Computing and Engineering

Department of Engineering

University of HUDDERSFIELD

NME3523

Signal Analysis and Processing

Engineering Control Systems and Instrumentation

Date: May 2017

Time allowed: 3 hours

Instructions to Candidates:

This is an unseen closed book examination.

Candidates should answer four out of six questions. All questions are marked out of 25.

Materials provided: Tables of transforms on pages 7 and 8

Materials allowed: None

A scientific calculator may be used in this exam.

Unannotated paper versions of general bi-lingual dictionaries only may be used by overseas students whose first language is not English. Subject-specific bi-lingual dictionaries are not permitted. Electronic dictionaries may not be used.

Access to any other materials is not permitted.

Wait to be told to Turn Over

Question 1

A second order system is defined by the differential equation given below, where y(t) is the output and u(t) is the input:

$$\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 41y(t) = 4u(t)$$

Assuming that the initial conditions are y(0) = 0 and $\dot{y}(0) = 1$:

(a) Find the total response of the system. What is the forced-response component and what is the free-response component?

[20 marks]

(b) Find the steady-state response component and transient response component.

[5 marks]

Question 2

A system is defined by the difference equation given below, where y_n is the output and x_n is the input:

$$y_{n+2} - 4y_{n+1} + 3y_n = x_{n+1}$$

(a) Find the system transfer function and impulse response.

[13 marks]

(b) Find the system response for $y_0 = 2$, $y_1 = 1$ and $x_n = 0$.

[12 marks]

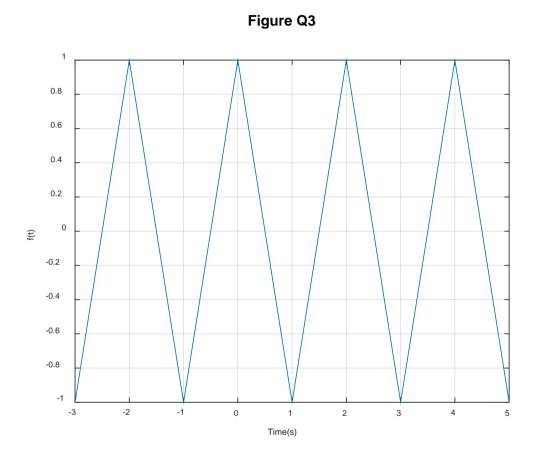
[Hint: you must use both the Z-Transform and Inverse Z-Transform for the correct solution in (a)].

Question 3

A signal f(t) is periodic with period 2 seconds. For $0 \le t \le 1$ seconds, the signal is defined as f(t) = 1 - |t|. The signal is shown in Figure Q3.

Express f(t) as an infinite Fourier series.

[Hint: you can first determine if f(t) is even or odd in order to avoid calculating both the a_k and b_k terms].



[25 marks]

Question 4

(a) A digital high pass filter is represented by the expression $y(n) = 0.5\{x(n) - x(n-1)\}$. Derive an expression for the gain of this filter as a function of ωT .

Using the expression you derived above, produce a graph of gain versus of for the filter and use this graph to explain why any low frequency components to be attenuated and any high frequency components to be passed must all lie below the Nyquist frequency.

[16 marks]

(b) If a sampled cosine wave with a frequency of one eighth of the sampling frequency is applied to the high pass filter given above what is the phase angle, in degrees, between the output cosine wave and the input cosine wave?

Does the output cosine wave lead or lag the input cosine wave?

[9 marks]

Question 5

(a) A processing system is monitored by two temperature sensors. S1 measures the input temperature and S2 measures the output. The axial separation of S1 and S2 is 75 cm. Samples of the output voltages (in mV) from S1 and S2 are taken every 0.2 seconds as shown in Table Q5. Calculate and plot the cross correlation function for the sampled outputs from S1 and S2. Use this cross correlation function to determine the mean rate of heat transfer between the two sensors.

[16 marks]

Table Q5

Time (seconds)	Output from S1 (mV)	Output from S2 (mV)	
0	13	4	
0.2	10	7	
0.4	4	14	
0.6	0	13	
0.8	-2	9	
1.0	-1	5	
1.2	1	0	
1.4	5	-2	
1.6	13	3	
1.8	15	0	
2.0	17	4	
2.2	12	11	
2.4	4	13	
2.6	0	20	
2.8	-6	13	
3.0	-5	1	
3.2	1	-2	
3.4	6	-8	
3.6	1	-6	
3.8	-4	2	

(b) It is required to produce a random signal x containing 512 evenly spaced points in the time interval 0 to 1 seconds. It is also required to produce a second random signal y which is identical to x but delayed by $\frac{20}{512}$ seconds. It is then

required to cross correlate the signals x and y using the MATLAB 'xcorr' function. Write an 'm' file which will perform the above functions and which will also enable the signals x, y and the resultant cross correlation function to be plotted.

