1 Introduction

LEARNING OBJECTIVES FOR THIS CHAPTER

1–1 To work comfortably with the engineering concept of a "system" and its interaction with the environment through inputs and outputs.

1–2 To distinguish among various types of mathematical models used to represent and predict the behavior of systems.

1–3 To recognize through (T-type) variables and across (A-type) variables when examining energy transfer within a system.

1–4 To recognize analogs between corresponding energy-storage and energy-dissipation elements in different types of dynamic systems.

1–5 To understand the key role of energy-storage processes in system dynamics.

1.1 SYSTEMS AND SYSTEM MODELS

The word "system" has become very popular in recent years. It is used not only in engineering but also in science, economics, sociology, and even in politics. In spite of its common use (or perhaps because of it), the exact meaning of the term is not always fully understood. A system is defined as a combination of components that act together to perform a certain objective. A little more philosophically, a system can be understood as a conceptually isolated part of the universe that is of interest to us. Other parts of the universe that interact with the system comprise the system environment, or neighboring systems.

All existing systems change with time, and when the rates of change are significant, the systems are referred to as dynamic systems. A car riding over a road can be considered as a dynamic system (especially on a crooked or bumpy road). The limits of the conceptual isolation determining a system are entirely arbitrary. Therefore any part of the car given as an example of a system – its engine, brakes, suspension, etc. – can also be considered a system (i.e., a subsystem). Similarly, two cars in a passing maneuver or even all vehicles within a specified area can be considered as a major traffic system.

The isolation of a system from the environment is purely conceptual. Every system interacts with its environment through two groups of variables. The variables in the first group originate outside the system and are not directly dependent on what happens in the system. These variables are called input variables, or simply inputs. The other group comprises variables generated by the system as it interacts with its

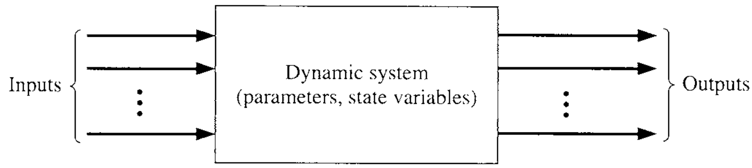

Figure 1.1. A dynamic system.

environment. Those dependent variables in this group that are of primary interest to us are called output variables, or simply outputs.

In describing the system itself, one needs a complete set of variables, called state variables. The state variables constitute the minimum set of system variables necessary to describe completely the state of the system at any given instant of time; and they are of great importance in the modeling and analysis of dynamic systems. Provided the initial state and the input variables have all been specified, the state variables then describe from instant to instant the behavior, or response, of the system. The concept of the state of a dynamic system is discussed in more detail in Chap. 3. In most cases, the state-variable equations used in this text represent only simplified models of the systems, and their use leads to only approximate predictions of system behavior.

Figure 1.1 shows a graphical presentation of a dynamic system. In addition to the state variables, parameters also characterize the system. In the example of the moving car, the input variables would include throttle position, position of the steering wheel, and road conditions such as slope and roughness. In the simplest model, the state variables would be the position and velocity of the vehicle as it travels along a straight path. The choice of the output variables is arbitrary, determined by the objectives of the analysis. The position, velocity, or acceleration of the car, or perhaps the average fuel flow rate or the engine temperature, can be selected as the output(s). Some of the system parameters would be the mass of the vehicle and the size of its engine. Note that the system parameters may change with time. For instance, the mass of the car will change as the amount of fuel in its tank increases or decreases or when passengers embark or disembark. Changes in mass may or may not be negligible for the performance of a car but would certainly be of critical importance in the analysis of the dynamics of a ballistic missile.

The main objective of system analysis is to predict the manner in which a system will respond to various inputs and how that response changes with different system parameter values. In the absence of the tools introduced in this book, engineers are often forced to build prototype systems to test them. Whereas the data obtained from the testing of physical prototypes are very valuable, the costs, in time and money, of obtaining these data can be prohibitive. Moreover, mathematical models are inherently more flexible than physical prototypes and allow for rapid refinement of system designs to optimize various performance measures. Therefore one of the early major tasks in system analysis is to establish an adequate mathematical model that can be used to gain the equivalent information that would come from several different physical prototypes. In this way, even if a final prototype is built to verify the mathematical model, the modeler has still saved significant time and expense.

