The Wavelet Bispectrum

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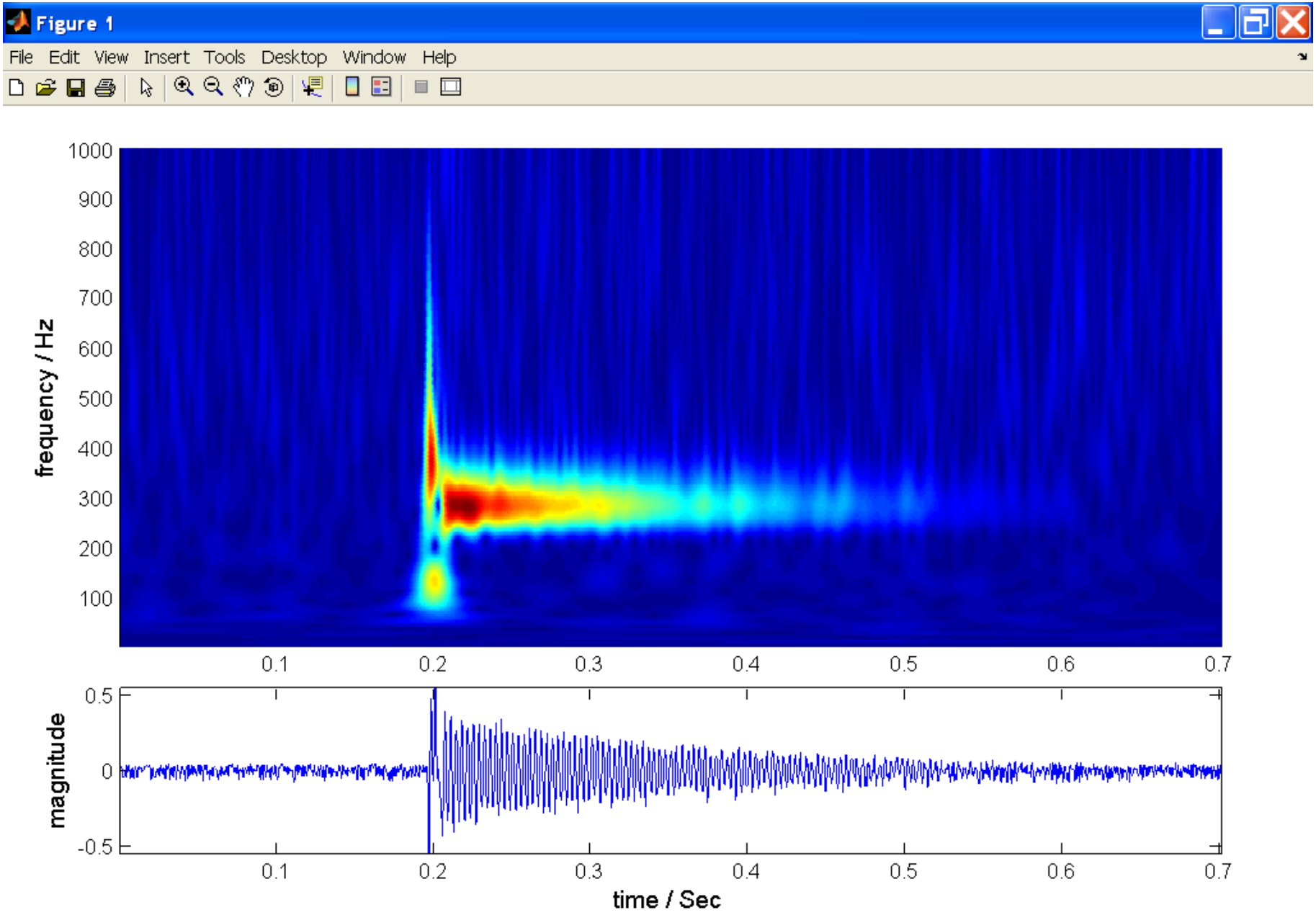
Practical application: impact experiments with piles

