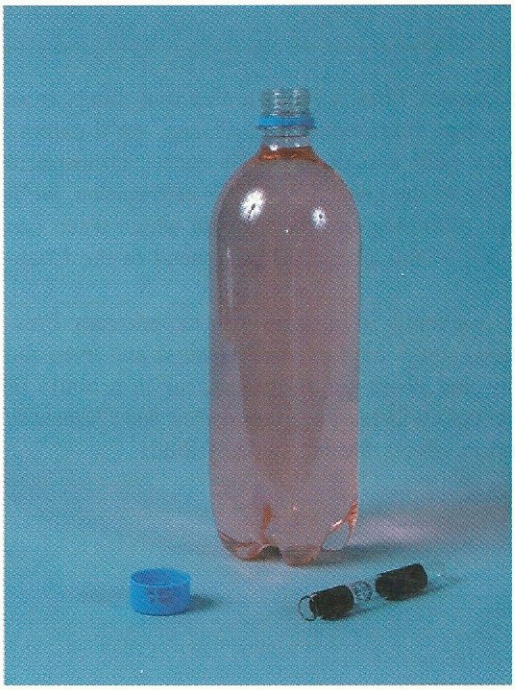
CHAPTER 5

Fluids

So far, all of the everyday objects we've examined have been solids. However, since gases and liquids are also important parts of the world around us—as the air we breathe, the water we swim in, and even the blood we pump through our veins—we'll now turn to objects that, unlike solids, don't have well-defined shapes. These objects are called fluids, and the study of fluid behavior and motion is a broad field, extending across the sciences and engineering. Fluid dynamics, often called hydrodynamics, is as important to an oil-well engineer as to an animal physiologist or a stellar astrophysicist. The tools used to analyze fluids are somewhat more complicated than for solids because fluids themselves are more complicated: it's hard to exert a force directly on them, and, even if we could, they usually don't move as a single rigid object. In this chapter, we'll look at some of the concepts and tools needed to understand their complex behaviors.

EXPERIMENT: A Cartesian Diver

One of these concepts is buoyancy: an object immersed in a fluid experiences an upward force from that fluid. This buoyant force is what lifts a helium balloon into the sky and



Courtesy Lou Bloomfield

suspends a boat on the surface of water. How the object responds to buoyancy depends on the relative densities of the object and the fluid surrounding it, where density is the ratio of mass to volume. As we'll see in this chapter, an object that's more dense than the surrounding fluid sinks, while one that's less dense than that fluid floats.

To see the importance of density in determining whether an object sinks or floats, you can construct a simple toy called a Cartesian diver. This once-popular parlor gadget consists of a small, air-filled vial floating in a sealed container of water. Normally, the air bubble inside the vial keeps it floating at the surface of the water, but whenever you squeeze the container, the vial sinks.

To make a Cartesian diver, you'll need only a plastic soda bottle and a small vial that's open at one end. The vial can be made of almost anything—plastic, metal, or glass—as long as it's dense enough to sink in water. Fill the plastic soda bottle full of water and float the vial in it upside down; air trapped inside the vial should keep the vial afloat. Now slowly reduce the size of the air bubble inside the vial until the vial barely floats. You can make this adjustment by tipping the vial to let some of its air escape or by removing it from the bottle and pouring water into it. Once you have the vial floating only a few millimeters out of the water at the top of the soda bottle, cap the bottle and prepare to test your diver.