A mathematical model is a set of equations that completely describes the relationships among the system variables. It is used as a tool in developing designs or control algorithms, and the major task for which it is to be used has basic implications for the choice of a particular form of the system model.

In other words, if a model can be considered a tool, it is a specialized tool, developed specifically for a particular application. Constructing universal mathematical models, even for systems of moderate complexity, is impractical and uneconomical. Let us use the moving automobile as an example once again. The task of developing a model general enough to allow for studies of ride quality, fuel economy, traction characteristics, passenger safety, and forces exerted on the road pavement (to name just a few problems typical for transportation systems) could be compared to the task of designing one vehicle to be used as a truck, for daily commuting to work in New York City, and as a racing car to compete in the Indianapolis 500. Moreover, even if such a supermodel were developed and made available to researchers (free), it is very likely that the cost of using it for most applications would be prohibitive.

Thus, system models should be as simple as possible, and each model should be developed with a specific application in mind. Of course, this approach may lead to different models being built for different uses of the same system. In the case of mathematical models, different types of equations may be used in describing the system in various applications.

Mathematical models can be grouped according to several different criteria. Table 1.1 classifies system models according to the four most common criteria: applicability of the principle of superposition, dependence on spatial coordinates as well

Table 1.1. Classification of system models

Type of model	Classification criterion	Type of model equation
Nonlinear	Principle of superposition does not apply	Nonlinear differential equations
Linear	Principle of superposition applies	Linear differential equations
Distributed	Dependent variables are functions of spatial coordinates and time	Partial differential equations
Lumped	Dependent variables are independent of spatial coordinates	Ordinary differential equations
Time-varying	Model parameters vary in time	Differential equations with time-varying coefficients
Stationary	Model parameters are constant in time	Differential equations with constant coefficients
Continuous	Dependent variables defined over continuous range of independent variables	Differential equations
Discrete	Dependent variables defined only for distinct values of independent variables	Time-difference equations

as on time, variability of parameters in time, and continuity of independent variables. Based on these criteria, models of dynamic systems are classified as linear or nonlinear, lumped or distributed, stationary time invariant or time varying, continuous or discrete, respectively. Each class of models is also characterized by the type of mathematical equations employed in describing the system. All types of system models listed in Table 1.1 are discussed in this book, although distributed models are given only limited attention.

1.2 SYSTEM ELEMENTS, THEIR CHARACTERISTICS, AND THE ROLE OF INTEGRATION

The modeling techniques developed in this text focus initially on the use of a set of simple ideal system elements found in four main types of systems: mechanical, electrical, fluid, and thermal. Transducers, which enable the coupling of these types of system to create mixed-system models, will be introduced later.

This set of ideal linear elements is shown in Table 1.2, which also provides their elemental equations and, in the case of energy-storing elements, their energy-storage equations in simplified form. The variables, such as force F and velocity v used in mechanical systems, current i and voltage e in electrical systems, fluid flow rate Q_f and pressure P in fluid systems, and heat flow rate Q_h and temperature T in thermal systems, have also been classified as either T-type (through) variables, which act through the elements, or A-type (across) variables, which act across the elements. Thus force, current, fluid flow rate, and heat flow rate are called T variables, and velocity, voltage, pressure, and temperature are called A variables. Note that these designations also correspond to the manner in which each variable is measured in a physical system. An instrument measuring a T variable is used in series to measure what goes *through* the element. On the other hand, an instrument measuring an A variable is connected in parallel to measure the difference *across* the element. Furthermore, the energy-storing elements are also classified as T-type or A-type elements, designated by the nature of their respective energy-storage equations: for example, mass stores kinetic energy, which is a function of its velocity, an A variable; hence mass is an A-type element. Note that although T and A variables have been identified for each type of system in Table 1.2, both T-type and A-type energy-storing elements are identified in mechanical, electrical, and fluid systems only. In thermal systems, the A-type element is the thermal capacitor but there is no T-type element that would be capable of storing energy by virtue of a heat flow through the element.

