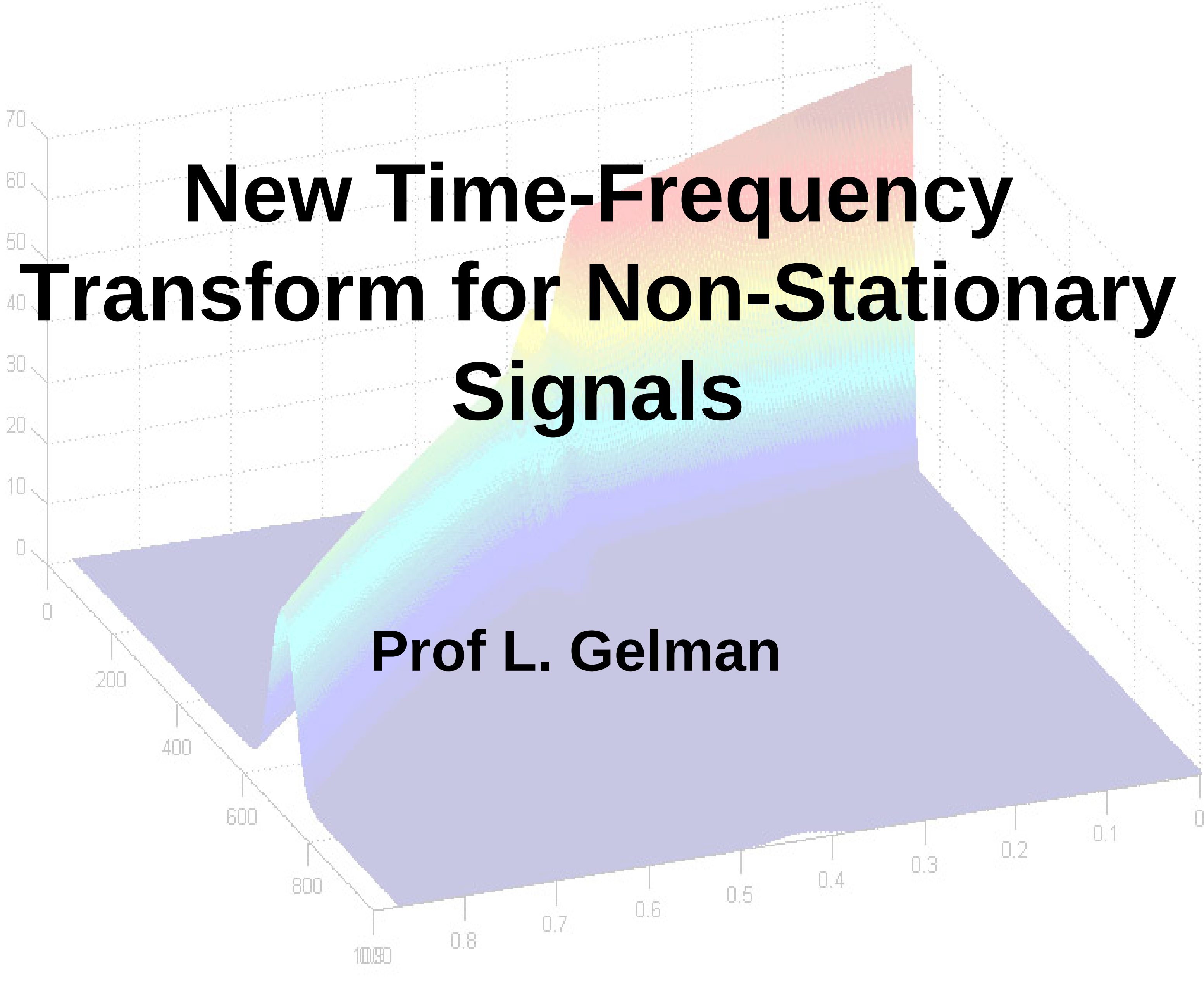
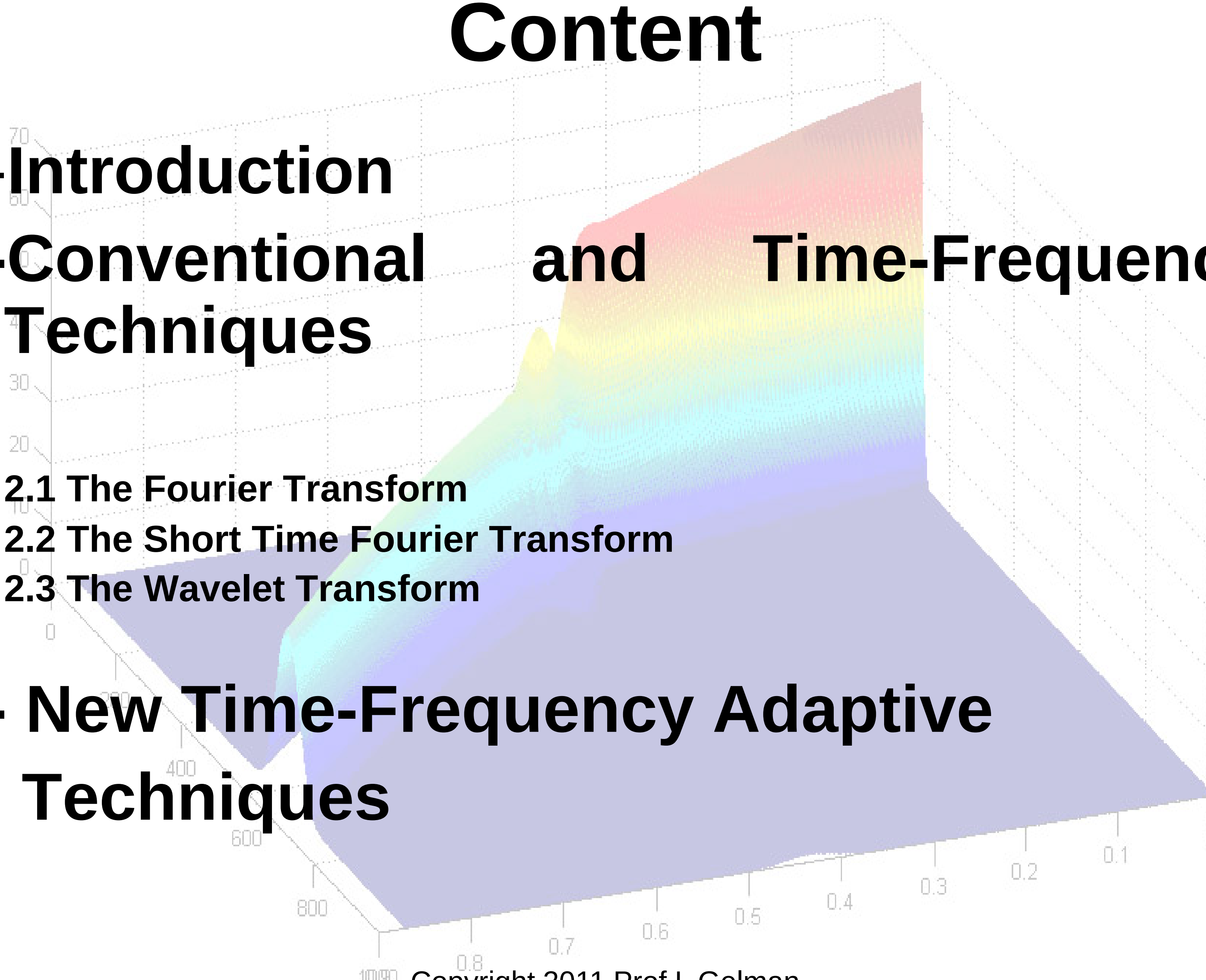
New Time-Frequency Transform for Non-Stationary Signals



Prof L. Gelman

Content



1-Introduction

2-Conventional and Time-Frequency Techniques

2.1 The Fourier Transform

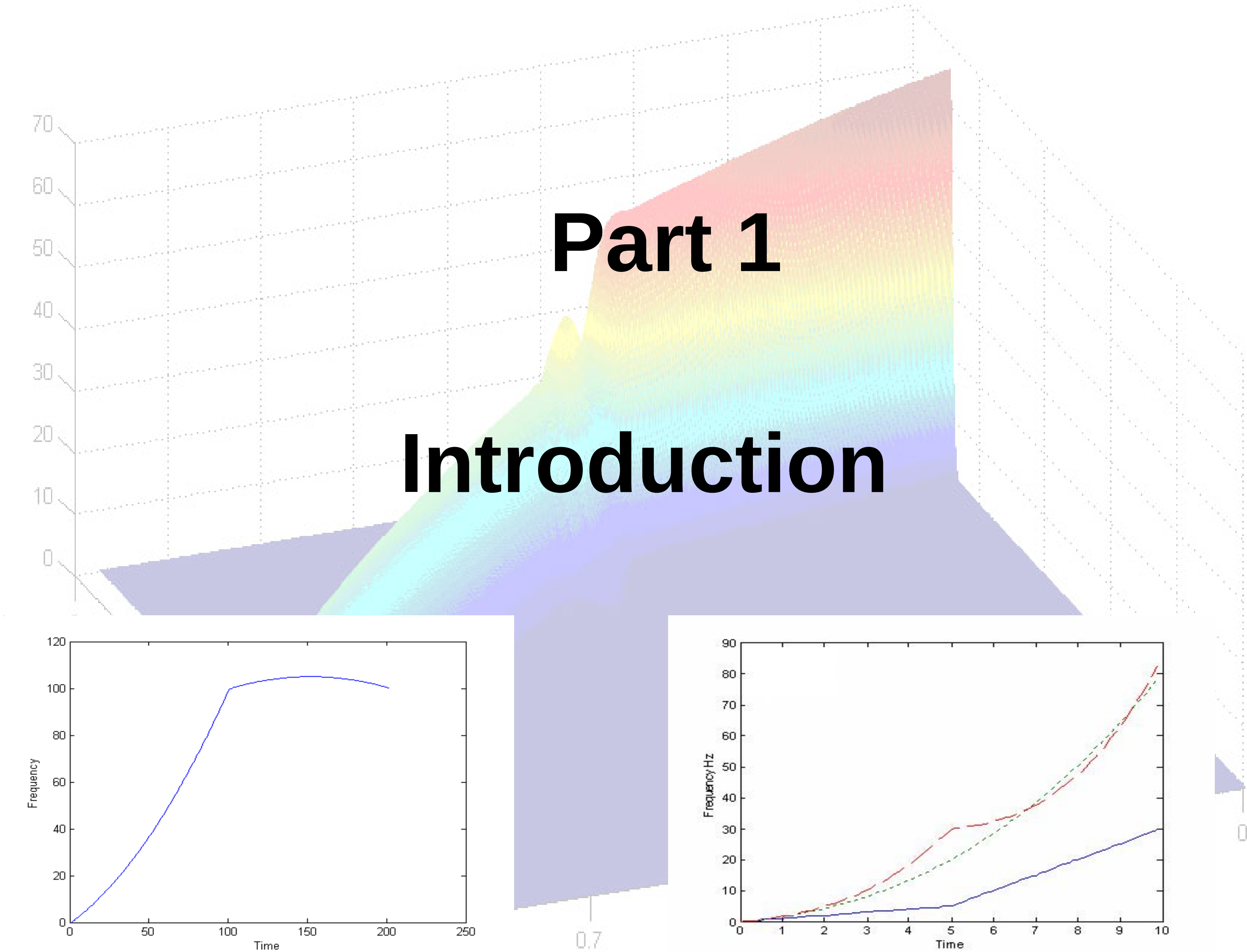
2.2 The Short Time Fourier Transform

2.3 The Wavelet Transform

3- New Time-Frequency Adaptive Techniques

Part 1

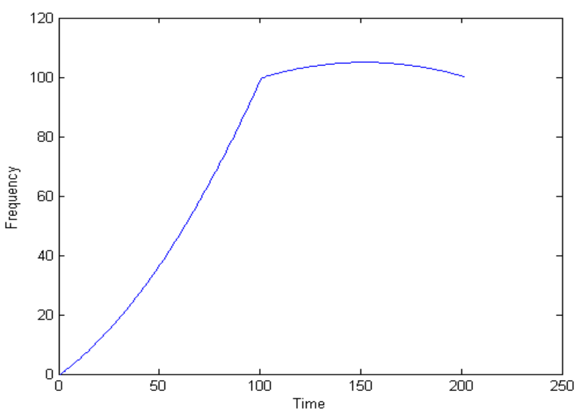
Introduction

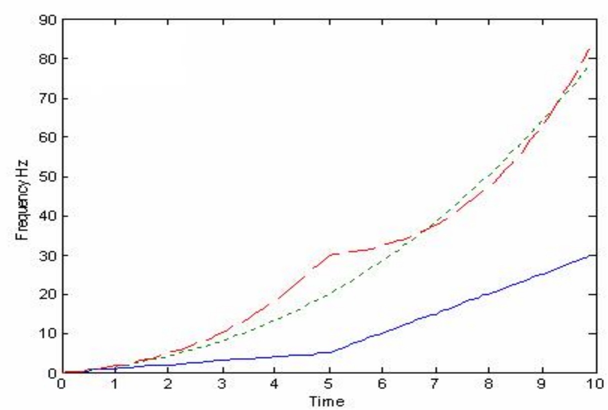


Introduction

- **For some important applications (e.g. radar, sonar and mechanical systems with transient shaft rotation, etc.) it is necessary to process transient signals with any (i.e. linear and *nonlinear)* polynomial variation of the instantaneous frequency.**

- **Examples of those signals are as follows: changes of the shaft frequency during aircraft engine operation, start up and shut down of engines, radar and sonar signals, electromagnetic interference, etc.**





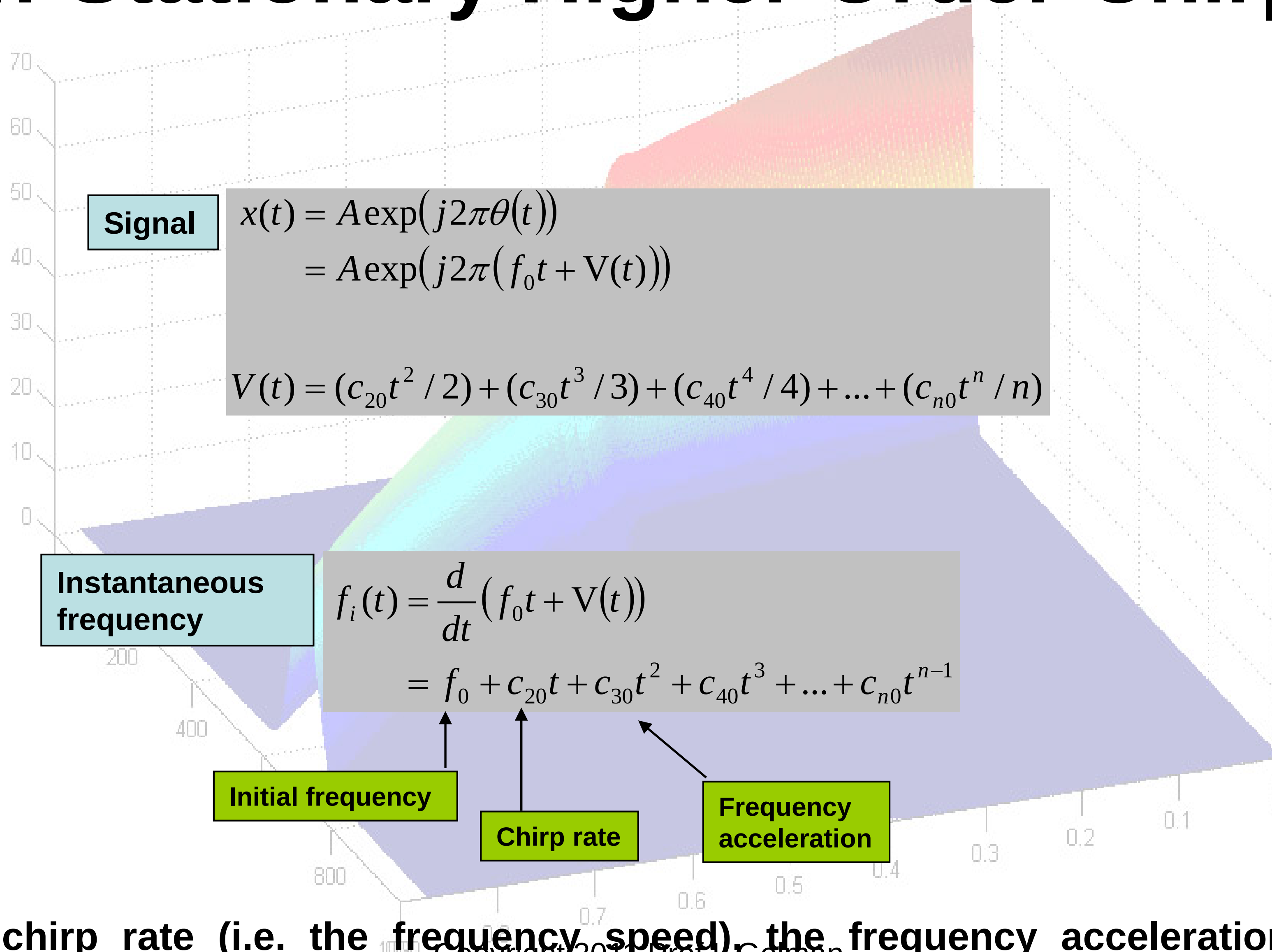
Problem Statement

•**Normally, for transient signals from complex mechanical systems (e.g. turbines, gearboxes, etc.) time variations of the instantaneous frequency are *known* from independent synchronous measurements (e.g. from tachometer signals, etc.).**

• **Sometimes, these frequency variations are even known *a priori*.**

• **Therefore, the main problem for these systems is *amplitude estimation* for transient signals (on the background of an interference) with *known* nonlinear time variation of the instantaneous frequency.**

Non-Stationary Higher Order Chirps I



Signal

$$x(t) = A\exp\left(j2\pi\theta(t)\right) = A\exp\left(j2\pi\left(f_0 t + \mathrm{V}(t)\right)\right)$$

$$V(t) = (c_{20}t^2/2) + (c_{30}t^3/3) + (c_{40}t^4/4) + \ldots + (c_{n0}t^n/n)$$

Instantaneous frequency

$$f_i(t) = \frac{d}{dt}\left(f_0 t + \mathrm{V}(t)\right) = f_0 + c_{20}t + c_{30}t^2 + c_{40}t^3 + \ldots + c_{n0}t^{n-1}$$

