Signal Classification

Prof. L. Gelman

Introduction

- ❑ **Real life signals are generally analogue**

- ❑ **Main limitations of analogue signal processing are:**

- **restricted accuracy**
- **sensitivity to noise**
- **restricted dynamic range**
- **poor repeatability due to component variations**
- **inflexibility to alter or adjust the processing functions**
- **limited speed**
- **high cost of storage for analogue signals**

Analogue vs. Digital Signal Processing

The main advantages of digital vs. analogue signal processing can be listed as follows:

- ***Flexibility***

  Analogue systems require hardware redesign if changes are needed; whereas, a digital system can be re-programmed

- ***Accuracy***

  The accuracy of a digital system can be controlled by word length, floating-point/fixed point arithmetic, etc. Analogue accuracy is determined by the tolerances of components

- ***Storage***

  Digital signals are easily stored via CDs, RAM, etc, therefore allowing of-line processing

Main Operations in Digital Systems

The main operations of many digital systems include:

- **converting analogue signals into a sequence of digital binary numbers, which requires both sampling and analogue-to-digital (A/D) conversion**
- **performing numerical manipulations in digital processor**
- **converting the digital information back to analogue signals by digital-to-analogue (D/A) conversion**

Digital Signals

- **A continuous-time signal with continuous amplitude is called an *analogue* signal**

- **A discrete-time signal with discrete-valued amplitudes represented by a finite number of digits is referred to as a *digital* signal**

- **A discrete-time signal with continuous-valued amplitudes is called a discrete-time *signal***

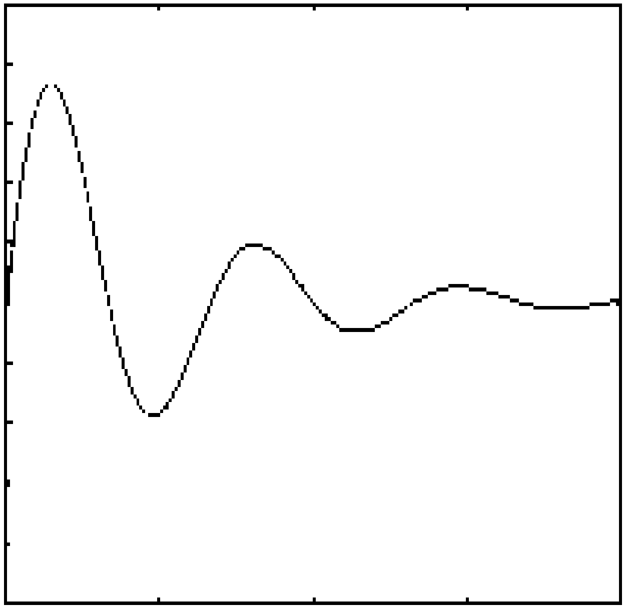
- **Each member of a discrete-time signal is called a *sample***

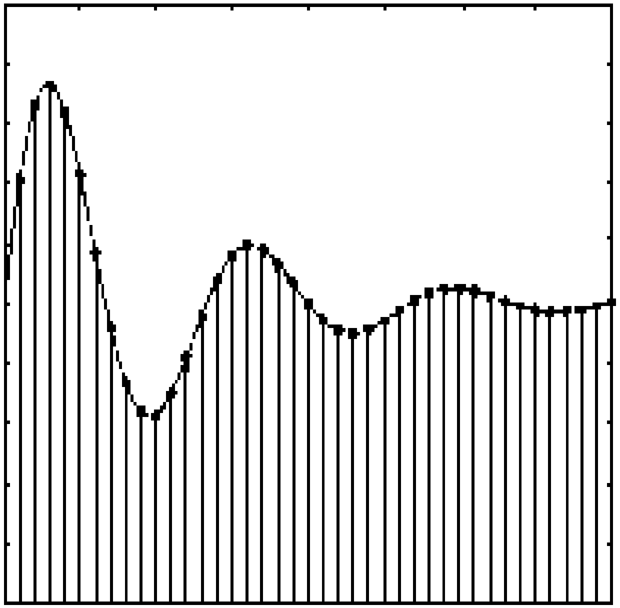
- **The usual signals in a digital signal processing are digital signals**

Digital Signals

- **In digital signal processing, signals are represented as sequences of samples**
- **A sample value of a typical discrete-time signal is denoted as $x[n]$ with the sample number n being an integer in the range $-\infty$ and ∞, in practice from 0 to *Nma*x**
- **A sequence $x[n]$ is generated by sampling and quantization of continuous time signals**