[9 marks]

Question 6

(a) Briefly explain the difference between the "Fourier Transform", "Short-time Fourier Transform" and "Wavelet transform." Thus highlight any advantages of using the wavelet technique.

[13 marks]

- (b) Sketch an example of the form of the following wavelet families:
 - i. Morlet wavelet
 - ii. Ricker (Mexican Hat) wavelet.

[6 marks]

(c) A motor drives a wind turbine. A feedback system measures the angular position of the turbine as the system rotates with a sample rate of 1 kHz. Figure Q6 is the result of a continuous wavelet transform analysis on the sampled system, which highlights three "fault" signals, all with the same amplitude. By visual inspection, estimate the location in time, frequency and relative duration for the three signals.

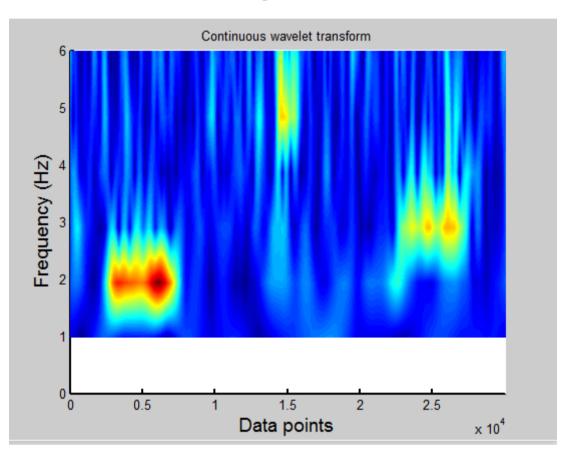


Figure Q6

[6 marks]

DATA SHEET

Fourier Series

If f(t) is a periodic waveform with period T then f(t) can be expressed in the form

$$f(t) = a_0 + \sum_{k=1}^{\infty} \left\{ a_k \cos\left(\frac{2\pi kt}{T}\right) + b_k \sin\left(\frac{2\pi kt}{T}\right) \right\}$$

where;

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

and;

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(\frac{2\pi kt}{T}\right) dt$$

and;

$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(\frac{2\pi kt}{T}\right) dt$$

DATA SHEET

	DATA SHEET						
	f(t)	F(s)	f(k)	F(z)			
1	Unit impulse	1	$\delta(k)$	1			
2	Unit step	$\frac{1}{s}$	u(k)	$\frac{z}{z-1}$			
3	Unit ramp t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$			
4	t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2z(z+1)}{(z-1)^3}$			
5	<i>t</i> ³	$\frac{6}{s^4}$	$(kT)^3$	$\frac{T^3z(z^2+4z+1)}{(z-1)^4}$			
6	e ^{-at}	$\frac{1}{s+a}$	$(e^{-aT})^k$	$\frac{z}{z - e^{-aT}}$			
7	$1 - e^{-at}$	$\frac{a}{s(s+a)}$	$(e^{-aT})^{k}$ $1 - (e^{-aT})^{k}$ $kT(e^{-aT})^{k}$	$\frac{z(1 - e^{-aT})}{(z - 1)(z - e^{-aT})}$			
8	<i>t</i> e ^{-at}	$\frac{1}{(s+a)^2}$	$kT(e^{-aT})^k$	$\frac{Tz e^{-aT}}{\left(z - e^{-aT}\right)^2}$			
9	$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$(1-akT)(e^{-aT})^k$	$\frac{z[z - e^{-aT}(1 + aT)]}{(z - e^{-aT})^2}$			
10	$e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$	$\left(e^{-aT}\right)^k - \left(e^{-bT}\right)^k$	$\frac{z(e^{-aT} - e^{-bT})}{(z - e^{-aT})(z - e^{-bT})}$			
11	Item 6 with e	$a^{T} = c$	c^k	$\frac{z}{z-c}$			
12	Item 8 with e	aT = c	kTc^k	$\frac{kTz}{(z-c)^2}$			
13	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\sin k\omega T$	$\frac{z\sin\omega T}{z^2 - 2z\cos\omega T + 1}$			
14	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\cos k\omega T$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$			
15	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\left(e^{-aT}\right)^k \sin k\omega T$	$\frac{z e^{-aT} \sin \omega T}{z^2 - 2z e^{-aT} \cos \omega T + e^{-2aT}}$			
16	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\left(e^{-aT}\right)^k \sin k\omega T$	$\frac{z(z-e^{-aT}\cos\omega T)}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$			
17	sinh ωt	$\frac{\omega}{s^2 - \omega^2}$	$\sinh k\omega T$	$\frac{z \sinh \omega T}{z^2 - 2z \cosh \omega T + 1}$			
18	cosh ωt	$\frac{s}{s^2-\omega^2}$	$\cosh k\omega T$	$\frac{z(z-\cosh\omega T)}{z^2-2z\cosh\omega T+1}$			

Note: T is the sampling period.

End of Exam Paper