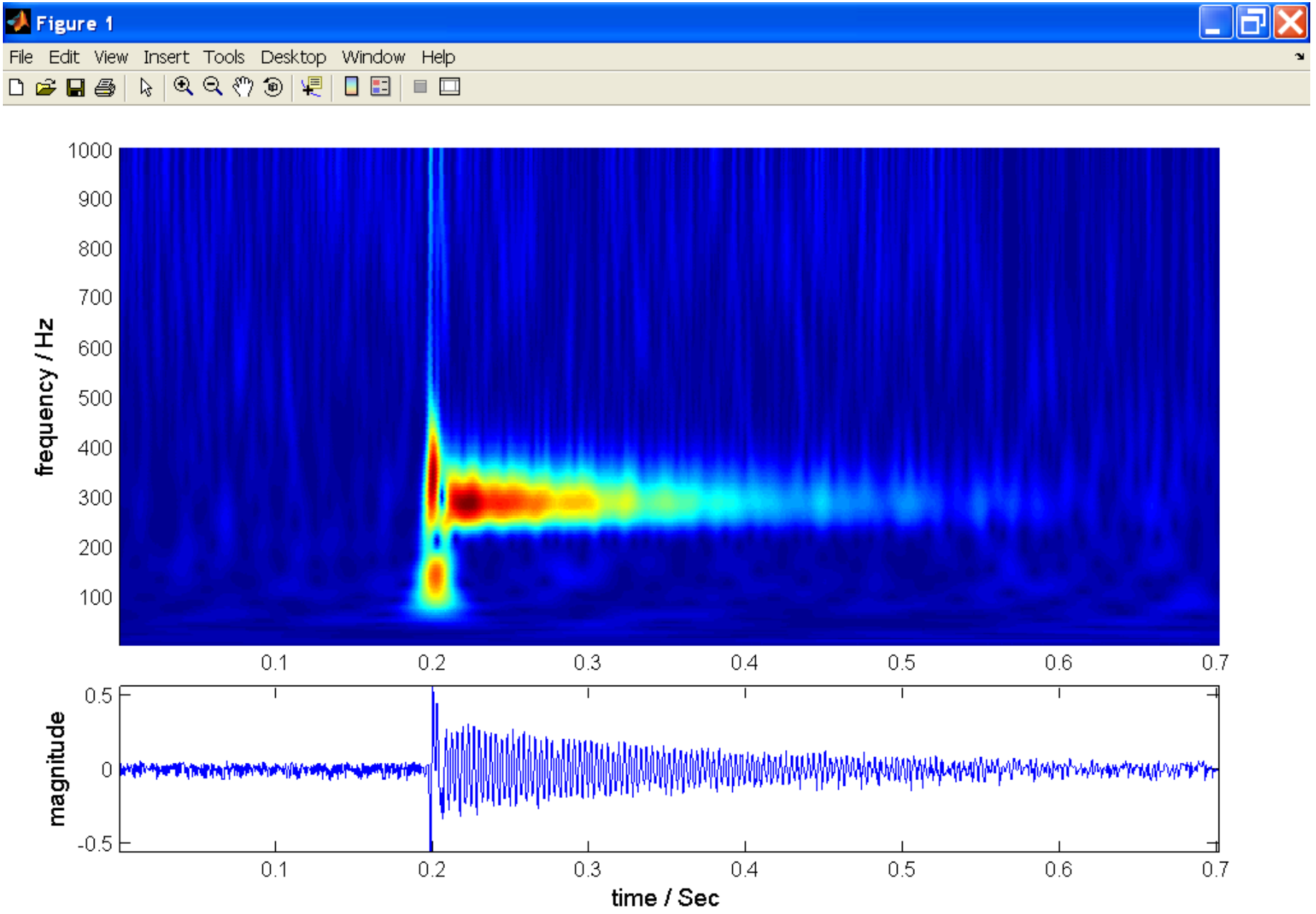
Vertical Pile Excitation

Horizontal Pile Excitation

