PHYSICS 272
Electric & Magnetic Interactions

Lecture 21 (4/3/2013)
Magnetic force [EMI 21]

Magnetic force on a moving charge

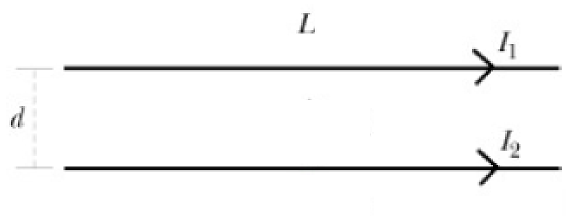
$$\vec{F}_{mag} = q\,\vec{v} \times \vec{B}$$

Magnetic force on a current-carrying wire

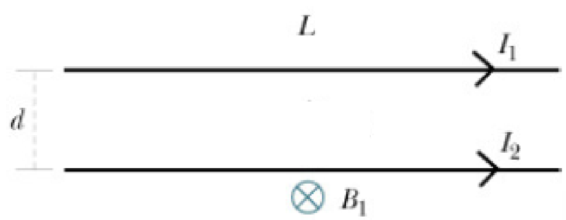
$$\Delta\vec{F} = I\,\Delta\vec{\ell} \times \vec{B}$$ Short wire

$$\vec{F} = I\,\vec{L} \times \vec{B}$$ Long wire

Application: Forces between parallel wires

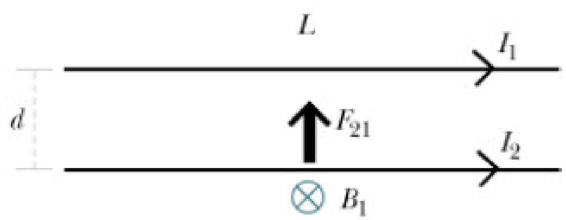


Magnetic field due to upper wire



$$B_1 \approx \frac{\mu_0}{4\pi}\frac{2I_1}{d}$$

Magnetic force on lower wire



$$\vec{F} = I\,\vec{L} \times \vec{B}$$

$$F_{21} = I_2 L B_1 \sin 90^\circ = I_2 L\left(\frac{\mu_0}{4\pi}\frac{2I_1}{d}\right) = \frac{\mu_0 I_1 I_2 L}{2\pi d}$$

Attractive if current same direction;
Repulsive if current opposite direction;

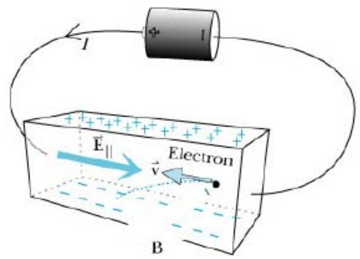
The Hall Effect

- *Work done as a graduate student, 1879*

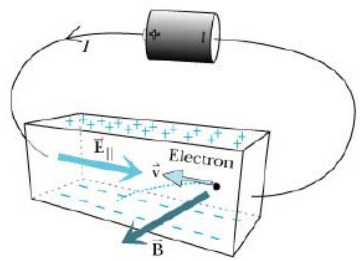
Edwin H. Hall
1855 - 1938

Klaus von Klitzing (1985) : integer quantum Hall effect
Robert B. Laughlin, Horst L. Stormer, Daniel C. Tsui (1998): fractional quantum Hall effect

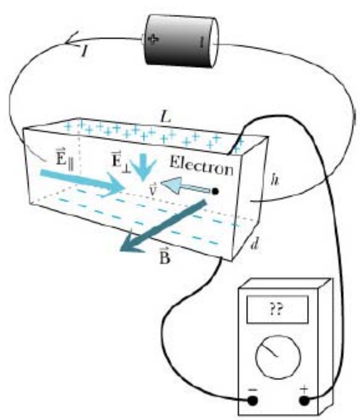
The Hall Effect



Initially, there is a steady state current.



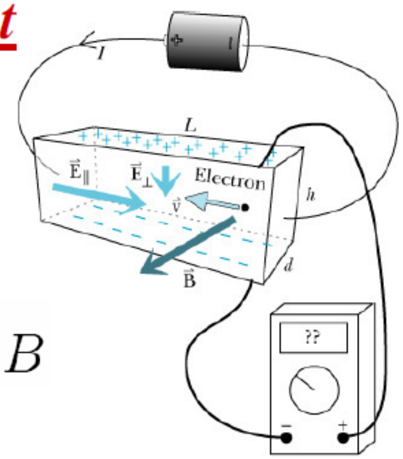
A B-field is applied in a direction perpendicular to the current. The mobile charge carriers are deflected by the magnetic force.



Eventually, a new steady state is established when the induced E-field exerts a force equal and opposite to the magnetic force.

