Week 4

Core Mathematics

Learning Outline

i. Simple Interest
ii. Compound Interest
iii. Proportions and Ratios

1.0 SIMPLE INTEREST

Interest is money paid for the use of money. For example, if you borrow from the bank to buy a car, the bank will charge you interest for giving you money to use. Also, if you open a savings account at the bank, the bank will pay you interest for as long as the account is open.
Note: Banks usually charge compound interest not simple interest.

The interest (I) is the amount earned or owed. The interest rate (R) is per year (T) unless otherwise noted.

If the time is in months, T can be found using the ratio $\text{Time} = \frac{\text{Number of month}}{12}$

The principal (P) is the amount borrowed or deposited and it is calculated using the following formula; $I = \frac{PRT}{100}$

where I = interest as percentage per annum (year), P = Principal, R = Rate of Interest, T = time in years

Worked Example 1

If £4000 is invested at an annual(yearly) interest rate of 5% for 4 years. Calculate the simple interest on the principal amount invested.

Solution

P=£4000; R=5%; T=4years; I=?

$$I = \frac{4000 \times 5 \times 4}{100} = £800$$

The interest to pay back on the principal after four years is £800

The total amount owed after 4 years will now be P+I i.e. £4000+£800=£4800.

Worked Example 2

Suzanne invests $450 for 5 years at a rate of 3.5% per year simple interest. How much money will she have after those 8 months?

Solution

P=£450; R=3.5%; T=8month ; I=?

Firstly, we need to convert the time in months to years to correctly solve this question. i.e

$$T = \frac{8}{12} years \Rightarrow \frac{2}{3} years$$

$$\mathrm{I} = \frac{450 \times 3.5 \times \frac{2}{3}}{100} = £10.5$$

The interest received by Suzanne on the principal after 8 months is £10.5

After 8 months, Suzanne's total money will now be P+I i.e. £450+£10.5=£460.5.

Exercise 1

1. A brand new car is bought at £22,399 with an annual interest rate of 4.6% over 7 years. What is the total cost?
2. A couple have just finished paying £357,650 in total to buy their house. They bought the house in 1988 with an annual interest rate of 3.4%. What was the initial house cost?
3. Tammy earns 3.5% interest per year on the money he has saved in his bank account. He starts off with £2000. How much is in his account after 5 years?
4. Elizabeth has £8000. She deposits it in a bank account that pays 2.7% interest per annum. She leaves it in the bank for 3 years. How much money does Elizabeth have now?
5. Gordon puts £22000 into an account which pays interest at a rate of 6.3% p.a. How much money would he have after two years?
6. Kristina puts £545 into an account which pays interest at a rate of 2.2% p.a. How much money would she have after a year?
7. Tim saves £7410 and deposits it in the same high interest account which pays interest at a rate of 9.5% per annum. How much interest does Tom get if he leaves his money in the account for three years?
8. Delphine has £6200 in her Clydeside Bank account. Clydeside Bank pays interest at 2.5% per annum. Highland Bank pays interest at 3.7% per annum. How much more money would Delphine get in interest if she moved her £6200 to the Highland Bank for one year?
9. You get a student loan from the government to pay for your educational expenses this year. Find the interest on the loan if you borrowed £2,000 at 8% for 1 year.
10. You are starting your own small business in Leeds. You borrow £30,000 from the bank at a 4.5% rate for 5 years. Find the interest you will pay on this loan.
11. You are tired at the end of the term and decide to borrow £500 to go on a trip to Whatever Land. You go to the bank and borrow the money at 11% for 2 years.

a) Find the interest you will pay on the loan.
b) How much will you have to pay the bank at the end of the two years?

12. Find the interest on a loan of $2500 that is borrowed at 9% for 7 months. How much would it cost to repay the loan?

2.0 COMPOUND INTEREST

This is interest earned not only on the original principal but also on interest earned previously. Consider the following examples to see how this works.

Worked Example 3

A builder is going to build six houses on a plot of land in Japan. He borrows £1,000,000 at 10% and will pay off the loan in full after three years.