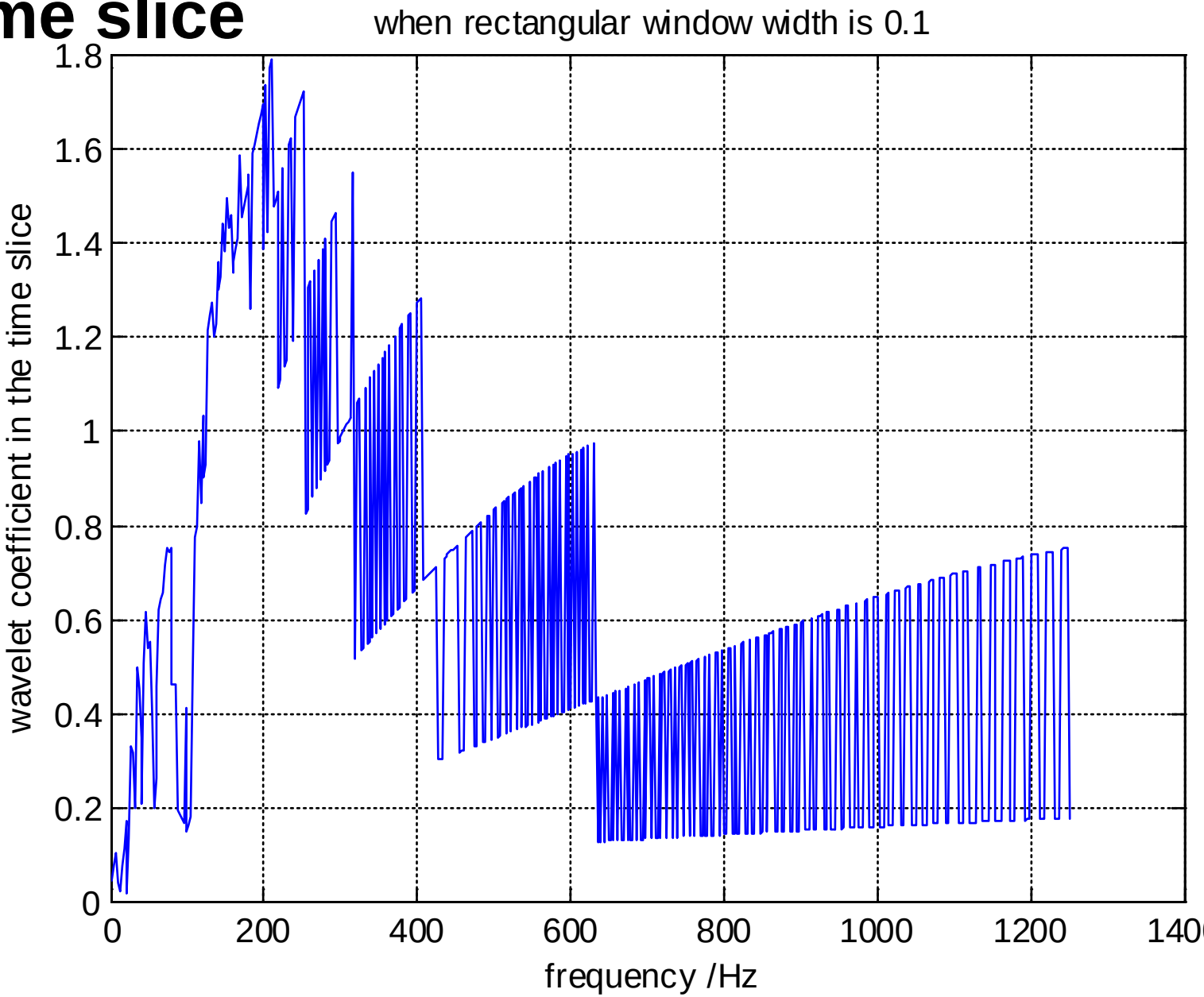
The Wavelet Transform of an Impact