Often easier to just think about force on current

Consequences of the Hall Effect



Eventually, a new steady state is established when the induced E-field exerts a force equal and opposite to the magnetic force.

$$q\,\vec{E}_\perp = q\,\vec{v}\times\vec{B} \qquad \Rightarrow \qquad E_\perp = \overline{v}\,B$$

The “Hall Voltage” is $\Delta V_{Hall} = E_\perp\,h$

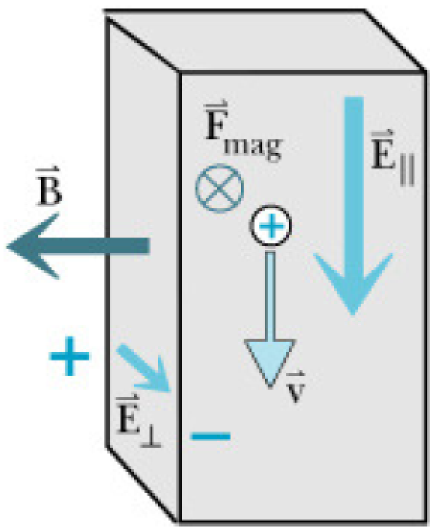
The carrier density?

$$\Delta V_{Hall} = E_\perp\,h = v\,B\,h = \left(\frac{I}{|q|\,n\,A}\right)(B\,h)$$

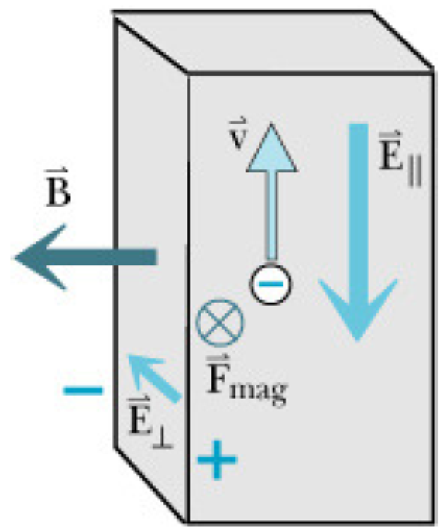
$$\boxed{n = \frac{I\,B\,h}{|q|\,\Delta V_{Hall}\,A}}$$

Consequences of the Hall Effect

The sign of the Hall voltage let's us determine the sign of the mobile charge carriers. Surprise: for some materials, the carriers are positive!!