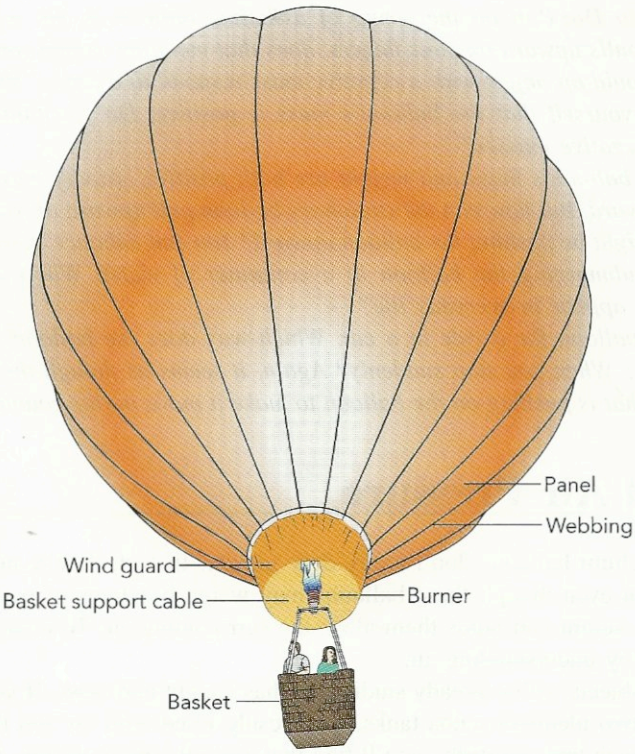
Before you squeeze the soda bottle, think about how squeezing it will affect the air bubble. Now squeeze the bottle gently and watch the air bubble. How does the bubble's size depend on how hard you squeeze the bottle? Why should the two be related? Squeeze the bottle hard enough that the vial sinks. Why is there a relationship between the size of the air bubble and the diver's height in the water?

As you release the pressure on the bottle, the diver will float back up to the surface. Why does the sunken diver suddenly become buoyant again? By carefully squeezing the bottle, you can even make the diver hover in the middle of the bottle. Try making the diver hover while your eyes are closed. Why is hovering so difficult to sustain? Why must you watch the diver to make it hover?

Chapter Itinerary

We'll return to the diver at the end of the chapter. First, we'll examine two things from the world around us: (1) *balloons* and (2) *water distribution*. In *balloons*, we'll explore how the concepts of pressure and buoyancy help explain how the earth's atmosphere keeps hot air and helium balloons from falling to the ground. In *water distribution*, we'll see both how pressure propels water through plumbing and the ways in which water can contain energy. For a more complete preview of this chapter, jump ahead to the Chapter Summary at the end of the chapter.

The issues we'll be looking at crop up frequently in our everyday experiences. Pressure plays an important role in aerosol cans, steam engines, firecrackers, and even the weather; buoyancy supports ships on water and keeps oil above vinegar in a bottle of salad dressing. Just as important, these concepts will lay the groundwork for Chapter 6, where we'll examine objects in which motion affects the behavior of fluids.



SECTION 5.1 Balloons

Because gravity gives every object near the earth's surface a weight proportional to its mass, objects fall when you drop them. Why then does a helium-filled balloon—which, after all, is just another object with a mass and a weight—sail upward into the sky when you let go of it? Does the balloon have a negative mass and a negative weight, or are we forgetting something?

We're forgetting air—specifically, the layer of air that sits atop the earth's surface and is held in place by gravity. Since this air is difficult to see and moves out of our way so easily, we often forget that it's there. But air sometimes makes itself noticeable. When you ride a bicycle, you feel its forces; when you blow up a beach ball, you see that it takes up space. And when you release a helium balloon, air lifts the balloon upward.

Questions to Think About: *Since most objects fall to the ground through the atmosphere, why doesn't the atmosphere itself fall downward? Why is the air "thinner" in the mountains than at sea level? If you suck all of the air out of a plastic bag, what squeezes the bag into a thin sheet? Why does blowing air into the bag make it inflate? What happens to the bag's total mass when you fill it with air? With hot air? With helium? If you took a sturdy helium balloon to the moon, where there is no air, and then released it, which way would it move?*

Experiments to Do: *Pull on the string of a helium balloon to get a feel for how it behaves. If it pulls upward on your fingers, does that mean its weight (and mass) is negative? How would an object with a negative mass respond to a force? Shake the balloon and convince yourself that the balloon's mass is positive. Do you think there are any objects with negative masses?*

Since the balloon's mass and weight are both positive, gravity must be pulling the balloon downward. But how can the stationary balloon pull upward on your finger? What other forces might be pushing the balloon upward? You can enhance these upward forces by partially submerging the balloon in a container of water. Where else do similar upward forces appear in everyday life?

Take the balloon for a ride in a car. Which way does the balloon move when you start suddenly? When you stop suddenly? Again, it seems as though the balloon's mass is negative. What is pushing on the balloon to make it move in this counterintuitive way?