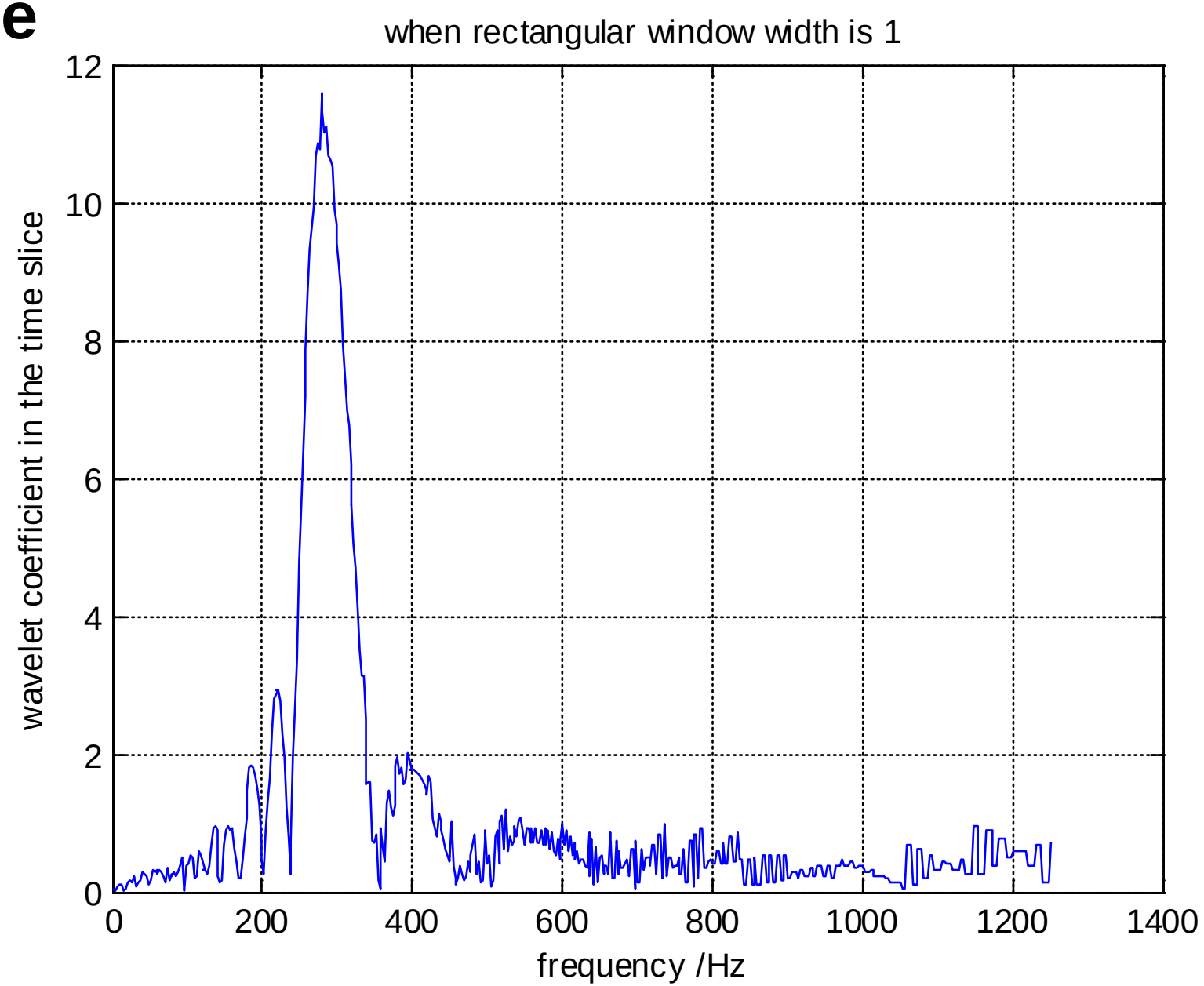
The Wavelet Transform of an Impact



