School of Computing & Engineering

(Engineering and Technology)

University of HUDDERSFIELD

NME3523

Signal Analysis & Processing

Date: May 2014

Time allowed: 3 hours

Instructions to Candidates:

This is an unseen examination.

Candidates should answer 4 out of 6 questions. All questions are marked out of *25*.

Materials provided: Data sheet and Table of transforms attached at the end of the paper.

Materials allowed: None

A scientific calculator may be used in this exam.

Unannotated paper versions of general bi-lingual dictionaries only may be used by overseas students whose first language is not English. Subject-specific bi-lingual dictionaries are not permitted.

Access to any other materials is not permitted.

Turn Over

Should a candidate have a query about this paper and the module leader is not in attendance, please call 01484 472150 or 472450

Question 1

(i) The following command when entered into MATLAB to give the transfer function for a filter produces the results as shown.

>> [b,a] = butter(4,6,'s')

b = 0   0   0   0   1.296e+3

a = 1   1.5679e+1   1.2291e+2   5.6444e+2   1.296e+3

What is the filter type and order? What is the -3dB frequency in Hz?

Write down the filter transfer function.

Sketch gain and phase Bode plots of the filter transfer function (show the gain 'roll off' in dB/decade).

[10 marks]

(ii) A simple passive high pass filter consists of a 100nF capacitor and a $10\,\mathrm{k\Omega}$ resistor. The output from the filter appears across a very high impedance load which draws negligible current. The input signal to the filter consists of three components (a) a 1V dc component; (b) a sine wave of 25Hz and $\pm 2\mathrm{V}$ peak to peak; and (c) a sine wave of 500Hz and $\pm 3\mathrm{V}$ peak to peak.

What are the amplitudes of the three components at the output from the filter?

[8 marks]

What is the attenuation of the 25Hz component in dB?

[3 marks]

What is the phase of the 25Hz component at the output relative to its phase at the input?

[4 marks]

Turn Over

Question 2

The input to an underdamped second order system undergoes a step change from 0 to *A* at time $t = 0$. The Laplace Transform of the resultant system output $X(s)$ is given by

$$X(s) = \frac{A}{Ms\{s^2 + 2c\omega_n s + \omega_n^2\}}$$

where $c < 1$.

Giving all of your working, show that the system output $x(t)$ can be expressed as

$$x(t) = \frac{A}{K}\left\{1 - \frac{e^{-\omega_n ct}\sin(\omega_n t\sqrt{1-c^2} + \alpha)}{\sqrt{1-c^2}}\right\}$$

where $\alpha = \cos^{-1}(c)$ and where $\omega_n = \sqrt{\frac{K}{M}}$

[25 marks]

Turn Over

Question 3

A signal $f(t)$ is periodic with period 1 second. For $0 \le t \le 1$ seconds, the signal is defined as $f(t) = e^t$.

Express $f(t)$ as an infinite Fourier series.

[Hint: you must calculate a_0 and <u>both</u> the a_k and b_k terms for the correct solution].

[25 marks]

Turn Over

Question 4

An open loop system is at rest with the input and the output both equal to zero. The system has a transfer function $G(s)$ where:

$$G(s) = \frac{K}{1+\tau\, s}$$

An input pulse $x(t)$ is applied to the system such that:

$x(t) = 0$ for $-\infty < t < -T_0$

$x(t) = -V$ for $-T_0 \le t < 0$

$x(t) = +V$ for $0 \le t < T_0$

$x(t) = 0$ for $T_0 \le t < \infty$

Show that the Fourier Tranform $Y(f)$ of the output from the system is given by:

$$Y(f) = \frac{VK(\cos(2\pi f T_0) - 1)(2\pi f \tau + j)}{\pi f + 4\pi^3 f^3 \tau^2}$$

[25 marks]

Turn Over

Question 5

An analogue filter has the transfer function $g(s) = \frac{12}{4s+1}$. What is the z-transfer function of the equivalent digital filter?

What is the sampling interval T for this equivalent digital filter?