Air and Air Pressure

Hot-air and helium balloons don't really defy gravity. In fact, they're not even close to weightless. But even though these balloons may weigh hundreds of pounds, something supports their weight and holds them aloft: the surrounding air. To understand balloons, we must start by understanding air.

Like the objects we've already studied, air has weight and mass. If you don't believe me, compare two identical scuba tanks, one freshly filled with air and the other empty. If you weigh each tank carefully, you'll find that the full tank is slightly heavier than the empty tank—an indication that air has weight. If you shake each tank, you'll find that the full tank is slightly harder to accelerate than the empty tank—an indication that air has mass. So air really is like any other object…or almost.

What makes air different from our previous objects is that air has no fixed shape or size. You can mold 1 kg of air into any form you like and it can occupy a wide range of **volumes**. Air is **compressible**—that is, you can squeeze a certain mass of it into almost any space. For example, 1 kg of air could fill a single scuba tank or a whole basketball arena.

This flexibility of size and shape originates in the microscopic nature of air. Air is a **gas**, a substance consisting of tiny, individual particles that travel around independently. These individual particles are atoms and molecules. An **atom** is the smallest portion of an element that retains all of the chemical characteristics of that element; a **molecule**, an assembly of two or more atoms, is the smallest portion of a chemical compound that retains all of the characteristics of that compound. A molecule's atoms are held together by **chemical bonds**, linkages formed by electromagnetic forces between the atoms.

Air particles are extremely small, less than a millionth of a millimeter in diameter. Most are nitrogen and oxygen molecules, but others include carbon dioxide, water, methane, and hydrogen molecules, and argon, neon, helium, krypton, and xenon atoms. Those atoms, which don't make strong chemical bonds and rarely form molecules, are called **inert gases** because of their chemical inactivity.

Like tiny marbles, these air particles have sizes, masses, and weights. But while marbles quickly settle to the ground when you spill them from a bag, air particles don't seem to fall at all. Why don't they pile up on the earth's surface?

The answer has to do with air's internal energy (i.e., its thermal energy), specifically the portion of that energy contained in the motions of the air particles. This **internal kinetic energy** keeps the tiny air particles moving, spinning, and away from the earth's surface. In contrast, real marbles are too massive and heavy to be moved noticeably by internal energy. The air's internal kinetic energy per particle is measured as its **temperature**; the

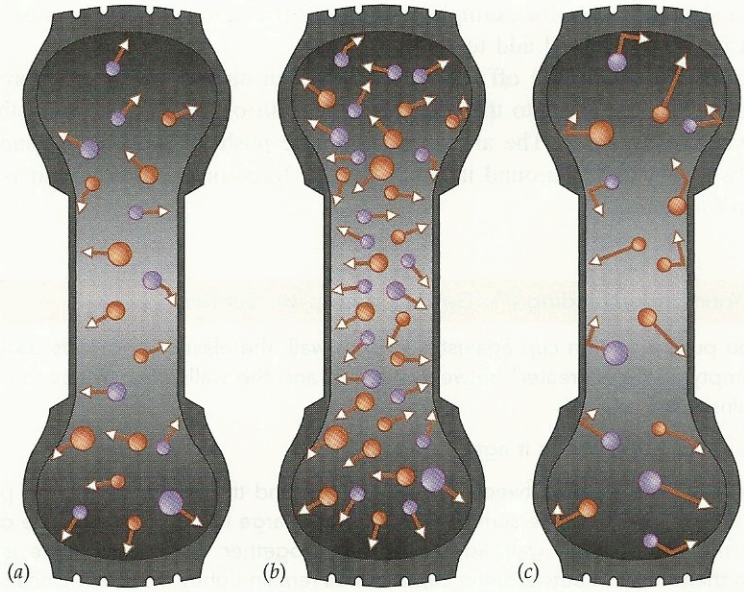


Fig. 5.1.1 (*a*) As air particles bounce off surfaces in this tire, they exert pressure on those surfaces; the amount of pressure depends on the air's temperature and on how densely its particles are packed. (*b*) Packing the air particles more densely increases the number of particles that hit the surfaces each second. (*c*) Increasing the temperature of the air increases the speed of the particles (shown by the arrows) so that they hit the tire surfaces harder and more frequently. Either change, in speed or collision frequency, increases the air's pressure.

greater that energy per particle, the hotter the air. While air's internal energy also includes a portion stored in the forces between particles, this **internal potential energy** is negligible because the average forces between air particles are so weak.