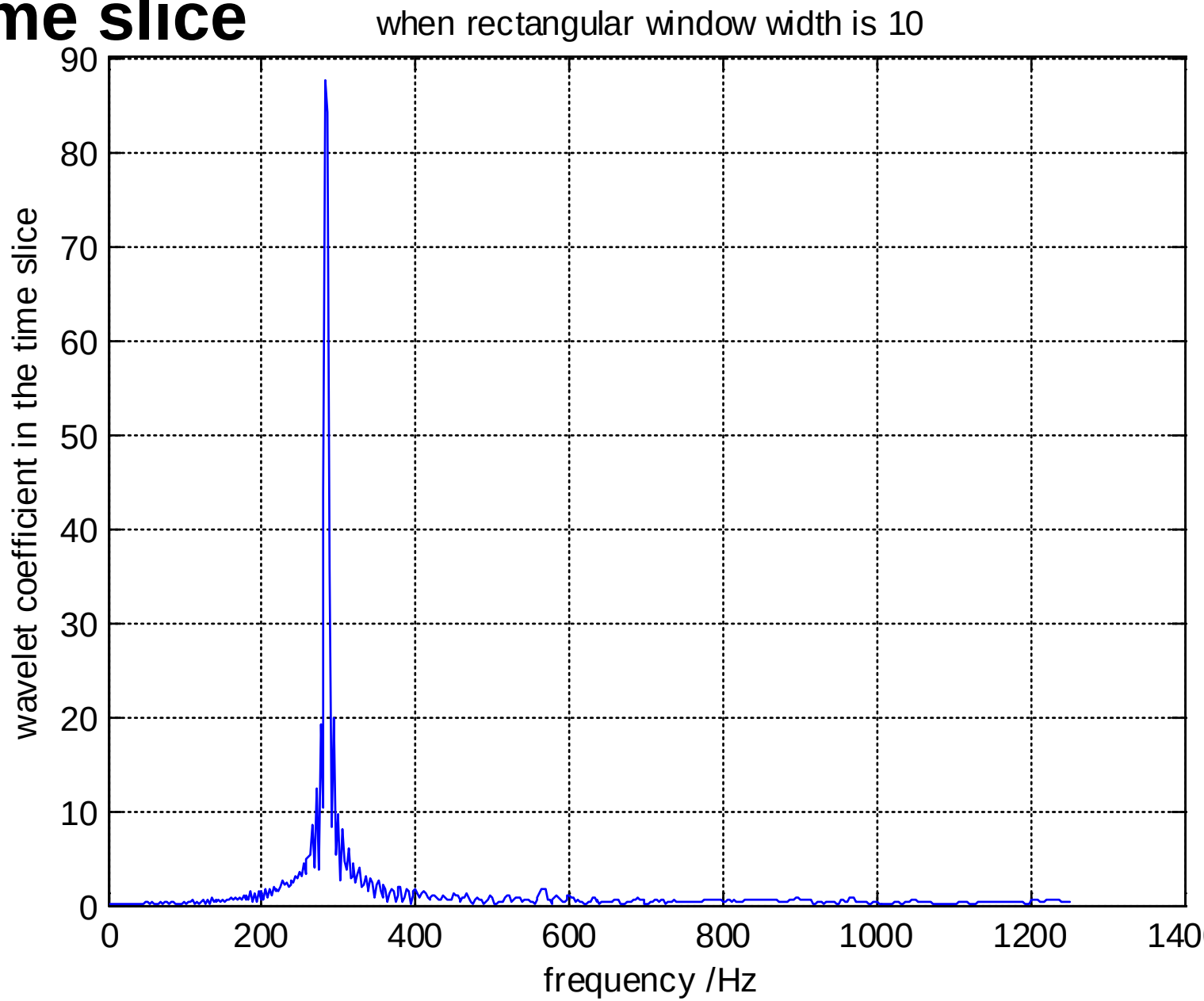
The wavelet transform of an impact: a time slice