In developing mathematical models of dynamic systems, it is very important not only to identify all energy-storing elements in the system but also to determine how many energy-storing elements are independent or, in other words, in how many elements the process of energy storage is independent. The energy storage in an element is considered to be independent if it can be given any arbitrary value without changing any previously established energy storage in other system elements. To put it simply, two energy-storing elements are not independent if the amount of energy stored in one element completely determines the amount of energy stored in the other element. Examples of energy-storing elements that are not independent are rack-and-pinion gears, and series and parallel combinations of springs, capacitors, inductors,

Table 1.2. Ideal system elements (linear)

System type	Mechanical translational	Mechancial rotational	Electrical	Fluid	Thermal
A-type variable	Velocity, v	Velocity, Ω	Voltage, e	Pressure, P	Temperature, T
A-type element	Mass, m	Mass moment of inertia, J	Capacitor, C	Fluid Capacitor, C_f	Thermal capacitor, C_h
Elemental equations	$F = m\frac{dv}{dt}$	$T = J\frac{d\Omega}{dt}$	$i = C\frac{de}{dt}$	$Q_f = C_f\frac{dP}{dt}$	$Q_h = C_h\frac{dT}{dt}$
Energy stored	Kinetic	Kinetic	Electric field	Potential	Thermal
Energy equations	$\mathscr{E}_k = \frac{1}{2}mv^2$	$\mathscr{E}_k = \frac{1}{2}J\Omega^2$	$\mathscr{E}_e = \frac{1}{2}Ce^2$	$\mathscr{E}_p = \frac{1}{2}C_f P^2$	$\mathscr{E}_t = \frac{1}{2}C_h T^2$
T-type variable	Force, F	Torque, T	Current, i	Fluid flow rate, Q_f	Heat flow rate, Q_h
T-type element	Compliance, $1/k$	Compliance, $1/K$	Inductor, L	Inertor, I	None
Elemental equations	$v = \frac{1}{k}\frac{dF}{dt}$	$\Omega = \frac{1}{K}\frac{dT}{dt}$	$e = L\frac{di}{dt}$	$P = I\frac{dQ_f}{dt}$	
Energy stored	Potential	Potential	Magnetic field	Kinetic	
Energy equations	$\mathscr{E}_P = \frac{1}{2k}F^2$	$\mathscr{E}_P = \frac{1}{2K}T^2$	$\mathscr{E}_m = \frac{1}{2}Li^2$	$\mathscr{E}_k = \frac{1}{2}IQ_f^2$	
D-type element	Damper, b	Rotational damper, B	Resistor, R	Fluid resistor, R_f	Thermal resistor, R_h
Elemental equations	$F = bv$	$T = B\Omega$	$i = \frac{1}{R}e$	$Q_f = \frac{1}{R_f}P$	$Q_h = \frac{1}{R_h}T$
Rate of energy dissipated	$\frac{dE_D}{dt} = Fv = \frac{1}{b}F^2 = bv^2$	$\frac{dE_D}{dt} = T\Omega = \frac{1}{B}T^2 = B\Omega^2$	$\frac{dE_D}{dt} = ie = Ri^2 = \frac{1}{R}e^2$	$\frac{dE_D}{dt} = Q_f P = R_f Q_f^2 = \frac{1}{R_f}P^2$	$\frac{dE_D}{dt} = Q_h$

Note: A-type variable represents a spatial difference across the element.

etc. As demonstrated in the following chapters, the number of independent energy storing elements in a system is equal to the order of the system and to the number of state variables in the system model.

The A-type elements are said to be analogous to each other; T-type elements are also analogs of each other. This physical analogy is also demonstrated mathematically by the same form of the elemental equations for each type of element. The general form of the elemental equations for an A-type element in mechanical, electrical, fluid, and thermal systems is

$$V_T = E_A \frac{dV_A}{dt}, \tag{1.1}$$

where V_T is a T variable, V_A is an A variable, and E_A is the parameter associated with an A-type element. The general form of the elemental equations for a T-type element in mechanical, electrical, and fluid systems is

$$V_A = E_T \frac{dV_T}{dt}. \tag{1.2}$$

Equation (1.2) does not apply to thermal systems because of lack of a T-type element in those systems.

