourier bicoherence and short time bicoherence based on the Fourier transform were critically studied and compared. Only the short time bicoherence based on the Fourier transform is utilised for the signals with frequency variable in time because the classical bicoherence is not good for

signals with variable frequency as it is based on the Fourier transform. However, the bicoherence based on the short time high order chirp spectra (HOCS) proposed by Gelman and Petrunun in 2007 is good because the non-traditional transient kernel of the higher order chirp Fourier transform is good for signals varying in frequency (Gelman & Petrunun, 2007). Gelman et al. utilised the short time chirp bicoherence to detect short rumbles.

“The short time chirp Fourier bicoherence used here is suitable for detection of short rumble events due to the time-domain windowing, where the time resolution of the detection will be limited by the external window size” (Gelman, Parrish, Pentrunin & Walters, 2017). Gelman et al. discovered that the bicoherence based on the short time chirp Fourier transform is more effective for detecting combustion instability compared to the bicoherence based on the Fourier transform. They added that “the implemented short time chirp Fourier bicoherence is an effective technique for in-service rumble detection because it is capable to suppress Gaussian noise, normalized and deal with signals with varying frequency of the rumble event” (Gelman, Parrish, Pentrunin & Walters, 2017). They adapted the Fisher criterion to determine which of the two techniques were effective for damage detection. I also made use of this Fisher criterion technique to determine the most effective of the techniques used for this project work as their magnitudes did not give a clear representation of the most effective technique and it yielded good results.

A new second and higher order spectral technique is proposed by Gelman in 2014 for condition monitoring of structure and machinery. This new technique is the normalized cross-covariance of complex spectral components. Gelman wanted to compare this new technique he proposed by simulation with the normalized HOS. He proposed the use of the normalized cross-covariance of n^{th} order between n complex spectral components, $n = 2, \dots, N$; where N is

the maximum value of the order of the proposed cross- covariance and N has no limitation (Gelman, 2014) . He explained that the complex spectral components have non-zero cross covariance if the complex spectral components appear as a result of damage. He could define this technique he proposed in continuous and discrete forms. He clarified that normalized cross-covariance between 3rd, 4th, and n^{th} complex spectral components cannot be estimated by the traditional HOS because the normalized HOS of n^{th} order do not show the exact normalized cross-covariance between n complex spectral components (Gelman, 2014). The whole time should be divided into overlapped segments in order to estimate this new technique. Based the Fourier transform, the generic expression of the proposed cross-covariance of n^{th} order by Gelman is defined by the expression;

$$ncov(f_1, f_2, \dots, f_{n-1}) = \frac{1}{M} \sum_{m=1}^M [\prod_{j=1}^n X_m(f_j) - \bar{X}(f_j)] \quad (5)$$

where $X_m(f_i)$ = the Fourier transform and $\bar{X}(f_i)$ = the mean of the variable $X(f_i)$.

I adopted this expression for my project work when trying to estimate the cross-covariance. This proposed technique by Gelman is complexed valued. Sachs explained in that in order to avoid misleading interpretation, the cross-covariance in the time domain should be normalized (Sachs,1984). The misleading interpretation can be as a result of variations of the signals power spectral density (PSD) (Gelman, 2014). Another advantage of the proposed technique is that it can be utilized for non-stationary signals which is a very important aspect of my research. Gelman stated that this when this technique is applied to an undamaged system for fault diagnosis, the values of the magnitude are close to zero while the magnitude

values are close to unity (1) when the system is damaged. The results of my research work validated this as it is proven that the magnitudes tend to unity when severely damaged.

Halim, Shah, Choudhury & Kadali applied the bicoherence analysis on vibration data for condition based monitoring of rotating machinery (Halim, Shah, Choudhury & Kadali, 2008). Halim *et al* decided to use the bicoherence to analyse vibration data from a rub-effected rotor-stator system. They utilised three different case studies to accomplish this; simulated case study, pilot plant case study and industrial case study. Halim *et al* simulated a rub-impact data of different damage size, performed a pilot plant case study, and detected impeller wear on oils-sands plant by utilizing the bicoherence in industrial data from final tailing pumps (Halim, Shah, Choudhury & Kadali, 2008). The bicoherence proved to be a useful tool for fault diagnosis as they were able to detect the amount of non-linearity present in the signal using this bicoherence analysis.

In 2017, the adaptation of the spectral kurtosis is investigated and proposed by Gelman, Kolbe, Shaw & Vaidhianathasamy for gearbox vibration diagnosis in non-stationary conditions. Raw signals from a single-stage gearbox were extracted by Gelman *et al* which were run in different speed and load combination. Following the collection of raw signals, the traditional residual signal after meshing harmonics were averaged to converge on stable values were removed by employing time synchronous averaging. Gelman *et al* used the short-time Fourier transform (STFT) to calculate the spectral kurtosis for each segment of the signal data after the signal data had been split into various segments based on the time synchronous average to converge on the stable values (Gelman, Kolbe, Shaw & Vaidhianathasamy, 2017). They established that irrespective of the signal data quality or the damage and undamaged separation, correct adaptation of the spectral kurtosis technique parameters aids correct

diagnostics of fault. This proposed adaptation by Gelman *et al* is a very important technique for fault diagnosis of gearboxes working in non-stationary conditions as it improved fault diagnosis of gearbox in non-stationary conditions like varying speed and load.

The bicoherence and tricoherence are the higher order coherences which were employed by Sinha, Balla, & Meher in 2007 for the demonstration of their effectiveness in early contact detection between two components. The importance of early contact detection between the components cannot be overemphasised as it is very essential for a good system working condition as well as good safety conditions. Sinha et al. highlighted that the advantage of the high order coherences is the fact that they only adapt measured responses to detect early contact and the elimination of the need for measurement of the force. (Sinha, Balla, & Meher, 2007). The usefulness of these high order coherences is demonstrated using experimental examples.