The Wavelet Transform of an Impact: a Time Slice



The Wavelet transform of an impact: a time slice



New Technique: the Wavelet Bispectrum

- **The author proposed, improved and successfully employed *for the first time* wavelet bispectrum for processing of impacts**

- **The proposed technique is non-traditional, our intensive literature search has showed that nobody used wavelet bispectrum for processing of impacts**

- **Moreover, a methodology for usage of wavelet bispectrum for impacts is not available in literature**

The Wavelet Bispectrum

- **The proposed bispectrum also depends on two frequencies and is defined by the wavelet transforms at three frequencies:**

$$Bisp\ (k_1, k_2, t_1) = \frac{1}{M} \sum_{m=1}^{M} W_m(k_1, t_1) W_m(k_2, t_1) W^*{}_m(k_1 + k_2,\ t_1)$$

where W*m (k, t)* is the wavelet transform of the *m*th impact of object at frequency *k* and time slice *t1*

The Wavelet Bispectrum

- **The proposed technique uses transient (not stationary) mother wavelet functions which are suitable for impacts**

- **The author has developed a novel methodology for usage of wavelet bispectrum for impacts**

The Wavelet Bispectrum: Principles of Methodology for Impact Processing

- **One wavelet bispectrum should be calculated for one impact**

- **Time slice of the impact should be found for wavelet bispectrum calculation**

- **Wavelet bispectrum should be averaged by consecutive sequence of impacts**

The Wavelet Bispectrum: Methodology for Impact Processing

The proposed technique should be employed by the following steps:

- **select the number of impacts (i.e. number of averaged bispectrums)**
- **select time window and place it at the beginning of impact**
- **estimate for the selected time window the time-frequency distribution of the wavelet transform for the fundamental harmonic of the selected mode**
- **estimate time slice provided the maximum of the scalogram at the fundamental harmonic of the selected vibration mode**
- **estimate the wavelet bispectrum (bicoherence) along the selected time slice**
- **average the wavelet bispectrum for the selected number of impacts**

New Technique: the Wavelet Bispectrum

- **The proposed technique is suitable for the impacts because the available in the literature mother wavelets are suitable for impacts**

- **However, all available wavelet bispectrums have two disadvantages for impact processing:**

- **variable frequency resolution for different harmonics; this creates a problem for estimation of coupling between harmonics**

- **all available mother wavelets have “cosine” type of wavelet function which is not perfectly matched with impacts**

The Improved Wavelet Bispectrum

The author has proposed two important improvements of wavelet bispectrum for impact processing:

- **Variable duration (e, g, variable time support) of mother wavelet for different fequencies**
- **New mother wavelet function based on sine wave and rectangular window**

The proposed technique is a generalization of the classical waveelet bispectrum for the case of impacts

Numerical Simulation:

the Improved Wavelet Bispectrum
vs.
the Classical Wavelet Bispectrum

Nonlinear Model

To compare the proposed and the classical techniques, an input impact has been passed via the following nonlinear (bilinear) model:

$$\begin{cases} \ddot{x} + 2h\dot{x} + \omega_S x = A(t) \;\; x \geq 0, \\ \\ \ddot{x} + 2h\dot{x} + \omega_C x = A(t), \;\; x < 0, \end{cases}$$