The Discrete-Time Signals: Sampling

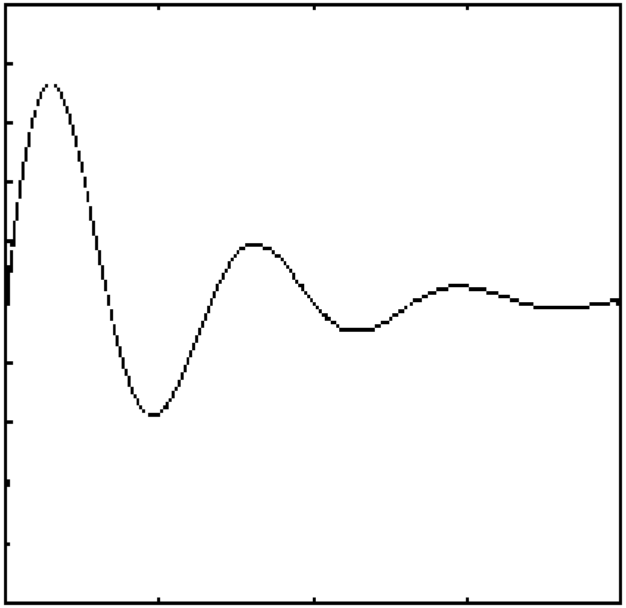


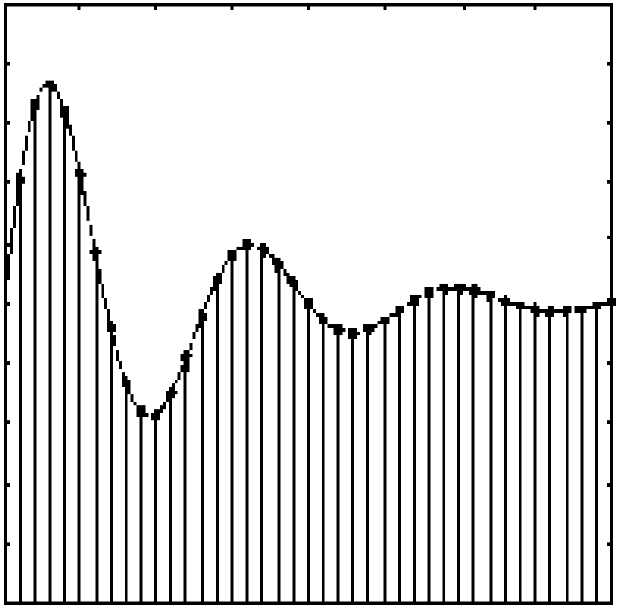


- **Consider an analogue signal *x(t)* that can be viewed as a continuous function of time, as shown above**

- **We can represent this signal as a discrete-time signal by using values of *x(t)* at intervals of nT_s to form *x[n]*, as shown above**