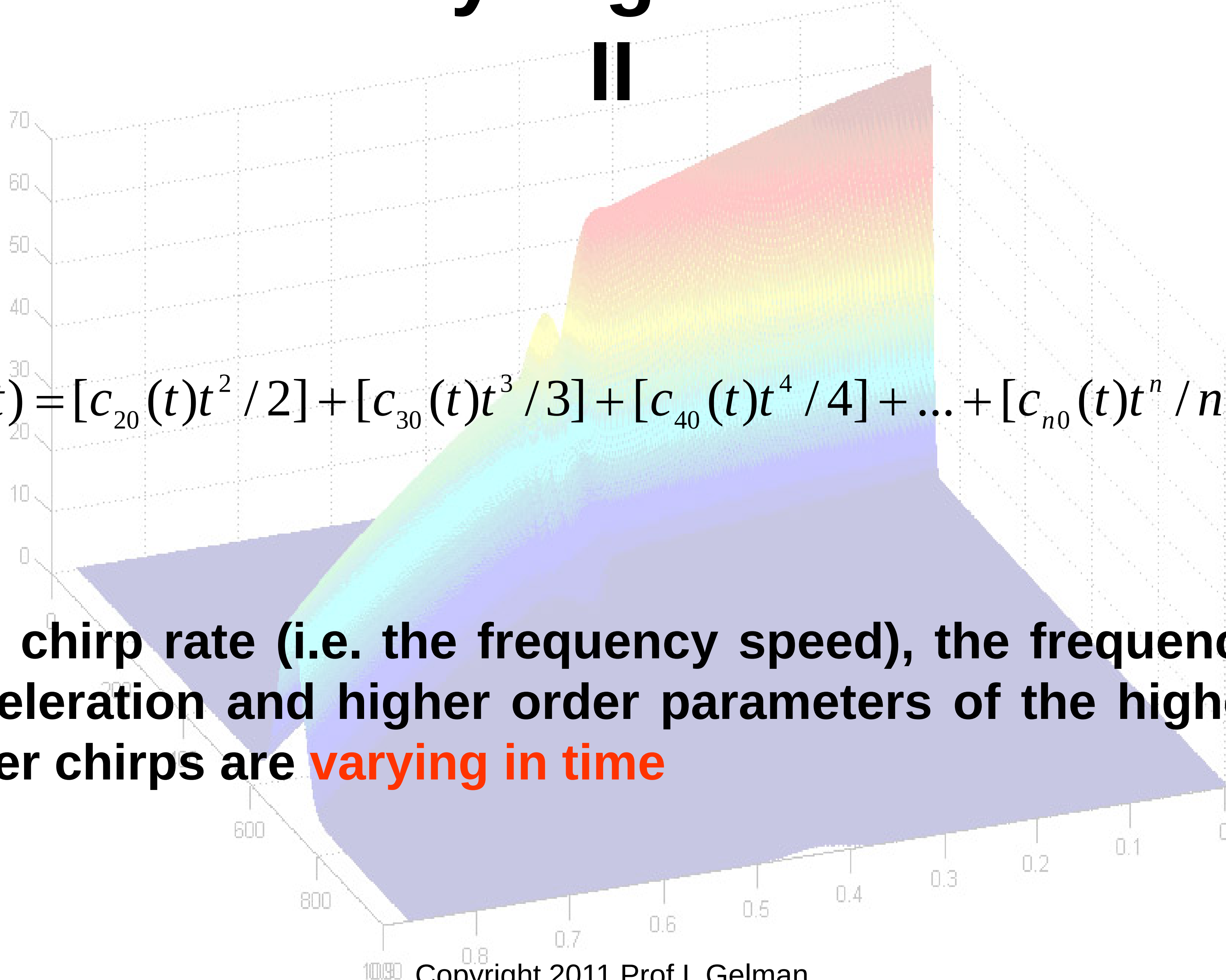
Initial frequency

Chirp rate

Frequency acceleration

The chirp rate (i.e. the frequency speed), the frequency acceleration and higher order parameters of the higher order chirps are not varying in time

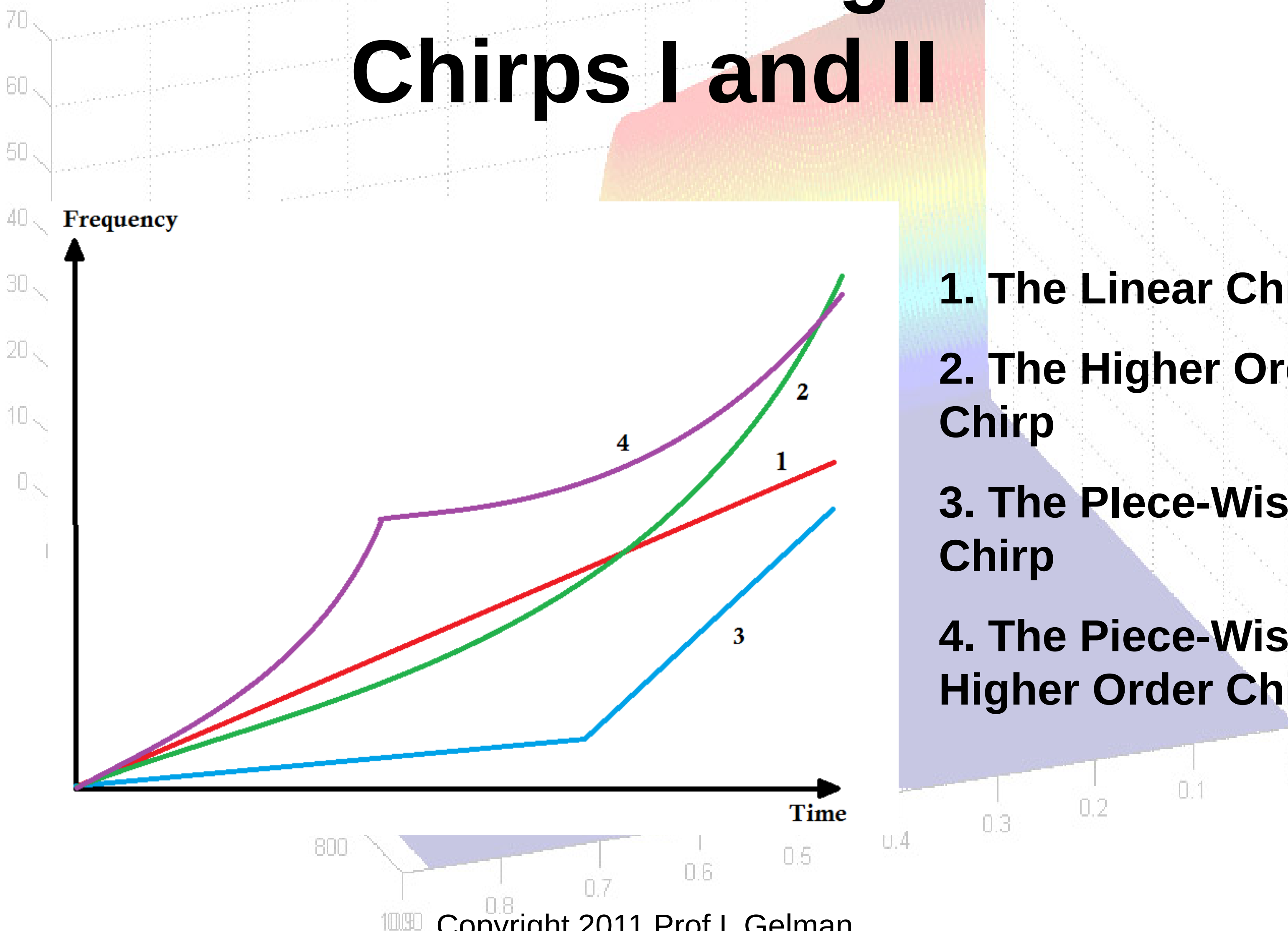
Non-Stationary Higher Order Chirps II



$$V_0(t) = [c_{20}(t)t^2/2] + [c_{30}(t)t^3/3] + [c_{40}(t)t^4/4] + ... + [c_{n0}(t)t^n/n]$$

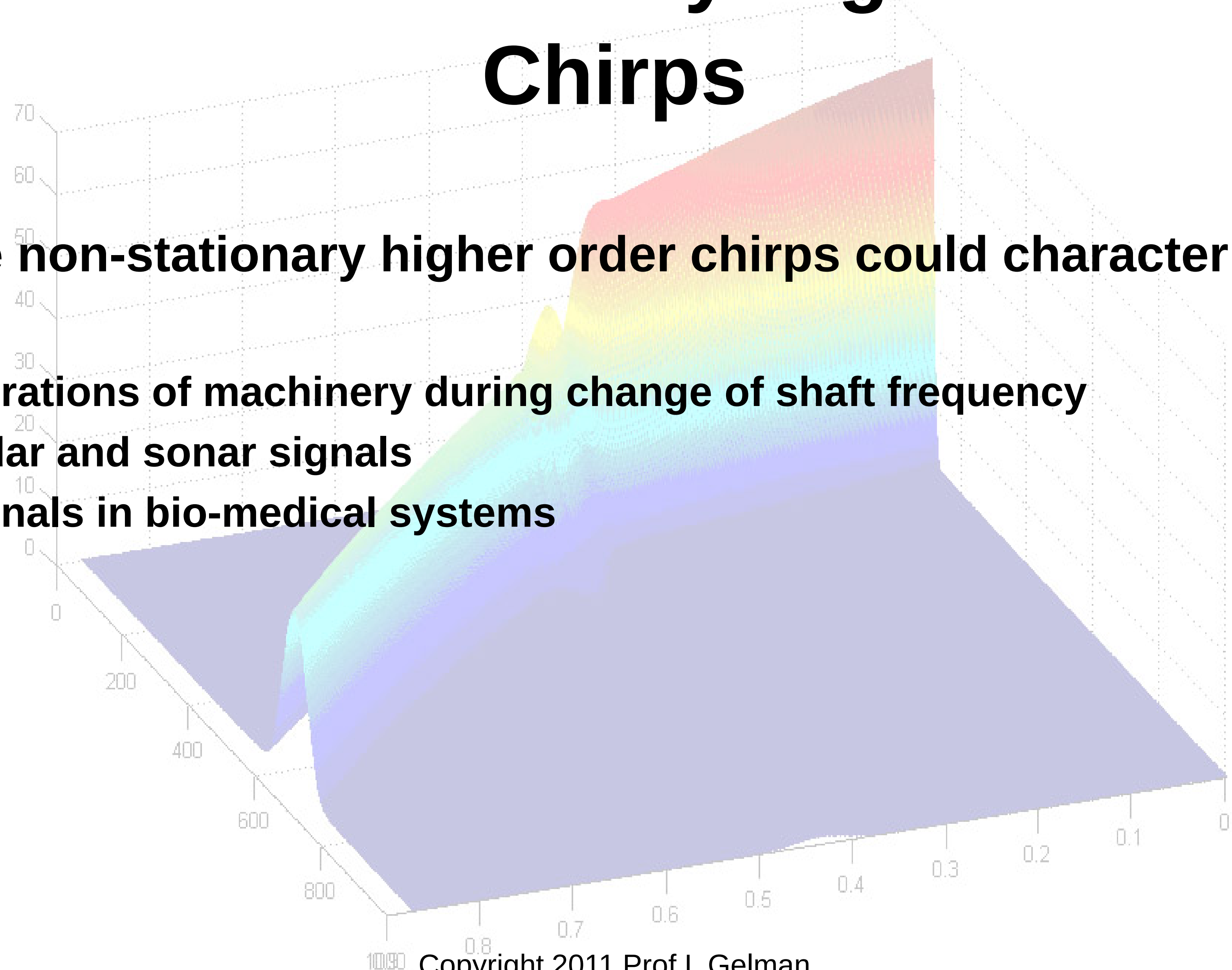
The chirp rate (i.e. the frequency speed), the frequency acceleration and higher order parameters of the higher order chirps are varying in time

Instantaneous Frequency-Time Variations for the Higher Order Chirps I and II



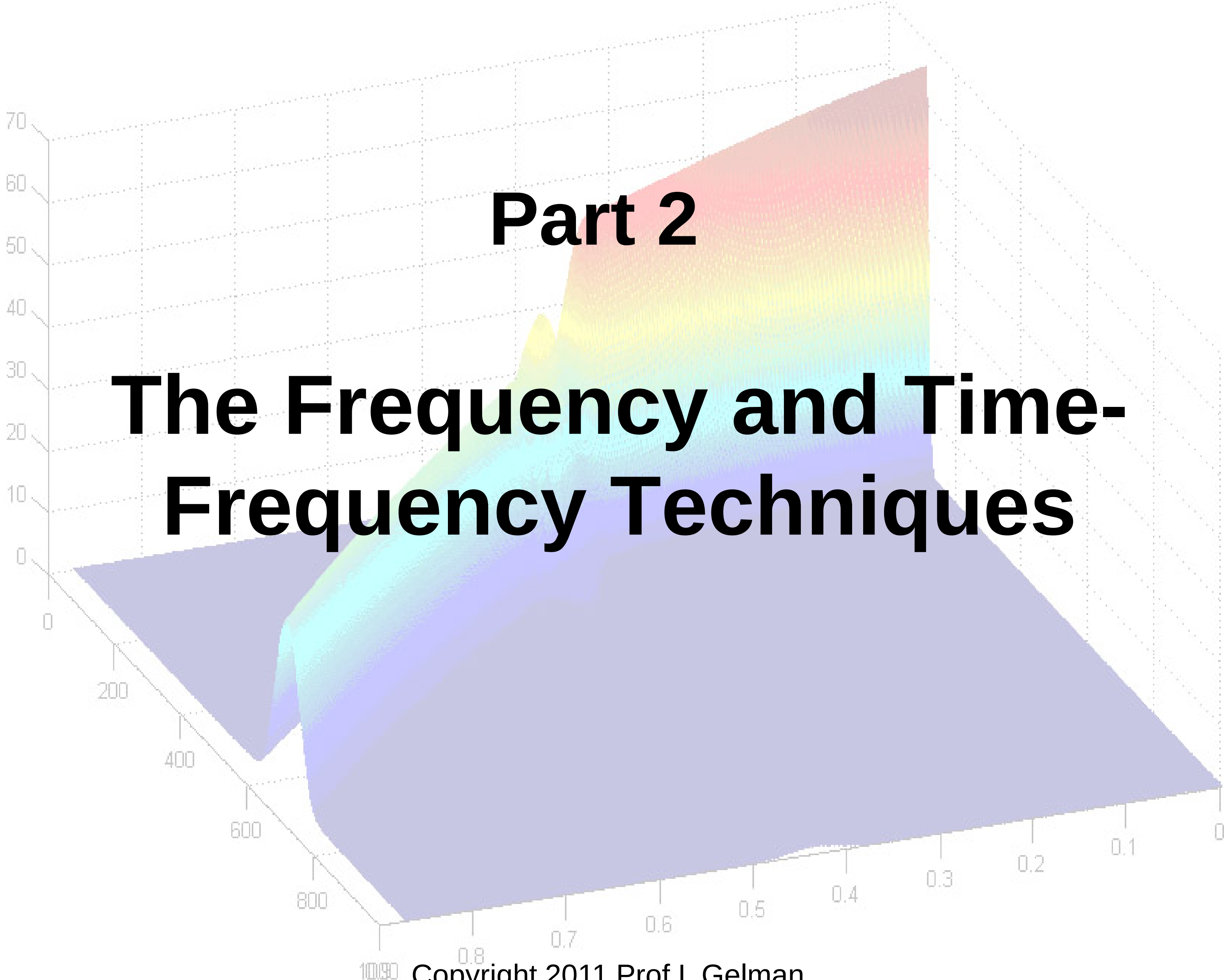
1. The Linear Chirp
2. The Higher Order Chirp
3. The Piece-Wise Chirp
4. The Piece-Wise Higher Order Chirp

The Non-Stationary Higher Order Chirps



The non-stationary higher order chirps could characterize:

- **vibrations of machinery during change of shaft frequency**
- **radar and sonar signals**
- **signals in bio-medical systems**