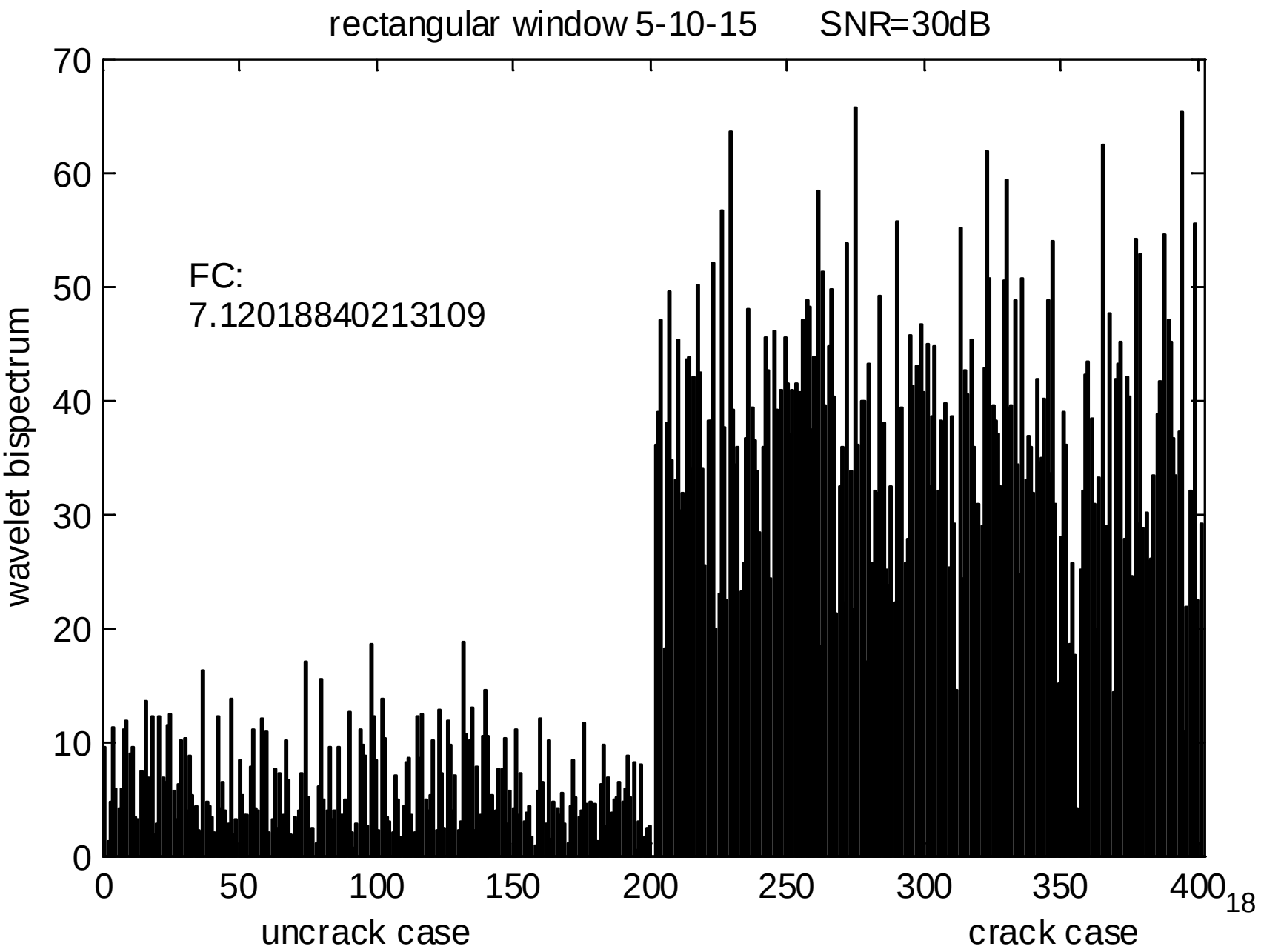
Impact Processing by the Wavelet Bispectrum

- **Two different mother wavelet functions were investigated:**

- **the improved mother wavelet proposed by the author**
- **the classical Morlet wavelet**

- **10000 impacts were simulated for class (i.e. linear and nonlinear classes)**

The Improved Morlet Mother Wavelet



The Classical Morlet Mother Wavelet