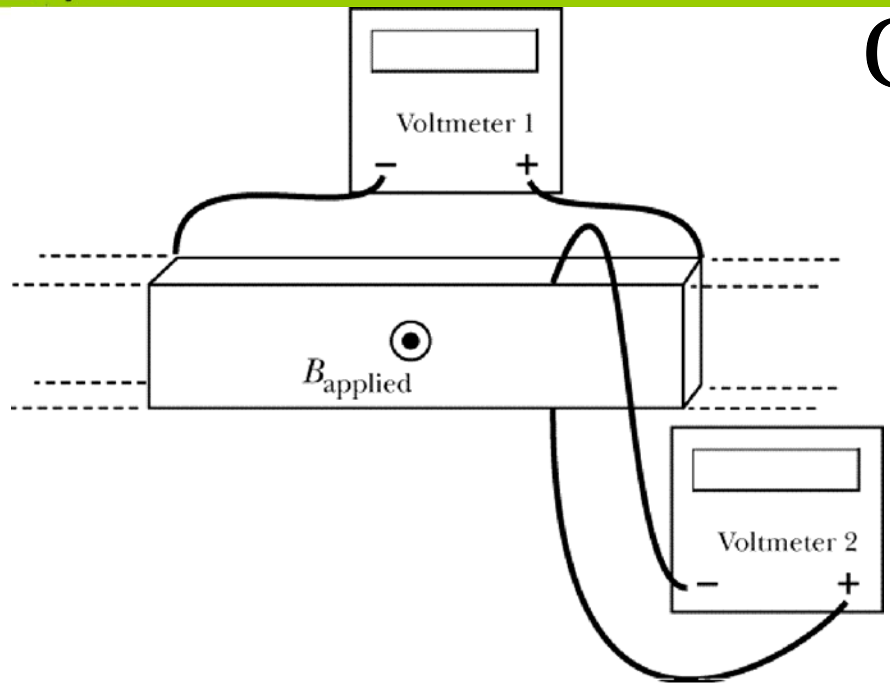


the back of the bar becomes positively charged



the back of the bar becomes negatively charged

Clicker Question

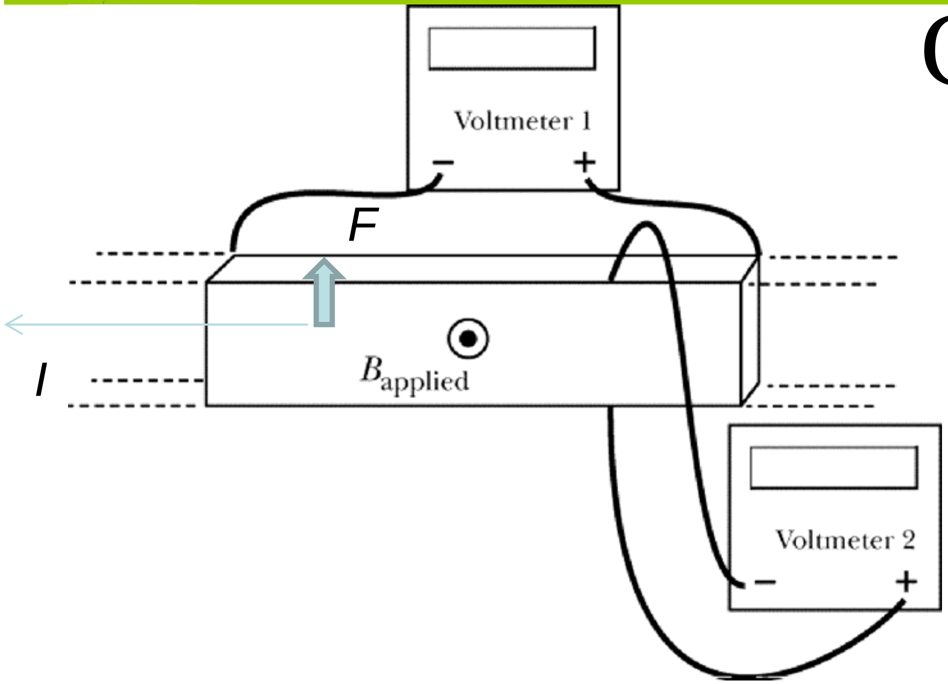


Voltmeter 1 reading is POSITIVE
Voltmeter 2 reading is POSITIVE

Mobile charges are:

A) Positive (holes)
B) Negative (electrons)
C) Not enough information

Clicker Question



Voltmeter 1 reading is POSITIVE
Voltmeter 2 reading is POSITIVE

Mobile charges are:

A) Positive (holes)
B) Negative (electrons)
C) Not enough information

B

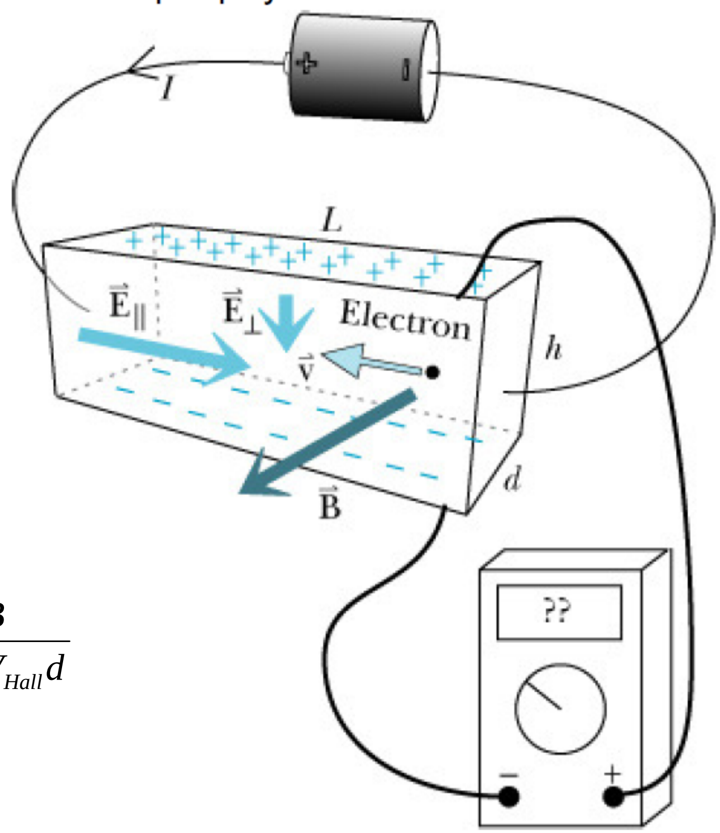
1. The Hall Effect: When a B-field is applied to a current-carrying conductor, the system develops an induced E-field.

2. Measuring this induced E-field allows one to determine a key (and fundamental and surprising) piece of microscopic physics.

$$E_{\perp} = \overline{v}\, B$$

$$\Delta V_{Hall} = E_{\perp}\, h$$

$$n = \frac{I\, B\, h}{|q|\, \Delta V_{Hall}\, A} = \frac{IB}{|q|\, \Delta V_{Hall} d}$$