Because differentiation is seldom, if ever, encountered in nature, whereas integration is very commonly encountered, the essential dynamic character of each energy-storage element is better expressed when its elemental equation is converted from differential form to integral form. Thus general elemental equations (1.1) and (1.2) in integral form are

$$V_A(t) = V_A(0) + \frac{1}{E_A} \int_0^t V_T dt, \tag{1.3}$$

$$V_T(t) = V_T(0) + \frac{1}{E_T} \int_0^t V_A dt. \tag{1.4}$$

To better understand the physical significance of integral equations (1.3) and (1.4), consider a mechanical system. The A-type element in a mechanical system is mass, and the equation corresponding to Eq. (1.3) is

$$v(t) = v(0) + \frac{1}{m} \int_0^t F dt. \tag{1.5}$$

This equation states that the velocity of a given mass m increases as the integral (with respect to time) of the net force applied to it. This concept is formally known as Newton's second law of motion. It also implicitly says that, lacking a very, very large (infinite) force F, the velocity of mass m cannot change instantaneously. Thus the kinetic energy $\mathscr{E}_k = (m/2)v^2$ of the mass m is also accumulated over time when the force F is finite and cannot be changed in zero time.

The integral equation for a T-type element in mechanical systems, compliance $(1/k)$, corresponding to Eq. (1.4) is

$$F_k(t) = F_k(0) + k \int_0^t v_{21} dt, \tag{1.6}$$

where F_k is the force transmitted by the spring k and v_{21} is the velocity of one end of the spring relative to the velocity at the other end. This equation states that the spring force F_k cannot change instantaneously and thus the amount of potential energy stored in the spring $\mathscr{E}_p = (1/2k)F_k^2$ is accumulated over time and cannot be changed in zero time in a real system. Although Eq. (1.6) might seem to be a particularly clumsy statement of Hooke's law for springs ($F = kx$), it is essential for the purposes of system dynamic analysis that the process of storing energy in the spring as one of a cumulative process (integration) over time.

Similar elemental equations in integral form may be written for all the other energy-storage elements, and similar conclusions can be drawn concerning the role of integration with respect to time and how it affects the accumulation of energy with respect to time. These two phenomena, integration and energy storage, are very important aspects of dynamic system analysis, especially when energy-storage elements interact and exchange energy with each other.

The energy-dissipation elements, or D elements, store no useful energy and have elemental equations that express instantaneous relationships between their A variables and their T variables, with no need to wait for time integration to take effect. For example, the force in a damper is instantaneously related to the velocity difference across it (i.e., no integration with respect to time is involved).

Furthermore, these energy dissipators absorb energy from the system and exert a "negative-feedback" effect (to be discussed in detail later), which provides damping and helps ensure system stability.

EXAMPLE 1.1

Consider a simplified diagram of one-fourth of an automobile, often referred to as a "quarter-car" model, shown schematically in Fig. 1.2. Such a model of vehicle dynamics is useful when only bounce (vertical) motion of the car is of interest, whereas both pitch and roll motions can be neglected.

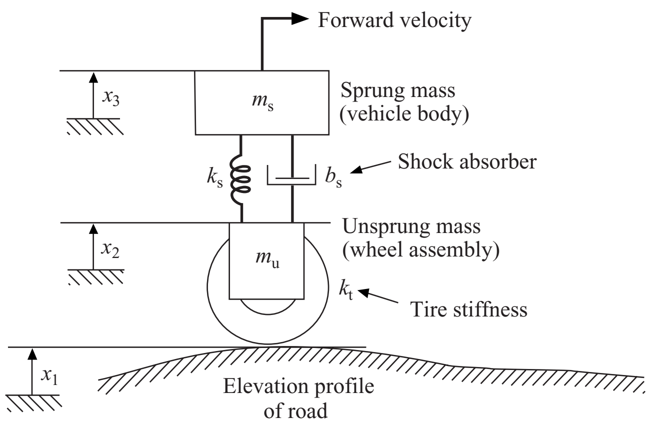

Figure 1.2. Schematic of a quarter-car model.