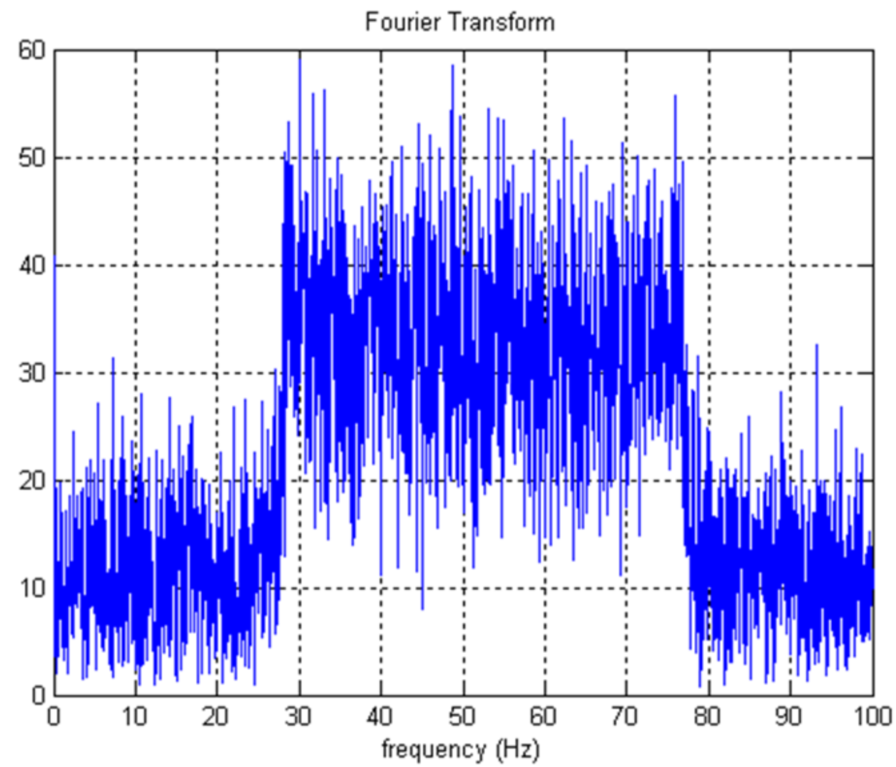


Part 2

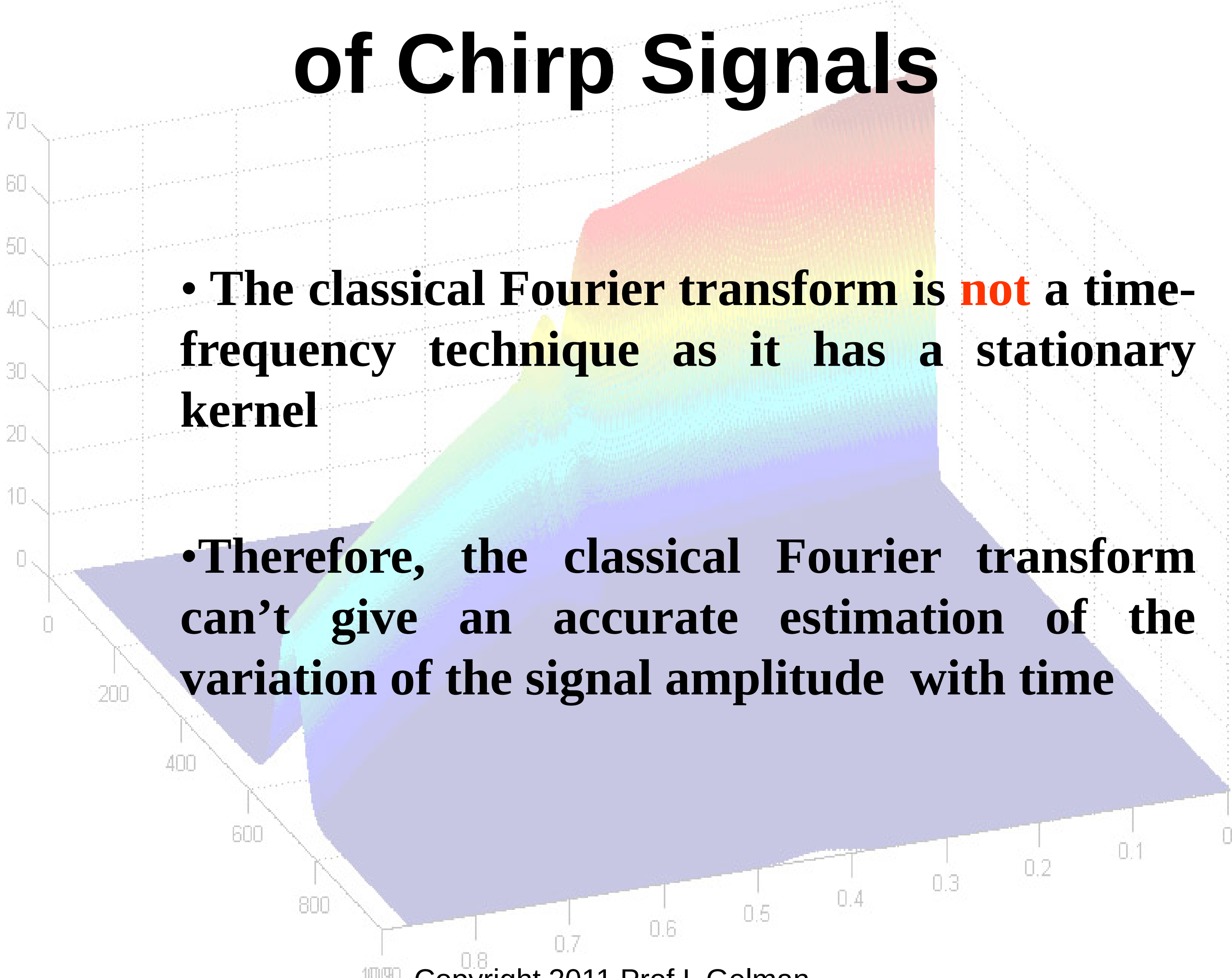
The Frequency and Time-Frequency Techniques

The Classical Fourier Transform of the Chirp Signal

$$X(f) = \int x(t) \cdot e^{-2\pi \cdot i \cdot f \cdot t} dt$$



The Classical Fourier Transform of Chirp Signals

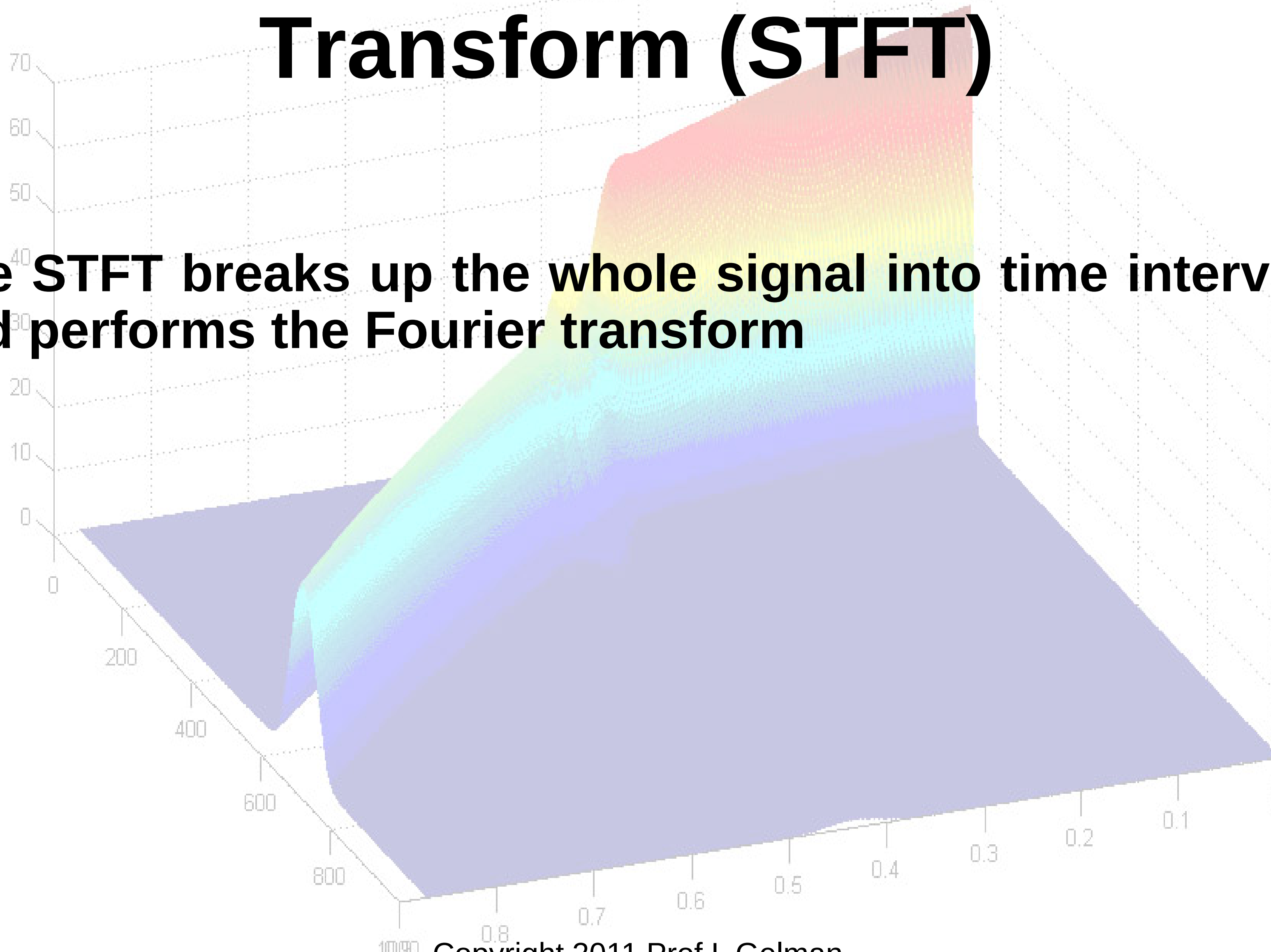


- **The classical Fourier transform is not a time-frequency technique as it has a stationary kernel**

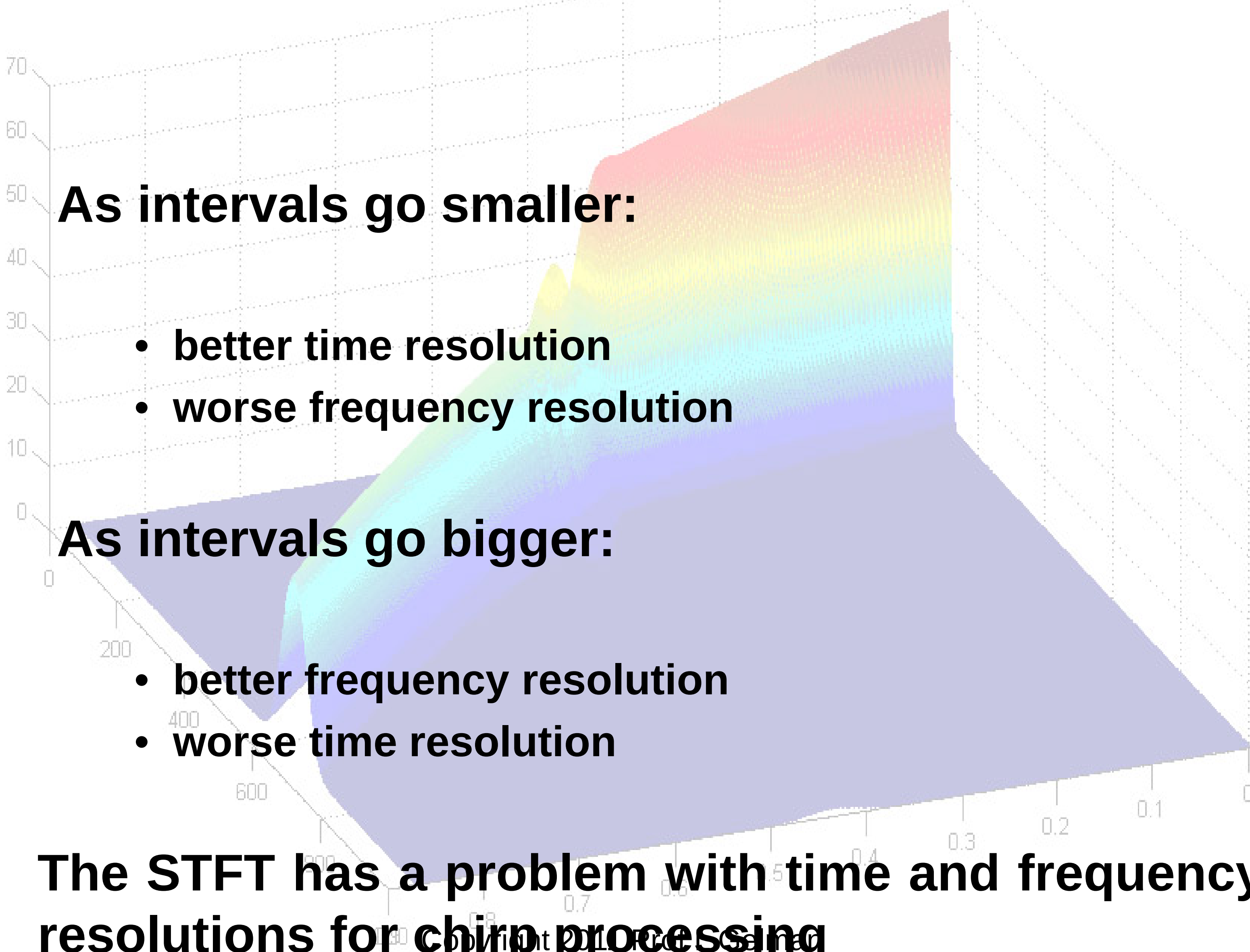
- **Therefore, the classical Fourier transform can't give an accurate estimation of the variation of the signal amplitude with time**

The Short Time Fourier Transform (STFT)

The STFT breaks up the whole signal into time intervals and performs the Fourier transform



The Short Time Fourier Transform



As intervals go smaller:

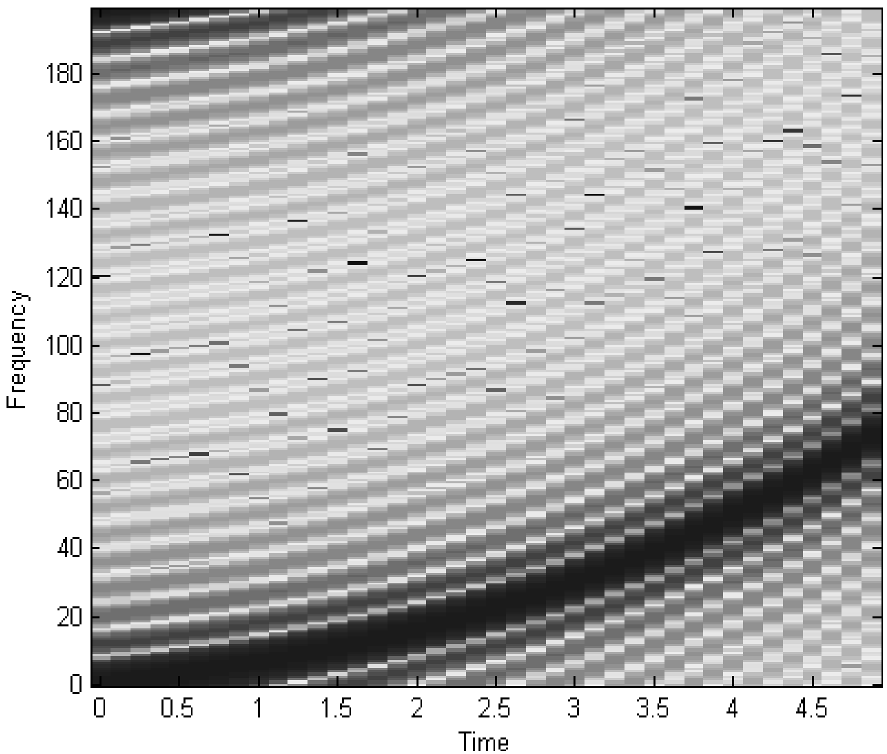
- better time resolution
- worse frequency resolution

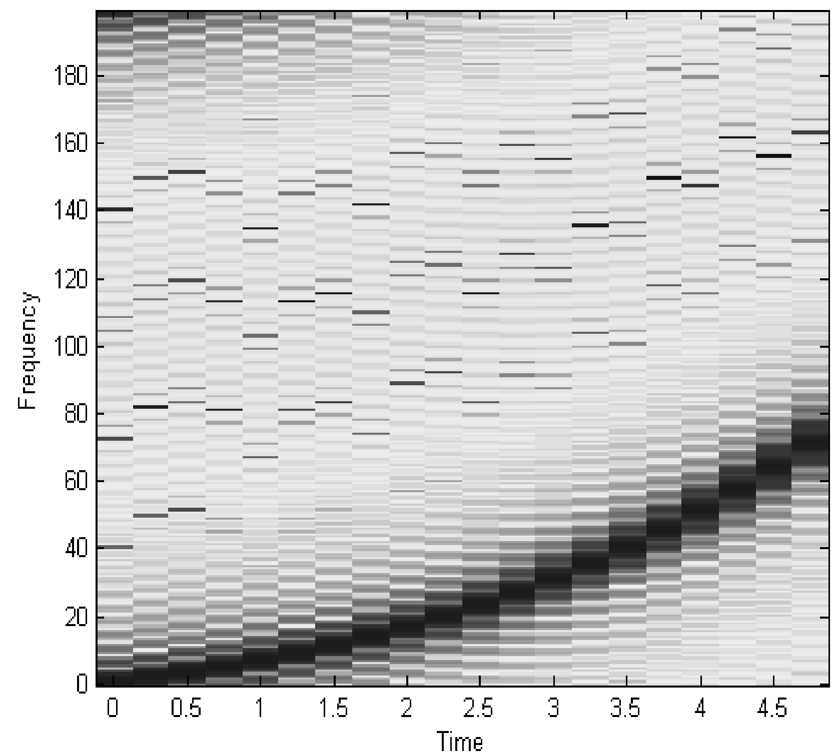
As intervals go bigger:

- better frequency resolution
- worse time resolution

- **The STFT has a problem with time and frequency resolutions for chirp processing**

Case Study: the Short Time Fourier Transform of the Quadratic Chirp



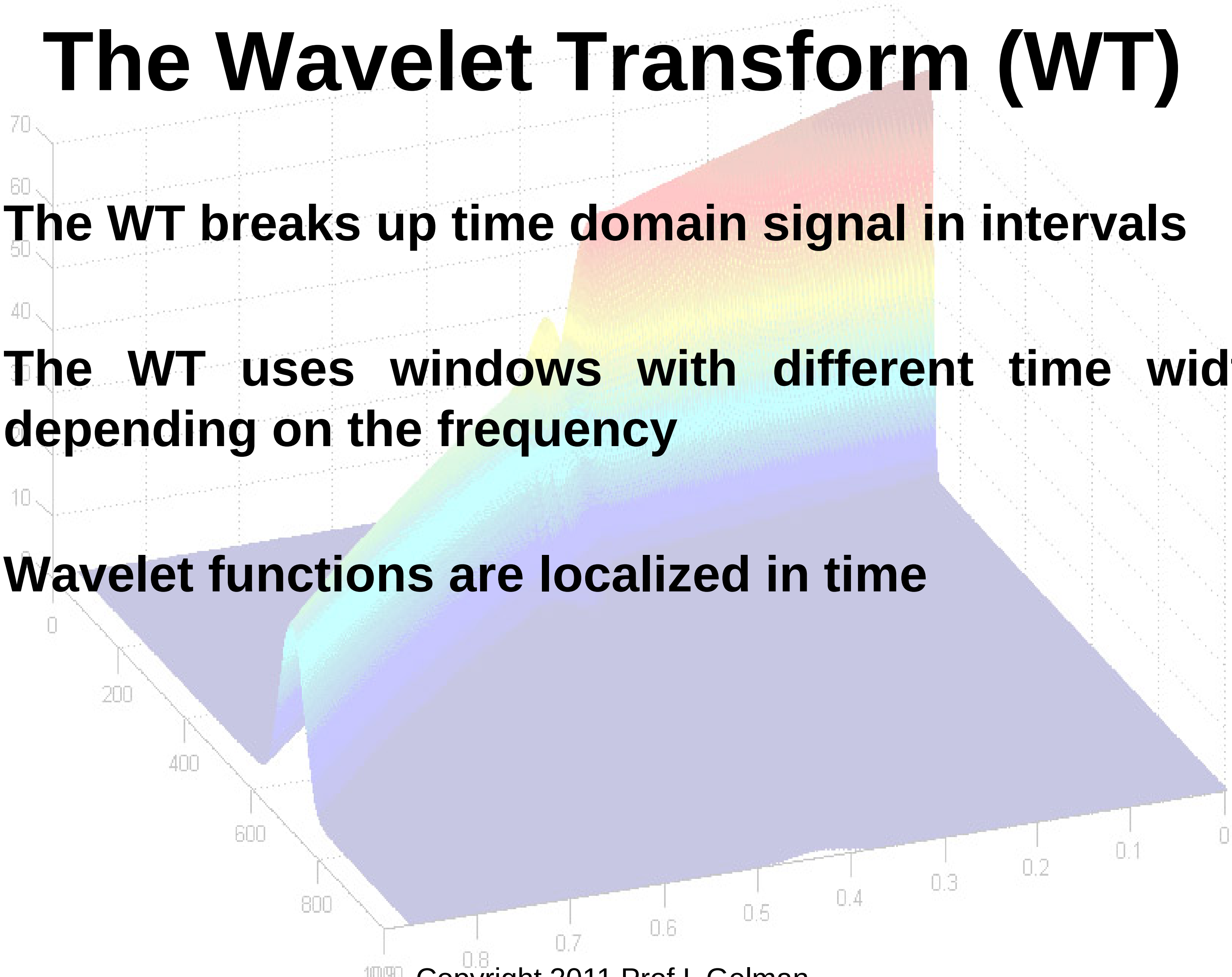


The Wavelet Transform (WT)