At the end of the first year he will owe

Amount borrowed + interest on amount borrowed

$£1{,}000{,}000 + 10\% \quad of \quad £1{,}000{,}000$

$£1{,}000{,}000 + £100{,}000 = £1{,}100{,}000$

At the end of the second year he will owe

$£1{,}100{,}000 + 10\% \quad of \quad £1{,}100{,}000$

$£1{,}100{,}000 + £110{,}000 = £1{,}210{,}000$

At the end of the third year he will owe

$£1{,}210{,}000 + 10\% \quad of \quad £1{,}210{,}000$

$£1{,}210{,}000 + £121{,}000 = £1{,}331{,}000$

His interest will be $£1{,}210{,}000 - £1{,}000{,}000 = £210{,}000$

Worked Example 4

If £4000 is invested at an annual rate of interest of 5% for 4 years

Interest at end of first year is $\frac{4000 \times 5 \times 4}{100} = £200$

Interest at end of second year is $\frac{4200 \times 5 \times 4}{100} = £210$ (Principal is now £4000+£200)

Interest at end of third year is $\frac{4410 \times 5 \times 4}{100} = £220.50$ (Principal is now £4200+£210)

Interest at end of fourth year is $\frac{4630.50 \times 5 \times 4}{100} = £231.53$

Here the interest amounts to £862.03 making the total come to £4862.03

Rather than computing the compound interest yearly as shown in example (3) and (4), it can be directly computed using the formula;

$$FV = PV\left(1+\frac{R}{100}\right)^{T}$$

where FV = Future value (Total amount at the end of the interest period), PV = Present value (principal), R = Rate of interest, T = time in years

Applying this formula to the examples (3) and (4) above, we have:

For example 3

$$FV = £1{,}000{,}000\left(1+\frac{10}{100}\right)^{3}$$

$$FV = £1{,}000{,}000(1.10)^{3}$$

$$FV = £1{,}331{,}000$$

Interest=FV-PV

$$\text{Interest} = £1{,}331{,}000 - £1{,}000{,}000$$

$$\text{Interest} = £331{,}000$$

For example 4

$$FV = 4000\left(1+\frac{5}{100}\right)^{4}$$

$$FV = 4000(1.05)^{4}$$

$$FV = 4000 \times 1.2155$$

$$FV = £4862.03$$

Interest=FV-PV

$$\text{Interest} = £4{,}862.03 - £4{,}000$$

$$\text{Interest} = £862.03$$

Exercise 2

1. Frederic buys 1000 shares of £10 from a start-up company in 1990. Every year his shares value increase by 10%. He uses this money to reinvest on new shares. How much money have he invested in this company in 2000.
2. Lisa borrows £10000 from a bank with 14% annual rate of interest rate on a compound interest based. She has to pay back between second and fourth year. How much does she have to pay by the end of each year?
3. Toby invested £4500 for 2 years in a savings account.He was paid 4% per annum compound interest. How much did Toby have in his savings account after 2 years?
4. The value of a car depreciates by 35% each year. At the end of 2007 the value of the car was £5460 Work out the value of the car at the end of 2006
5. Mario invests £2000 for 3 years at 5% per annum compound interest.Calculate the value of the investment at the end of 3 years

6. Derek invests £154 500 for 2 years at 4% per year compound interest. Work out the value of the investment at the end of 2 years.

7. Henry invests £4500 at a compound interest rate of 5% per annum. At the end of n complete years the investment has grown to £5469.78.Find the value of n.
8. Liam invests £6200 for 3 years in a savings account. He gets 2.5% per annum compound interest. How much money will Liam have in his savings account at the end of 3 years?
9. Jaspir invested £2400 for n years in a savings account. He was paid 7.5% per annum compound interest. At the end of the n years he had £3445.51 in the savings account. (b) Work out the value of n.
10. If a student invested £2000 in a savings account when he was 18 years and just about starting University, how much would it be worth if he accessed this money on his 22nd birthday given that the compound interest rate is 3.5% per year? i.e. calculate the compound interest.
11. Viv wants to invest £2000 for 2 years in the same bank.

