Differentiation of common functions is demonstrated in the following worked problems.

Problem 2. Find the differential coefficients of

(a) $y=12x^3$ (b) $y=\dfrac{12}{x^3}$

If $y=ax^n$ then $\dfrac{dy}{dx}=anx^{n-1}$

(a) Since $y=12x^3$, $a=12$ and $n=3$ thus $\dfrac{dy}{dx}=(12)(3)x^{3-1}=\mathbf{36x^2}$

(b) $y=\dfrac{12}{x^3}$ is rewritten in the standard ax^n form as $y=12x^{-3}$ and in the general rule $a=12$ and $n=-3$.

Thus $\dfrac{dy}{dx}=(12)(-3)x^{-3-1}=-36x^{-4}=-\dfrac{\mathbf{36}}{\mathbf{x^4}}$

Problem 3. Differentiate (a) $y=6$ (b) $y=6x$.

(a) $y=6$ may be written as $y=6x^0$, i.e. in the general rule $a=6$ and $n=0$.

Hence $\dfrac{dy}{dx}=(6)(0)x^{0-1}=\mathbf{0}$

In general, **the differential coefficient of a constant is always zero.**

(b) Since $y=6x$, in the general rule $a=6$ and $n=1$.

Hence $\dfrac{dy}{dx}=(6)(1)x^{1-1}=6x^0=\mathbf{6}$

In general, the differential coefficient of kx, where k is a constant, is always k.

Problem 4. Find the derivatives of

(a) $y=3\sqrt{x}$ (b) $y=\dfrac{5}{\sqrt[3]{x^4}}$

(a) $y=3\sqrt{x}$ is rewritten in the standard differential form as $y=3x^{\frac{1}{2}}$.

In the general rule, $a=3$ and $n=\dfrac{1}{2}$

Thus $\dfrac{dy}{dx}=(3)\left(\dfrac{1}{2}\right)x^{\frac{1}{2}-1}=\dfrac{3}{2}x^{-\frac{1}{2}}$

$$=\frac{3}{2x^{\frac{1}{2}}}=\frac{\mathbf{3}}{\mathbf{2\sqrt{x}}}$$

(b) $y=\dfrac{5}{\sqrt[3]{x^4}}=\dfrac{5}{x^{\frac{4}{3}}}=5x^{-\frac{4}{3}}$ in the standard differential form.

In the general rule, $a=5$ and $n=-\frac{4}{3}$

Thus $\dfrac{dy}{dx}=(5)\left(-\dfrac{4}{3}\right)x^{-\frac{4}{3}-1}=\dfrac{-20}{3}x^{-\frac{7}{3}}$

$$=\frac{-20}{3x^{\frac{7}{3}}}=\frac{\mathbf{-20}}{\mathbf{3\sqrt[3]{x^7}}}$$

Problem 5. Differentiate, with respect to x,

$y=5x^4+4x-\dfrac{1}{2x^2}+\dfrac{1}{\sqrt{x}}-3.$

$y=5x^4+4x-\dfrac{1}{2x^2}+\dfrac{1}{\sqrt{x}}-3$ is rewritten as

$y=5x^4+4x-\dfrac{1}{2}x^{-2}+x^{-\frac{1}{2}}-3$

When differentiating a sum, each term is differentiated in turn.

Thus $\dfrac{dy}{dx}=(5)(4)x^{4-1}+(4)(1)x^{1-1}-\dfrac{1}{2}(-2)x^{-2-1}$

$$+(1)\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}-0$$

$$=20x^3+4+x^{-3}-\frac{1}{2}x^{-\frac{3}{2}}$$

i.e. $\mathbf{\dfrac{dy}{dx}=20x^3+4+\dfrac{1}{x^3}-\dfrac{1}{2\sqrt{x^3}}}$

Problem 8. Find the gradient of the curve $y=3x^4-2x^2+5x-2$ at the points $(0, -2)$ and $(1, 4)$.

The gradient of a curve at a given point is given by the corresponding value of the derivative. Thus, since $y=3x^4-2x^2+5x-2$

Then the gradient $=\dfrac{dy}{dx}=12x^3-4x+5$

At the point $(0, -2)$, $x=0$

Thus the gradient $=12(0)^3-4(0)+5=\mathbf{5}$

At the point $(1, 4)$, $x=1$

Thus the gradient $=12(1)^3-4(1)+5=\mathbf{13}$.

Problem 9. Determine the co-ordinates of the point on the graph $y=3x^2-7x+2$ where the gradient is -1.

The gradient of the curve is given by the derivative.

When $y=3x^2-7x+2$ then $\frac{dy}{dx}=6x-7$

Since the gradient is -1 then $6x-7=-1$, from which, $x=1$

When $x=1$, $y=3(1)^2-7(1)+2=-2$

Hence the gradient is -1 at the point $(1, -2)$.

Now try the following exercise

Exercise 115 Further problems on differentiating common functions

In Problems 1 to 6 find the differential coefficients of the given functions with respect to the variable.

1. (a) $5x^5$ (b) $2.4x^{3.5}$ (c) $\frac{1}{x}$

   $\left[\text{(a) } 25x^4 \text{ (b) } 8.4x^{2.5} \text{ (c) } -\frac{1}{x^2}\right]$

2. (a) $\frac{-4}{x^2}$ (b) 6 (c) $2x$ $\left[\text{(a) } \frac{8}{x^3} \text{ (b) } 0 \text{ (c) } 2\right]$

3. (a) $2\sqrt{x}$ (b) $3\sqrt[3]{x^5}$ (c) $\frac{4}{\sqrt{x}}$

   $\left[\text{(a) } \frac{1}{\sqrt{x}} \text{ (b) } 5\sqrt[3]{x^2} \text{ (c) } -\frac{2}{\sqrt{x^3}}\right]$

7. Find the gradient of the curve $y=2t^4+3t^3-t+4$ at the points (0, 4) and (1, 8).

   $[-1, 16]$

8. Find the co-ordinates of the point on the graph $y=5x^2-3x+1$ where the gradient is 2. $\left[\left(\frac{1}{2}, \frac{3}{4}\right)\right]$