An expanded view of air would reveal countless individual particles in frenetic **thermal motion** (Fig. 5.1.1*a*). At room temperature, these particles travel at bulletlike speeds of roughly 500 m/s (1100 mph), but they collide so often that they make little progress in any particular direction. Between collisions, air particles travel in nearly straight-line paths because gravity has too little time to make them fall very far.

Let's ignore gravity for the moment and consider what happens in a truck tire containing 1 kg of air. The air particles whiz around inside the tire, and each time a particle bounces off a wall of the tire it exerts a force on that wall. Although the individual forces are tiny, the number of particles is not, and together they produce a large average force. The size of this total force depends on the wall's **surface area**; the larger its surface area, the more average force it experiences. In order to characterize the air, however, we don't really need to know the wall's surface area; instead, we can refer to the average force the air exerts on each unit of surface area, a quantity called **pressure**:

$$\text{pressure} = \frac{\text{force}}{\text{surface area}}.$$

Pressure is measured in units of force-per-area. Since the SI unit of surface area is the **meter^2** (abbreviated m^2 and often referred to as the square meter), the SI unit of pressure is the **newton-per-meter^2**. This unit is also called the **pascal** (abbreviated Pa), after French mathematician and physicist Blaise Pascal. One pascal is a small pressure; the air around you has a pressure of about 100,000 Pa (2100 lbf/ft^2 or 15 lbf/inch^2), so that it exerts a force of about 100,000 N on a 1-m^2 surface. Since 100,000 N (22,500 lbf) is about the weight of a city bus, air pressure can exert enormous forces on large surfaces.

Besides pushing on the walls of the tire, air also pushes on any object immersed in it. Its particles bounce off the object's surfaces, pushing them inward. As long as the object can withstand these compressive forces, the air won't greatly affect it, since the uniform air pressure ensures that the forces on all sides of the object cancel one another

perfectly. A sheet of paper, for example, will experience zero net force because the forces exerted on its two sides will add to zero.

Air particles also bounce off one another, so that air pressure exerts forces on air, too. A cube of air inserted into the tire experiences all of the inward forces that a cube of metal would experience. The air around the cube pushes inward on it, and the cube pushes outward on the air around it. Since the net force on the cube of air is zero, the cube doesn't accelerate.

Check Your Understanding #1: Getting a Grip on Suction

After you push a suction cup against a smooth wall, the elastic cup bends back and a small, empty space is created between the cup and the wall. What keeps the suction cup against the wall?

Answer: Air pressure keeps it against the wall.

Why: Because the space between the suction cup and the wall is empty, the pressure there is zero. Pressure of the surrounding air exerts large inward forces on the outsides of both the cup and the wall, squeezing them together. As long as there is no air between them to push outward, the cup and wall remain tightly attached. Once air leaks into the suction cup, it's easily detached from the wall.

Pressure, Density, and Temperature

Since air pressure is produced by bouncing air particles, it depends on how often, and how hard, those particles hit a particular region of surface. The more frequent or harder the impacts, the greater is the air pressure.

To increase the rate at which air particles hit a surface, we can pack them more tightly. If we use an air pump to add another 1 kg of air to the tire, we double the number of air particles in the same volume, which doubles the rate at which they hit each surface and therefore doubles the pressure (Fig. 5.1.1*b*). Air's pressure is thus proportional to its **density**—that is, its mass per unit of volume:

$$\text{density} = \frac{\text{mass}}{\text{volume}}.$$

Since the SI unit of volume is the **meter^3** (abbreviated m^3 and often referred to as the cubic meter), the SI unit of density is the **kilogram-per-meter^3** (abbreviated kg/m^3). The air around you has a density of about 1.25 kg/m^3 (0.078 lbm/ft^3). Water, in contrast, has a much greater density of about 1000 kg/m^3 (62.4 lbm/ft^3).

We can also increase the rate at which air particles hit a surface by speeding them up (Fig. 5.1.1*c*). If we double the internal kinetic energy of the air in the tire, we double the average kinetic energy of each particle. Because a particle's kinetic energy depends on the square of its speed, doubling its kinetic energy increases its speed by a factor of $\sqrt{2}$. As a result, each particle hits the surface $\sqrt{2}$ times as often and exerts $\sqrt{2}$ times as much average force when it hits. With each particle exerting $\sqrt{2} \cdot \sqrt{2}$ or two times as much average force, the pressure doubles. Air's pressure is thus proportional to the average kinetic energy of its particles—to their average internal kinetic energies.