The International Bank	The Friendly Bank
Compound Interest 4% for the first year. 1% for each extra year	Compound Interest 5% for the first year 0.5% for each extra year

At the end of 2 years, Viv wants to have as much money as possible. Which bank should she invest her £2000 in?

3.0 RATIO

In this section you will learn how to: (i) simplify a ratio (ii) express a ratio as a fraction (iii) divide amounts according to ratios (iv) complete calculations from a given ratio and partial information.

We use ratio as a way of comparing the sizes of two or more quantities. Ratios may be expressed in any of the following three ways:

(i) 1 : 4 or (ii) 1 to 4 or (iii) as a fraction 1/4

For example, Joy is 5 years old and James is 20 years old. The ratio of their ages is:

	Joy's age	:	James's age
which is	5	:	20
Divide by 5 to simplify	1	:	4

Units

When working with ratio involving different units, it is important to always change them to a common unit.

Worked Example 5:

Express 125 g to 2 kg as a simple ratio.

Solution

The ratio of 125 g to 2 kg must be changed to 125 g to 2000 g .

Ratio 125 : 2000

Divide by 125 to simplify to 1 : 16

Ratios as fractions

A ratio in its simplest form can be expressed as a fraction by changing the whole numbers in the ratio into fractions with the same denominator (bottom number).

Worked Example 6:

A garden is divided into lawn and shrubs in the ratio of 3 : 4. What fraction of the garden is covered by lawns and what fraction is covered by shrubs.

Solution

The lawn covers 3/7 and the shrubs cover 4/7 . The common denominator 7 comes from adding the numbers in the ratio.

Exercise 3

(1) Express each of the following ratios in its simplest form.

i. 6 : 18
ii. 15 : 20
iii. 16 : 24
iv. 24 : 36
v. 20 to 50
vi. 12 to 30
vii. 25 to 40
viii. 125 to 30
ix. 15 : 10
x. 32 : 12
xi. 28 to 12
xii. 100 to 40
xiii. 0.5 to 3
xiv. 1.5 to 4
xv. 2.5 to 1.5
xvi. 3.2 to 4

(2) Express each of the following ratios of quantities in its simplest form. (Remember always to express both parts in a common unit before you simplify.)

i. £5 to £15
ii. £24 to £16
iii. 125 g to 300 g
iv. 40 minutes : 5 minutes
v. 34 kg to 30 kg
vi. £2.50 to 70p
vii. 3 kg to 750
viii. 50 minutes to 1 hour
ix. 1 hour to 1 day
x. 12 cm to 2.5 mm
xi. 1.25 kg : 500 g
xii. 75p : £3.50
xiii. 4 weeks : 14 days
xiv. 600 m: 2 km
xv. 465 mm : 3 m
xvi. 15hours:1day

3. A length of wood is cut into two pieces in the ratio 3 : 7. What fraction of the original length is the longer piece?
4. Jack and Thomas find a bag of marbles that they share between them in the ratio of their ages. Jack is 10 years old and Thomas is 15. What fraction of the marbles did Jack get?
5. Dave and Sue share a pizza in the ratio 2 : 3. They eat it all. (a) What fraction of the pizza did Dave eat? (b) What fraction of the pizza did Sue eat?
6. A camp site allocates space to caravans and tents in the ratio 7 : 3. What fraction of the total space is given to: a the caravans b the tents?
7. Two sisters, Amy and Katie, share a packet of sweets in the ratio of their ages. Amy is 15 and Katie is 10. What fraction of the sweets does each sister get?
8. The recipe for a fruit punch is 1.25 litres of fruit crush to 6.75 litres of lemonade. What fraction of the punch is each ingredient?
9. In a safari park at feeding time, the elephants, the lions and the chimpanzees are given food in the ratio 10 to 7 to 3. What fraction of the total food is given to: a the elephants b the lions c the chimpanzees?
10. Three brothers, James, John and Joseph, share a huge block of chocolate in the ratio of their ages. James is 20, John is 12 and Joseph is 8. What fraction of the bar of chocolate does each brother get? The recipe for a pudding is 125 g of sugar, 150 g of flour, 100 g of margarine and 175 g of fruit. What fraction of the pudding is each ingredient?