Table 1.3. Elements of the quarter-car model

Element	Element type	Type of energy stored	Energy equation
m_s	A-type energy storing	Kinetic	$\mathscr{E}_k = \frac{1}{2} m_s v_3^2$
m_u	A-type energy storing	Kinetic	$\mathscr{E}_k = \frac{1}{2} m_u v_2^2$
k_s	T-type energy storing	Potential	$\mathscr{E}_p = \frac{1}{2} k_s (x_2 - x_3)^2$
k_t	T-type energy storing	Potential	$\mathscr{E}_p = \frac{1}{2} k_t (x_1 - x_2)^2$
b_s	D-type energy dissipating	None	$\frac{d\mathscr{E}_D}{dt} = b_s (v_2 - v_3)^2$

List all system elements, indicate their type, and write their respective energy equations. Draw input–output block diagrams, such as that shown in Fig. 1.1, showing what you consider to be the input variables and output variables for two cases:

(a) in a study of passenger ride comfort, and
(b) in a study of dynamic loads applied by vehicle tires to road pavement.

SOLUTION

There are four independent energy-storing elements, m_s, m_u, k_s, and k_t. There is also one energy-dissipating element, damper b_s, representing the shock absorber. The system elements, their respective types, and energy-storage or -dissipation equations are given in Table 1.3.

The input variable to the model is the history of the elevation profile, $x_1(t)$, of the road surface over which the vehicle is traveling. In most cases, the elevation profile is measured as a function of distance traveled, and it is then combined with vehicle forward velocity data to obtain $x_1(t)$.

In studies of ride comfort, the main variable of interest is usually acceleration of the vehicle body,

$$a_3 = \frac{dv_3}{dt}.$$

In studies of dynamic tire loads, on the other hand, the variable of interest is the vertical force applied by the tire to the road surface:

$$F_t = k_t (x_1 - x_2).$$

Simple block diagrams for the two cases are shown in Fig. 1.3. There is an important observation to make in the context of this example. When a given physical system is modeled, different output variables can be selected as needed for the modeling task at hand.

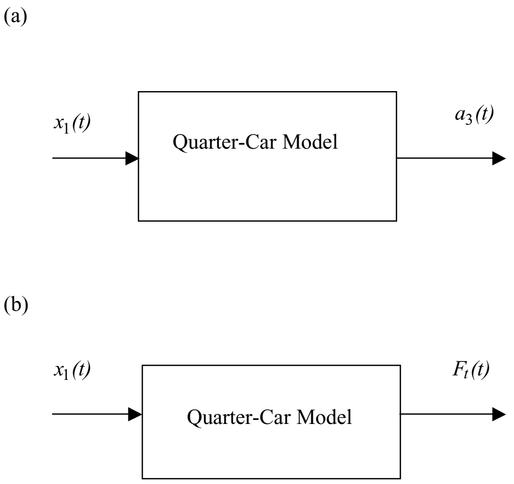

Figure 1.3. Block diagrams of the quarter-car models used in (a) ride comfort and (b) dynamic tire load studies.

PROBLEMS

1.1 Using an input–output block diagram, such as that shown in Fig. 1.1. show what you consider to be the input variables and the output variables for an automobile engine, shown schematically in Fig. P1.1.

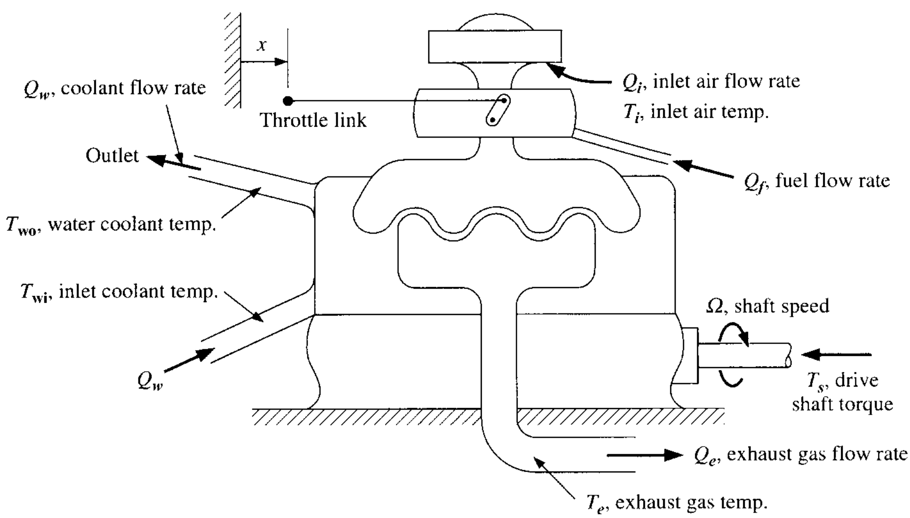

Figure P1.1.

