A Generalized Model for the Conversion from CT Numbers to Linear Attenuation Coefficients

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Abstract

We have developed a generalized model for accurate conversion from CT numbers to linear attenuation coefficients (LAC) by introducing a material-dependent conversion factor (CF). When assuming that a material x is a uniform mixture of water and another material A (we call this the "water-A assumption" in this paper), we show that the conversion from CT number of x (HU_x) to LAC is linear. The slope of the linear function is determined by the attenuation property of material A, namely, its CT number (HU_A) at a given kVp and density (ρ_A). This generalized model can be applied to the conversion from CT images to attenuation maps for combined CT/PET and CT/SPECT imaging. When HU_x is less than zero, we use "water-air assumption," otherwise, we us "water-cortical bone assumption." These assumptions lead to different slopes for the linear conversion when CT number is below and above 0. In practice, for each CT system, a cylindrical phantom with a small cortical bone cylinder in the center is filled with water and scanned once for each kVp. The CT number of the cortical bone (HU_{CB}) at each kVp is then measured and used for the conversion. Experiments performed on a Philips CT AURA system show that, for a spongiosa bone sample with known LAC, the errors in LAC's converted from CT images at all the kVp's are 0% for PET and less than 1.5% for SPECT. ***Conclusions**:* The proposed model illustrates the linearity for the conversion from CT numbers to LAC at energy of interest under the water-A assumption. Its application to the conversion from CT images to PET/SPECT attenuation maps is accurate and convenient. In addition, the proposed technique can be used to characterize CT systems by obtaining the effective CT energy at each operating kVp. This allows for absolute attenuation measurement using CT systems instead of the relative measurement given by CT-numbers.

I. INTRODUCTION

In combined CT/PET and CT/SPECT imaging, the use of CT images for PET/SPECT attenuation correction requires accurate conversion from CT numbers to linear attenuation coefficients (LAC's) at PET/SPECT energies, e.g., 511/140 keV. To make the conversion accurate, one should take into account two crucial facts, namely,

(a) Polychromatic X-ray spectrum: X-ray has a broad energy spectrum, CT number thus reflects the average LAC of a material at different energies.
(b) Material-dependent energy scaling: The mass attenuation coefficient (MAC) of a high-Z material decreases from CT energy range to PET/SPECT energy more quickly than a low-Z material, due to the fact that high-Z materials have more photoelectric absorption than low-Z value materials. For example, from 80 keV to 511 keV, the MAC (cm^2/g) of water decreases by a factor of 1.90, from 0.184 to 0.096, but the MAC of cortical bone decreases by a factor of 2.40, from 0.221 to 0.092.

Fact (a) is the culprit of the well-known beam-hardening effect in CT imaging that makes the CT number of the same tissue smaller when it is deeper inside of the patient body. Fact (a) also undermines the application of CT systems for absolute attenuation measurement. For non-uniform objects, fact (b) suggests that simple scaling of the CT images may result in large bias when converting CT images to attenuation maps at energies of interest.

For decades, efforts have been made to include the effect of polychromatic x-ray spectrum in CT imaging [1-5]. These efforts can potentially lead to CT images that are nearly free from beam-hardening effect, i.e., CT images are obtained as if the X-ray spectrum is mono-energetic. These techniques can thus simplify the conversion from CT number to LAC significantly.

With the emergence of combined CT/SPECT and CT/PET imaging systems, accurate conversion from CT numbers to LAC becomes important for accurate CT-based attenuation correction. LaCroix *et al.* [6] showed that conversion from CT number to LAC at SPECT energy using a uniform scaling factor introduced large error for materials with high Z values, such as bones. Hasegawa *et al.* [7] used the technique proposed by Alvarez and Macovski [1] to determine the photoelectric and Compton components of the LAC of materials at CT energy. Then they scaled the two components to SPECT energy differently. This technique is more accurate but requires CT scans at two different kVp's, significantly increasing patient dose.

