Part 1 of the PSD

The Power Spectral Density (PSD)

Prof L. Gelman

Direct Method of PSD Estimation

- **The oldest and the most popular approach, a direct method, involves computing the Fourier transform of signal and then**

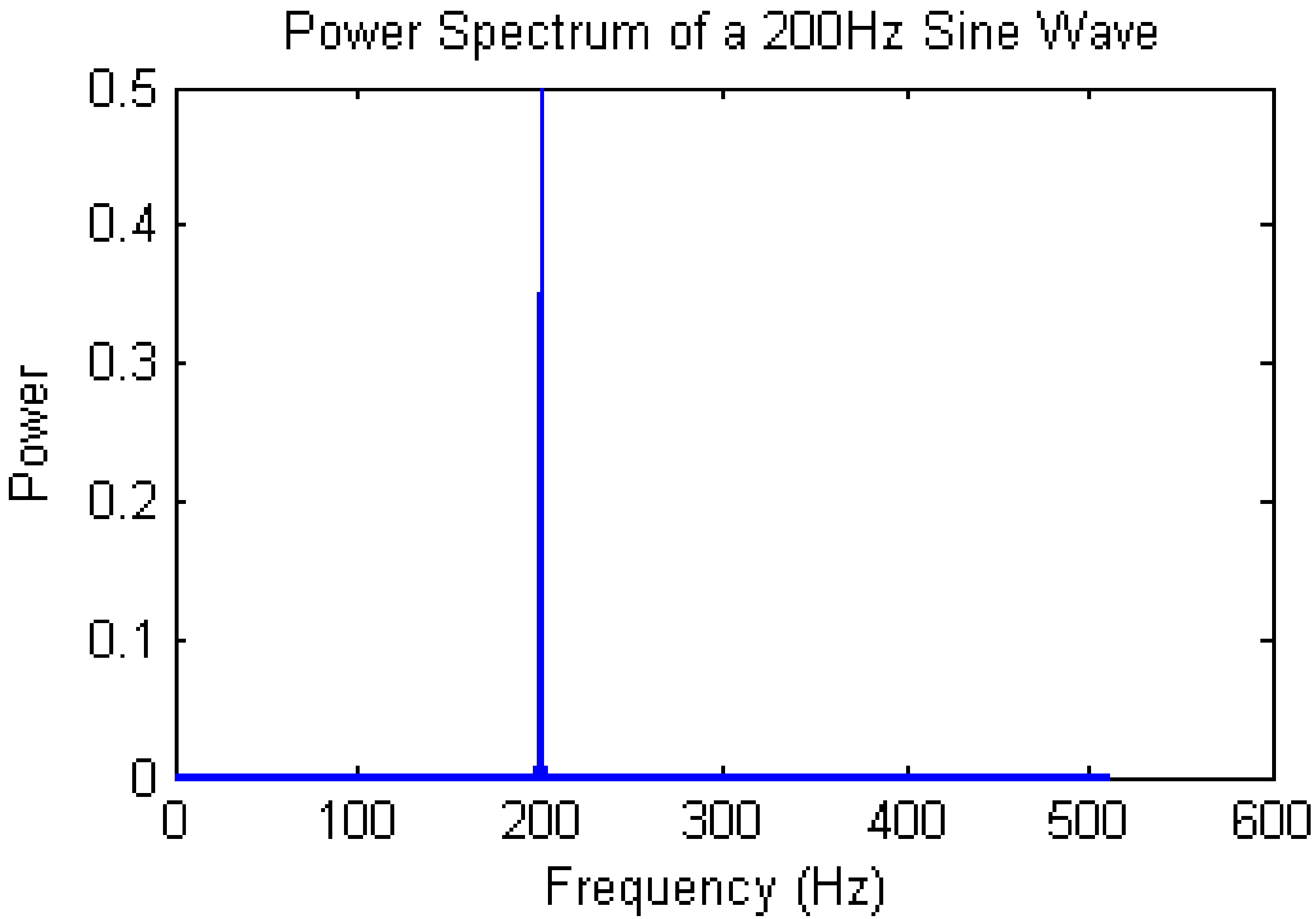
$$\hat{S}_{xx}^{(d)} = \frac{1}{2T}\left|\int_{-T}^{T} x(t)e^{-i2\pi ft}\,dt\right|^2$$