- **The WT breaks up time domain signal in intervals**

- **The WT uses windows with different time width depending on the frequency**

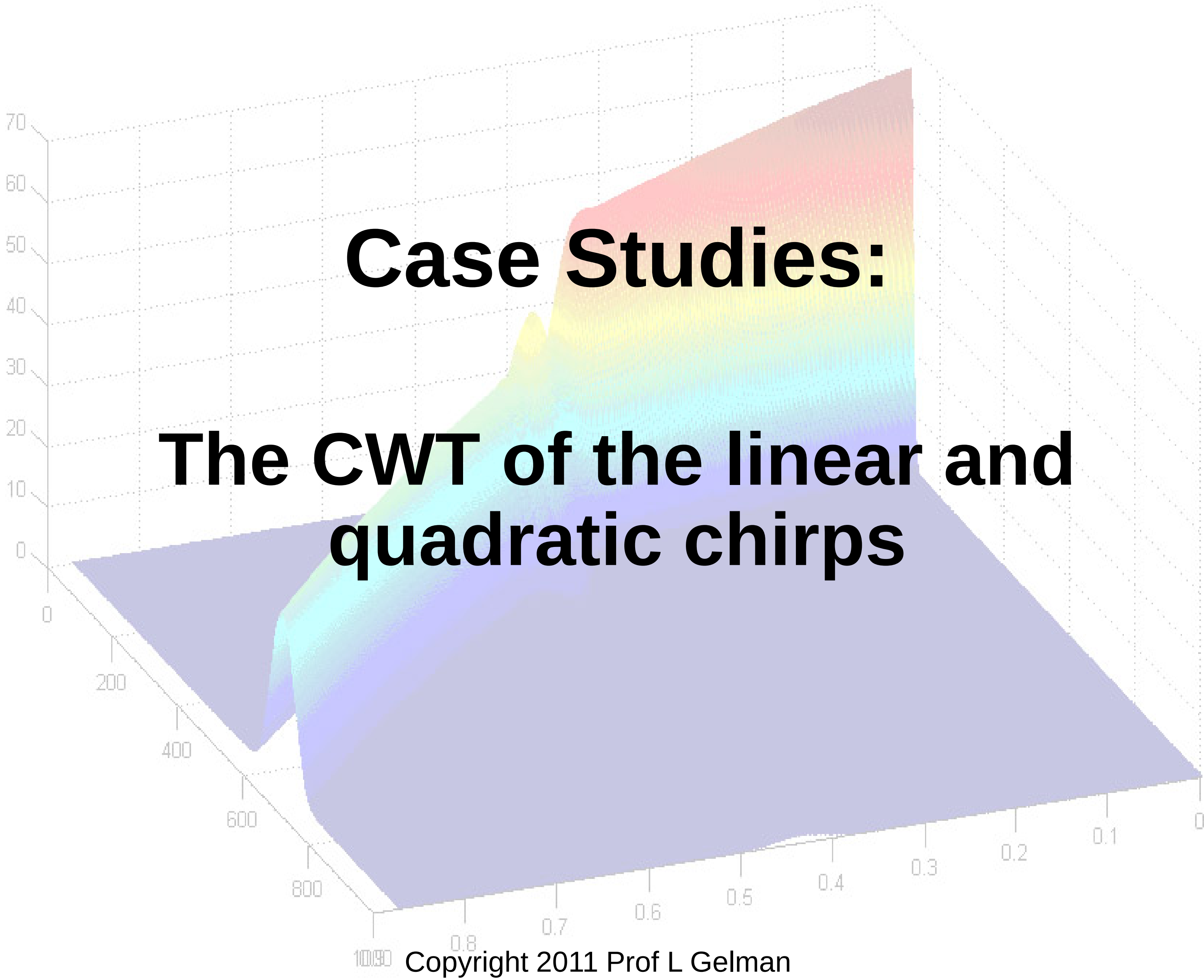
- **Wavelet functions are localized in time**



The Morlet Wavelet

$$\psi(t) = \left(\frac{2}{f_b \pi}\right)^{1/4} \left(\exp(j2\pi f_0 t) - \exp(-(2\pi f_0)^2/2)\right)\exp\left(-\frac{t^2}{f_b}\right)$$

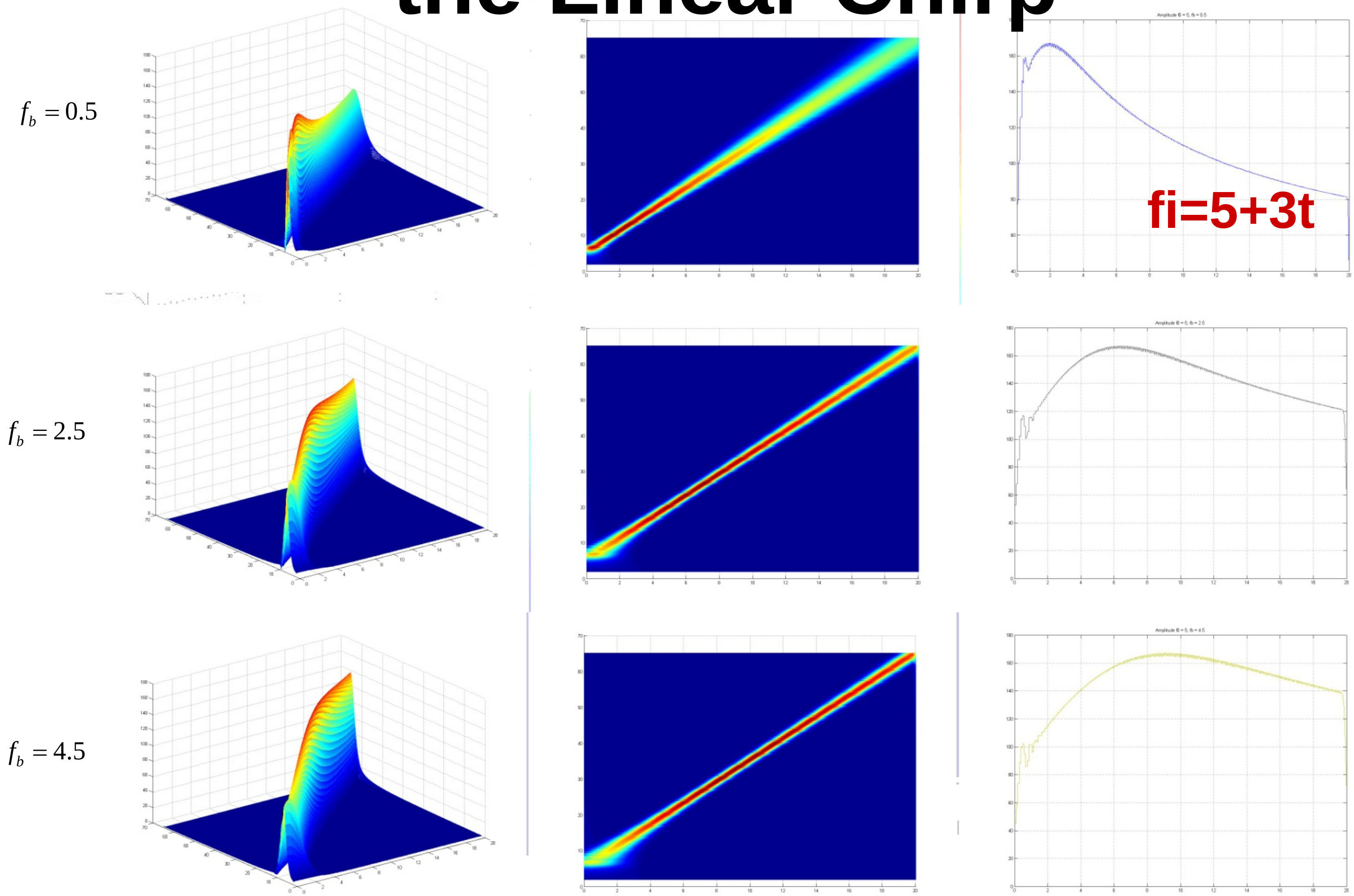
- **The bandwidth parameter fb defines a width of the Gaussian window that modulates the compelx exponential signal**
- **Gaussian window is wider with bigger values of fb**
- **The optimum fb provides**
  - **thinnest distribution in the frequency-time plane**
  - **smallest amplitude errors**



Case Studies:

The CWT of the linear and quadratic chirps

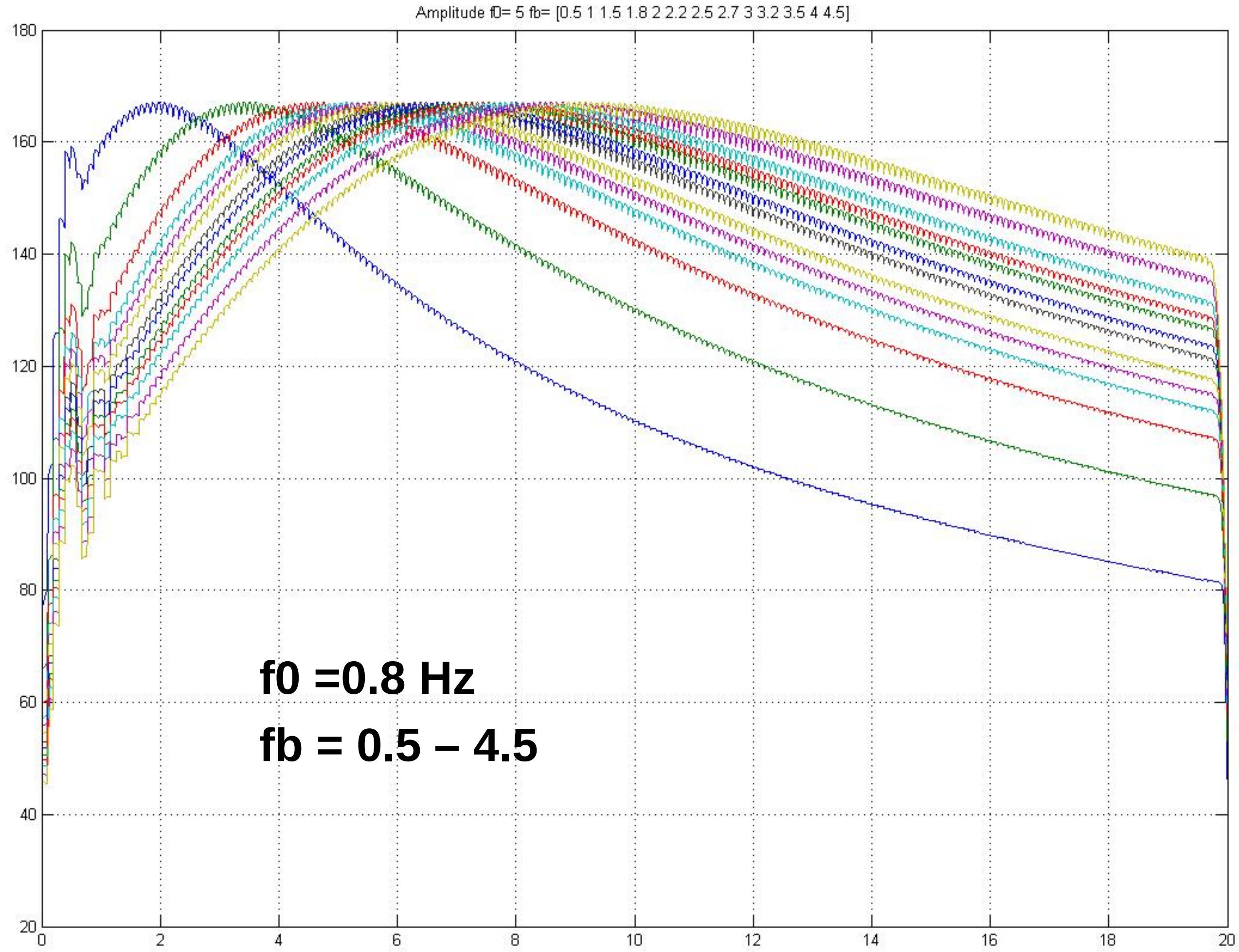
The Morlet Wavelet Transform for the Linear Chirp



The wavelet follows the instantaneous frequency variation with amplitude errors

The Morlet Wavelet Transform for the Linear Chirp

Amplitude Profile

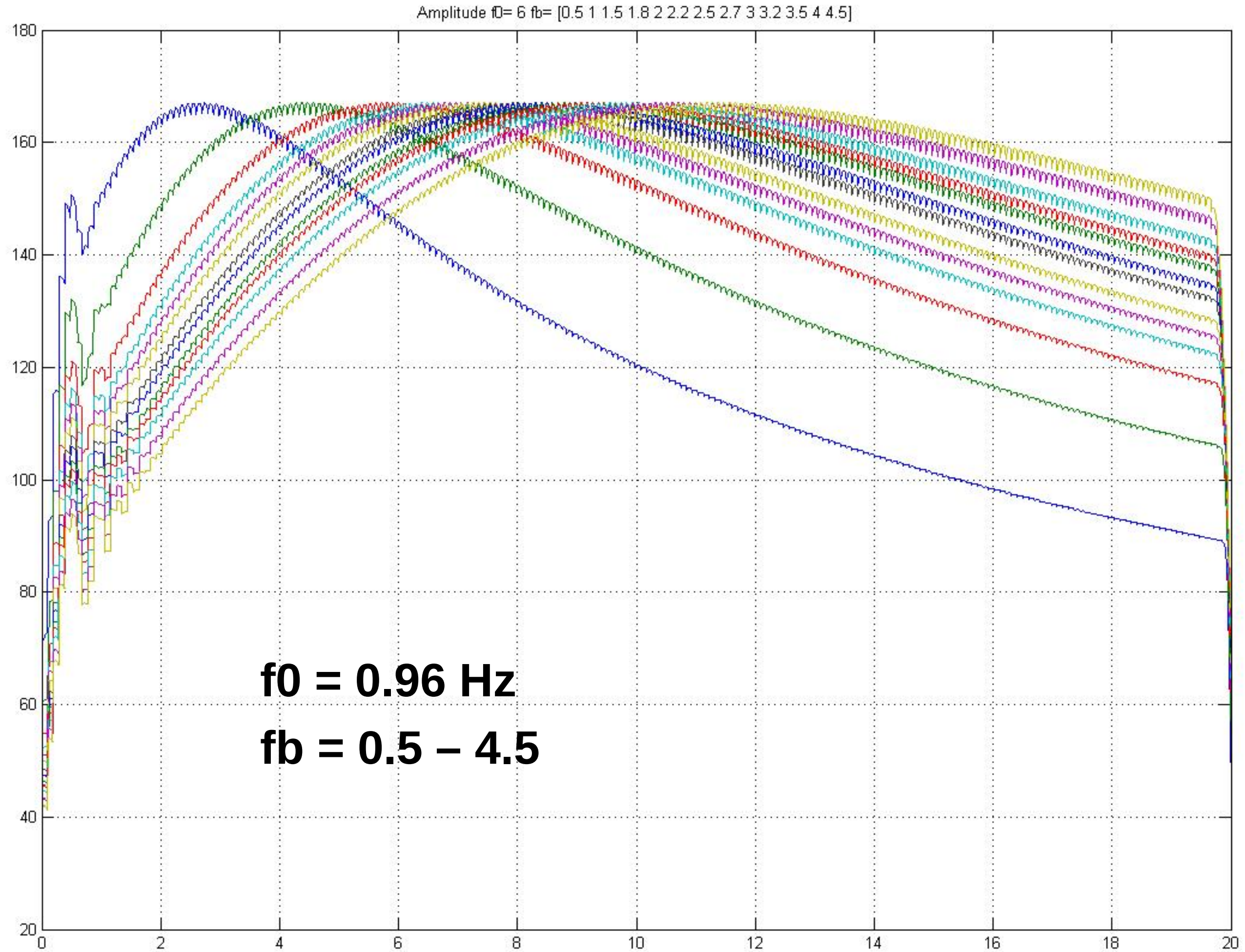


fi=5+3t

Low values of the bandwidth parameter give a maximum of the transform at low frequencies