Magnetic force on a moving charge

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Magnetic force on a current-carrying wire

$$\Delta\vec{F} = I\,\Delta\vec{\ell} \times \vec{B}$$

Short wire

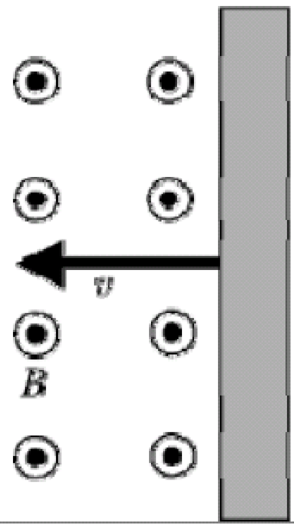
$$\vec{F} = I\,\vec{L} \times \vec{B}$$

Long wire

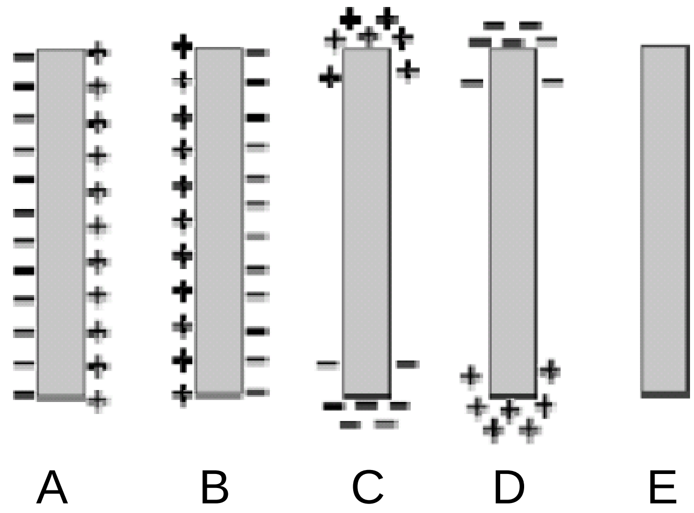
What's next?

Clicker Question

A neutral copper bar is dragged at speed *v* through a region with magnetic field *B*.

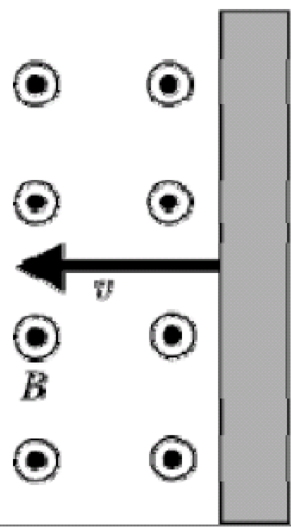


Which diagram best shows the state of the bar?



Clicker Question

A neutral copper bar is dragged at speed v through a region with magnetic field B.



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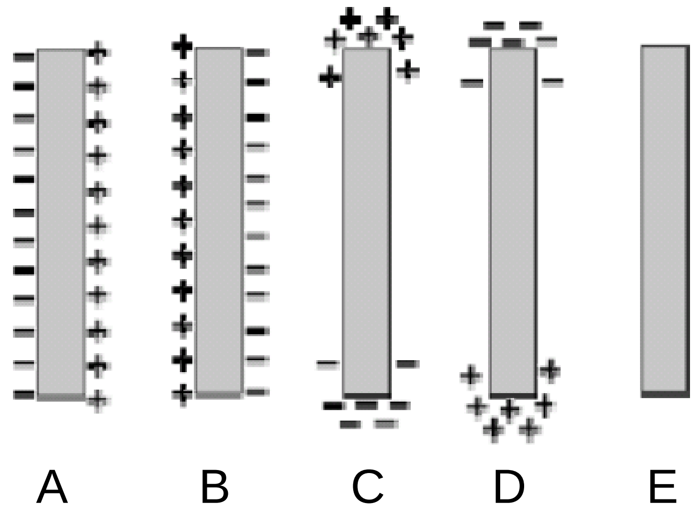
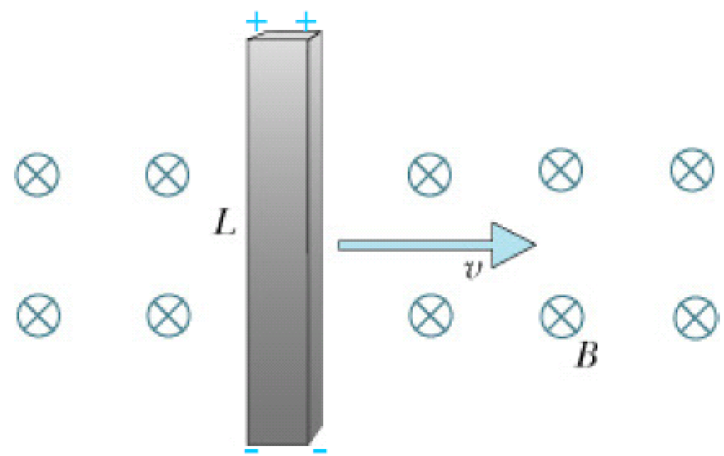