In 2018 Candon, Carrese, Ogawa, Marzocca, Mouser, Levinski, & Silva collaborated to characterize three degree of freedom (3DOF) aeroelastic system with freeplay and aerodynamic nonlinearities using the higher-order spectra. Free play here talks about the structural linearities. The problem of identifying these nonlinearities in aeroelasticity that is frequently hindered by the complex nature of aeroelastic response information which may contain different types of nonlinearity is an issue that led Candon et al. to collaborate on this project (Candon, Carrese, Ogawa, Marzocca, Mouser, Levinski, & Silva, 2018). Nonlinear inviscid aerodynamics and linearized inviscid aerodynamics are two aerodynamic models Candon et al. used for this investigation. Candon et al. established that when the response is gotten at an area which is away from the nonlinear source and or if the response is contaminated by noise, portraying and evaluating the nonlinearities is hindered. They utilised signal analysis tools based on the higher order spectra techniques to bring out the nonlinear

mechanism features resulting from freeplay with derived Euler nonlinear inviscid aerodynamic phenomena in isolation and features derived from freeplay in isolation. Their decision to use higher order spectra analysis is due to the fact that it is a worthwhile tool to examine and process nonlinear aeroelastic systems, and its ability to detect the strength of nonlinearities as well as its functional form and presence (Candon, Carrese, Ogawa, Marzocca, Mouser, Levinski, & Silva, 2018). Candon et al. made use of bispectral density analysis and tripsectral density analysis.

High order spectra have also been used in medical and biomedical industries. In 2016 Hossain, Jassim, & Zilany used bispectrum of an auditory neurogram for reference-free assessment of speech intelligibility. The bispectrum is proposed by Hossain et al. to extract features from the auditory neurogram. Most auditory processing stages respond in a nonlinear way and thus making the bispectral analysis a good tool for feature extraction (Hossain, Jassim, & Zilany, 2016). “The application of HOS to the simulated neural responses (with all nonlinearities) is expected to provide results that reflect the pattern of human behavioural performance (subjective scores) in response to acoustic stimuli” (Hossain, Jassim, & Zilany, 2016). Phase entropy and mean magnitude are amongst the features which were extracted from the bispectrum.

Signal components of a rolling element bearing relating to rotational speed cannot be extracted from the time-frequency representation and as a result of this in 2013 Wang, Liang, Li, & Cheng proposed a solution to this problem. They explained that the repetition frequency of impulses could be detected as a result of the high amplitude in the envelope spectrum of a fault signal. Wang *et al* proposed a technique for the detection of the presence of

instantaneous fault characteristic frequency (IFCF) after bearing component is compared with the associated fault characteristic coefficients due to the IFCF not being able to be directly extracted from the time-frequency representation. The proposed technique is based on the Fault characteristic order approach to find a solution to the fault diagnosis of bearings under varying rotational speed without a tachometer (Wang, Liang, Li, & Cheng, 2014). The principles of resonance demodulation and order analysis were combined by Wang *et al.* This technique involved utilizing the fast spectral kurtosis for signal filtering. The time-frequency representation of the filtered signal with damage revealing thread lines is as a result of the signal filtering along with the short time Fourier transform. They utilized an amplitude-sum based spectral peak search algorithm to extract the IFCF from the time-frequency representation. The third step the took to achieve this proposed technique is to resample signal to convert the non-stationary domain signal into the stationary fault phase angle domain signal which is transformed into the fault characteristic order domain. Wang *et al* carried out simulation tests to validate this method. However, this technique proposed proved to be unsuitable when there is a change in the rotational speed with large abrupt variations. (Wang, Liang, Li, & Cheng, 2014).

CHAPTER 2 – METHODOLOGY

2.1 BICOHERENCE

Complex quantities like the bispectrum and the trispectrum have been used for signal analysis and processing by several researchers. The classical bispectrum depends on two frequencies. It analyses interactions of frequencies between the two frequency components. However, the bispectrum and the trispectrum are dependent on the spectral characteristics of a signal and due to this dependency, many methods have been used to normalize the bispectrum and the trispectrum. The bicoherence and the skewness are two popular and well-developed methods to normalize the bispectrum. Just like the bispectrum and trispectrum which are complex quantities, their normalized functions are also complex valued. The bicoherence can detect non-linearities in a signal and it is also capable of quantifying them. The severity of the machine fault is shown by these non-linearities. The functions of the bicoherence are bounded between 0 and 1.

The frequency domain representation of the third order cumulants is the bispectrum. The classical bispectrum is defined by the Fourier transform of a signal at three frequencies and it is dependent on two frequencies. Mathematically it is defined as

$$Bisp(f_1, f_2) = \frac{1}{M} \sum_{m=1}^M X_m(f_1) X_m(f_2) X_m^*(f_1 + f_2) \quad (6)$$

where $X_m(f_1)$ = the discrete Fourier transform of the m^{th} segment of a signal and

f = discrete frequency

The bispectrum is a complex quantity which has both magnitude and phase and it can be seen in the equation above.

The bicoherence is a well-developed normalization method for the bispectrum which is normalized to remove dependencies on the spectral properties of the signal.

It is a normalized bispectrum and it is very sensitive to the presence of nonlinearity. The bicoherence is defined by the mathematical expression;

$$b(f_1, f_2) = \frac{\sum_{m=1}^M X_m(f_1) X_m(f_2) X_m^*(f_1 + f_2)}{\sqrt{\sum_{m=1}^M |X_m(f_1) X_m(f_2)|^2} \sqrt{\sum_{m=1}^M |X_m(f_1 + f_2)|^2}} \quad (7)$$

where $b(f_1, f_2)$ = the bicoherence

Bicoherence contain amplitude and phase information between the fundamental and higher harmonics of signals. The bicoherence between the fundamental and the second harmonics of the characteristic frequency is normally used to detect nonlinearity.

2.2 TRICOHERENCE

The classical trispectrum depends on three frequencies and it analyses interactions of frequencies between the three frequency components. The trispectrum is the third member of the polyspectra family. The frequency domain representation of the fourth order cumulants is the trispectrum. Two methods for the normalization of the trispectrum are the tricoherence and the kurtosis.

The classical trispectrum is defined by the Fourier transform of a signal at three frequencies and it is dependent on three frequencies. Mathematically the tricoherence is expressed as

$$Trisp(f_1, f_2, f_3) = \frac{1}{M} \sum_{m=1}^M X_m(f_1) X_m(f_2) X_m(f_3) X_m^*(f_1 + f_2 + f_3) \quad (8)$$

Much the same as the bicoherence, the elements of the tricoherence varies somewhere in the range of 0 and 1. Amplitude and phase information between the fundamental and higher harmonics of signals are contained in the tricoherence.