1.2 For the automotive alternator shown in Fig. P1.2, prepare an input–output diagram showing what you consider to be inputs and what you consider to be outputs.

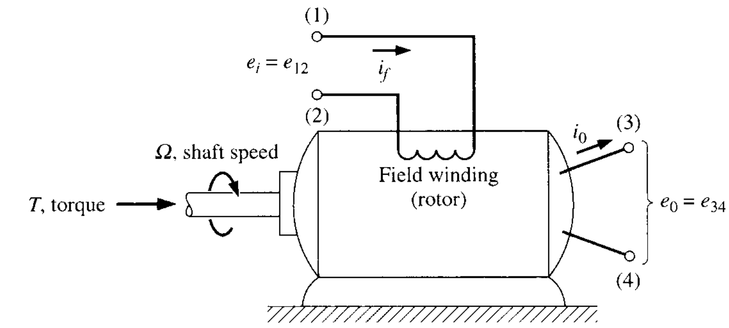

Figure P1.2.

1.3 Prepare an input–output block diagram showing what you consider to be the inputs and the outputs for the domestic hot water furnace shown schematically in Fig. P1.3.

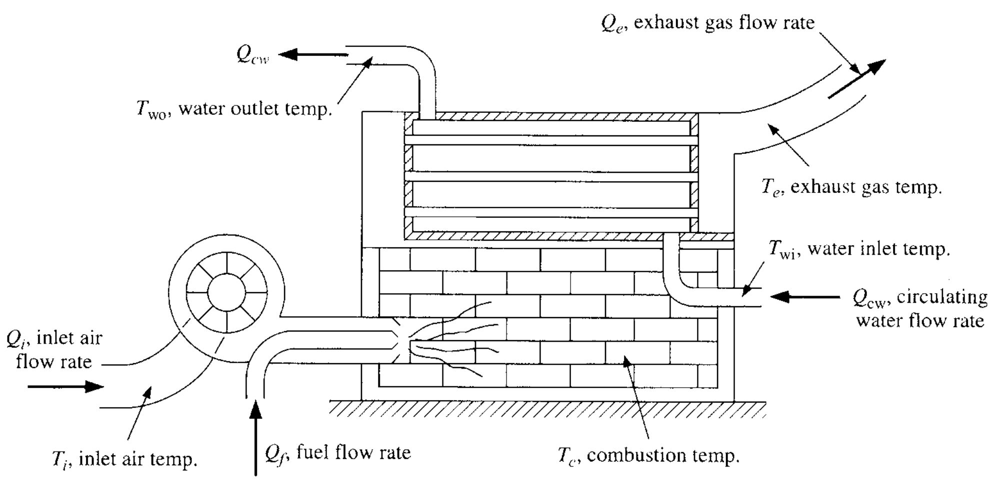

Figure P1.3.

1.4 A simple drawing of a hand-powered egg beater is shown in Fig. P1.4. The handle driven by torque T_i turns a large double-sided crown wheel, which in turn drives two bevel pinions to spin the beaters. The diameter of the crown wheel is much larger than the diameter of the bevel pinions, so the rotational velocity of the pinions (and the beaters), Ω_b, is much greater the rotational velocity of the crown wheel, Ω_c. The shafts that connect the bevel pinions to the beaters are slightly compliant, and the beaters experience a frictional resistance as they spin while beating the eggs. Make reasonable simplifying assumptions and list all elements that you would include in a mathematical model of the egg beater, indicate their types, and write their corresponding energy equations. How many independent energy-storing elements are in the system?

1.5 Figure P1.5 is a schematic representation of a wind turbine used for irrigation. The turbine is located on the rim of a canyon where the wind speed (V_w) is highest. The velocity of the rotor (the blade assembly) is Ω_r, and the electrical generator runs at Ω_g because of a gearbox in the nacelle of the wind turbine. Electrical power (V_e and I_e) is supplied to the motor and pump, located on the riverbank at the bottom of the canyon.