C

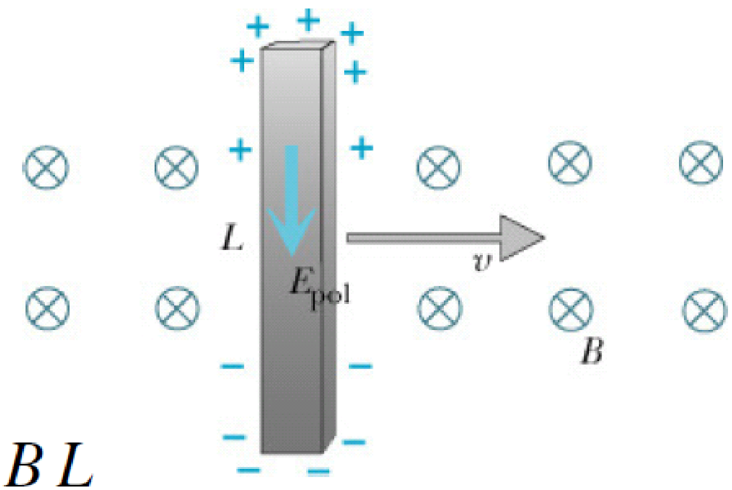
Motional emf

Initial transient: polarization develops



+

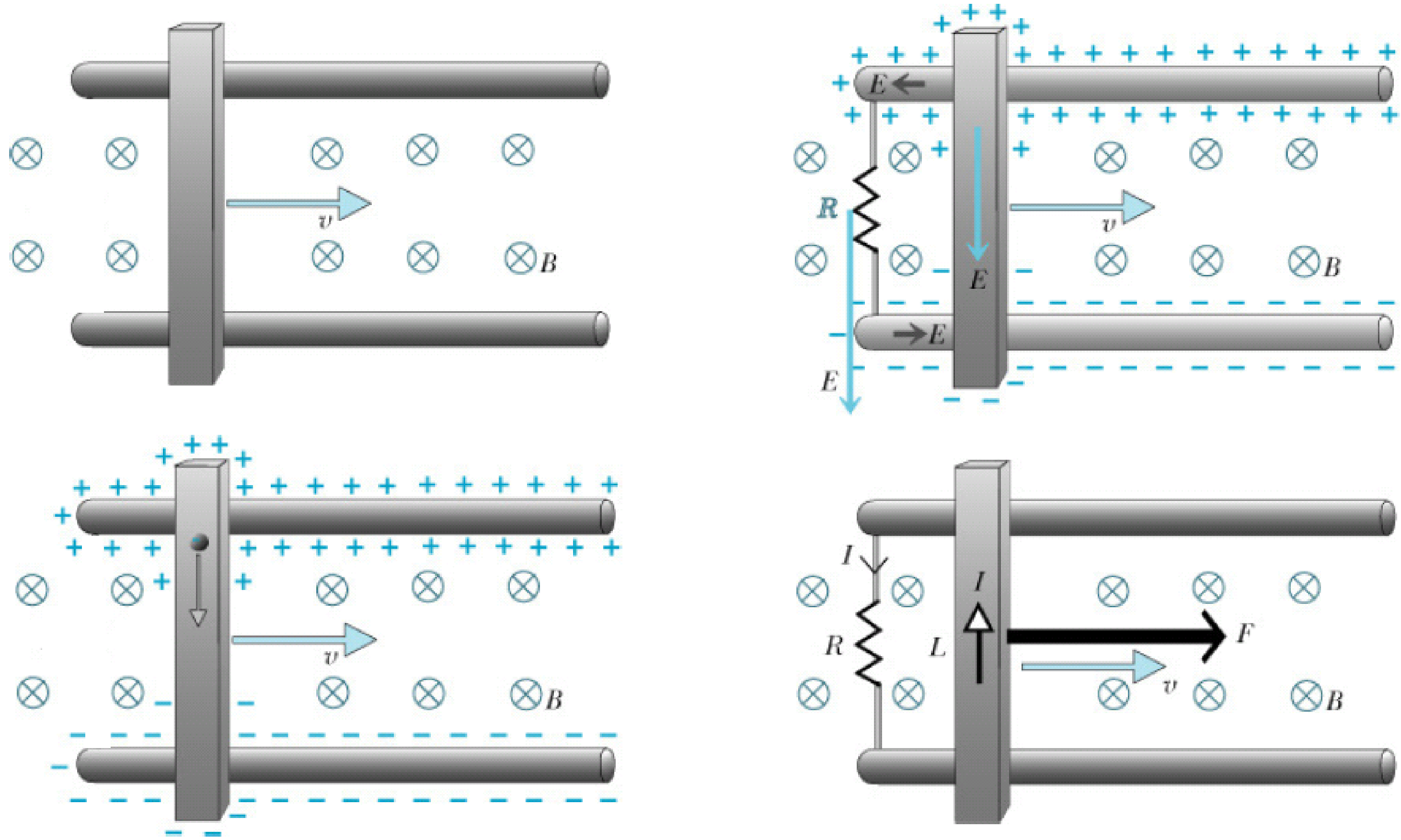
Steady state: magnetic force balanced by electric force



$$q\,\vec{E}_{pol} + q\,\vec{v} \times \vec{B} = 0$$

$$\Rightarrow \quad E_{pol} = v\,B \quad \text{and} \quad \underline{\Delta V = v\,B\,L}$$