The piecewise-linear conversion technique developed by Blankespoor *et al.* [8] uses a series of CT scans of K_2HPO_4 solution of different concentrations at a given CT kVp. The LAC of the solution at each concentration at SPECT energy can be calculated from tables of atomic attenuation coefficients, thus can be used to fit a LAC-CT number curve for CT number greater than 0. For CT number less than or equal to 0, the curve fitting uses air, a fat-equivalent solution of 60% ethanol, and water. This technique was shown to be accurate for the studies performed but required complicated calibration studies for each kVp. The technique proposed by Burger *et al.* [9] is also a curve-fitting technique, that fits the LAC-CT number pairs to a quadratic function.

Another technique developed by *Kinahan et al.* [10] uses a hybrid segmentation and scaling during the conversion. For CT number great than a threshold value (e.g., 300 as was used in [10]), the material is assumed to be bone and the scaling uses the scaling factor of cortical bone from effective CT energy to PET energy. For CT number below the threshold value, the scaling uses the scaling factor of water. This technique was shown to be effect for PET /CT reconstruction using CT-based attenuation correction. However, this technique has two drawbacks. Phantom emission scans and emission reconstruction using CT-based attenuation correction are required to obtain the effective CT energy used to determine the scaling factors and there is a discontinuity for the conversion for CT numbers at the threshold.

The technique developed in this work starts with the general physics principles for the conversion from CT number to any energy of interest. By using the "water-A assumption," an accurate conversion technique can be reached.

II. METHODS

1. Conversion Theory

For a material x the CT number is given by

$$HU_x = \frac{\mu_x(E_{CT}) - \mu_W(E_{CT})}{\mu_W(E_{CT})} \times 1000 \qquad (1)$$

in Hounsfield unit, where $\mu_x(E_{CT})$ and $\mu_W(E_{CT})$ are the LAC's of x and water at the effective CT energy E_{CT}, respectively. E_{CT} is defined so that materials have the same HU's if X-ray photons are of the mono-energy E_{CT}. The LAC of x at E_{CT} is then

$$\mu_x(E_{CT}) = \left(1 + \frac{HU_x}{1000}\right) \times \mu_W(E_{CT}) \cdot \qquad (2)$$

The LAC at E_{CT} can be converted to the LAC at any energy of interest E by using a scaling factor $S_x(E_{CT} \to E)$ that is defined as the ratio of the LAC of material x at E_{CT} to that at E. Using Equation (2), the LAC of x at E can be written as

$$\mu_x(E) = \left(1 + \frac{HU_x}{1000}\right) \times \mu_W(E) \times \left[\frac{S_W(E_{CT} \to E)}{S_x(E_{CT} \to E)}\right] \qquad (3)$$

We define the term in the square brackets in Equation (3) as the **<u>conversion factor</u>** of material x **<u>(CF_x)</u>** at energy E, i.e.,

$$CF_x(E, E_{CT}) = \frac{S_W(E_{CT} \to E)}{S_x(E_{CT} \to E)} = \frac{\mu_W(E_{CT})}{\mu_W(E)} / \frac{\mu_x(E_{CT})}{\mu_x(E)} . \qquad (4)$$

Equation (3) is then simplified as Equation (5), in which the LAC of water at energy E is known and the only unknown is CF_x. For different materials CF_x is listed in Table 1 and plotted in Figure 1 as a function of E_{CT} when $E=511$ keV.

$$\mu_x(E) = \left[\left(1 + \frac{HU_x}{1000}\right) \times CF_x(E, E_{CT})\right] \times \mu_w(E) \qquad (5)$$

Table 1. Conversion factors (CF's) of materials from different effective CT energies (in keV) to 511keV.