The Morlet Wavelet Transform for the Linear Chirp

Amplitude Profile



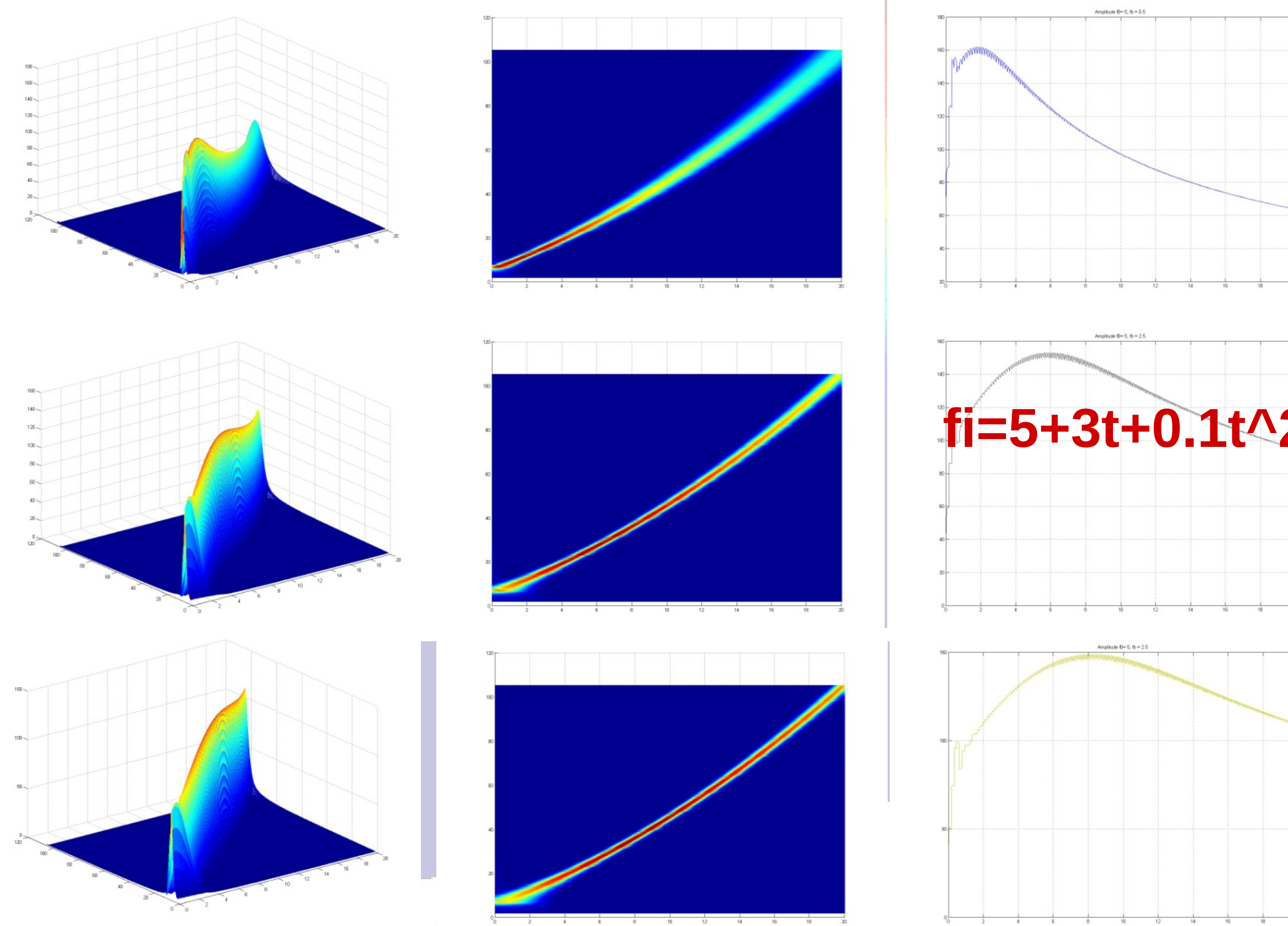
Low values of the bandwidth parameter give a maximum of the transform at low frequencies

The Morlet Wavelet Transform for the Quadratic Chirp

$f_b = 0.5$

$f_b = 2.5$

$f_b = 4.5$

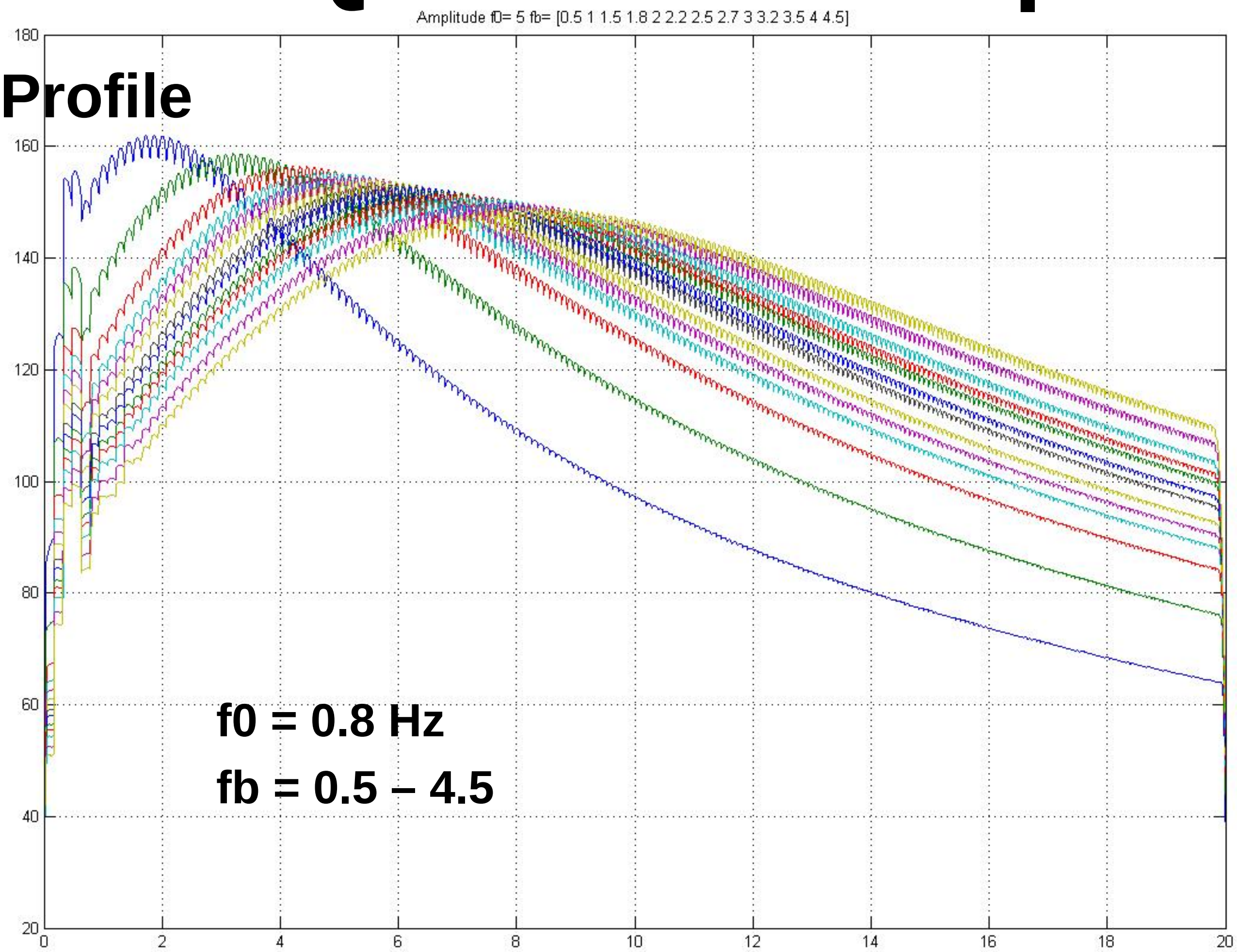


fi=5+3t+0.1t^2

The wavelet follows the instantaneous frequency with amplitude errors

The Morlet Wavelet Transform for the Quadratic Chirp

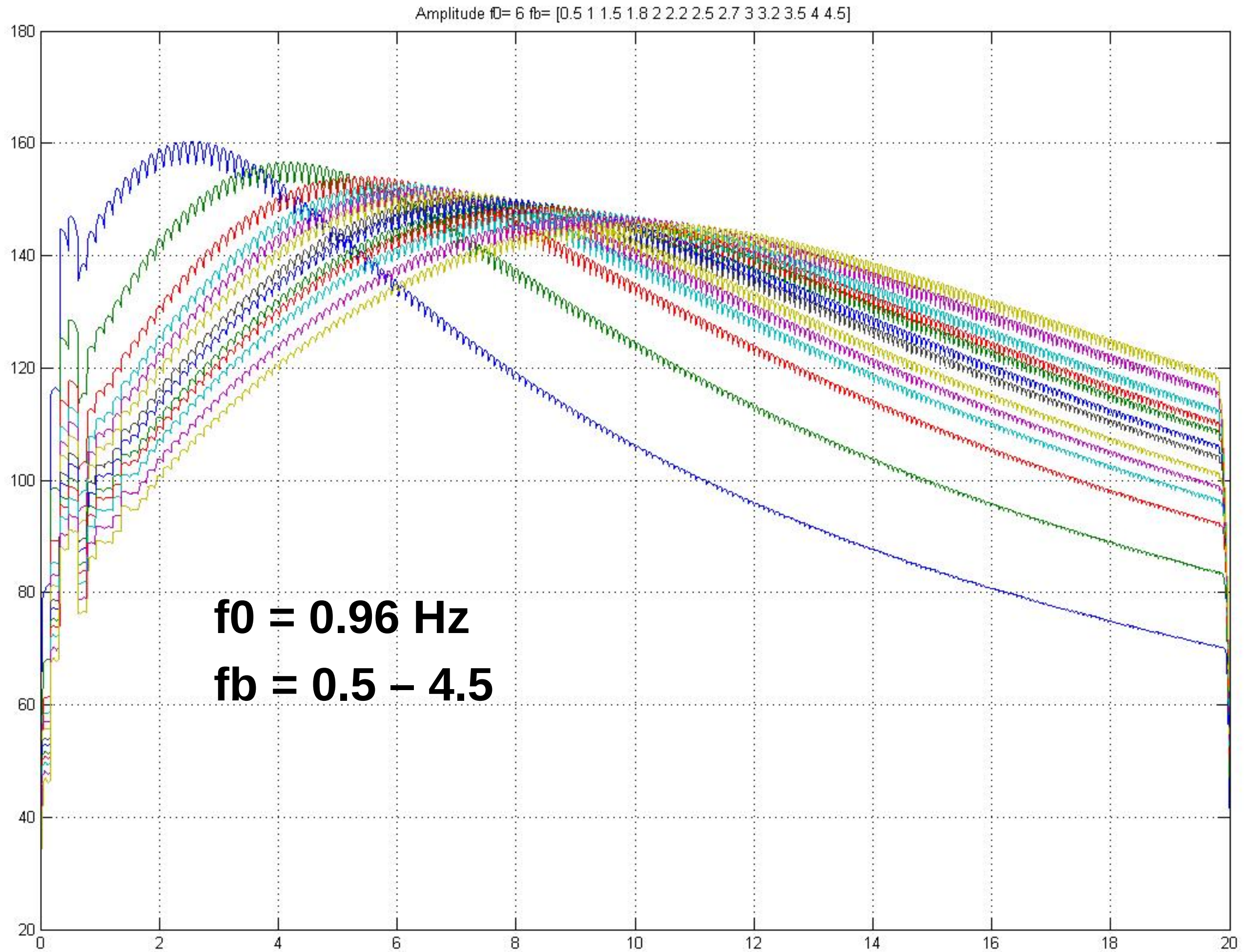
Amplitude Profile



Low values of the bandwidth parameter give a maximum of the transform at low frequencies

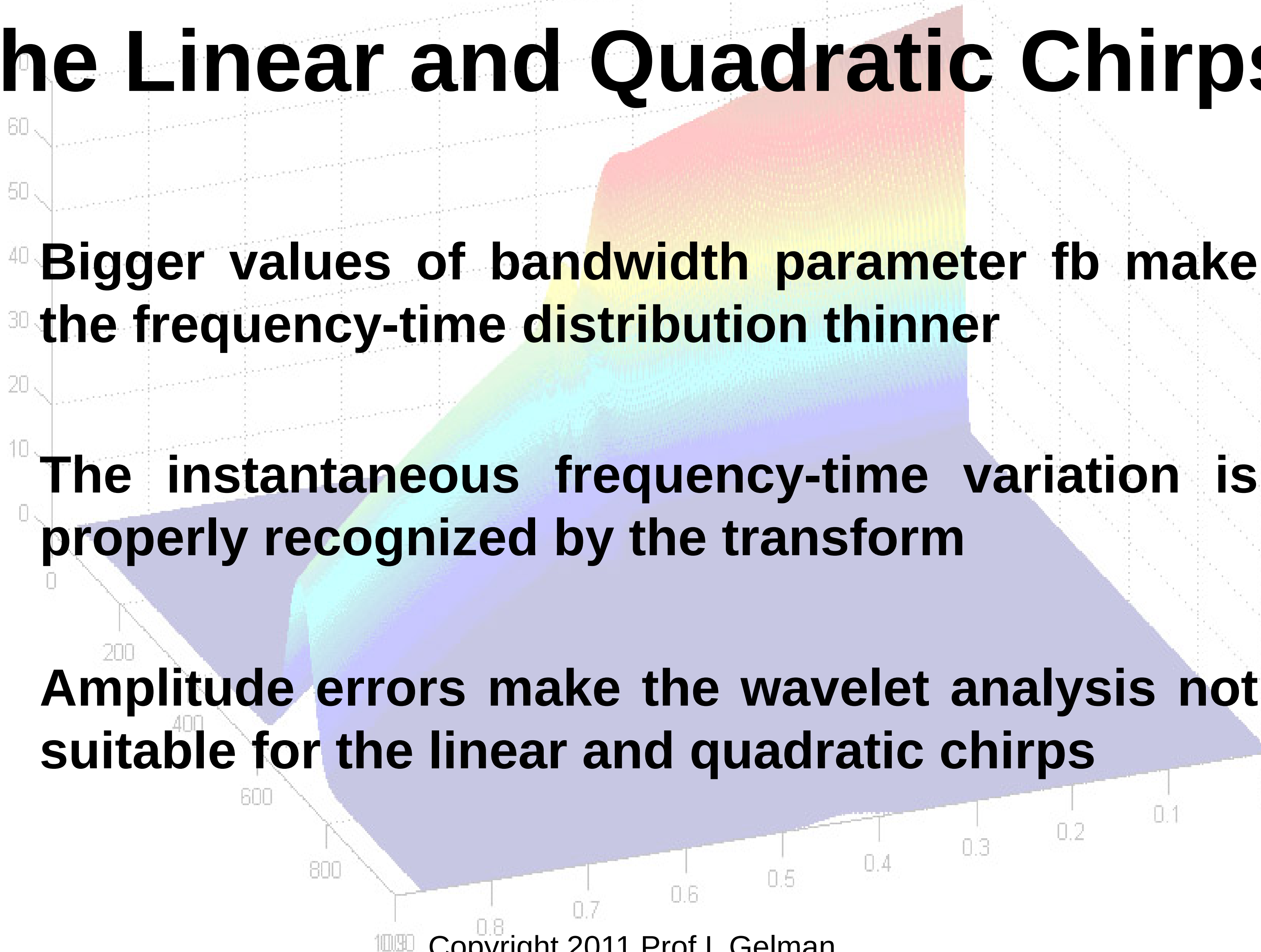
The Morlet Wavelet Transform for the Quadratic Chirp

Amplitude Profile



Low values of the bandwidth parameter give a maximum of the transform at low frequencies

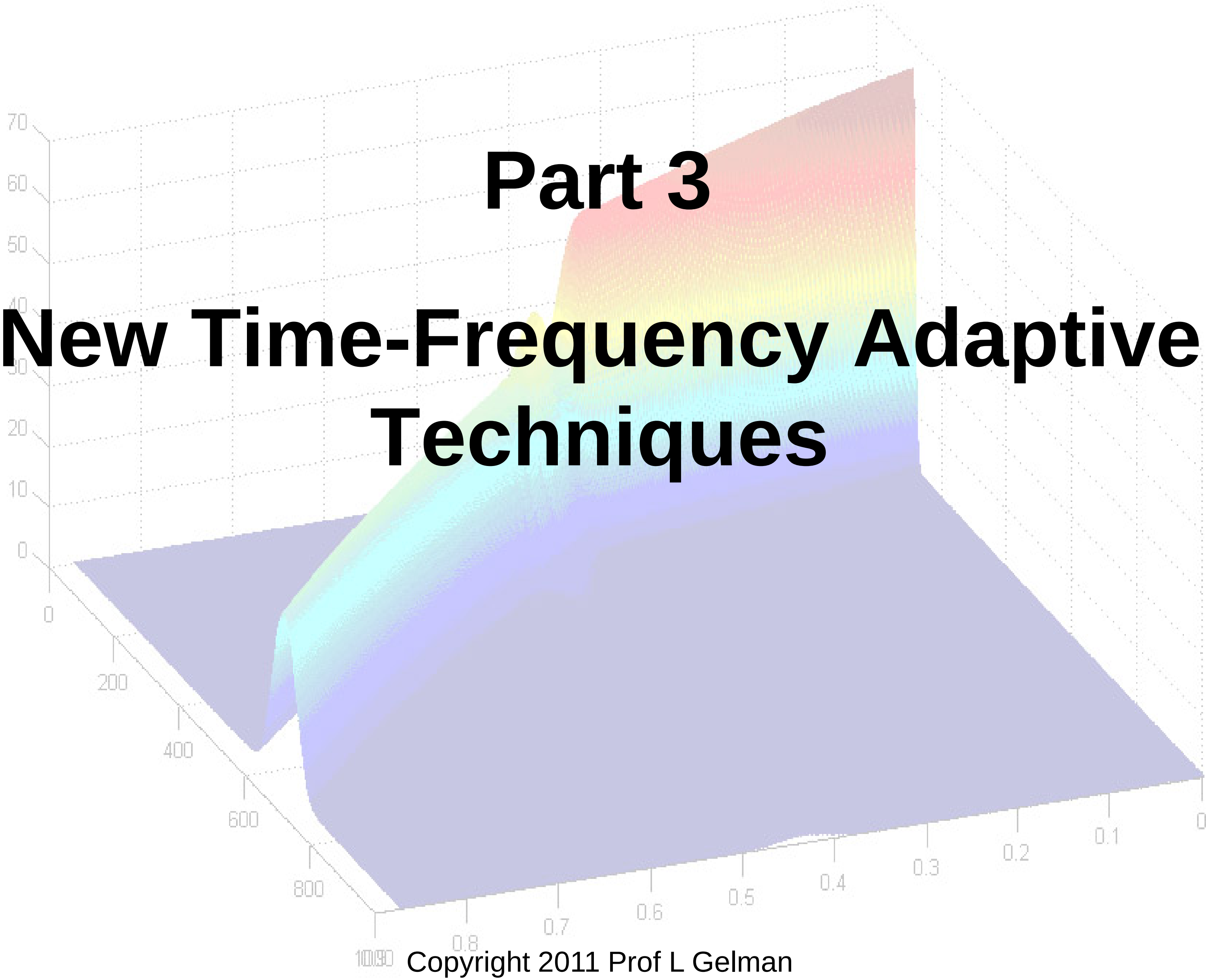
The Morlet Wavelet Transform for the Linear and Quadratic Chirps