The tricoherence is a well-developed normalization method for the trispectrum which is normalized to remove dependencies on the spectral properties of the signal. It is a normalized trispectrum and it is delicate to the nearness of nonlinearity. The tricoherence is characterized by the mathematically expression;

$$t(f_1, f_2, f_3) = \frac{\sum_{m=1}^M X_m(f_1) X_m(f_2) X_m^*(f_1 + f_2)}{\sqrt{\sum_{m=1}^M |X_m(f_1) X_m(f_2) X_m(f_3)|^2} \sqrt{\sum_{m=1}^M |X_m(f_1 + f_2 + f_3)|^2}} \quad (9)$$

2.3 CROSS-COVARIANCE

Cross-covariance for 3rd and 4th order are compared to the bicoherence and tricoherence respectively. The cross-covariance is a very sensitive signal processing technique which is very sensitive to the appearance of damage in a system. It is complex valued, and the complex spectral components contain non-zero cross-covariance due to damage in a system. The bicoherence cannot be used to measure the normalized cross-covariance. The cross-covariance can be used for any selected frequency component unlike the bicoherence which

is limited to selected frequency components. The cross covariance is evaluated in the frequency domain in order to perform restriction of the frequency segments. The bicoherence and other normalized high order spectra do not represent the normalized cross-covariance. Based the Fourier transform the cross-covariance of n^{th} order can be defined by the expression;

$$ncov(f_1, f_2, \dots, f_{n-1}) = \frac{1}{M} \sum_{m=1}^M [\prod_{j=1}^n X_m(f_j) - \bar{X}(f_i)] \quad (10)$$

where $X_m(f_i)$ = the Fourier transform and $\bar{X}(f_i)$ = the mean of the variable $X(f_i)$.

The normalization of the cross-covariance in the frequency domain prevents misinterpretation due to the dissimilarities in the signal's power spectral density. The cross-covariance normalization utilized for n^{th} order is given below;

$$C(f_1, f_2, \dots, f_{n-1}) = \frac{ncov(f_1, f_2, \dots, f_{n-1})}{\sqrt{[\prod_{j=1}^n var(X(f_j))]} \quad (11)$$

The cross-covariance normalization utilized for the 3rd order is given below

$$tcor(f_1, f_2, f_3) = \frac{\frac{1}{M} \sum_{m=1}^M Y_m(f_1) \times Y_m(f_2) \times Y_m^*(f_3)}{\sqrt{var[X(f_1)]var[X(f_2)]var[X(f_3)]}} \quad (12)$$

The cross-covariance normalization utilized for 4th order is given below;

$$fcor(f_1, f_2, f_3, f_4) = \frac{\frac{1}{M} \sum_{m=1}^M Y_m(f_1) \times Y_m(f_2) \times Y_m^*(f_3) \times Y_m^*(f_4)}{\sqrt{var[X(f_1)]var[X(f_2)]var[X(f_3)]var[X(f_4)]}} \quad (13)$$

where var = variance and $Y_m(f_i) = X(f_i) - \bar{X}(f_i); i = 1,2,3,4$.

CHAPTER 3 – SIMULATION MODEL

3.1 BRIEF INTRODUCTION TO THE SIMULATION SYSTEM

To establish that the bicoherence and cross-covariance techniques discussed in the methodology of this project work is effective for fault diagnosis and to compare the two different techniques with each other, a simulation system is designed to carry out simulation test. A linear and bilinear system were designed to demonstrate this. A nonstationary random sinusoidal signal input is modelled to have a random initial phase, a constant amplitude and an instantaneous frequency in time which is linearly changed (i.e. linear chirp) and the modelled signal passed through the following bilinear system which describes the system with or without crack/damage. The following generic nonlinear system is considered;

$$\frac{d^2y}{dt^2} + 2h\frac{dy}{dt} + \omega_s^2 x(t) = A\cos(\omega(t)t + \varphi), \quad x \geq 0$$

$$\frac{d^2y}{dt^2} + 2h\frac{dy}{dt} + \omega_c^2 x(t) = A\cos(\omega(t)t + \varphi), \quad x < 0$$

(14)

with

$$x(t) = \frac{X(t)}{m}, \quad h = \frac{c}{2m}, \quad \omega_s = \sqrt{\frac{k_s}{m}}, \quad \omega_c = \sqrt{\frac{k_c}{m}}, \quad A = \frac{A_1}{m}$$

where;

X = displacement, m = mass, h = damping, c = damping coefficient, k_s = stiffness of positive displacement, k_c = stiffness of negative displacement, A_1 = constant amplitude of input sinusoidal signal, $\omega(t)$ = linearly changed instantaneous frequency, φ = random initial phase

of the signal. ω_s and ω_c are the resonance frequencies at the positive and negative displacement respectively.

The equation (1) above is used to describe the system with and without any damage. The stiffness of positive displacement, k_s and the stiffness of negative displacement

, k_c are equal for an undamaged system while for a system with damage the stiffness of positive displacement k_s and the stiffness of negative displacement k_c are different and they are characterized by the stiffness ratio $k^* = \frac{k_c - k_s}{k_c}$.

3.2 INTRODUCTION TO THE SIGNAL MODEL

MATLAB is used to simulate this signal using the SIMULINK toolbox. The power present in the simulated signal is described by the power spectral density. The power spectral density shows which frequencies variations of the signal are strong and weak. The peak values represent the strength of the signal. The damping, h is a value which describes the amount of damping in the system is one of the given parameters for the model signal. Another parameter, the frequency without damage which is the fundamental frequency is single degree of freedom without crack/damage. For the transient part of the system response from the signal to be cut off, the simulation is programmed to start from 0.1s.

The simulated signal without damage (linear system) has input signal frequency of 100Hz and damping of 10. The duration of the signal duration 50s and the self-frequency of the signal without damage is 100Hz.