CT Energy	40	60	70	80
Carbon	1.155	1.057	1.025	1.016
Oxygen	0.921	0.968	0.977	0.982
Water	**1.000**	**1.000**	**1.000**	**1.000**
Muscle*	1.039	1.024	1.035	1.020
Soft Tissue	1.027	1.015	1.012	1.010
Aluminum	0.408	0.639	0.721	0.781
Cortical Bone	0.397	0.637	0.724	0.788
Iodine	0.0117	0.0265	0.0381	0.0513

* Excluding the coherent scatter

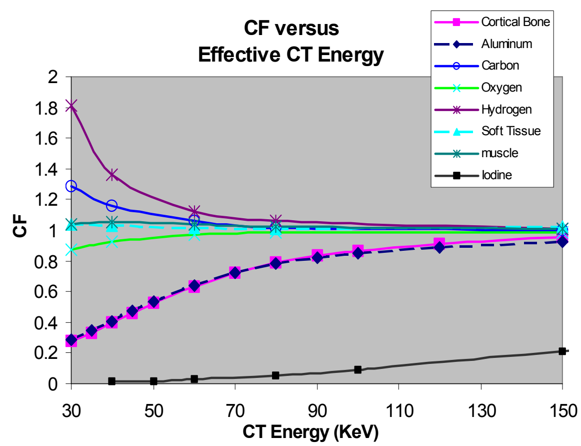

Figure 1. The conversion factor (CF) as a function of the CT energy for different tissues (materials).

Equation (5) illustrates the conversion from CT numbers to LAC's using the conversion factors. In the next section, we will show the relationship between CF_x and HU_x by using a "water-A assumption." This assumption says material x is a uniform mixture of water and material A, given that the CT number of A is greater than that of x at the same kVp's.

2. Linear Conversion Model

Under the "water-A assumption," if x is a uniform mixture of water and A and the volume partition of A is *v*, then, theoretically, at any energy

$$\begin{aligned}\mu_x &= v \times \mu_A + (1 - v) \times \mu_W \\ &= \mu_W + v \times (\mu_A - \mu_W)\end{aligned} . \qquad (6)$$

Thus, if we assume that the X-ray energy can be approximated using the mono-energetic effective CT energy E_{CT}, from equation (1), the CT number of x is

$$HU_x = v \times (\mu_A - \mu_w)/\mu_w \times 1000 \\ = v \times HU_A \quad . \tag{7}$$

Therefore, we can obtain the volume partition of A using the CT numbers of x and A by the inverse of Equation (7):

$$v = HU_x / HU_A \ . \tag{8}$$

Now, plug Equation (6) into Equation (4) and consider the fact that $\mu_x(E) = \rho_x \cdot \mu'_x(E)$, where ρ_x is the density of material x and prime indicates mass attenuation coefficients (MAC), we can show that the conversion factor CF_x is:

$$CF_x(E, E_{CT}) = \frac{v \times \rho_A + (1-v) \times \mu'_W(E)/\mu'_A(E)}{v \times \rho_A / CF_A(E, E_{CT}) + (1-v) \times \mu'_W(E)/\mu'_A(E)} \tag{9}$$

where ρ_A, μ_A, μ'_A are the density, LAC, and MAC of A, respectively.

Now we denote the MAC ratio of A and x as:

$$R_A(E) = \frac{\mu'_A(E)}{\mu'_W(E)} \ , \tag{10}$$

and express the conversion factor of A using its CT number by the inverse of Equation (5),

$$CF_A(E, E_{CT}) = \left(1 + \frac{HU_A}{1000}\right)^{-1} \times \frac{\mu_A(E)}{\mu_W(E)} \ . \tag{11}$$

By using Equations (8), (10), and (11) we can greatly simplify Equation (9) as:

$$CF_x(E, E_{CT}) = \left(1 + \frac{HU_x}{1000}\right)^{-1} \times \left[1 + \left(\rho_A \times R_A(E) - 1\right) \times \frac{HU_x}{HU_A}\right] . \tag{12}$$

Therefore, the conversion from CT numbers to LAC's shown by equation (5) becomes

$$\mu_x(E) = \left[1 + \left(\rho_A \times R_A(E) - 1\right) \times \frac{HU_x}{HU_A}\right] \times \mu_w(E) \tag{13}$$

Equation (13) demonstrates the general model for the conversion from CT numbers to LAC's. The conversion is linear. The Appendix shows an easier way to derive this equation. However, the derivation in this section introduces an important concept, the conversion factor [shown in Equations (4), (11) and (12)], that is the key to the analysis of the conversion accuracy of different materials using this model.