The Discrete-Time Signals: Sampling



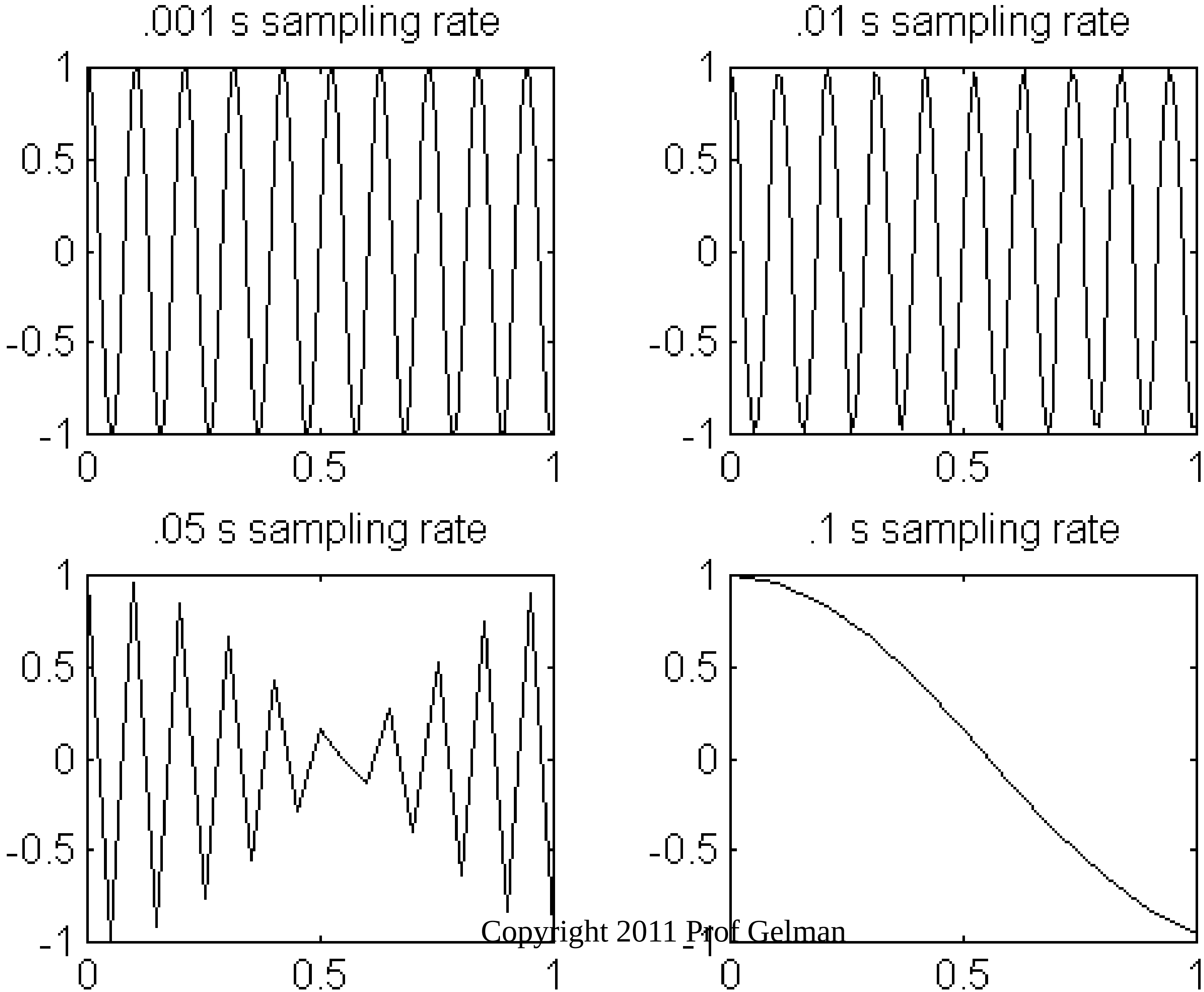


- We are "grabbing" points from the function *x(t)* at regular intervals of time, T_s , called the ***sampling period.***

- It is usual to specify a **sampling rate or frequency** f_s rather than the sampling period.

- The sampling frequency is given by $f_s = 1/T_s$, where f_s is in Hertz.

The Discrete-Time Signals: Sampling



Deterministic and Random Digital Signals

- **A signal that can be determined by a mathematical expression or rule, or table, is called a *deterministic signal***
- **A signal that is generated in a random fashion and cannot be predicted ahead of time is called a *random signal***
- ***Example: sinusoid signal with deterministic and random phase***

Periodic Digital Signals

- **A sequence $\tilde{x}[n]$ satisfying**

$$\tilde{x}[n] = \tilde{x}[n + kN] \qquad \textbf{for all} \quad n$$

is called a periodic sequence with a period N where N is a *positive* integer and k is any integer

- **The smallest value of N for which this expression holds is called the *fundamental period***

Basic Discrete-Time Signals

There are a number of basic signals that appear often and play an important role in signal processing. These signals are defined below

1. **The unit sample (impulse) sequence is defined as**

$$\delta(n) = \begin{cases} 1, & for \quad n = 0 \\ 0, & for \quad n \neq 0 \end{cases}$$

2. **The unit step signal is defined as**

$$u\ (n) = \begin{cases} 1, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$

3. **The unit ramp signal is defined as**

$$u_r(n) = \begin{cases} n, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$

Discrete-Time Exponential Signal

4. The exponential signal is defined as

$$x(n) = a^n$$

- **If the parameter** a **is real, then** $x(n)$ **is a real signal**

- **If the parameter** a **is complex, it can be expressed as**

$$a = re^{i\theta}$$

where r **and** θ **are the modulus and the phase**

- **Hence, we can express complex valued exponential signal as**

$$x(n) = r^n e^{i\theta n} = r^n(\cos\theta n + i\sin\theta n)$$

Discrete-Time Sinusoids

5. A discrete-time sinusoidal signal may be expressed as

$$x[n] = A\cos(\omega n + \theta), -\infty \leq n \leq \infty$$

where A is the amplitude; ω is the circular frequency in radians per sample, and θ is the phase in radians

