

OR Mini Project

- Q. A factory manufactures chairs, tables and bookcases each requiring the use of three operations: Cutting, Assembly and Finishing. The first operation can be used at most 600 hours; the second at most 500 hours and the third at most 300 hours. A chair requires 1 hr of cutting, 1 hr of assembly and 1 hr of finishing; a table needs 1 hr of cutting, 2 hrs of assembly and 1 hr of finishing and a bookcase requires 3 hrs of cutting, 1 hr of assembly and 1 hr of finishing. If the profit is \$ 20 per unit for a chair, \$ 30 for a table and \$ 25 for a bookcase, how many units of each should be manufactured to maximize profit?

$$\text{Max } Z = 20x_1 + 30x_2 + 25x_3$$

Subject to constraints:

$$x_1 + x_2 + 3x_3 \leq 600$$

$$x_1 + 2x_2 + x_3 \leq 500$$

$$x_1 + x_2 + x_3 \leq 300$$

Let x_4, x_5, x_6 be slack variables.

$$\text{Max } Z = 20x_1 + 30x_2 + 25x_3 + 0x_4 + 0x_5 + 0x_6$$

$$\therefore x_1 + x_2 + 3x_3 + x_4 + 0x_5 + 0x_6 \leq 600$$

$$x_1 + 2x_2 + x_3 + 0x_4 + x_5 + 0x_6 \leq 500$$

$$x_1 + x_2 + x_3 + 0x_4 + 0x_5 + 0x_6 \leq 300$$

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 600 \\ 500 \\ 300 \end{bmatrix}$$

		C_j	20	30	25	0	0	0	
BV	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	MR
x_4	0	600	1	1	3	1	0	0	600
x_5	0	500	1	2	1	0	1	0	250 ←
x_6	0	300	1	1	1	0	0	1	300
		A_j	-20	-30	-25	0	0	0	

		C_j	20	30	25	0	0	
BV	C_B	X_B	x_1	x_2	x_3	x_4	x_6	M.R
x_4	0	350	$1/2$	0	$5/2$	1	0	140
x_2	30	250	$1/2$	1	$1/2$	0	0	500
x_6	0	50	$1/2$	0	$1/2$	0	1	100 ←
		A_j	-5	0	-10	0	0	

↑

$$\begin{aligned}
 R_2 &\rightarrow R_2 \times 2 \\
 R_3 &\rightarrow R_3 - R_2 \\
 R_1 &\rightarrow R_1 - R_2
 \end{aligned}$$

		C_j	20	30	25	0	
BV	C_B	X_B	x_1	x_2	x_3	x_4	MR
x_4	0	100	-2	0	0	1	
x_2	30	200	0	1	0	0	
x_3	25	100	1	0	1	0	
		A_j	5	0	0	0	

$$\begin{aligned}
 R_3 &\rightarrow R_3 \times 2 \\
 R_2 &\rightarrow R_2 - \frac{1}{2} R_3 \\
 R_1 &\rightarrow R_1 - \frac{5}{2} R_3
 \end{aligned}$$

$$\begin{aligned}\text{Max } z &= 20x_1 + 30x_2 + 25x_3 \\ &= 20(0) + 30(200) + 25(100) \\ &= 6000 + 2500 \\ &= \underline{\underline{8500}}\end{aligned}$$