3. Application in CT/PET and CT/SPECT Imaging—Piecewise-Linear Conversion

By definition [Equation (4)], CF for water is 1.0. Table 1 and Figure 1 show that CF is approximately 1.0 for muscle, soft tissue, and low Z-number elements when E_{CT} >60 keV. For cortical bone (CB) and aluminum, the CF is significantly less than 1.0. This is due to the fact that cortical bone and aluminum have relatively higher Z values. Therefore, for patient studies without the introduction of CT contrast agents, one can assume that tissues with CT number less than 0 are uniform mixtures of water and air and tissues with CT number greater than 0 are uniform mixtures of water and cortical bone.

Under the "water-air assumption" for $HU_x \le 0$, with the approximation that the density of air is 0 and its CT number is –1000, Equation (13) becomes:

$$\mu_x(E) = \left[1 + \frac{HU_x}{1000}\right] \times \mu_w(E) \ . \tag{14a}$$

Under the "water-cortical bone assumption" for $0 < HU_x < HU_{CB}$, Equation (13) becomes

$$\mu_x(E) = \left[1 + \left(\rho_{CB} \times R_{CB}(E) - 1\right) \times \frac{HU_x}{HU_{CB}}\right] \times \mu_w(E) \tag{14b}$$

Equations (14a) and (14b) show that the conversion from CT number to LAC at energy E is piecewise linear. The slope for CT number above 0 is determined by the attenuation property (density, MAC) of the cortical bone phantom.

For the conversion from CT images to attenuation map in CT/PET and CT/SPECT imaging, a cortical bone phantom with MAC known in a wide range of energy will be scanned once at each different operation kVp to measure the CT number of cortical bone. The MAC ratio R_{CB} can be calculated using Equation (10). Then the density, R_{CB}, and the measured CT number of the cortical bone can be used for the piecewise linear conversion in Equation (14).

III. RESULTS

1. Piecewise Linear Conversion

A cortical bone phantom (cylindrical, 5 cm in diameter, 10 cm in height) was put in the center of a large cylindrical phantom (20 cm in diameter and 20 cm in height). The cortical bone phantom has density of 1.88 g/cm^3, and MAC of 0.150 cm^2/g at 140 keV and 0.092 cm^2/g at 511 keV. The large phantom was then filled with water and positioned in the center of the transaxial FOV of a Philips CT AURA system. One CT scan was performed for each operational kVp, with slice thickness 5 mm, pitch 1.5, current 100 mA, and 1 second per rotation.

CT number of the cortical bone is measured from the CT images obtained at each kVp. The conversion from CT number for $0 < HU_x < HU_{CB}$ to PET/SPECT energies is shown in Tables 2 and 3.

2. Spongiosa Bone Phantom Studies

CT scans were performed at each kVp using the same setup as above but replacing the cortical bone phantom with a spongiosa bone (SB) phantom (same dimension as the cortical bone phantom). The spongiosa phantom has density of 1.17 g/cm^3, and

MAC of 0.150 cm^2 /g at 140 keV and 0.096 cm^2 /g at 511 keV. Table 4 shows the converted LAC of the spongiosa bone and the percentage error compared with the true values using the conversion listed in Tables 2 and 3.