[13 marks]

The first 5 input values to the digital filter are: 1.0, 1.1, 1.2, 1.2 and 1.1. The first output value from the digital filter is 12.0.
What are the second, third, fourth and fifth output values from the digital filter?

[12 marks]

Turn Over

Question 6

(i) With the aid of diagrams describe the design of a cross correlation flow meter that could be used for measuring the mean velocity of a vertical 'air-in-water' two phase flow. Your description should also include any relevant electronic circuitry. Explain why the signal from the downstream sensor would not be expected to be identical to a delayed version of the signal from the upstream senor.

[10 marks]

(ii) A 'two phase' cross correlation flow meter consists of an upstream sensor **X** and a downstream sensor **Y** separated by an axial distance of 0.1m. The sampled outputs from **X** and **Y,** and the times at which the samples were taken, are given in 'TABLE QUESTION 6' below.

Calculate and plot the cross correlation function for the sampled outputs from **X** and **Y**. What is the flow velocity (in ms^{-1}) that is obtained from this cross correlation flow meter?

[15 marks]

Time (seconds)	Output from **X**	Output from **Y**
0	**-4**	**9**
0.1	**-17**	**-21**
0.2	**1**	**0**
0.3	**3**	**19**
0.4	**-11**	**-9**
0.5	**12**	**12**
0.6	**12**	**0**
0.7	**0**	**-13**
0.8	**3**	**8**
0.9	**2**	**6**
1.0	**-2**	**-6**
1.1	**7**	**6**
1.2	**-6**	**12**
1.3	**22**	**-1**
1.4	**-1**	**-5**
1.5	**1**	**3**
1.6	**11**	**-7**
1.7	**1**	**14**
1.8	**-1**	**-10**
1.9	**-8**	**24**

<u>TABLE QUESTION 6</u>

Turn Over

DATA SHEET

Fourier Series

If $f(t)$ is a periodic waveform with period T then $f(t)$ can be expressed in the form

$$f(t) = a_0 + \sum_{k=1}^{\infty}\left\{a_k \cos\left(\frac{2\pi kt}{T}\right) + b_k \sin\left(\frac{2\pi kt}{T}\right)\right\}$$

where;

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

and;

$$a_k = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi kt}{T}\right)dt$$

and;

$$b_k = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi kt}{T}\right)dt$$

	$f(t)$	$F(s)$	$f(k)$	$F(z)$
1	Unit impulse	1	$\delta(k)$	1
2	Unit step	$\frac{1}{s}$	$u(k)$	$\frac{z}{z-1}$
3	Unit ramp t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
4	t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2z(z+1)}{(z-1)^3}$
5	t^3	$\frac{6}{s^4}$	$(kT)^3$	$\frac{T^3z(z^2+4z+1)}{(z-1)^4}$
6	e^{-at}	$\frac{1}{s+a}$	$(e^{-aT})^k$	$\frac{z}{z-e^{-aT}}$
7	$1-e^{-at}$	$\frac{a}{s(s+a)}$	$1-(e^{-aT})^k$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
8	$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$kT(e^{-aT})^k$	$\frac{Tz\,e^{-aT}}{(z-e^{-aT})^2}$
9	$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$(1-akT)(e^{-aT})^k$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$
10	$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$	$(e^{-aT})^k-(e^{-bT})^k$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
11	Item 6 with $e^{-aT}=c$		c^k	$\frac{z}{z-c}$
12	Item 8 with $e^{-aT}=c$		kTc^k	$\frac{kTz}{(z-c)^2}$
13	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	$\sin k\omega T$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$
14	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	$\cos k\omega T$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$
15	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$(e^{-aT})^k\sin k\omega T$	$\frac{z\,e^{-aT}\sin\omega T}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2aT}}$
16	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$(e^{-aT})^k\sin k\omega T$	$\frac{z(z-e^{-aT}\cos\omega T)}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2aT}}$
17	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$	$\sinh k\omega T$	$\frac{z\sinh\omega T}{z^2-2z\cosh\omega T+1}$
18	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$	$\cosh k\omega T$	$\frac{z(z-\cosh\omega T)}{z^2-2z\cosh\omega T+1}$

Note: T is the sampling period.

End of Exam Paper

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