Using motional emf to drive a current

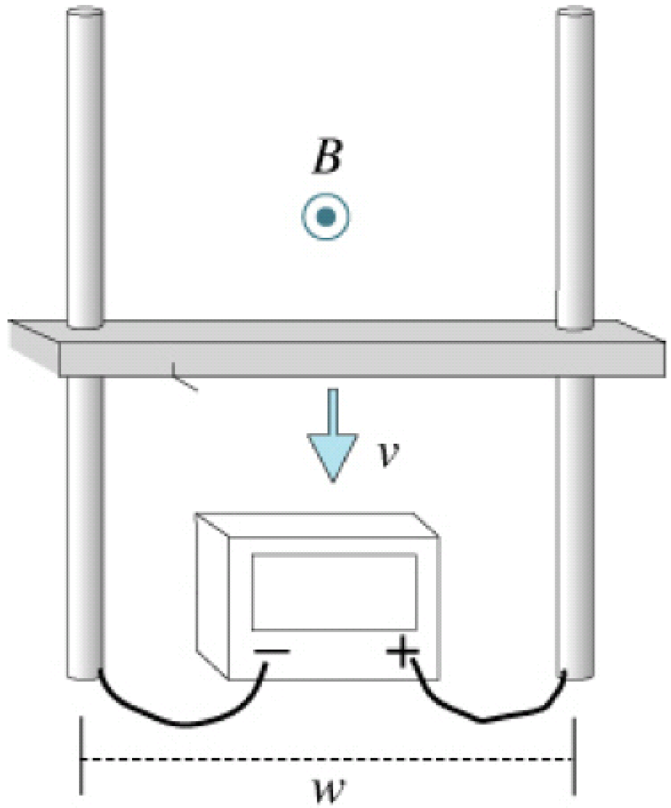


Loop Rule: $vBL - IR = 0$

mechanical power supplied: $F\,v = (I\,L\,B)v$

electric power dissipated: $I^2 R = I(I\,R) = I(v\,B\,L)$

Worked Example: A metal bar moves downward along conducting rails (see figure). There is a uniform B-field throughout the region, it points perpendicular to the direction of motion. If the instantaneous speed of the bar is *v*, what does the voltmeter read (sign and magnitude)?



First, figure out the sign. Since $\vec{v} \times \vec{B}$ points to the left, positive charge will pile up on the left and negative charge on the right. You can see that the voltmeter is hooked up with the opposite polarity, so it will read negative.

To get the magnitude of ΔV, note that the charge buildup will stop when there is no net force inside the bar, so that $q\,E = q\,v\,B$ (since $\vec{v}$ and $\vec{B}$ are perpendicular). Thus:

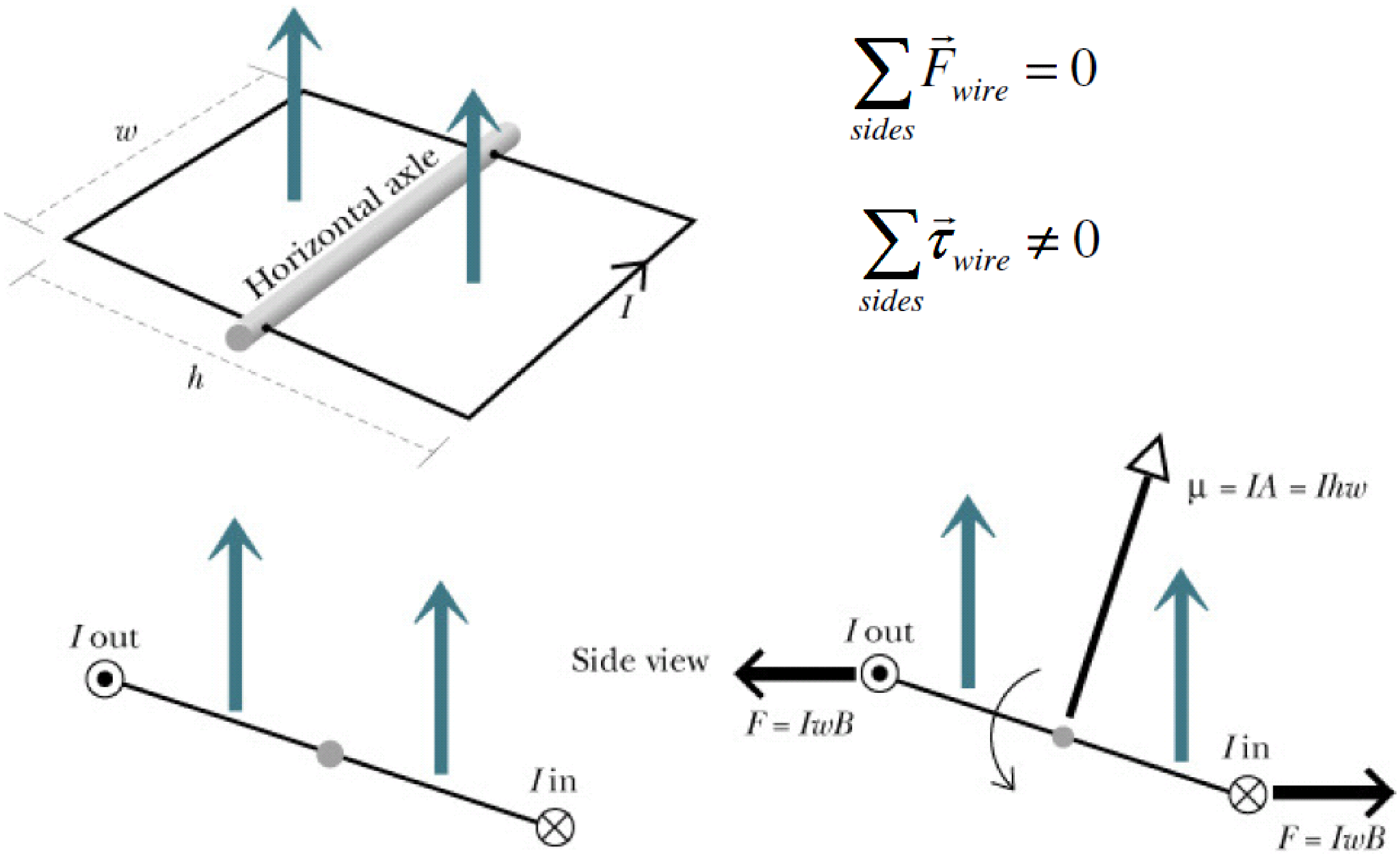
$$|\Delta V| = E\,w = v\,B\,w$$

Magnetic Torque

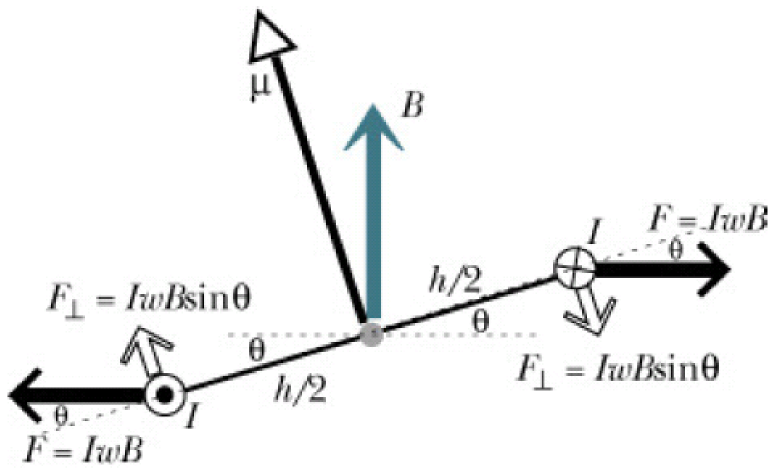
$$\Delta \vec{F}_{mag} = I(\Delta \vec{l} \times \vec{B})$$