- **Bigger values of bandwidth parameter fb make the frequency-time distribution thinner**
- **The instantaneous frequency-time variation is properly recognized by the transform**
- **Amplitude errors make the wavelet analysis not suitable for the linear and quadratic chirps**

Part 3

New Time-Frequency Adaptive Techniques



The Short Time Chirp-Fourier Transform

- **Let's consider a signal with the linear and piece-wise dependencies of the instantaneous frequency in time**

- **For these signals a new technique, the short-time chirp-Fourier transform, is proposed and defined as follows:**

$$S(f,T,c_2)=\frac{1}{T_i}\int_{-\infty}^{\infty} h_i(t-T)\,x_1(t)\,e^{-j2\pi\left(ft+\frac{c_2(t)}{2}t^2\right)}dt$$

The Short Time Chirp-Fourier Transform

$$S(f,T,c_2)=\frac{1}{T_i}\int_{-\infty}^{\infty} h_i(t-T)\,x_1(t)\,e^{-j2\pi\left(ft+\frac{c_2(t)}{2}t^2\right)}dt$$

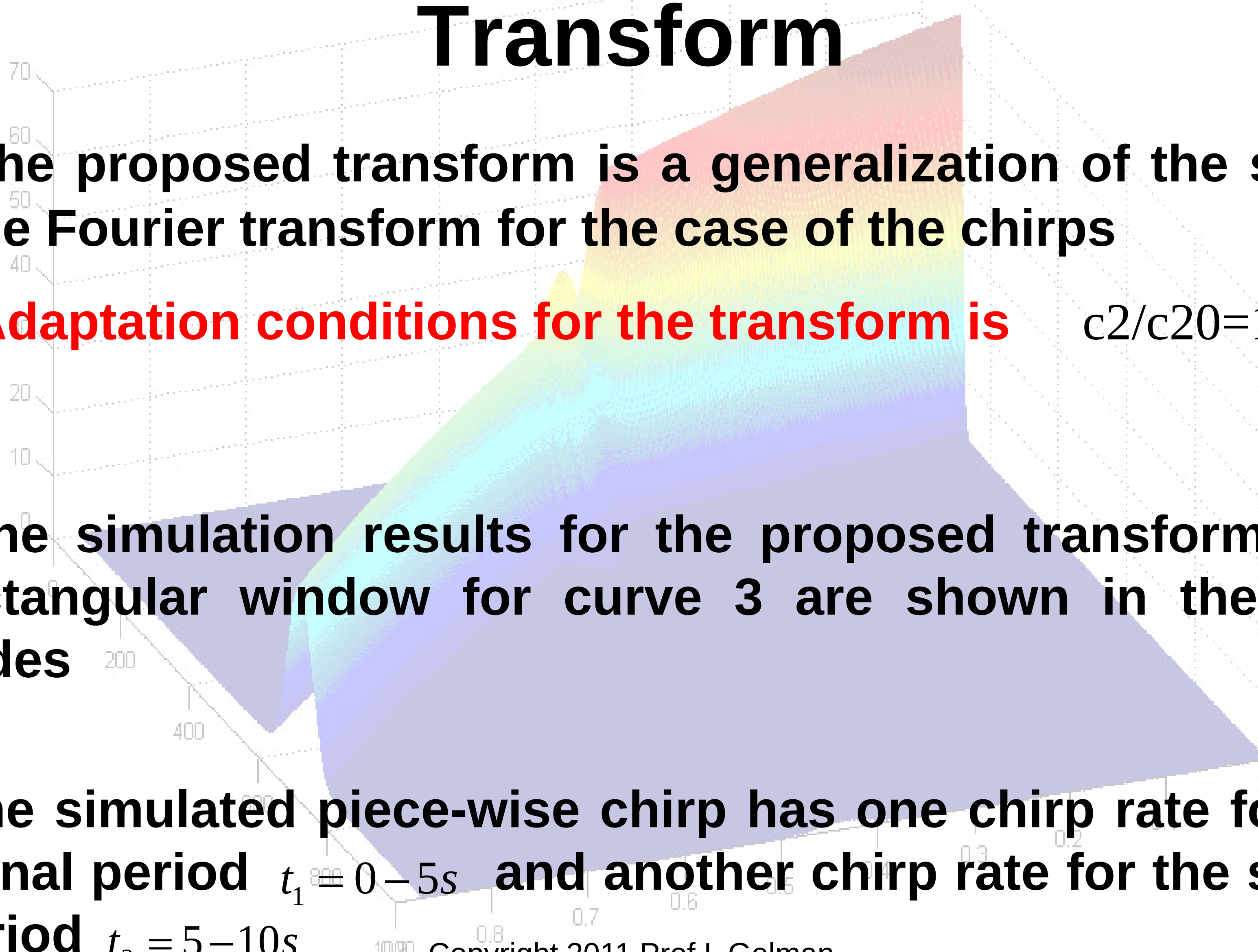
where $h(t)$ **is a time window,** T_i **is the window duration;** $i=1,2,\ldots N$

f **is frequency;** $c_2(t)$ **is the variable frequency speed of the transform;**

T **is window centre.**

The frequency speed of the transform is constant during duration of a constant frequency speed of a signal

The Short Time Chirp-Fourier Transform

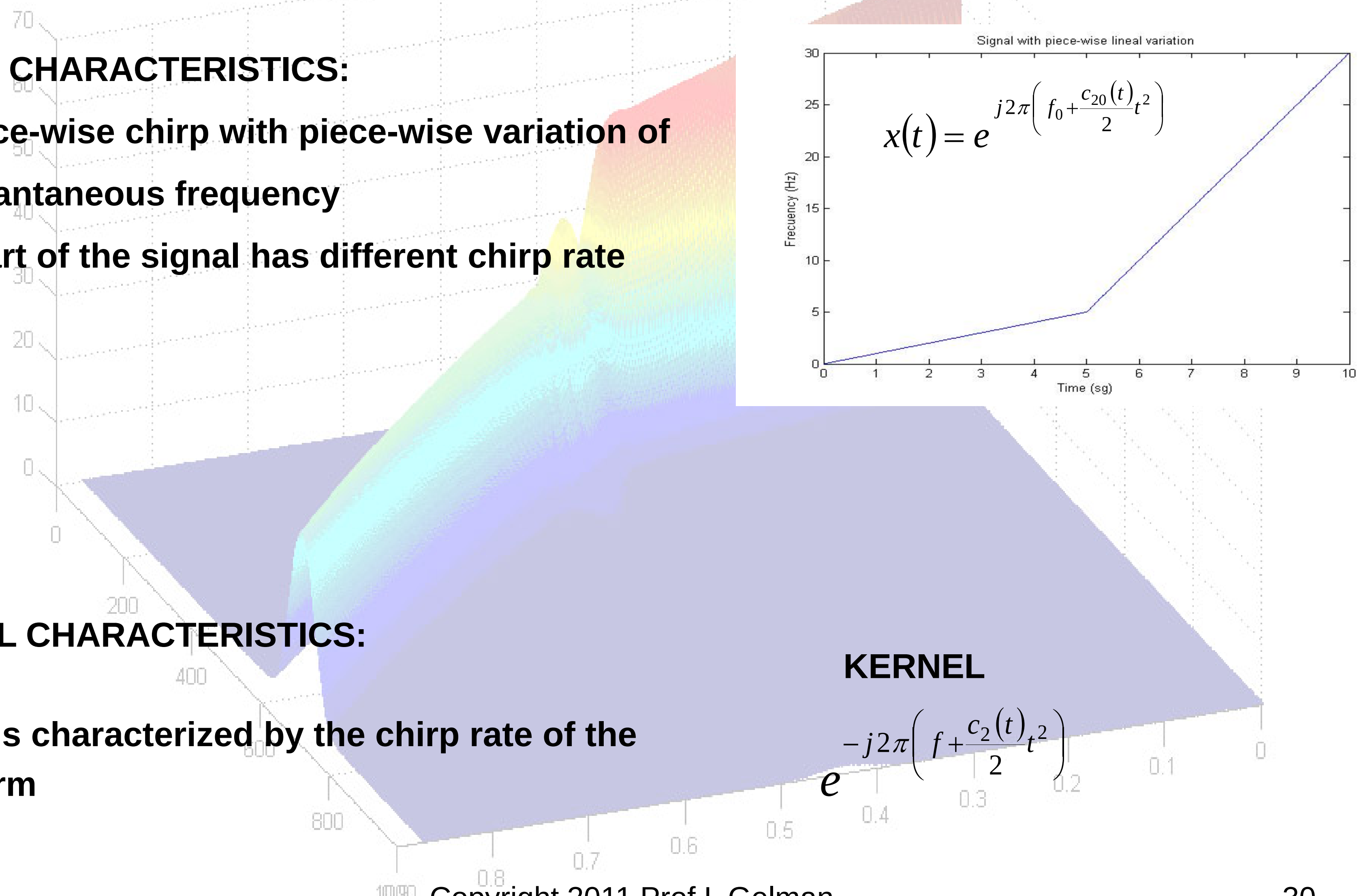


- **The proposed transform is a generalization of the short-time Fourier transform for the case of the chirps**
- **Adaptation conditions for the transform is** c2/c20=1

- **The simulation results for the proposed transform with rectangular window for curve 3 are shown in the next slides**

- **The simulated piece-wise chirp has one chirp rate for the signal period** $t_1 = 0 - 5s$ **and another chirp rate for the signal period** $t_2 = 5 - 10s$

THE PIECE-WISE CHIRP: A SIGNAL AND A KERNEL



SIGNAL CHARACTERISTICS:

The piece-wise chirp with piece-wise variation of the instantaneous frequency

Each part of the signal has different chirp rate

$$x(t) = e^{j2\pi\left(f_0 + \frac{c_{20}(t)}{2}t^2\right)}$$

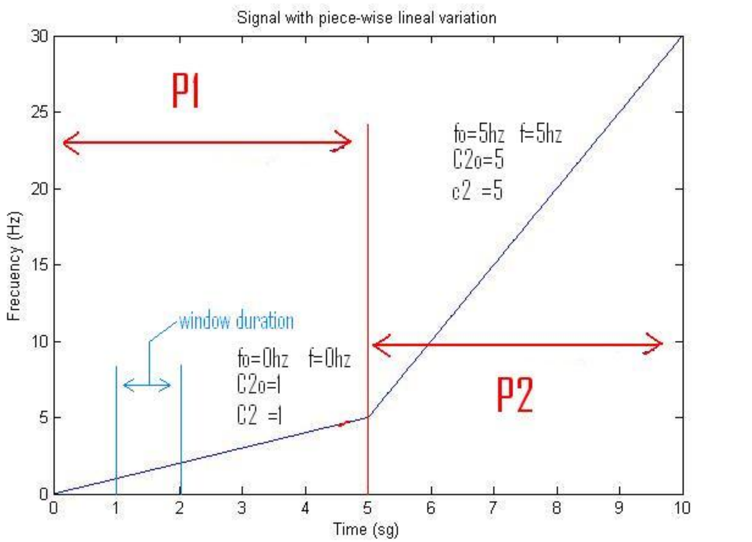
KERNEL CHARACTERISTICS:

Kernel is characterized by the chirp rate of the transform

KERNEL

$$e^{-j2\pi\left(f + \frac{c_2(t)}{2}t^2\right)}$$

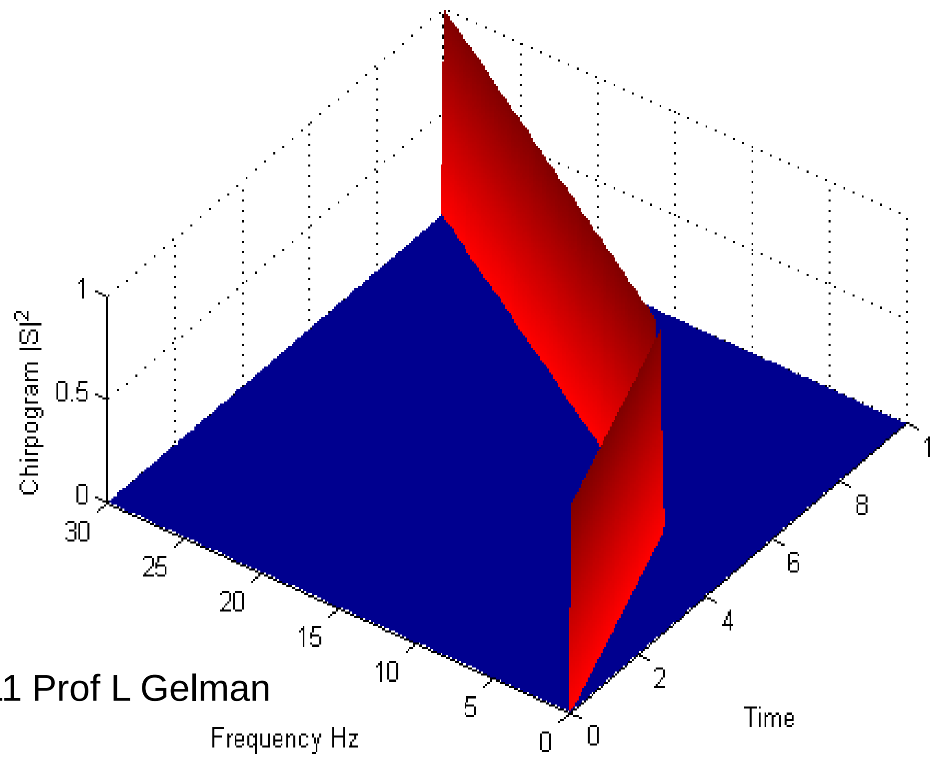
THE PIECE-WISE CHIRP: THE KERNEL IS MATCHED



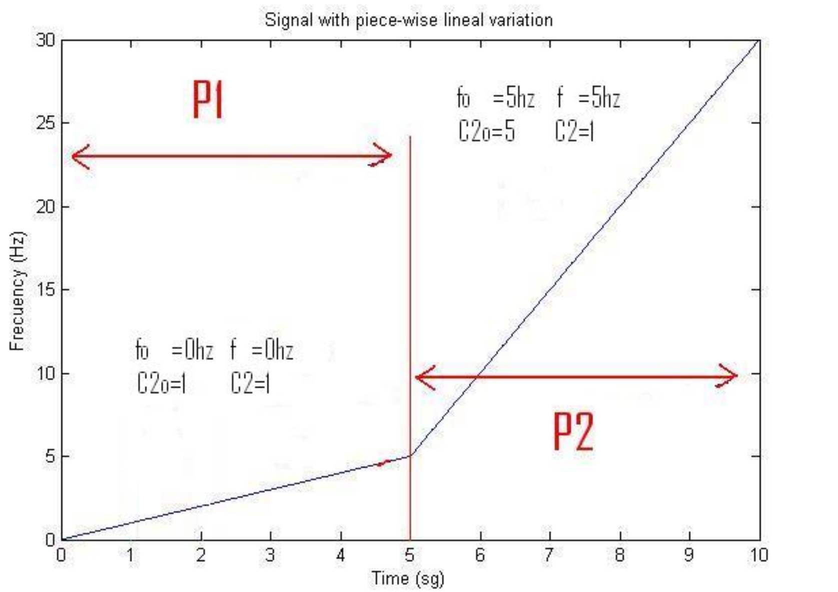
PARAMETERS OF THE SIGNAL AND KERNEL ARE MATCHED

- **Sampling frequency is 200 Hz**
- **P1=0-5s**
- **P2=5-10s**

ONE CAN EVALUATE THE PIECE-WISE FREQUENCY-TIME DEPENDENCY OF THE SIGNAL AND CHIRP RATE FOR EACH PART OF THIS DEPENDENCY AND EMPLOY THE CHIRP FOURIER TRANSFORM WITH APPROPRIATE CHIRP RATES ON EACH SIGNAL PART.