Table 2. Conversion from CT numbers to LAC's at 511 keV.

kVp	HU_{CB}	Conversion for $0<HU_x< HU_{CB}$ *
140	1250	$(1+6.40\times10^{-4}\times HU_x)\times0.096$
130	1325	$(1+6.05\times10^{-4}\times HU_x)\times0.096$
120	1390	$(1+5.76\times10^{-4}\times HU_x)\times0.096$
100	1574	$(1+5.09\times10^{-4}\times HU_x)\times0.096$

* For $HU_x \leq 0$: $(1+1.00\times10^{-3}\times HU_x)\times0.096$

Table 3. Conversion from CT numbers to LAC's at 140 keV.

kVp	HU_{CB}	Conversion for $0<HU_x< HU_{CB}$ *
140	1250	$(1+7.04\times10^{-4}\times HU_x)\times0.150$
130	1325	$(1+6.65\times10^{-4}\times HU_x)\times0.150$
120	1390	$(1+6.33\times10^{-4}\times HU_x)\times0.150$
100	1574	$(1+5.59\times10^{-4}\times HU_x)\times0.150$

* For $HU_x \leq 0$: $(1+1.00\times10^{-3}\times HU_x)\times0.150$

Table 4. LAC of the spongiosa bone converted from CT numbers obtained from CT scans at different kVp's. The true LAC is 0.112/cm at 511 keV and 0.176 at 140 keV.

	140	130	120	100
HU_{SB}	263	276	288	324
$\mu(511)$	0.112	0.112	0.112	0.112
% error	0.0	0.0	0.0	0.0
$\mu(140)$	0.178	0.176	0.177	0.177
% error	+1.1	0.0	+0.9	+0.9

IV. DISSCUSSIONS

1. Conversion Accuracy

This proposed model is accurate if the "water-A assumption" is satisfied. The piecewise linear fitting in [8] is the application of this model under the "water- K_2HPO_4 assumption." The fitting result shown in [8] is equivalent to $(1+5.17\times10^{-4}\times HU_x)\times0.150$ at 140 keV for 140 kVp CT scans.

For soft tissue, under the "water-cortical bone assumption," the conversion slopes shown in Tables 2 and 3 are smaller than shown by Equation (5). The converted LAC is systematically less than the true value when its CT (HU_{ST}) number is above 0. This suggests that the "water-air assumption" is more accurate for the conversion of soft tissue when $HU_x>0$. For PET/CT and SPECT/CT application, the result for "water-air assumption" in Equation (14a) can be extended to $0<HU_x<HU_{ST}$ for approximation. This results in the same conversion slope for soft tissue as in [10]. As a comparison, for soft tissue with $HU_x=50$, the LAC converted using "water-cortical assumption" is 0.099/cm at 511 keV and 0.155/cm at 140 keV. By using conversion in [10] or the slope from "water-air assumption," the converted LAC is 1.7% higher at 511 keV and 1.4% higher at 140 keV.

2. Conversion of CT Contrast Agents

The proposed technique uses the "water-cortical bone assumption" for the conversion with CT number above 0. In CT scans where CT contrast is used, this assumption is incorrect for tissues with contrast uptake. Rather, a "water-iodine assumption" should be used. The application of the proposed model to the conversion for 140 kVp CT scans under the new assumption is $(1+4.82\times10^{-5}\times HU_x)\times\mu_w(E)$. Comparing this with the results from "water-cortical bone assumption" in Tables 2 and 3, it is obvious that the slope for the linear conversion is one magnitude less under the new assumption. Therefore, the same CT number 263 that results in LAC of 0.112/cm at 511 keV and 0.178/cm at 140 keV under the "water-cortical bone assumption" now only results in LAC of 0.097/cm at 511 keV and 0.152/cm at 140 keV. In other words, the tissue with CT number of 263 has LAC about 1.18 time that of water under the "water-cortical bone assumption" now is only 1.01 times that of water under the "water-iodine assumption." This result is consistent with the data reported in [11]. For the conversion from CT images to PET/SPECT attenuation map, if the region with contrast uptake can be identified, the proposed technique can be applied to the region and the conversion can be accurate.