Input frequency for linear system	100Hz
Damping, h	10
Signal duration, t	50s
Self-frequency of signal without damage	100Hz
Input Signal frequency of 10% damage	97.4Hz
Input Signal frequency of 50% damage	82.84Hz

Table 1 –List the parameters of the simulated signals

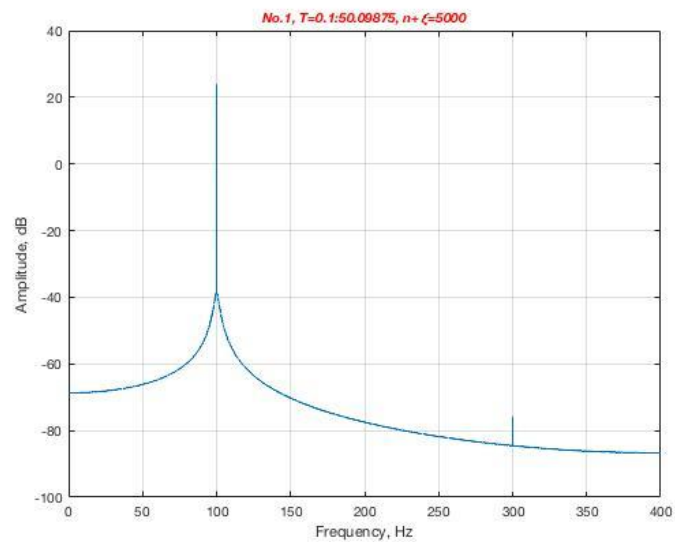


Figure 1 – Signal of the linear system with crack 0

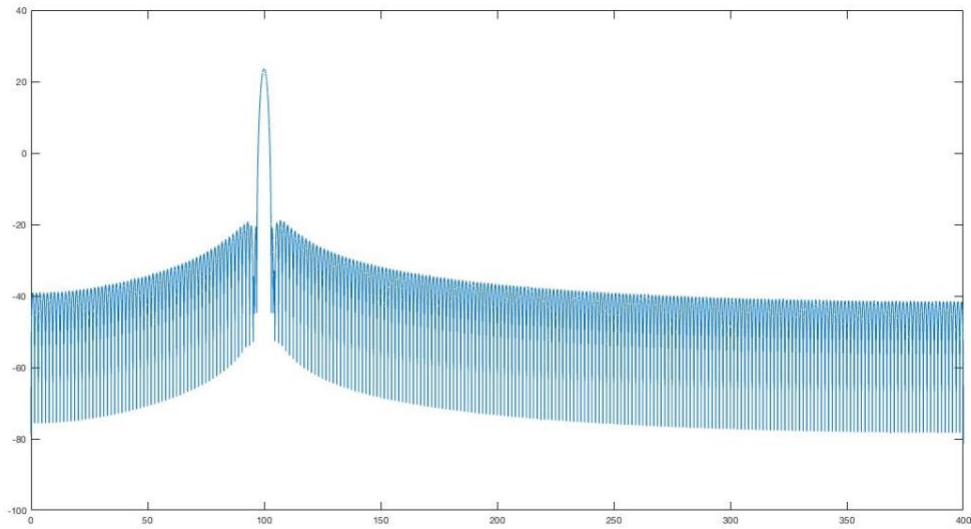


Figure 2 – PSD of linear system with crack 0

Simulated signal for the bilinear system with 10% damage has an input signal frequency of 97.4 Hz and damping of 10. The duration of the signal is 50 seconds and the self-frequency of the signal without damage is 100Hz.

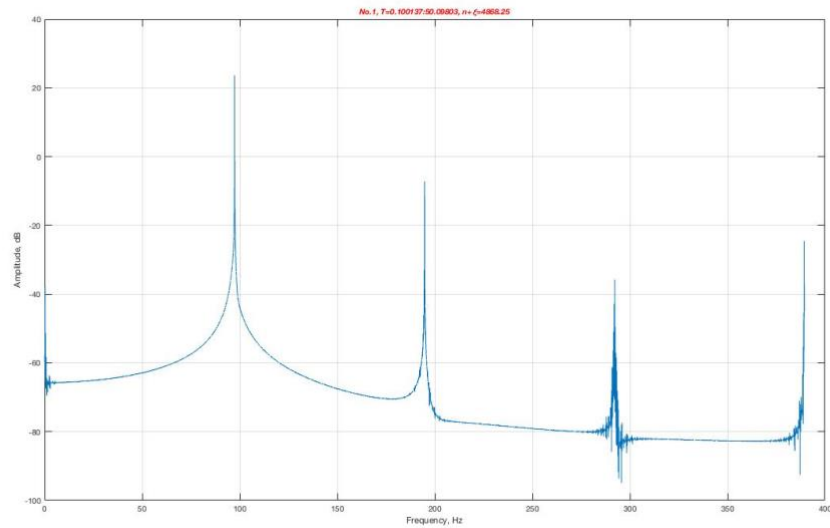


Figure 3 – Signal of the bilinear system with crack 0.1

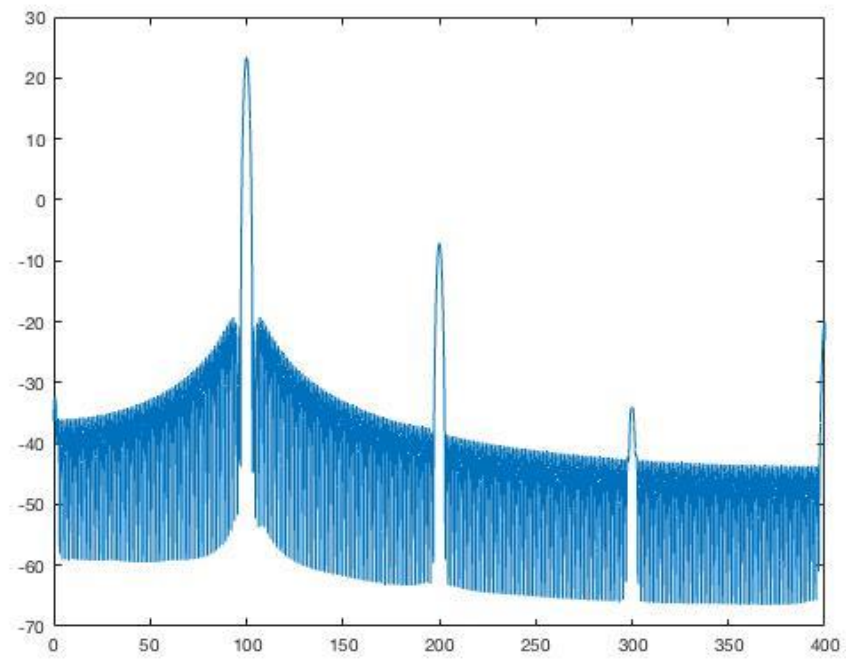


Figure 4 – PSD of the bilinear system with crack 0.1 and white Gaussian noise of 30 dB

The simulated signal for the second bilinear system with damage of 50% has input signal frequency of 82.84Hz and damping of 10. The duration of this signal is 50 seconds and the self-frequency of the signal is 100Hz.