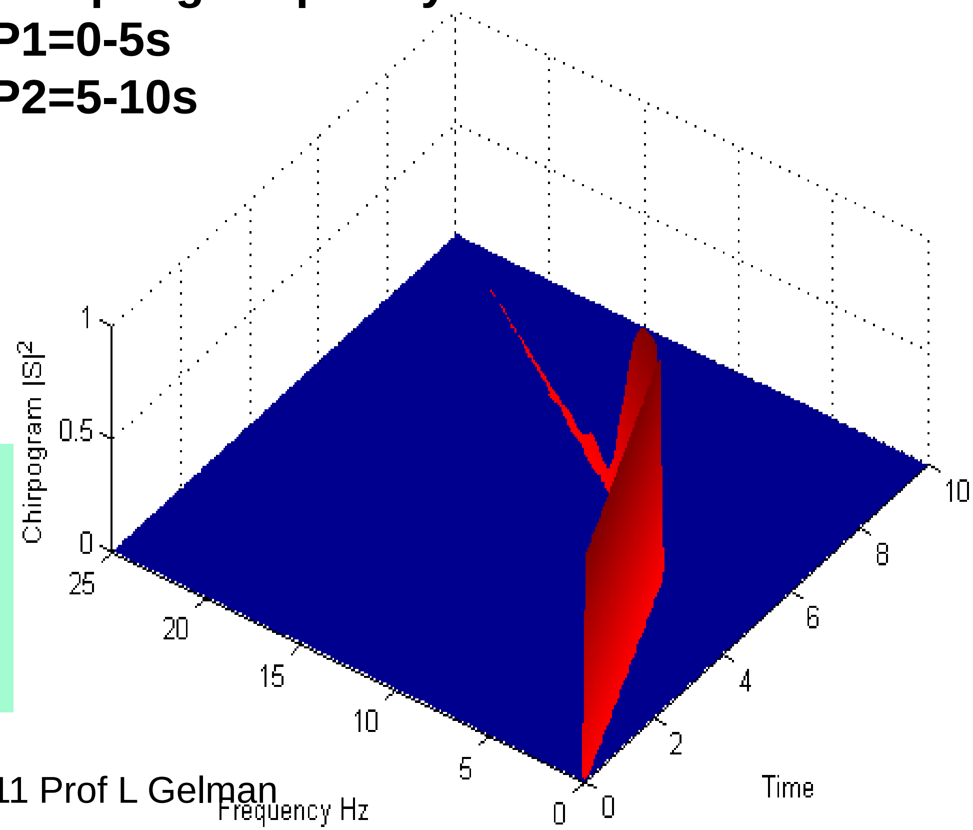


THE PIECE-WISE CHIRP: THE KERNEL IS UNMATCHED



PARAMETERS OF SIGNAL AND KERNEL ARE UNMATCHED FOR PERIOD 2

- **Sampling frequency is 200 Hz**
- **P1=0-5s**
- **P2=5-10s**



- **THE TRANSFORM DOES NOT WORK PROPERLY WHEN KERNEL AND SIGNAL ARE UNMATCHED**
- **THE SIGNAL AMPLITUDE OF THE FIRST PERIOD IS ESTIMATED WITH HIGH ACCURACY**

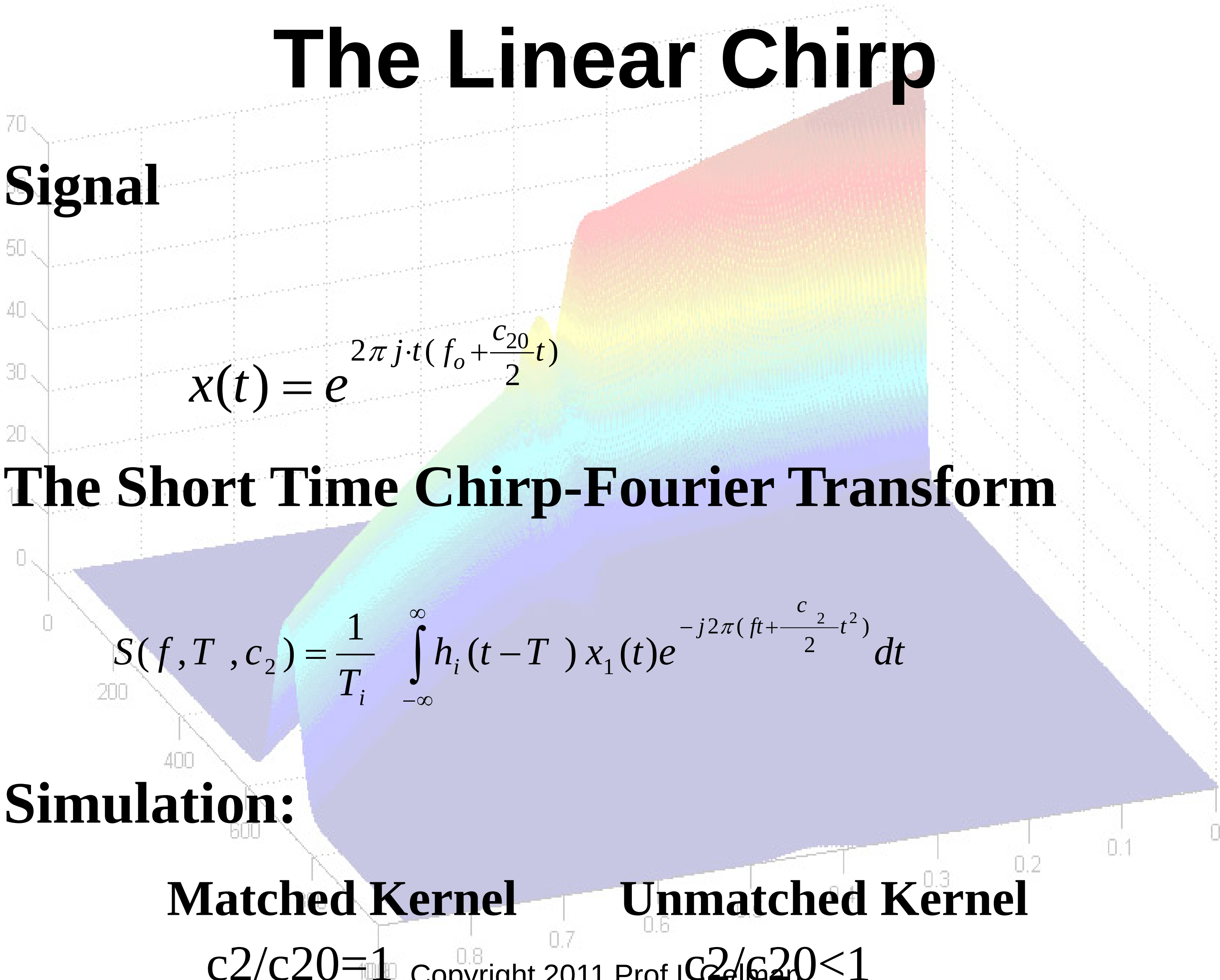
The Short Time Chirp-Fourier Transform: Adaptation

- **Results in the previous slides show the essential advantage of the proposed transform: it could be used for selective detection of the linear chirps and piece-wise chirps**

- **It could be done by the following two step adaptation procedure:**

- **to evaluate the frequency-time dependency of the chirps and chirp rates**
- **to employ the short time chirp-Fourier transform with the adapted chirp rates following adaptation conditions for the transform c2/c20=1**

The Linear Chirp



Signal

$$x(t) = e^{2\pi j \cdot t (f_o + \frac{c_{20}}{2} t)}$$

The Short Time Chirp-Fourier Transform

$$S(f, T, c_2) = \frac{1}{T_i} \int_{-\infty}^{\infty} h_i(t - T)\, x_1(t) e^{-j2\pi (ft + \frac{c_2}{2} t^2)} dt$$

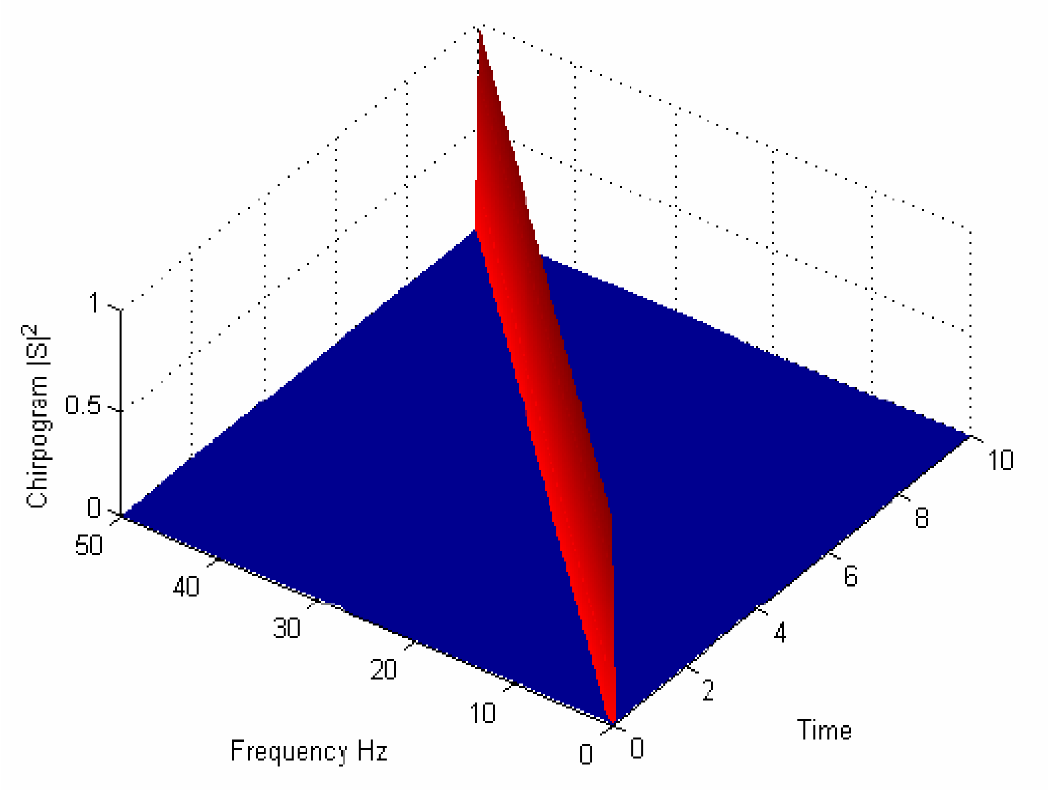
Simulation:

Matched Kernel c2/c20=1

Unmatched Kernel c2/c20<1

The Linear Chirp: the Kernel is Matched

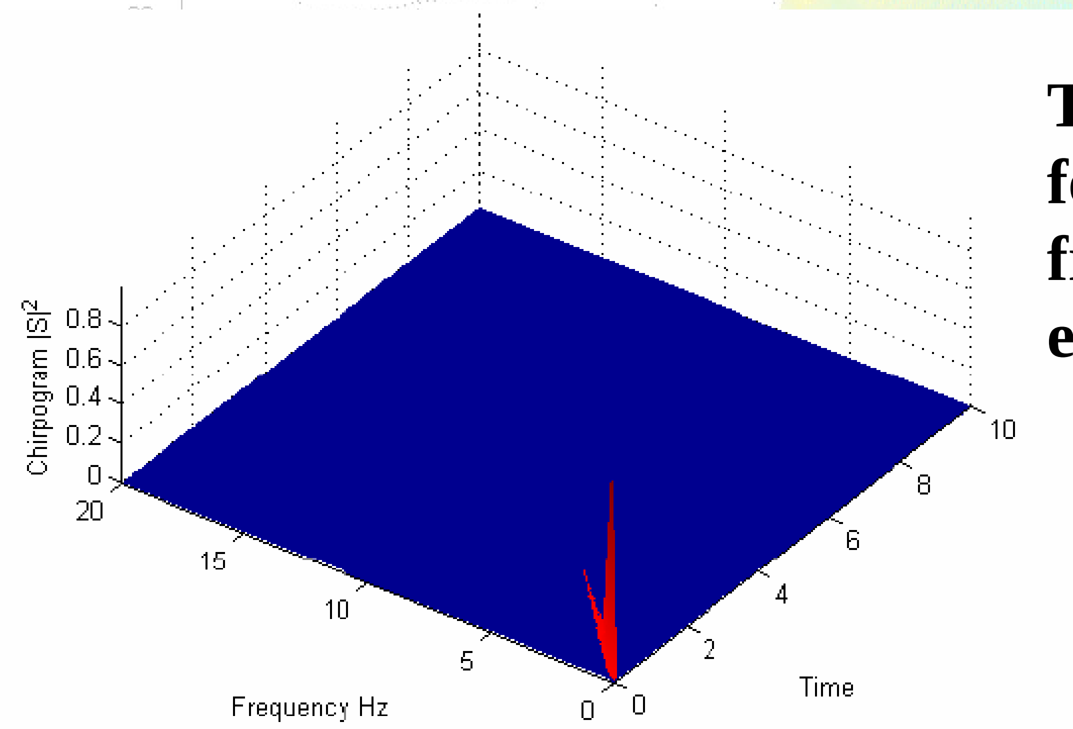
- **Chirp rate ratio c20/c2=1**
- **c20=5Hz/s**
- **Constant signal amplitude**



The chirpogram (i.e. the squared magnitude of the chirp-Fourier transform) ideally follows the instantaneous frequency without amplitude errors

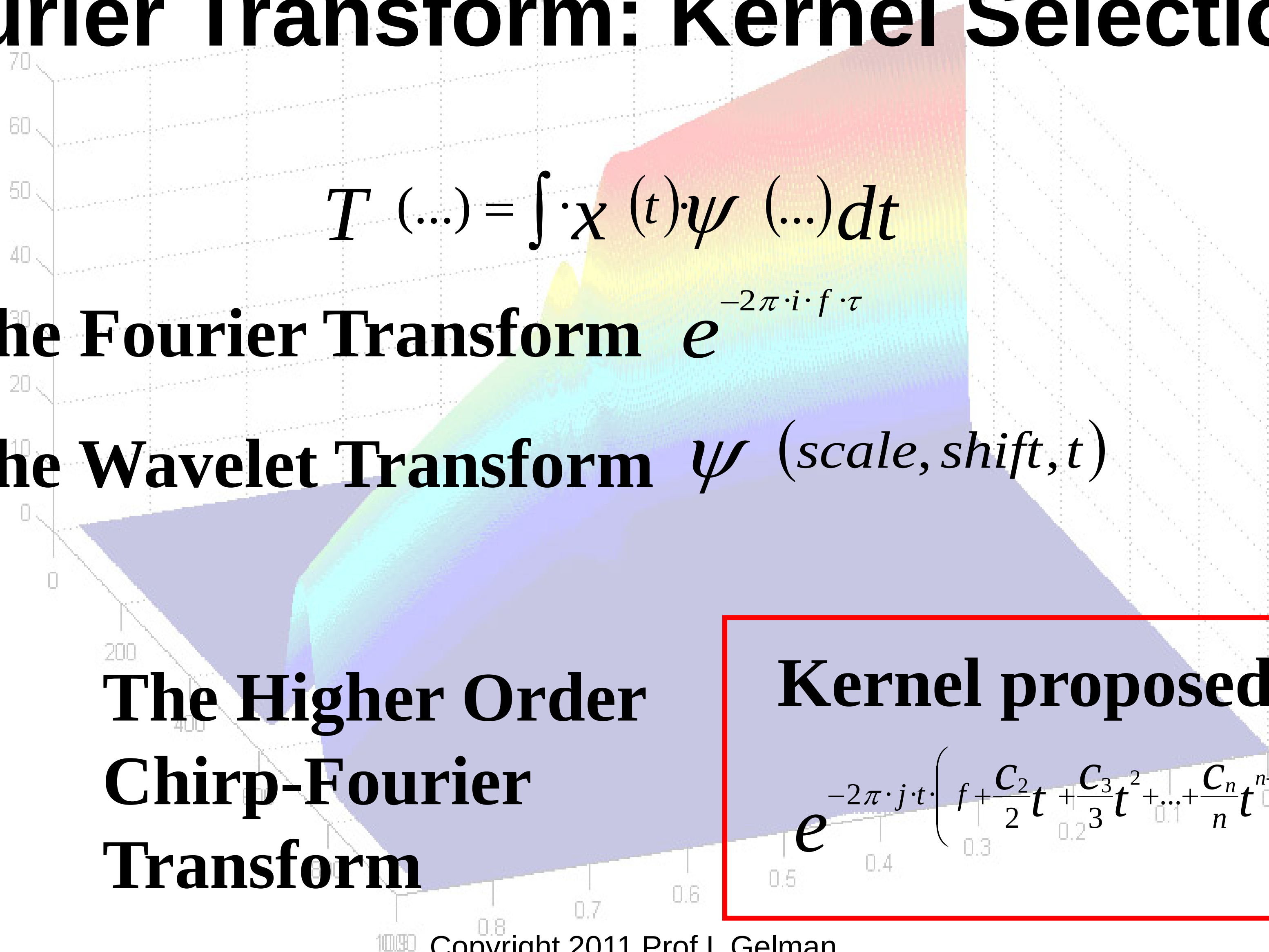
The Linear Chirp: the Kernel is Unmatched

- **Chirp rate ratio c2/c20=0.8**
- **c20=5Hz/s c2=4Hz/s**
- **Constant signal amplitude**



The chirpogram does not follow the instantaneous frequency; there are essential amplitude errors

The Short Time Higher Order Chirp-Fourier Transform: Kernel Selection



$$T(\dots) = \int \cdot x(t) \cdot \psi(\dots) dt$$

The Fourier Transform $e^{-2\pi \cdot i \cdot f \cdot \tau}$

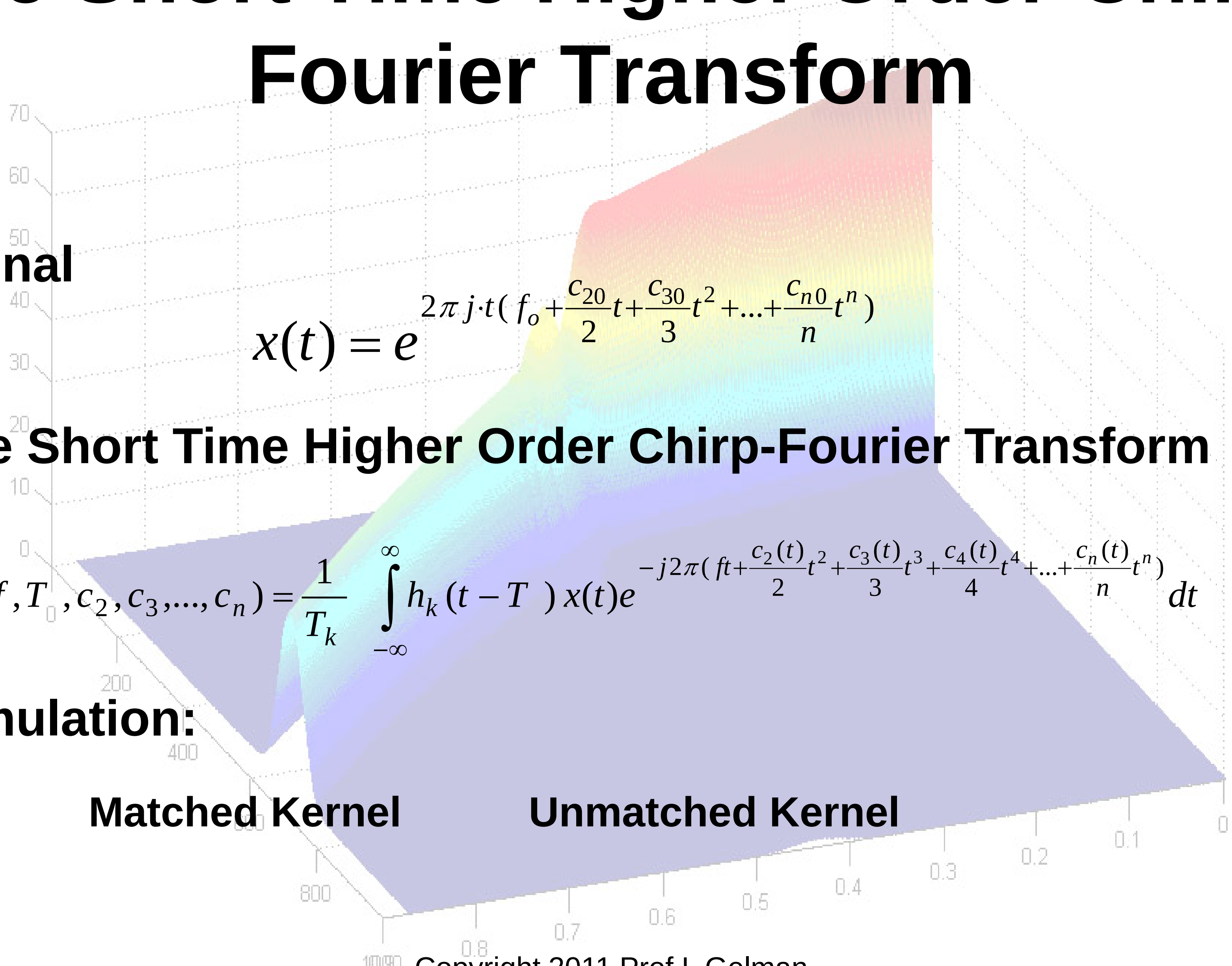
The Wavelet Transform $\psi(scale, shift, t)$