3. Beam-Hardening Effect

Equation (7) is based on the assumption that beam-hardening effect has been perfectly compensated for. With imperfect beam-hardening compensation, Equation (7) becomes an approximation. the CT number of the cortical bone phantom varies when it has different type and amount of surrounding materials. This will affect the slope of the linear conversion for CT number above 0. To address how serious the problem could be, we replaced the spine insert of an anthropomorphic phantom with our cortical bone phantom and filled the chamber and the liver insert of the phantom with water. The CT number of the cortical bone was measured at CT slices with lungs and livers separately. We also scanned the cortical bone in air. Table 5 shows the converted LAC of the spongiosa bone and the percentage errors when using the different CT numbers of the cortical bone. It can be seen that the error caused by the beam-hardening effect on HU_{CB} measurement can be as high as 20% from measurement in air to those in phantoms. But when used for the spongiosa bone conversion with CT number of 263 at 140 kVp, the error is less than 2%.

4. Obtaining Effective CT Energy

Using the proposed model, a CT system can be characterized by specifying the effective CT energy at each operation kVp of the system. Once CF_{CB} is calculated from the measured HU_{CB} [Equation (11)], from Table 1 and Figure 1, the effective CT energy E_{CT} can be obtained. One can then directly use Equation (2) to calculate the LAC of an unknown material from its CT number. This allows for absolute attenuation measurement using CT systems instead of the relative measurement given by CT numbers.

The E_{CT} at each kVp of the CT AURA system calculated using the HU_{CB}'s in Table 2 are: 82 keV at 140 kVp, 78 keV at 130 kVp, 74 keV at 120 kVp, and 67 keV at 100 kVp. This result is consistent with the data reported in [10], where an E_{CT} of 70 keV was chosen for 120 kVp scans, considering that keV was chosen in [10] by observing the best match between attenuation-corrected PET emission images using PET transmission maps and attenuation maps converted from CT images.

Table 5. Converted LAC of the spongiosa bone and the percentage error from the true value, using HU_{CB} obtained with different surrounding materials at 140 kVp scans. C.Ph: the previous cylindrical phantom; A. Lung and A. Liver: Anthropomorphic phantom at the lung and liver regions.

	Air	C. Ph.	A. Lung	A. Liver
HU_{CB}	1452	1250	1245	1215
μ(511)	0.110	0.112	0.112	0.112
% error	-1.8	0.0	0.0	0.0
μ(140)	0.174	0.178	0.178	0.179
% error	-1.1	+1.1	+1.1	+1.7

V. CONCLUSION

The model proposed in this paper is novel and theoretically accurate under the "water-A assumption." Its application to the conversion from CT images to PET or SPECT attenuation maps leads to the piecewise linear conversion that has been observed by other investigators. The application only requires a simple CT scan at each operation kVp of a CT system, thus is easy and convenient. The technique can also be used to characterize a CT system by obtaining the effective CT energy for each kVp, so that the CT system can be used for absolute attenuation measurement.

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Appendix

By plugging Equation (8) into Equation (6), one can obtain the LAC of x at any energy of interest, explicitly,

$$\mu_x(E) = (1 - \frac{HU_x}{HU_A}) \times \mu_w(E) + \frac{HU_x}{HU_A} \times \mu_A(E) \cdot \quad \text{(A1)}$$

Consider that the LAC of A can be expressed by its density and MAC, we have

$$\mu_A(E) = \mu'_A(E) * \rho_A \cdot \quad \text{(A2)}$$

Plug Equation (A2) into (A1), we have

$$\mu_x(E) = \left[1 + \left(\rho_A \times R_A(E) - 1\right) \times \frac{HU_x}{HU_A}\right] \times \mu_w(E) \cdot \quad \text{(A3)}$$