This average kinetic energy per particle is measured by the air's temperature; the hotter the air, the larger the average kinetic energy per particle and the higher the air's pressure. That's why the tire's air pressure increases on hot days and when it heats up

during highway driving. The most convenient scale for relating the temperature of air to its pressure isn't the common **Celsius** (°C) or **Fahrenheit** (°F) scale; instead, it's a special **absolute temperature scale**. The SI scale of absolute temperature is the **Kelvin** scale (K). When the air's temperature is 0 K (−273.15 °C or −459.67 °F), it contains no internal kinetic energy at all and has no pressure; this temperature is called **absolute zero**. The Kelvin scale is identical to the Celsius scale, except that it's shifted so that 0 K is equal to −273.15 °C. In addition to associating the zero of temperature with the zero of internal kinetic energy, the Kelvin scale avoids the need for negative temperatures. Room temperature is about 293 K.

Since air pressure is proportional to both the air's density and its absolute temperature, we can express the relationship among these quantities in the following way:

$$\text{pressure} \propto \text{density} \cdot \text{absolute temperature}. \tag{5.1.1}$$

This relationship is useful because it allows us to predict what will happen if we change the temperature or density of a specific gas, such as air. The relationship does have its limitations; in particular, it doesn't work if we compare the pressures of two different gases, such as air and helium, which differ in their chemical compositions. To make such a comparison, we'll need to improve on Eq. 5.1.1. We'll do that later when we examine helium balloons.

Even in describing a specific gas, Eq. 5.1.1 has other shortcomings. The main problem is that real gas particles aren't completely independent of one another. If the temperature drops too low, the particles begin to stick together to form a liquid and Eq. 5.1.1 becomes invalid. Despite its limitations, however, this simple relationship between pressure, density, and temperature will prove useful in understanding how hot-air balloons float: it will help us understand the basic structure of the earth's atmosphere, the origins of the upward force that keeps a hot-air balloon aloft, and the reason why hot air rises.

Quantity	*SI Unit*	*English Unit*	*SI → English*	*English → SI*
Area	meter^2 (m^2)	foot^2 (ft^2)	1 m^2 = 10.764 ft^2	1 ft^2 = 0.092903 m^2
Volume	meter^3 (m^3)	foot^3 (ft^3)	1 m^3 = 35.315 ft^3	1 ft^3 = 0.028317 m^3
Pressure	pascal (Pa)	pound-force-per-foot^2 (lbf/ft^2)	1 Pa = 0.020885 lbf/ft^2	1 lbf/ft^2 = 47.880 Pa
Density	kilogram-per-meter^3 (kg/m^3)	pound-mass-per-foot^3 (lbm/ft^3)	1 kg/m^3 = 0.062428 1bm/ft^3	1 lbm/ft^3 = 16.018 kg/m^3

Check Your Understanding #2: Snacks That Go Pop in the Night

If you remove a partially filled container of food from the refrigerator and allow it to warm to room temperature, the lid will often bow outward and may even pop off. What has happened?

Answer: The pressure of the air trapped in the container increases as its temperature increases, causing the lid to bulge outward.

Why: Whenever a trapped quantity of gas changes temperature, it also changes volume or pressure or both. In this case, warming the air trapped in the container causes its pressure to increase. The unbalanced pressures inside and outside the container cause the lid to bow outward or even to pop off.

The Earth's Atmosphere

Most of the mass of the earth's atmosphere is contained in a layer less than 6 km (4 miles) thick. Since the earth is 12,700 km (7900 miles) in diameter, this layer is relatively thin—so thin that, if the earth were the size of a basketball, it would be no thicker than a sheet of paper.