- **In contrast to continuous-time sinusoids, the discrete-time sinusoids are characterized by the following properties**

Property 1 of the Discrete-Time Sinusoids

A discrete-time sinusoid is periodic only if its frequency is a rational number

- **For a sinusoid with frequency f_0 to be periodic, we should have**

$$\cos\left[2\pi f_0 (N+n)+\theta\right]=\cos(2\pi f_0 n+\theta)$$

- **The relation is true if and only if there exists an integer such that**

$$2\pi f_0 N = 2\pi k$$

or, equivalently,

$$f_0 = \frac{k}{N}$$

The Fundamental Period of the Discrete Sinusoid

- **To determine the fundamental period of a periodic sinusoid, we express its frequency as in the above slide and cancel common factors so that k and N are prime**

- **Then, the fundamental period of the sinusoid *is equal to* N**

- **A small change in frequency can result in a large change in period**

Property 2 of the Discrete-Time Sinusoids

Discrete-time sinusoids whose circular frequencies are separated by an integer multiple of 2π are identical

- **To prove this property, we consider the sinusoid** $\cos(\omega n + \theta)$

- **It easily follows that**

$$\cos\left[(\omega_0 + 2\pi)n + \theta\right] = \cos(\omega_0 n + 2\pi n + \theta) = \cos(\omega_0 n + \theta)$$

- **As a result, all sinusoidal sequences**

$$x_k(n) = A\cos(\omega_k n + \theta) \quad k = 0,1,2,\ldots$$

where $\omega_k = \omega_0 + 2k\pi,\ \ 0 \le \omega_0 \le 2\pi$ **are identical**

Property 2 of the Discrete Time Sinusoids

- **On the other hand, sinusoids with circular frequencies in the range** $-\pi \le \omega \le \pi$ **are distinct**

- **Consequently, discrete-time sinusoidal signals with frequencies in the range** $-\pi \le \omega \le \pi$ **are unique**

- **Any sequence resulting from a sinusoid with a frequency** $|\omega| \ge \pi$ **is identical to a sequence obtained from a sinusoid with frequency** $-\pi \le \omega \le \pi$

- **Because of this similarity, we call the sinusoid having the frequency** $|\omega| \ge \pi$ **an** ***alias*** **of the corresponding sinusoid with frequency** $-\pi \le \omega \le \pi$

Property 2 of the Discrete-Time Sinusoids

- **Hence the frequency range for discrete-time sinusoids is finite:** 2π

- **Usually, we choose the range** $0, 2\pi$ **or** $-\pi, \pi$ **which we call the fundamental range**

- **If we use the range** $-\pi, \pi$ **we employ negative frequencies**

Energy and Power Signals

- **The energy of a signal is defined as**

$$E = \sum_{n=-\infty}^{\infty} \left| x(n) \right|^2$$

- **The energy of a signal can be finite or infinite**

- **If E is finite, then a signal is called *an energy signal***

- **Many signals that possess infinite energy have a finite average power**

Energy and Power Signals

- **The average power of a signal is defined as**

$$P = \lim \frac{1}{2N+1} \sum_{n=-N}^{N} \left| x(n) \right|^2 \qquad N \to \infty$$

- **If we define the signal energy over the finite interval as**

$$E_N = \sum_{n=-N}^{N} \left| x(n) \right|^2$$

then we can express the average power of a signal as

$$P = \lim \frac{1}{2N+1} E_N \qquad N \to \infty$$

Obviously, if E_N **is finite , then** $P = 0$

Energy and Power Signals

- **On the other hand, if E_N is infinite, the average power may be either finite or infinite**

- **If the average power is finite (and nonzero), the signal is called a *power signal***

Ergodic Digital Signals

- **Although we have characterized a random sequence in terms of statistical averages, in practice, a finite portion of a single realization of a sequence is available, from which estimation of the statistical properties must be made**

- **Such an approach can lead to meaningful results only if the following ergodicity condition is satisfied:**

The random digital signal is ergodic if time averages obtained from a single realization are equal to the statistical (ensemble) averages