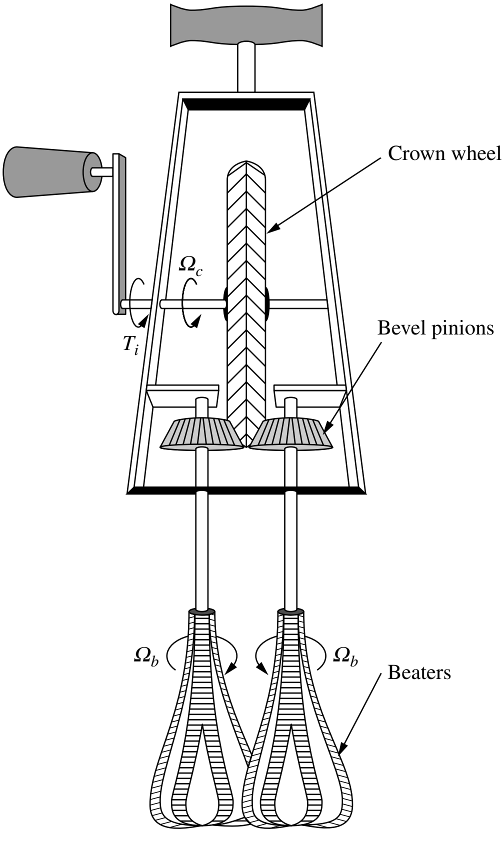

Figure P1.4.

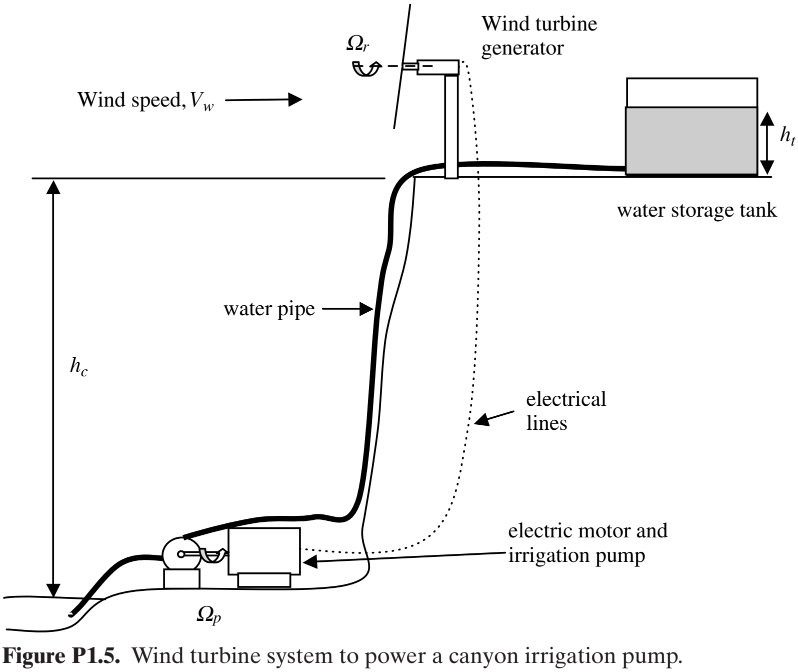

Figure P1.5. Wind turbine system to power a canyon irrigation pump.

The motor spins the pump at a fixed velocity, Ω_p, and the water is pumped to a holding pond above the canyon rim at a height of h_c above the river.

(a) Prepare an input–output block diagram of the wind turbine–generator as a system. Identify the energy-storing elements.

(b) Prepare an input–output block diagram of the motor–pump as a system.

(c) Prepare an input–output block diagram of this system and specify the energy-storing elements of the system. For each energy-storing element, chose the appropriate A- or T-type element from Table 1.2 that would be appropriate for each component of the system. Briefly explain your choices.

1.6 A schematic representation of an artificial human heart is shown in Fig. P1.6. The heart consists of two flexible chambers (blood sacs) enclosed in a rigid case. The chambers are alternately squeezed by flat plates that are, in turn, moved back and forth by a dc motor and rollerscrew arrangement. Two one-way valves (prosthetic heart valves) are used in each blood sac to ensure directional flow. The motor is powered by a battery pack through a controller circuit. The blood sac shown on the left-hand side in the figure (the right heart) takes blood returning from the body (through the vena cavae) and pumps it to the lungs (through the pulmonary artery). Oxygenated blood returns from the lungs (via the pulmonary veins) to the left heart where it is then pumped to the body through the aorta.