The atmosphere stays on the earth's surface because of gravity. Every air particle, as we've seen, has a weight. Just as a marble thrown upward eventually falls back to the ground, so the particles of air keep returning toward the earth's surface. Although the particles are moving too fast for gravity to affect their motions significantly over the short term, gravity works slowly to keep them relatively near the earth's surface. An air particle, like a rapidly moving marble, may appear to travel in a straight line at first, but it will arc over and begin to fall downward eventually. Only the lightest and fastest moving particles in the atmosphere—hydrogen molecules and helium atoms—occasionally manage to escape from earth's gravity and drift off into interplanetary space.

While gravity pulls the atmosphere downward, air pressure pushes the atmosphere upward. As the air particles try to fall to the earth's surface, their density increases and so does their pressure. It's this air pressure that supports the atmosphere and prevents it from collapsing into a thin pile on the ground.

To understand how gravity and air pressure structure the atmosphere, picture a 1-m^2 column of the atmosphere as though it were a tall stack of 1-kg air blocks (Fig. 5.1.2). These blocks support one another with air pressure to form a stack of about 10,000 blocks. The bottom block must support the weight of all the blocks above it and is tightly compressed, with a height of about 0.8 m, a density of about 1.25 kg/m^3, and a pressure of about 100,000 Pa. A block farther up in the stack has less weight to support and is less tightly compressed. The higher in the stack you look, the lower the density of the air and the less the air pressure.

The atmosphere has essentially the same structure as this stack of blocks. The air near the ground supports the weight of several kilometers of air above it, giving it a density of about 1.25 kg/m^3 and a pressure of about 100,000 Pa; at higher altitudes, however, the air's density and pressure are reduced, since there is less atmosphere overhead and the air doesn't have to support as much weight. High-altitude air is thus "thinner" than low-altitude air. Whatever the altitude, the pressure of the surrounding air is referred to as **atmospheric pressure**.

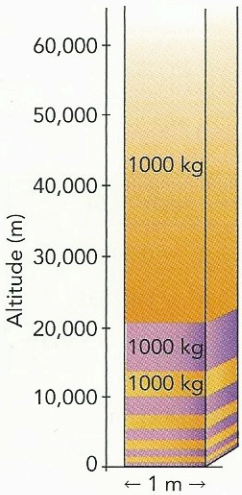


Fig. 5.1.2 The air in a 1-m^2 column of atmosphere has a mass of about 10,000 kg. The bottom 1000 kg is the most tightly compressed because it supports the most weight above it. At higher altitudes, the air is less tightly compressed because it has less weight above it.

Check Your Understanding #3: Mountain Travel Is a Pain in the Ears

As you drive up and down in the mountains, you may feel a popping in your ears as air moves to equalize the pressures inside and outside your eardrum. What causes these pressure changes?

Answer: As you change altitude, the atmospheric pressure changes.

Why: The air inside your ears is trapped, so that its temperature, density, and pressure are normally constant. As your altitude changes, the pressure outside your ear changes and your eardrum experiences a net force. It bows inward or outward, muting the sounds you hear and causing some discomfort. The pressure imbalance is relieved during swallowing, when air can flow into or out of your eardrum.

The Lifting Force on a Balloon: Buoyancy

So far we've examined air, air pressure, and the atmosphere. While it may seem that we've avoided dealing with balloons, these topics really are involved in keeping hot-air and helium balloons aloft. As we've seen, the air in the earth's atmosphere is a **fluid**, a shapeless substance with mass and weight. This air has a pressure and exerts forces on the surfaces it touches; that pressure is greatest near the ground and decreases with increasing altitude. Air pressure and its variation with altitude allow air to lift hot-air and helium balloons through an effect known as buoyancy.

Buoyancy was first described more than two thousand years ago by the Greek mathematician Archimedes (287–212 BC). Archimedes realized that an object partially or wholly immersed in a fluid is acted on by an upward **buoyant force** equal to the weight of the fluid it displaces. **Archimedes' principle** is actually very general and applies to objects floating or submerged in any fluid, including air, water, or oil. The buoyant force originates in the forces a fluid exerts on the surfaces of an object. We've seen that such forces can be quite large but tend to cancel one another. How then can pressure create a nonzero total force on an object, and why should that force be in the upward direction?

Archimedes' Principle
An object partially or wholly immersed in a fluid is acted on by an upward buoyant force equal to the weight of the fluid it displaces.