The Higher Order Chirp-Fourier Transform

Kernel proposed

$$e^{-2\pi \cdot j \cdot t \cdot \left(f + \frac{C_2}{2} t + \frac{C_3}{3} t^2 + \dots + \frac{C_n}{n} t^{n-1} \right)}$$

The Short Time Higher Order Chirp-Fourier Transform



Signal

$$x(t) = e^{2\pi j \cdot t\left(f_o + \frac{c_{20}}{2} t + \frac{c_{30}}{3} t^2 + \ldots + \frac{c_{n0}}{n} t^n \right)}$$

The Short Time Higher Order Chirp-Fourier Transform

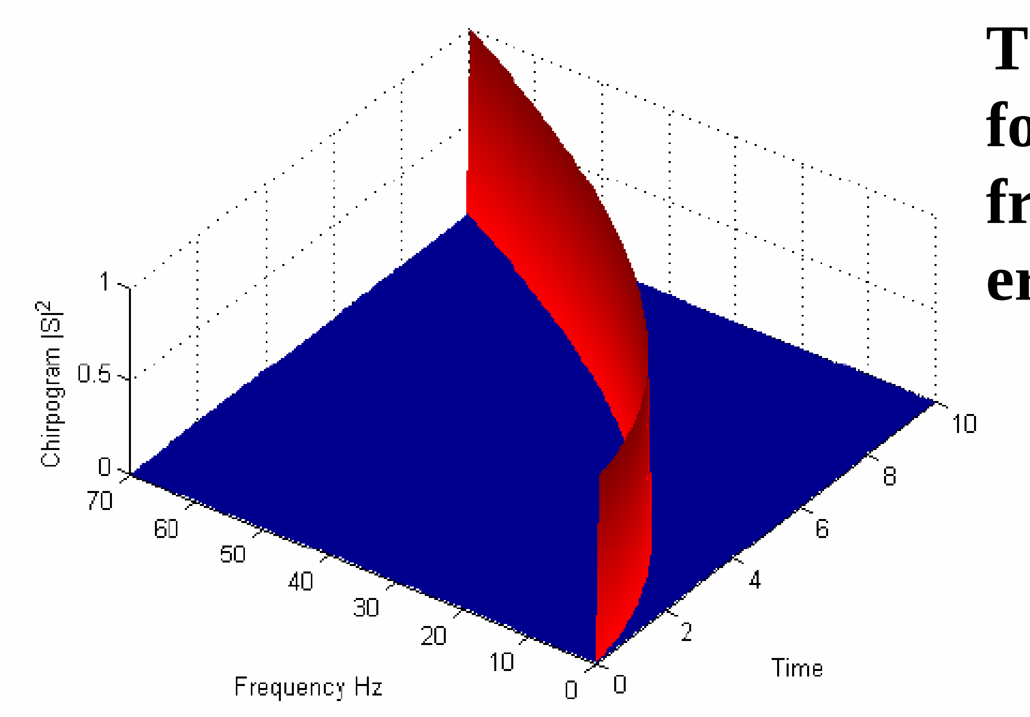
$$S(f, T, c_2, c_3, \ldots, c_n) = \frac{1}{T_k} \int_{-\infty}^{\infty} h_k(t - T)\, x(t) e^{-j2\pi\left(ft + \frac{c_2(t)}{2} t^2 + \frac{c_3(t)}{3} t^3 + \frac{c_4(t)}{4} t^4 + \ldots + \frac{c_n(t)}{n} t^n \right)} dt$$

Simulation:

Matched Kernel **Unmatched Kernel**

The Matched Kernel

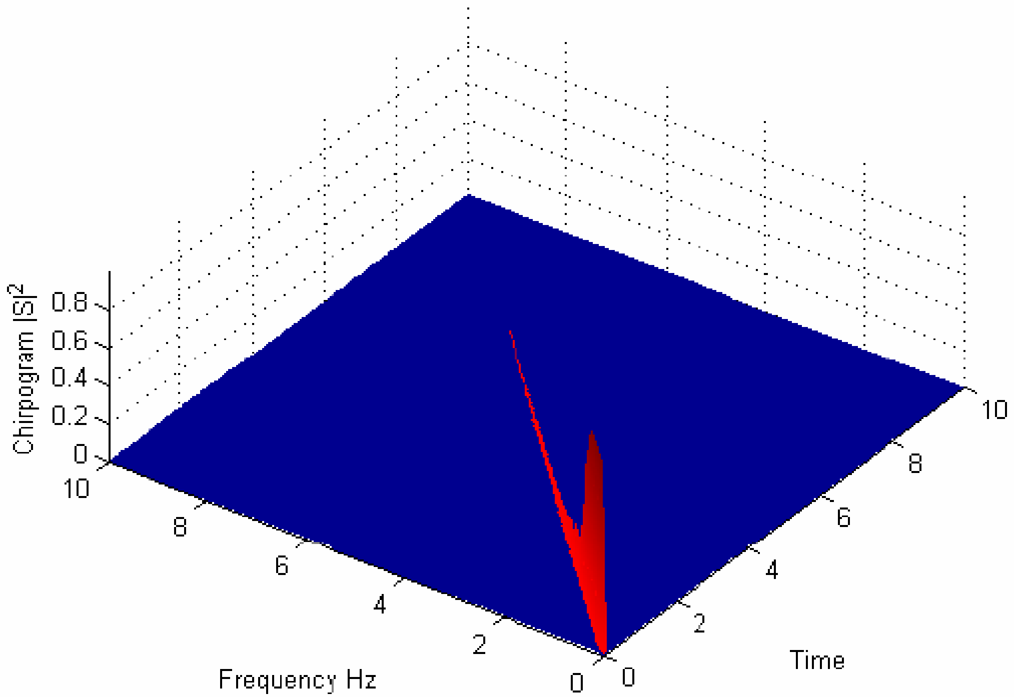
- **Signal f0=0; c20=1 Hz/s; c30=0.5 Hz/s2; c40=0.01 Hz/s3**
- **Kernel f=0; c2=1 Hz/s; c3=0.5 Hz/s2; c4=0.01Hz/s3**



The chirpogram ideally follows the instantaneous frequency without amplitude errors

The Unmatched Kernel

- Signal f0=0; c20=1 Hz/s; c30=0 Hz/s2; c40=0 Hz/s3
- Kernel f=0; c2=1 Hz/s; c3=0.5 Hz/s2; c4=0.01 Hz/s3



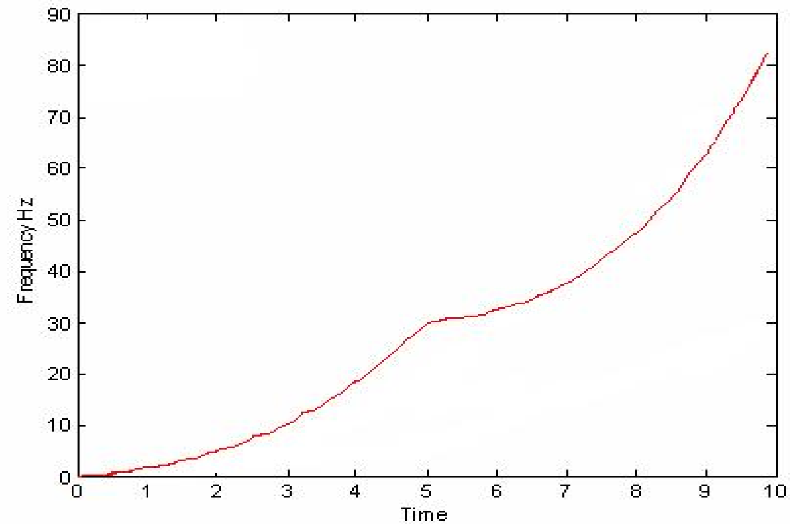
The chirpogram does not follow the instantaneous frequency; there are essential amplitude errors

THE HIGHER ORDER PIECE-WISE CHIRP: A SIGNAL AND TRANSFORM KERNEL

SIGNAL CHARACTERISTICS:

The piece-wise higher order chirp is characterized by:

- **C20: the chirp rate**
- **C30: the frequency acceleration**
- **C40, C50, ...: the higher order parameters**



THE KERNEL CHARACTERISTICS:

$$x(t) = e^{2\pi j \cdot t \left(f_o + \frac{c_{20}}{2} t + \frac{c_{30}}{3} t^2 + \ldots + \frac{c_{n0}}{n} t^n \right)}$$

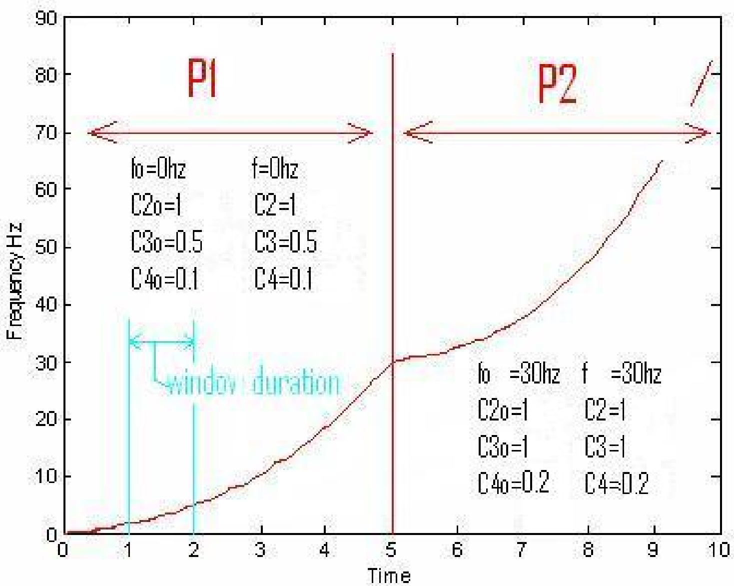
The kernel is characterized by

- **C2: the chirp rate of the transform**
- **C3: the frequency acceleration of the transform**
- **C4, C5, ...: the higher order parameters of the transform**

Kernel proposed

$$e^{-2\pi \cdot j \cdot t \cdot \left(f + \frac{C_2}{2} t + \frac{C_3}{3} t^2 + \ldots + \frac{C_n}{n} t^{n-1} \right)}$$

THE HIGHER ORDER PIECE-WISE CHIRP: KERNEL IS MATCHED

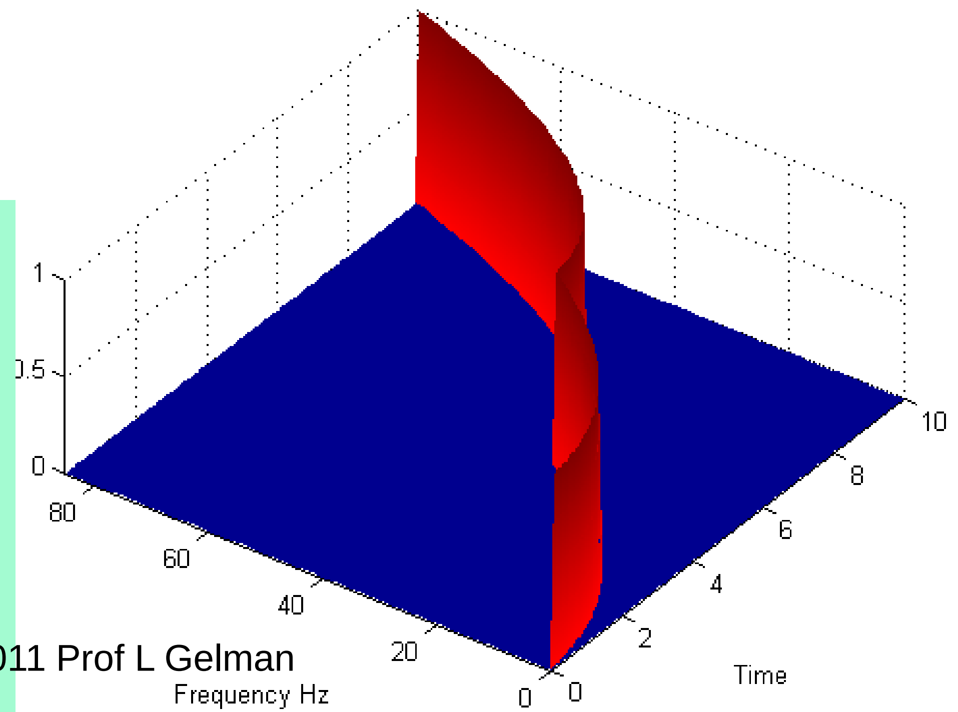


PARAMETERS OF THE SIGNAL AND KERNEL ARE MATCHED

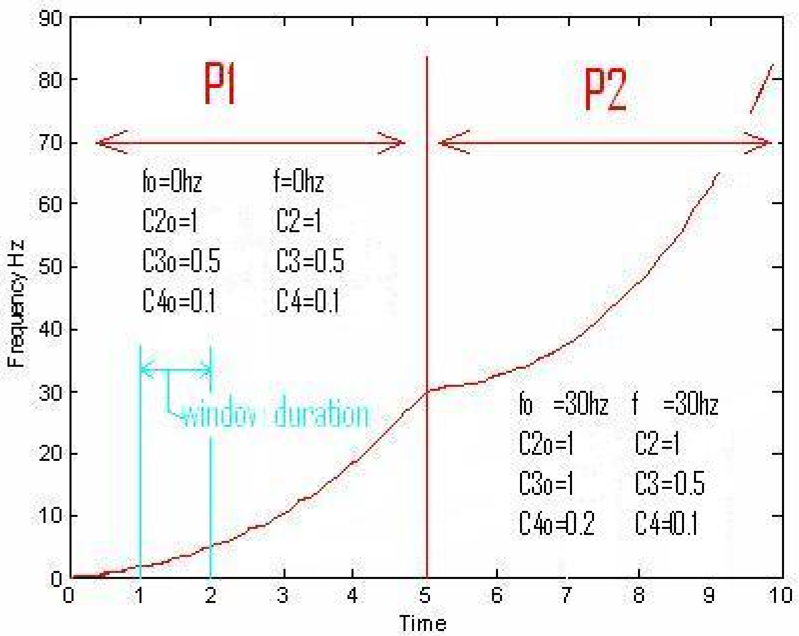
- **Sampling frequency is 200 Hz**
- **P1=0-5s**
- **P2=5-10s**

ONE CAN EVALUATE THE PIECE-WISE FREQUENCY-TIME DEPENDENCY OF THE SIGNAL AND PARAMETERS FOR EACH PART OF THIS DEPENDENCY AND

EMPLOY THE HIGHER ORDER CHIRP FOURIER TRANSFORM WITH THE ADAPTED PARAMETERS ON EACH SIGNAL PART.

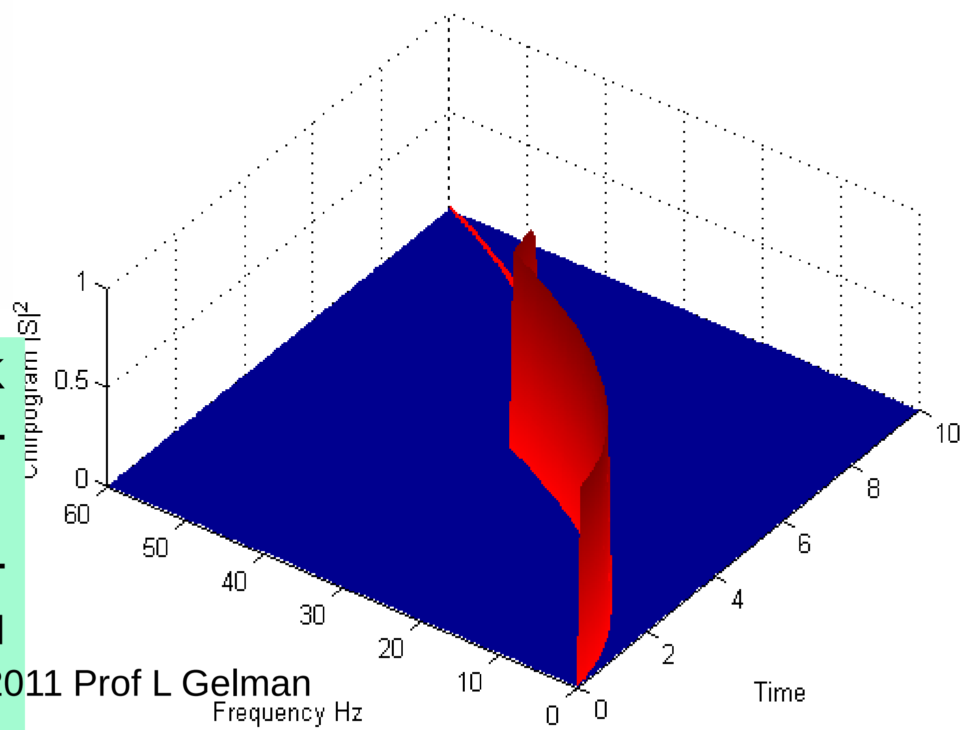


THE HIGHER ORDER PIECE-WISE CHIRP: KERNEL IS UNMATCHED



PARAMETERS OF THE SIGNAL AND KERNEL ARE UNMATCHED FOR PERIOD 2

- Sampling frequency is 200 Hz
- P1=0-5s
- P2=5-10s



- THE TRANSFORM DOES NOT WORK PROPERLY WHEN KERNEL AND SIGNAL ARE UNMATCHED

- THE SIGNAL AMPLITUDE ON THE FIRST PERIOD IS ESTIMATED WITH HIGH ACCURACY

The Adaptive Short Time Higher Order Chirp-Fourier Transforms

- **Results in the previous slides show the essential advantage of the proposed transforms: they could be used for selective detection of the higher order chirps**

- **It could be done by the following on-line wo step adaptation procedure:**

- **to evaluate the frequency-time dependency of the higher order chirp and parameters of this dependency**

- **to employ the short time higher order chirp-Fourier transforms with the adapted transform parameters: c2/c20=1, c3/c30=1, c4/c40=1… cn/cn0=1**