Name - Mosarrat yasmin
Class - MSc-CS-II
Roll No - 520
OR Mini project

Q. A farmer has 100 acres of land on which she plans to grow wheat and corn. Each acre of wheat require 4 hours of labor and \$20 of capital, and each acre of corn requires 16 hours of labor and \$40 of capital. The farmer has at most 800 hours of labor and \$2400 of capital available. If the profit from an acre of wheat is \$80 and from an acre of corn is \$100, how many acres of each crop should she plant to maximize her profit

Soln: Simplex method

Max of $z = 80x_1 + 100x_2 \leq 100$

Subject to

$4x_1 + 16x_2 \leq 800$

$20x_1 + 40x_2 \leq 2400$

Convert all the inequality constraints into equalities by adding slack variable

Let the slack variable be x_3 and x_4

Now,

Max of $z = 80x_1 + 100x_2 + 0x_3 + 0x_4$

Subject to

$4x_1 + 16x_2 + x_3 + 0x_4 = 800$

$20x_1 + 40x_2 + 0x_3 + x_4 = 2400$

$$\begin{bmatrix} 4 & 16 & 1 & 0 \\ 20 & 40 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 800 \\ 2400 \end{bmatrix}$$

			80	100	0	0	
BV	CB	XB	x_1	x_2	x_3	x_4	Minimum ratio
x_3	0	800	4	*16	1	0	50 ←
x_4	0	2400	20	40	0	1	60
		ΔJ	-80	-100	0	0	
				↑ incomin vector			

We remove x_3 and replace with pivot column because it's give 1 and 0

$$\Delta J = (CB_1 \times x_1) + (CB_2 \times x_{1 2}) - x_1$$
$$= (0 \times \cancel{800}^{4}) + (0 \times 20) - 80$$
$$= -80$$

Minimum Ratio = XB / pivot column

∴ Here we get ΔJ value in negative so the solution is not optimal.

Perform the solution until we get ΔJ ≥ 0

BV	CB	XB	~~x_1~~	~~x_2~~	x_4	Minimum Ratio
x_2	100	50	1/4 *	1	0	200 ←
x_4	0	400 ~~1600~~	10	0	1	—
		ΔJ	-55	0	0	
			↑ incoming vector			

$R_1 \rightarrow R_1 / 16$

$= 16/16 = 1$

$R_2 \rightarrow R_2 - 40(R_1)$

$40 - 40(1)$

$= 0$

		CJ	80	100	0	
BV	CB	XB	x_1	x_2	x_4	Minimum ratio
x_2	100	50	1/4	1	0	200
x_4	0	400	10*	0	1	40 ←
		ΔJ	-55	0	0	

↑ incoming vector

Now,

$R_2 \rightarrow R_2 / 10$

$10/10$

$= 1$

$R_1 \rightarrow R_1 - \frac{1}{4} R_2$

$\frac{1}{4} - \frac{1}{4}(1)$

$= 0$

			80	100	
~~BB~~ BV	CB	XB	x_1	x_2	Minimum ratio
x_2	100	40	0	1	
x_1	80	40	1	0	
	~~ΔJ~~	ΔJ	0	0	

Here all the value of $\Delta_J \geq 0$
So our Solution is optimal.

$$\text{Max } Z = 80x_1 + 100x_2$$
$$= 80 \times 40 + 100 \times 40$$
$$= 7200$$