$$\sum_{sides} \vec{F}_{wire} = 0$$

$$\sum_{sides} \vec{\tau}_{wire} \neq 0$$

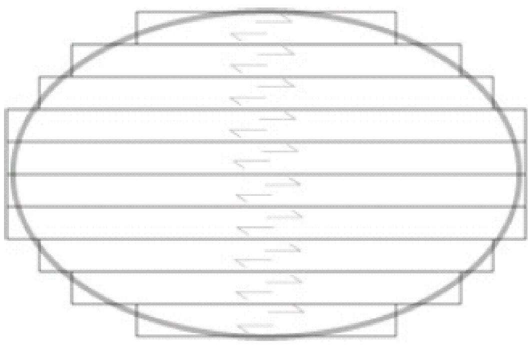


Small Current Loop: Magnetic Torque

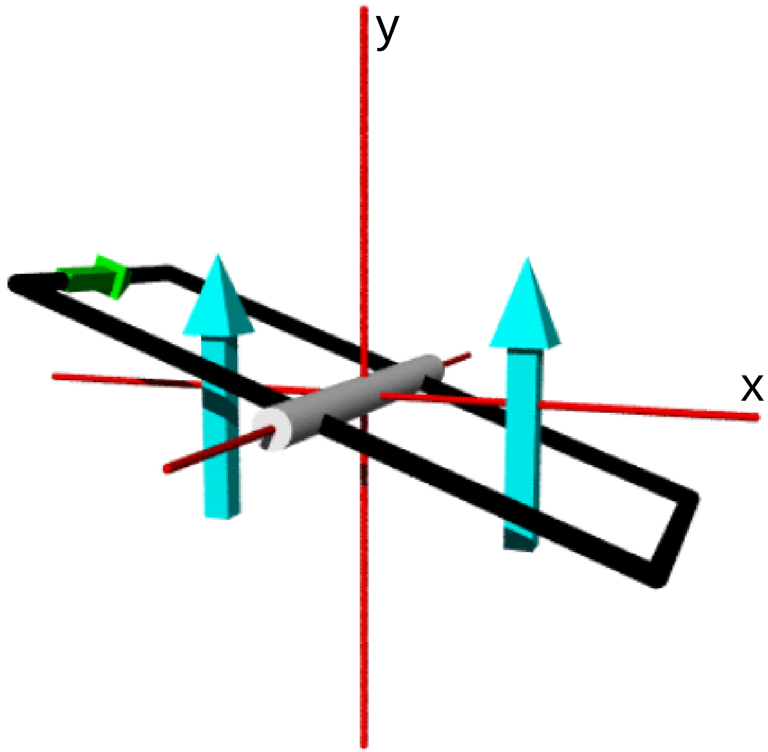


$$\begin{aligned} F_\perp &= IwB\sin\theta \\ \tau &= 2\times F_\perp \times \frac{h}{2} \\ &= IwB\sin\theta\, h \\ &= \mu B\sin\theta \end{aligned}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$



Clicker Q3

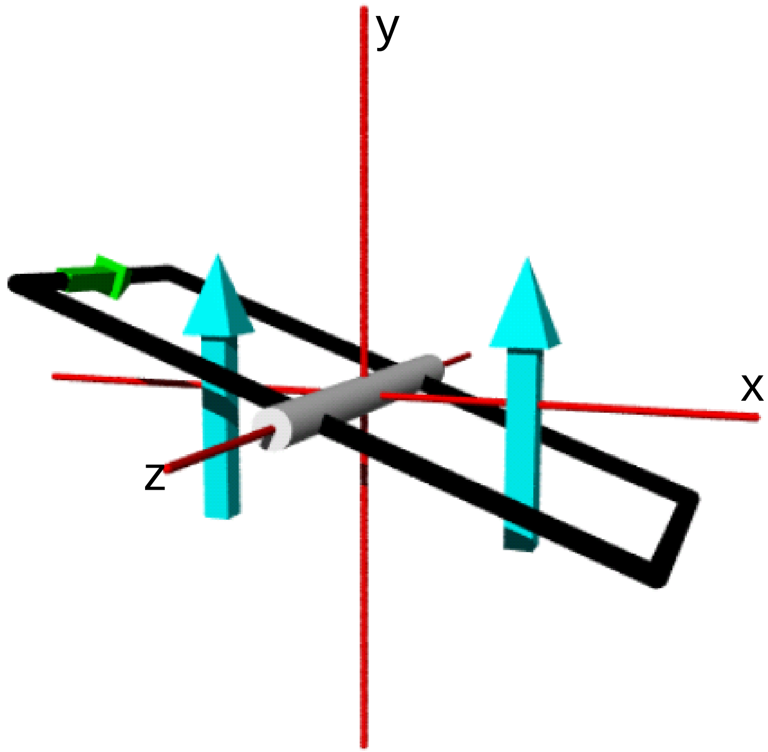


Cyan: applied magnetic field.
Green: conventional current

Direction of magnetic torque?

A) +z (counter-clockwise)
B) –z (clockwise)
C) zero magnitude
D) my answer doesn't match any of the above

Clicker Q3



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A) +z (counter-clockwise)
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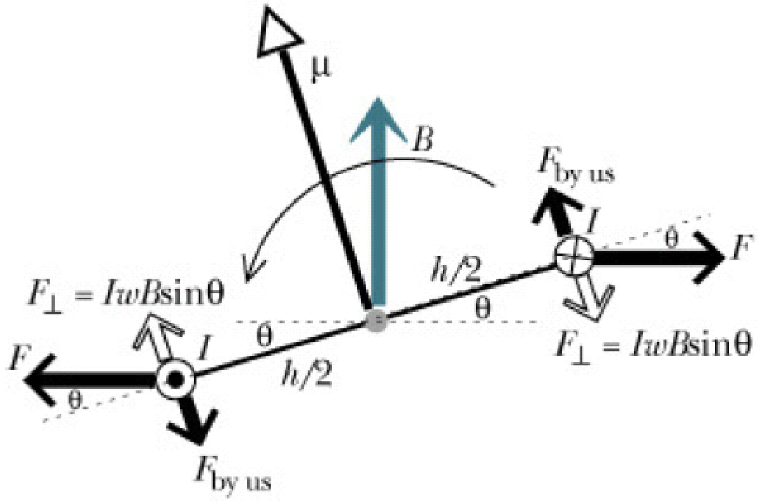
Reminder:
Lecture 22 (Wed 4/3) will be given by Prof. Yulia Pushkar

Small Current Loop: Magnetic Potential Energy



$$W + \Delta U_m = 0$$

$$\begin{aligned}
\text{work} &= -\Delta U_m \\
\text{work} &= -\int \tau \, d\theta \\
&= -\int IwhB\sin\theta \, d\theta \\
&= -IwhB \int d(\cos\theta) \\
&= +\mu B \Delta \cos\theta \\
&= \Delta(\mu B \cos\theta)
\end{aligned}$$

$$U_m = -\vec{\mu} \cdot \vec{B}$$

Lowest energy $\Leftrightarrow$ $\theta = 0$

(potential Energy of a magnetic dipote)