Dividing amounts by ratios

To divide an amount into portions according to a given ratio, you first change the whole numbers in the ratios into fractions with the same common denominator. Then multiply the amount by each fraction.

Worked Example 7

Share out £40 between Hasan and Halima in the ratio 2 : 3

First, change the ratio to fractions.

Fraction of Hasan's share = 2/5

Fraction of Halima's share = 3/5

Amount received by Hasan $£40 \times \frac{2}{5} = £16$

Amount received by Halima $£40 \times \frac{3}{5} = £24$

Exercise 4

(1) Divide the following amounts according to the given ratios.

i. 400 g in the ratio 2 : 3
ii. 280 kg in the ratio 2 : 5
iii. 500 in the ratio 3 : 7
iv. 1 km in the ratio 19 : 1
v. 5 hours in the ratio 7 : 5
vi. £100 in the ratio 2 : 3 : 5
vii. £240 in the ratio 3 : 5 : 12
viii. 600 g in the ratio 1 : 5 : 6
ix. £5 in the ratio 7 : 10 : 8
x. 200 kg in the ratio 15 : 9 : 1

(2) The ratio of female to male members of Lakeside Gardening Club is 5 : 3. The total number of members of the group is 256. (a) How many members are female? (b) What percentage of members are male?

(3) A supermarket aims to stock branded goods and their own goods in the ratio 2 : 5. They stock 350 kg of breakfast cereal. (a) What percentage of the cereal stock is branded? (b) How much of the cereal stock is their own?

(4) The Illinois Department of Health reported that, for the years 1981 to 1992 when they tested a total of 357 horses for rabies, the ratio of horses with rabies to those without was 1 : 16. (a) How many of these horses had rabies? (b) What percentage of the horses did not have rabies?

(5) The prize shared among the top three students in Core mathematics at the end of each academic year is £2,050. If this prize is shared in the following ratio 8:5:3 with the best student receiving the highest prize and the third best receiving the lowest prize. What does each student receive depending on their position?

4.0 Proportion

Proportion is a way of showing the relationship between two quantities. Suppose you buy 12 items, with each item costing the same. The total amount you spend is 12 times the cost of one item. That is, the total cost is in direct proportion to number of items bought. The cost of a single item is known as the unit cost. It is the constant factor that links the two quantities.

Proportion is concerned not only with costs. Any two related quantities can be in proportion to each other. We trying to find the relationship between two quantities, we can start by finding the single unit value.

Worked Example 8

If eight pens cost £2.54, what is the cost of five pens?

Cost of one pen is £2.64 ÷ 8 = £0.33

Cost of five pens is £0.33 x 5 = £1.65

Worked Example 9

Fourteen loaves of bread will make packed lunches for 18 people. How many packed lunches can be made from 20 loaves?

14 loaves = 18 packed lunches

One loaf will make 18 ÷ 14 = 1.286 lunches.

So, twenty loaves will make 1.286 x 20 = 26 lunches. (round up to the nearest whole number)

Exercise 5

1 If 30 matches weigh 45 g, what would 40 matches weigh?
2 Five bars of chocolate cost £2.90. Find the cost of nine bars.
3 Eight men can chop down 18 trees in a day. How many trees can 20 men chop down in a day?
4 Find the cost of 48 eggs when 15 eggs can be bought for £2.10.
5 Seventy maths textbooks cost £875. a How much will 25 maths textbooks cost? b How many maths textbooks can you buy for £100?
6 A lorry uses 80 litres of diesel fuel on a trip of 280 miles. a How much diesel would the same lorry use on a trip of 196 miles? b How far would the lorry get on a full tank of 100 litres of diesel?
7 During the winter, I find that 200 kg of coal keeps my open fire burning for 12 weeks. a If I want an open fire all through the winter (18 weeks), how much coal will I need to buy? b Last year I bought 150 kg of coal. For how many weeks did I have an open fire?
8 It takes a photocopier 16 seconds to produce 12 copies. How long will it take to produce 30 copies?