Identify the energy-storing elements of the system. If the intent of the model is to design a control system, what would be appropriate inputs and outputs of this system?

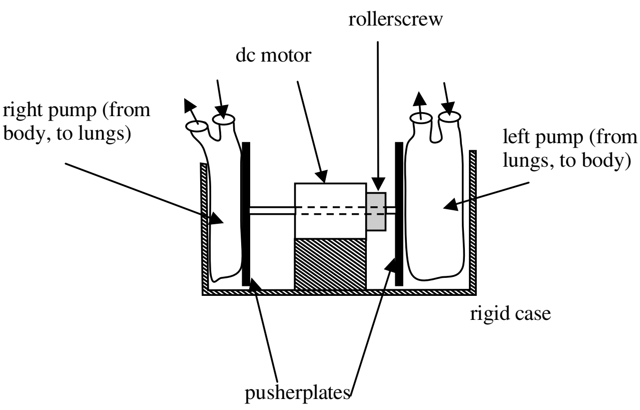

Figure P1.6. Schematic of a total artificial heart.

1.7 As mobile electronic systems become more efficient and require less energy, it becomes more attractive to generate power by scavenging energy from the motion inherent in moving these systems around. Analogous to the self-winding wristwatches that were popular in the 1970s, this concept makes use of a mechanical oscillator coupled with an electromagnetic generator to keep a battery charged.

Figure P1.7 shows a schematic of just such a system, proposed for a cell phone. The motion the phone experiences while in your backpack as you walk across campus is given as y_b in the figure. The mass (m) and spring (k) make up the mechanical oscillator. The mass is actually a permanent magnet that generates a time-varying (because of its motion) magnetic field in the vicinity of wire coil. The coil has both resistance (R) and inductance (L) and a current (i) is induced in the coil because of the motion of the magnet. This current supplies a charging circuit that maintains a set voltage (e) on the battery.

Identify the energy-storing elements of this system and draw an input–output block diagram. From Table 1.2, identify the A- and T-type elements appropriate for each energy-storing element and briefly discuss your choices.

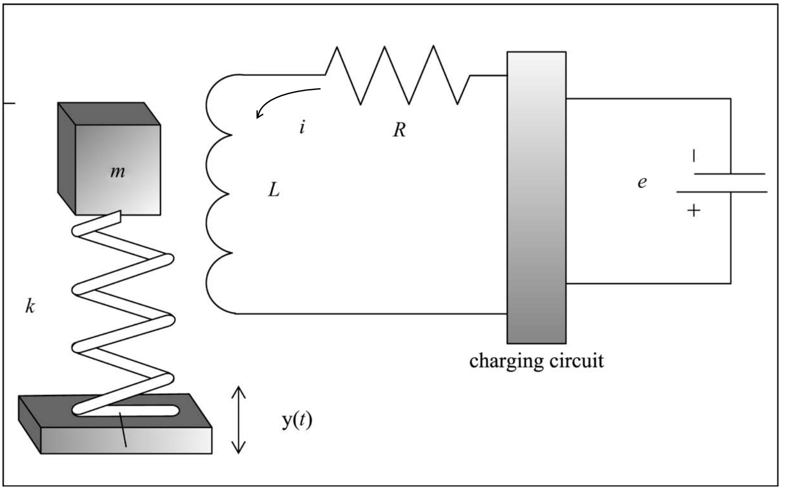

Figure P1.7. Schematic of energy-generating oscillator.

1.8 A motorized wheelchair uses a battery pack to supply two dc motors that, in turn, drive the left and right wheels through a belt transmission. The wheelchair is controlled through a joystick that allows the user to select forward and backward rotation of the wheels. The user accomplishes turning by running the two motors in opposite directions, thus rotating the chair about a vertical axis. The battery voltage is E_b, the internal moving parts of the motors have rotational inertia (J_a), and the electrical coils of the motors have both resistance (R_a) and inductance (L_a). The belt transmissions drive the wheels, which have rotational inertia (J_w) through a drive ratio given by the parameter N. The mass of the chair and rider is given by m_t and the grade is given by the angle γ.

Is the energy stored in the motor armatures, wheels, and chair (because of kinetic energy of the inertias) independent? Why or why not?