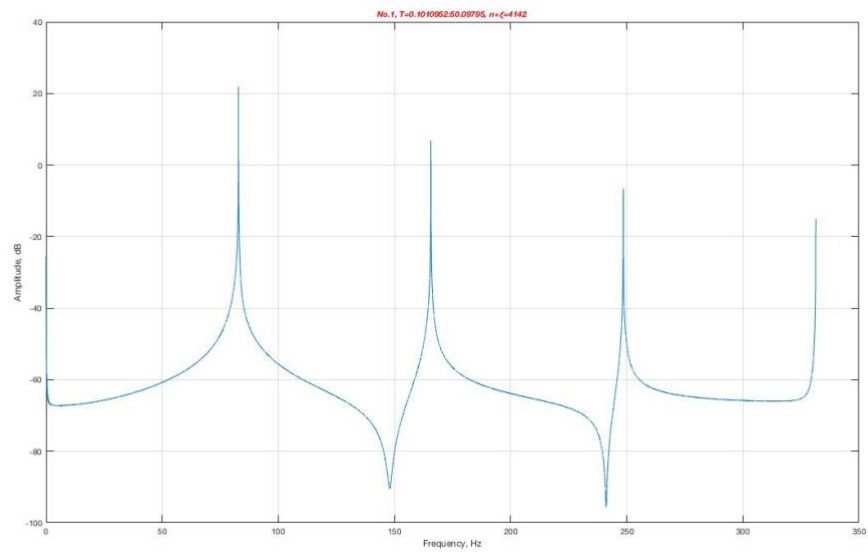


Figure 5 – Signal of the bilinear system with crack 0.5

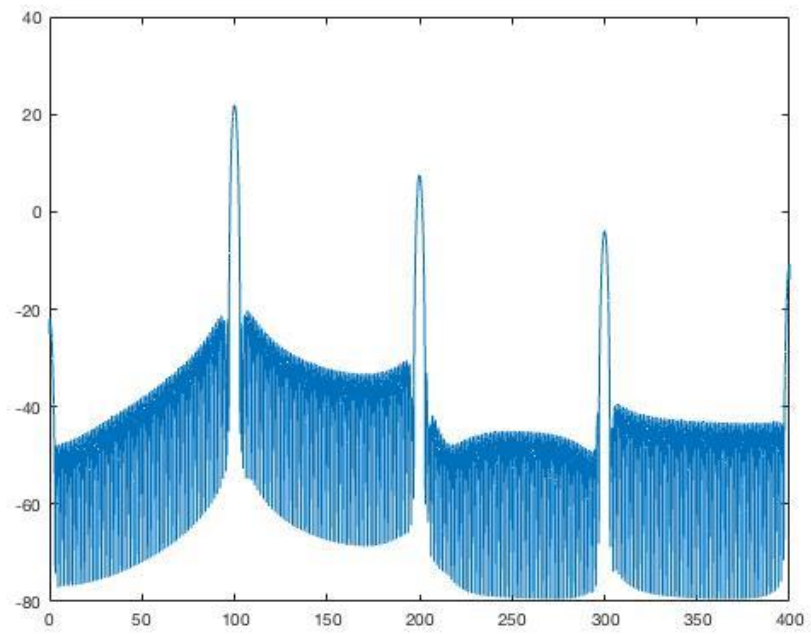


Figure 6 – PSD of the bilinear system with crack 0.5

CHAPTER 4 – RESULTS AND DISCUSSION

4.1 PARAMETERS OF THE METHODS

A total of 100 modelling tests were carried out for every crack size to compare the results of the magnitudes of the bicoherence and 3rd order spectral covariance. 100 signals from the undamaged system having a stiffness ratio of 0 and 100 signals from the damaged system (each with stiffness ratio of 0.1 and 0.2) were tested for fault diagnosis. Noise having signal to noise ratio of 30dB is applied to the signal. Hamming window is applied to reduce the effect of leakage and reduce the ripple present in the signal. The window is split into segments of 0.1s for good resolution, each with an overlap of 50 so as to increase the number of averaged segments. The resonance frequency for the undamaged signal is 100 Hz while resonance frequency for 10% damage and 20% damage are 97.37 Hz and 94.43 Hz respectively. These parameters were used to calculate the magnitudes of the bioherence and spectral coovariance. The window is split into little segments in order to get a better representation of the magnitude of the biocoherence and the spectral covariance. The duration of the signal is 50 seconds.

500 modelling tests were carried out for every crack size (0, 0.1, 0.2 & 0.3) to compare the tricoherence with the 4th order spectral covariance. 30dB of noise is added to the signal and hamming window of 50.1s is used to reduce the effect of leakage and reduce the ripple present in the signal just like the case of the bicoherence and the 3rd order spectral covariance. The window is split into 0.1 s segments. Every other parameter to simulate the signal for the tricoherence and the 4th order spectral covariance is the same with the parameter used for the bicoherence and the 3rd order spectral covariance.

4.2 RESULT OF THE BICOHERENCE

The number of the feature of bicoherence results that is obtained from the 100-simulation test for each damage size (0%, 10% and 20% damage) are utilized for fault diagnosis and are represented in the histogram below. The number of the features give a clear representation and meaning to effect of damage in the system. Estimates of the bicoherence magnitudes plotted against the damage size to get a clear representation for fault diagnosis is seen in the histogram below.