9 David read 40 pages of a book in 5 minutes. How many pages will he read in 80 minutes if he reads at a constant rate?
10 On a map, one inch represents 150 miles. If Las Vegas and Reno are five inches apart on the map, what is the actual distance between them?
11 Bob had 21 problems correct on a math test that had a total of 25 questions, what percent grade did he earn? (In other words, how many questions would we expect him to get correct if there were 100 questions on the test?)
12 If there should be three calculators for every 4 students in an elementary school, how many calculators should be in a classroom that has 44 students? If a new school is scheduled to open with 600 students, how many calculators should be ordered?
13 If your car can go 350 miles on 20 gallons of gas, at that rate, how much gas would you have to purchase to take a cross country trip that was 3000 miles long?

Best buys

In this section you will learn how to: (i) find the cost per unit weight (ii) find the weight per unit cost and (iii) use the above to find which product is the cheaper.

Given the variety of product choices we have to select from in a supermarket, it is rarely obvious which choice is the best buy. It is helpful to be able to compare the cost per unit weight or the weight per unit cost in order to determine the best buy.

For cost per unit weight, divide cost by weight.

For weight per unit cost, divide weight by cost.

Worked Example 10

A 300g tin of cocoa costs £1.20. Find the cost per unit weight and weight per unit cost of the tin of cocoa.

Solution

First change £1.20 to 120p. Then divide, using calculator, to get:

Cost per unit weight 120 ÷ 300 = 0.4p per gram

Weight per unit cost 300 ÷ 120 = 2.5g per penny

Worked Example 11

Which is the better buy, an 800 g pack of soap powder at £1.60 or a 2.5 kg pack for £4.75?

solution

Comparing figures.

Weight per unit cost of 800g soap powder; 800g÷ 160p = 5 g per penny.

Weight per unit cost of 2.5kg soap powder; 2500g ÷475p = 5.26 g per penny.

The better buy of the two options is the 2.5kg soap power because you get more quantity (0.26g more) for every penny spent.

Worked Example 12

A shop buys 1000 chocolate bars and sells them all. The amount realised from the sales of the chocolate bars is given by the equation $y-0.2x+100=0$ where y is amount realised in dollars and x is number of chocolate bars sold.

Solution

Amount realised is given by $y=0.2x-100$

(1) when $x=0$, $y=-100$. The negative sign indicates amount spent. Thus, the total cost of all the chocolate bars is £100.

(2) To break even means that the shop neither makes a gain or a loss. i.e. $y=0$

$0=0.2x-100$

$x=500$ chocolate bars

(3) Maximum profit from sales of 1000 chocolate bars

$x=1000$

$y=0.2(1000)-100$

$y=£100$

Exercise 6

1 A company employs 200 staff to manage a park and each staff earns a salary of £20,000 per year. If a visitor ticket to the park cost £60 per visit. (a) How many visitors ticket must the company sell in a year to cover the running cost of the park? (i.e. break-even point)? (b) What number of visitor tickets must be sold to make an annual profit of £0.5million?

2 Huddersfield town football club offer 20,000 season ticket seats at an average cost of £1,090 per seat for a season. The overall running cost per seat is £650 a year. (a) How many number of seats is required to cover the running cost of the football club? (i.e. break-even point)? (b) How many number of seats is required to make £5million profit?

3 Two telephone service providers have the following pricing structures for providing a customer with call minutes. Vodafone charges £15 per month for 2000mins plus £0.10 per min for any call made above this limit. T-Mobile offers £19.50 per month for

unlimited calls. If you use the telephone service for one month, for what number of call minutes made will the two companies have the same total cost?

4 Broadband is free for 6 months then is £30p/m, Plusnet is 4p per day and Sky broadband is £7.50 per week. Which is the best value broadband per year? (not a leap year).

5 Two internet service providers have the following pricing structures for providing a customer with internet data. Virgin Media charges £30 per month for 20GB of data plus £0.4 per GB for any data used above this limit. Sky charges £35 per month with unlimited data usage. If you use the internet service for one month, for what amount of data usage will the two companies have the same total cost?

6 Asda offers three sales option for a newly stocked tin of milk;

Option 1: 250g tin of milk for £3.40

Option 2: Two 450g tin of milk £7.00

Option 3: Five 200g tin of milk for £11.00. Which of these options is the best buy?