- **The magnitude squared of the Fourier transform (except for a constant multiplier) is the power spectral density**

- **This well-known expression is called the *periodogram***

- **The popularity of the approach lies in the ready availability of the fast Fourier transform to perform the Fourier transform computation**

PSD of Sine Wave



Symmetry Properties of the Fourier Transform

Suppose that the signal is real. Then it easily follows that for the appropriate Fourier transforms

$$X_R(-\omega) = X_R(\omega) \qquad \textbf{(even symmetry)}$$

$$X_I(-\omega) = -X_I(\omega) \qquad \textbf{(odd symmetry)}$$

$$X^*(\omega) = X(-\omega)$$

Symmetry Properties of the PSD

- **As a consequence, the modulus, phase of the DFT and power spectral density also posses the symmetry properties:**

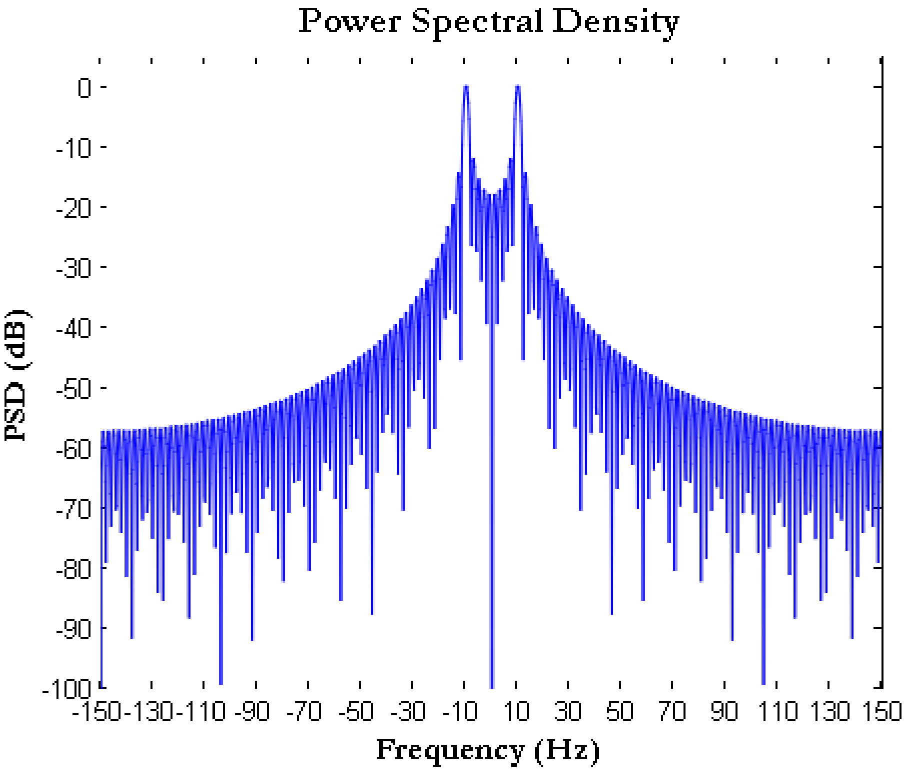
$$|X(-\omega)| = |X(\omega)|$$ **(even symmetry)**

$$\angle X(-\omega) = -\angle X(\omega)$$ **(odd symmetry)**

$$S_{xx}(-\omega) = S_{xx}(\omega)$$ **(even symmetry)**

- **The power spectral density of a real signal has the even symmetry**

Symmetry Properties of the PSD



Symmetry Properties of the PSD

- From these symmetry properties, we conclude that the frequency range of real digital signals **can be limited further to the range** $0 \le \omega \le \pi$ **(i. e . half of the period)**

- Indeed, if we know power spectral density in the range $0 \le \omega \le \pi$ we can determine it for the range $-\pi \le \omega \le 0$ using the symmetry property given above

- Therefore, the frequency-domain description of a real discrete-time signal **is completely specified by its power spectral density in the frequency range** $0 \le \omega \le \pi$

One-Sided Power Spectral Density

The one-sided estimate of the power spectral density is given by

$$S_{xx\ onesided} = 2S_{xx} \qquad f \geq 0$$