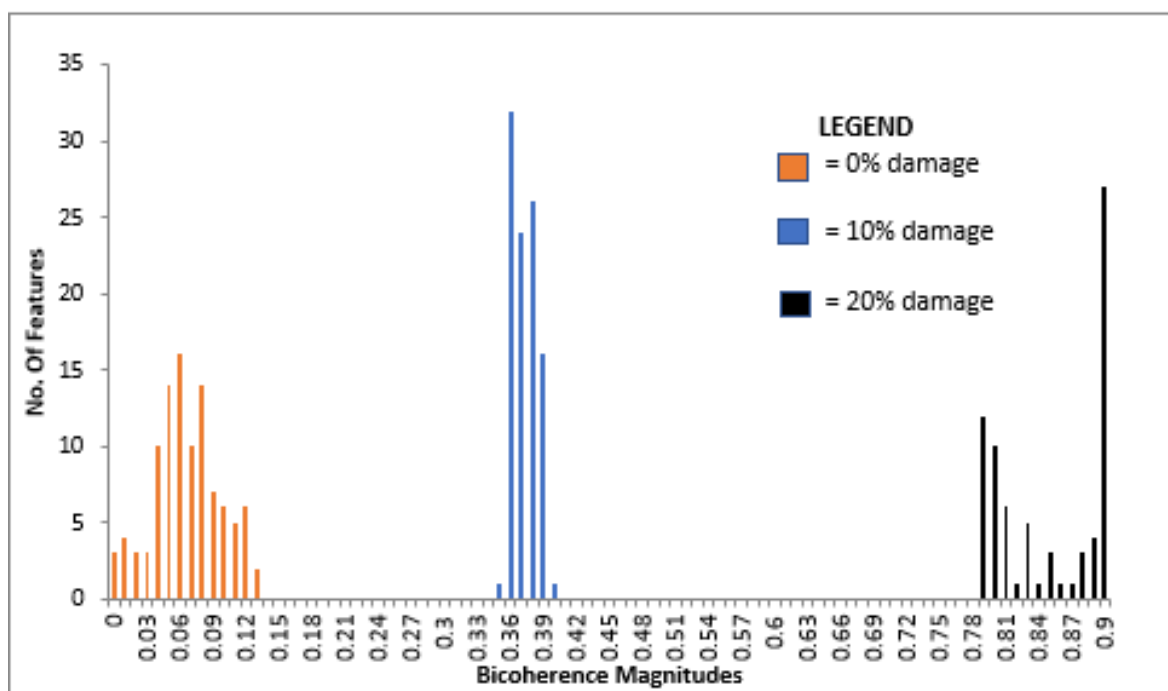


Figure 7 – Bicoherence result

From the histogram above it is observe that as the damage increase, the magnitude of the bicoherence increases. The number of features as seen in the histogram refers to the number of occurrences of the bicoherence magnitudes. The values close to 0 indicates the undamaged system while the values close to 1 indicates a high damage system. This shows that the magnitudes of the bicoherence are bounded between 1 and 0 which proves that the values of the bicoherence vary between 0 and 1. The peak amplitude of the bicoherence is directly

proportional to the damage size. i.e. the amplitude peak increases with increase in damage size.

4.3 RESULT OF THE CROSS COVARIANCE FOR ORDER 3

Just like the bicoherence, the cross covariance of order 3 magnitude of the results that is obtained from the 100 simulation test for each damage size (0%, 10% and 20% damage) are utilized for fault diagnosis and are represented in the histogram below. The magnitudes give a clear representation and clear meaning to effect of damage in the system. Estimates of the spectral covariance of order 3 magnitudes plotted against the damage size to get a clear representation for fault diagnosis is seen in the histogram below.

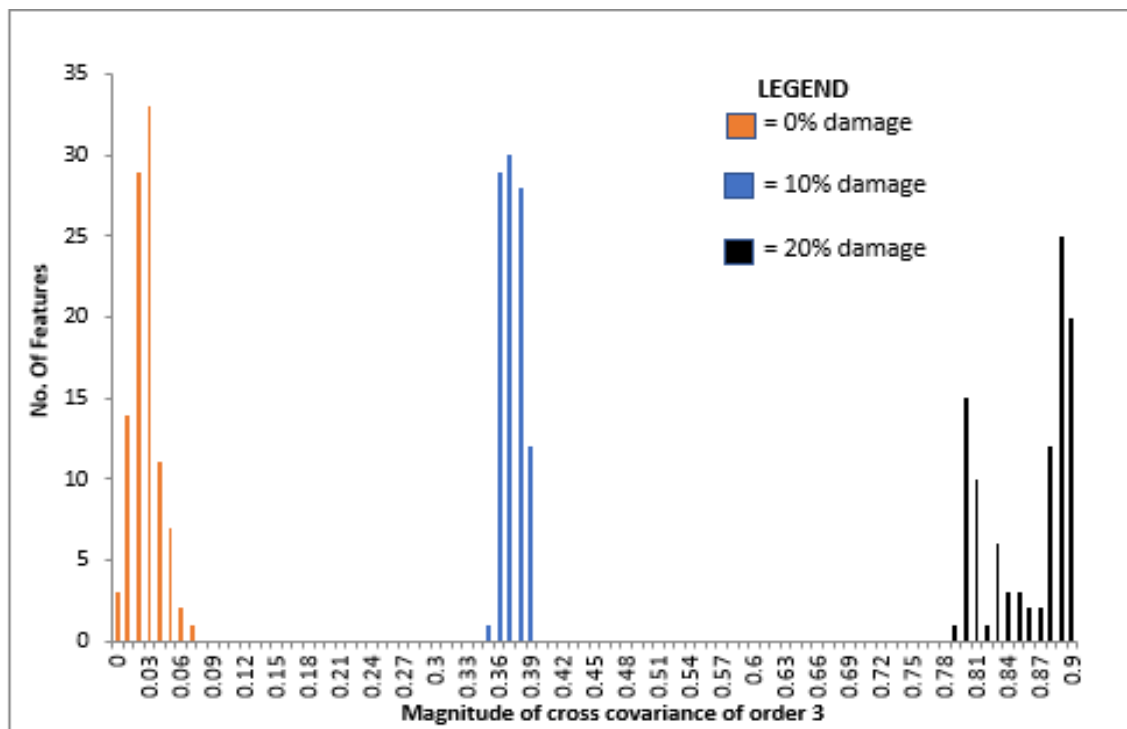


Figure 8 – Result of Cross covariance of order 3

It is observed from the histogram above that the magnitude of the cross covariance of order 3 is directly proportional to the damage size because as the damage increases, the magnitude

of the cross covariance increases. The values that tends to 0 indicates the undamaged system while the values close to 1 indicates a high damage system. This shows that the magnitudes of the cross covariance of order 3 are also bounded between 1 and 0 just like that of the bicoherence. The number of features as seen in the histogram refers to the number of occurrences of the bicoherence magnitudes.

4.4 RESULT OF THE TRICOHERENCE

500 simulations were carried out for each damage size in order to get a good result and the results of the magnitudes of the tricoherence are used to detect damage. The magnitudes give a good representation for detection of damage in the system and this can be seen in the histogram below. Estimates of the tricoherence magnitudes plotted against the damage size to get a clear representation for fault diagnosis is seen in the histogram below.

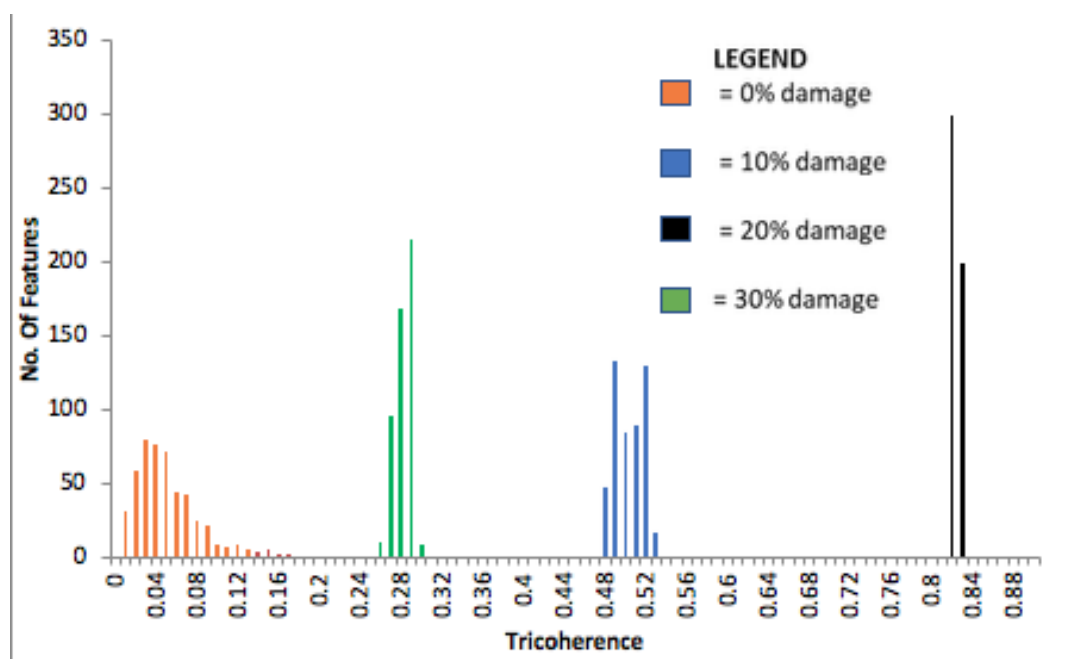


Figure 9 – Tricoherence result

Just like the case of the bicoherence and the cross covariance of order 3, the magnitude of the tricoherence is directly proportional to the damage size because as the damage increases,

the magnitude of the tricoherence increases. The property of tricoherence which explains that the magnitude of tricoherence is bounded between 0 and 1 is proven in the histogram above. The values close to 0 indicates the undamaged system while the values close to 1 indicates a high damage system. The peak amplitude of the tricoherence is directly proportional to the damage size. i.e. the amplitude peak increases with increase in damage size.

4.5 RESULT OF THE CROSS COVARIANCE FOR ORDER 4

The magnitudes of the cross covariance of order 4 is effective in representing damage and this can be seen in the histogram below. The histogram also confirms that the magnitudes are varying between 0 and 1. Estimates of the magnitudes of spectral covariance of order 4 plotted against the damage size to get a clear representation for fault diagnosis is seen in the histogram below.

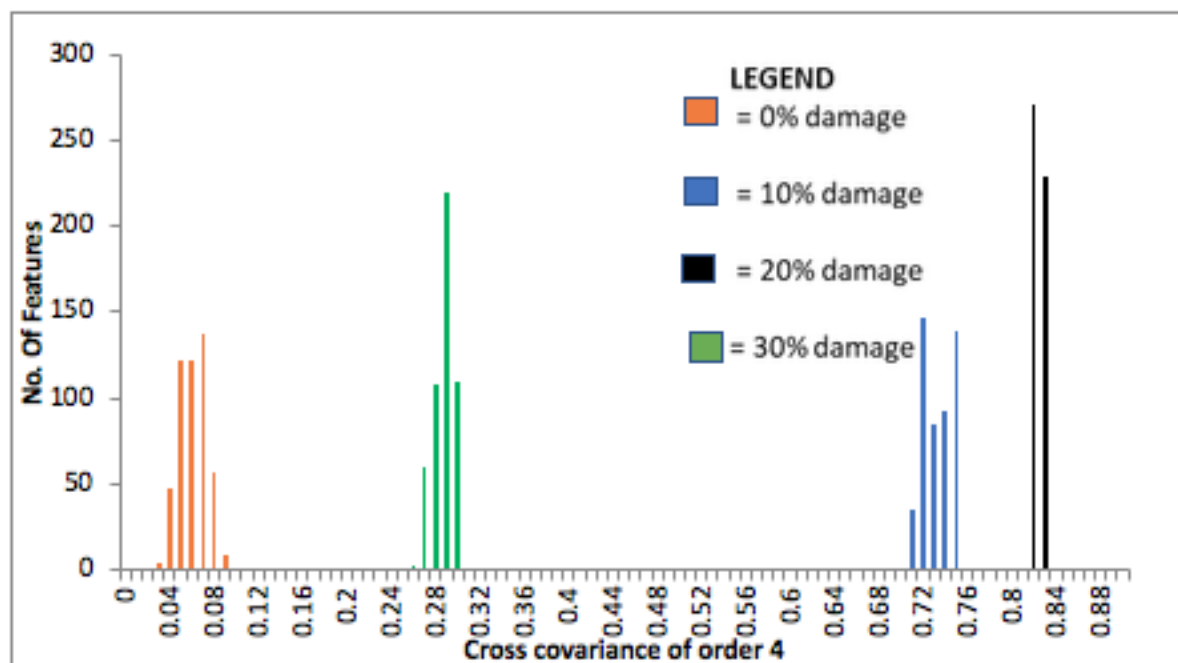


Figure 10 – Result of Cross covariance of order 4

4.6 FISHER CRITERION AND ITS RESULT

The Fisher criterion is a statistical technique based on mean and variance that gives information onto a line and classifies them in one-dimensional space. It is used to establish the most effective of the proposed methods for fault diagnosis. This technique utilizes the distance between the means of the damage and the undamaged conditions, and it minimizes the variance between the two conditions. The Fisher criterion is calculated as

$$F_c = \frac{(\mu_{damaged} - \mu_{undamaged})^2}{\sigma_{damaged} + \sigma_{undamaged}}$$

where $\mu_{damaged}$ and $\mu_{undamaged}$ are the mean values for the damaged and the undamaged conditions and the variance for the damaged and the undamaged conditions are represented by $\sigma_{damaged}$ and $\sigma_{undamaged}$ respectively.

4.6.1 FISHER CRITERION FOR THE CROSS COVARIANCE FOR ORDER 3

The results of the magnitudes of the bicoherence and the cross covariance of order 3 displayed on the histogram do not show which of the two techniques gives a better representation for fault diagnosis and as a result, the Fisher criterion is used to show which of the methods is effective.

The Fisher criterion for the result of the spectral covariance for the undamaged and 10% damage system is 463.

The Fisher criterion for the result of the bicoherence for the undamaged and 10% damage system is 96.

For the undamaged and 20% damage system, the Fisher criterion for the result of the 3rd order spectral covariance is 292.

For the undamaged and 20% damage system, the Fisher criterion for the result of the bicoherence is 210.

FISHER CRITERION	NORMALIZED CROSS- COVARIANCE OF ORDER 3	THE BICOHERENCE
Undamaged and 10% damage system	463	96
Undamaged and 20% damage system	292	210

Table 2 – Fisher Criterion for Bicoherence and Cross covariance for order 3

It can be observed from the following results that the proposed cross covariance of order 3 possesses a value of the Fisher criterion which is higher than that of the bicoherence. It shows that the cross covariance of order 3 has a larger value of effective detection of damage for the two cases, undamaged vs 10% damage and undamaged vs 20% damage. This observation proves that the proposed cross covariance of order 3 provides a clearer representation for fault diagnosis than the bicoherence.

The Fisher criterion's ratio for the bicoherence and the spectral covariance of order 3 is known as the effectiveness gain, G and it is mathematically expressed as

$$G = \frac{\text{Fisher criteria for spectral covariance}}{\text{Fisher criteria for bicoherence}}$$

The gain for the 10% damage system is 4.82 while the gain for the system with 20% damage is 1.39.

	THE GAIN
Undamaged and 10% damage system	4.82
Undamaged and 20% damage system	1.39

Table 3 – Gain for Bicoherence and the Cross covariance for order 3

4.6.2 FISHER CRITERION FOR THE 4TH ORDER CROSS-COVARIANCE

The results of the magnitudes of the tricoherence and the spectral covariance of 4th order is displayed on the histogram below. The histogram does not show which of the two techniques gives a better representation for fault diagnosis and as a result, the Fisher criterion is used to show which of the methods is effective.

The Fisher criterion for cross covariance of order 4 for the undamaged and 10% damage system is 215.4680. The Fisher criterion for cross covariance of order 4 for the undamaged and 20% damage system is 1334.1. The Fisher criterion for cross covariance of order 4 for the undamaged and 30% damage system is 3547.8. The Fisher criterion for the tricoherence for the undamaged and 10% damage system is 49.4228. The Fisher criterion for the tricoherence for the undamaged and 20% damage system is 371.3040. The Fisher criterion for the tricoherence for the undamaged and 30% damage system is 582.5270.

	NORMALIZED CROSS- COVARIANCE OF ORDER 4	THE TRICHERENCE
Undamaged and 10% damage system	215.4680	49.4228
Undamaged and 20% damage system	1334.1	371.3040
Undamaged and 30% damage system	3547.8	582.5270

Table 4 – Fisher Criterion for Tricoherence and Cross covariance of order 4

It can be observed from the following results that the proposed cross covariance of order 4 gives a value of the Fisher criterion, higher than that of the tricoherence. It shows that the cross covariance of order 4 has a larger value of effective detection of damage for the three cases; undamaged vs 10% damage, undamaged vs 20% damage and undamaged vs 30% damage, respectively. This observation proves that the proposed cross covariance of order 4 provides a much better representation for fault diagnosis than the tricoherence.

The effectiveness gain is estimated as the ratio for Fisher criteria for spectral covariance of order 4 to that of the Fisher criteria for tricoherence G and it is mathematically expressed as

$$G = \frac{\text{Fisher criteria for spectral covariance}}{\text{Fisher criteria for tricoherence}}$$

Using this expression, the Gain for the system with damage of 10% is 4.3597, Gain for system with 20% damage is 3.5930 while the Gain for 30% damaged system is 6.0903

	THE GAIN
Undamaged and 10% damage system	4.3597
Undamaged and 20% damage system	3.5930
Undamaged and 30% damage system	6.0903

Table 5 – Gain for Tricoherence and the Cross covariance of order 4

CHAPTER 5 – CONCLUSION

This project work proposed novel technique for fault diagnosis based on the spectral covariance for order 3 and order 4, which were tested by nonlinear signal generated using the modelled system. This has not been done before. This novel technique is compared to the traditional bicoherence and tricoherence. A paper based on my work was accepted by an international conference, Condition Monitoring and Diagnostic Engineering Management (COMADEM) and it was presented in the 32nd International Congress and Exhibition on Condition Monitoring and Diagnostic Engineering Management (COMADEM 2019) hosted by the University of Huddersfield. Other authors of the paper are L. Gelman, D. Zhao, & A. Ball. Linear and bilinear systems varying linearly in instantaneous frequency were modelled and simulated for this comprehensive study. The comprehensive study among the spectral covariance, the bicoherence and tricoherence revealed that spectral covariance of 3rd order provides a clearer representation for fault diagnosis than the bicoherence. SIMULINK is MATLAB-based design and simulation software which is used to simulated signals used for this project. Signals with 0%, 10%, 20 & 50% crack size are simulated. From the results it is seen that the spectral covariance of order 4 is a better tool for fault diagnosis than the tricoherence. The effectiveness gain calculated by the Fisher criterion is used to reveal which of the techniques is better for fault diagnosis as the results of their magnitudes did not give a clear representation of which of the techniques gave better result.

Based on my results and observations, as seen from the effectiveness gains, I recommend using the spectral covariance of order 4 over the tricoherence and using the spectral covariance of order 3 over the bicoherence to detect non linearities and faults.

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