Without gravity the forces would cancel each other perfectly because the pressure of a stationary fluid would be uniform throughout. But gravity causes a stationary fluid's pressure to decrease with altitude. For example, when nothing is moving, the air pressure beneath an object is always greater than the air pressure above it. Thus air pushes upward on the object's bottom more strongly than it pushes downward on the object's top, and the object consequently experiences an upward overall force from the air—a buoyant force.

How large is the buoyant force on this object? It's equal in magnitude to the weight of the fluid that the object displaces. To understand this remarkable result, imagine replacing the object with a similarly shaped portion of the fluid itself (Fig. 5.1.3*a*). Since

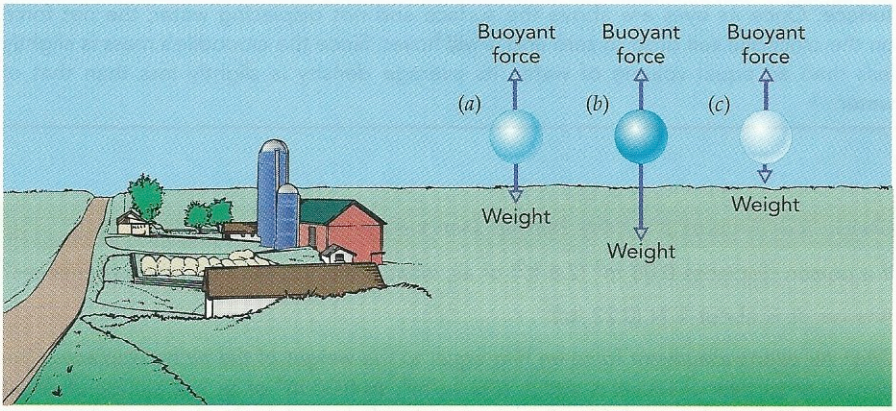


Fig. 5.1.3 (*a*) A portion of air immersed in that same air experiences an upward buoyant force equal to its weight and doesn't accelerate. (*b*) An object that is heavier than the air it displaces sinks, while (*c*) another object that is lighter than the air it displaces floats.

the buoyant force is exerted by the surrounding fluid, not the object, it doesn't depend on the object's composition. A balloon filled with helium will experience the same buoyant force as a similar balloon filled with water or lead or even air. So replacing the object with a similarly shaped portion of fluid will leave the buoyant force on it unchanged.

However, a portion of fluid suspended in more of the same fluid doesn't accelerate anywhere; it just sits there, so the net force on it is clearly zero. It has a downward weight, but that weight must be canceled by some upward force that can only come from the surrounding fluid. This upward force is the buoyant force, and it's always equal in magnitude to the weight of the object-shaped portion of fluid displaced by the object.

This buoyant principle explains why some objects float while others sink. An object placed in a fluid experiences two forces: its downward weight and an upward buoyant force. If its weight is more than the buoyant force, it will accelerate downward (Fig. 5.1.3*b*); if its weight is less than the buoyant force, it will accelerate upward (Fig. 5.1.3*c*). If the two forces are equal, it won't accelerate at all and will maintain a constant velocity. If the balloon in this latter case starts motionless, it will remain motionless and will hover at a constant velocity of zero.

Whether or not an object will float in a fluid can also be viewed in terms of density. An object that has an average density greater than that of the surrounding fluid sinks, while one that has a lesser average density floats. A water-filled balloon, for example, will sink in air because water and rubber are more dense than air. If you double the volume of the balloon, you double both its weight and the buoyant force on it, so it still sinks. The total volume of an object is less important than how its density compares to that of the surrounding fluid.

Check Your Understanding #4: **Only Eyes above the Surface**

Crocodiles sometimes swallow rocks, partly to aid their digestion but also to lower their heights in the water. To float with only its eyes above the water's surface, approximately what average density does a crocodile need?

Answer: It needs an average density just slightly less than the density of water.

Why: The crocodile should weigh a little less than the water it displaces when fully immersed. That way, it will experience a net upward force and will rise to the water's surface. Once its eyes are above the surface and not displacing water, the net force on the crocodile will drop to zero and it will hover. Since the crocodile's mass is slightly less than an equal volume of water, its average density is slightly less than that of water.

Check Your Figures #1: **Why People Don't Float in Air**

If a person displaces 0.08 m^3 (2.8 ft^3) of air, what is the buoyant force he experiences?

Answer: It is about 1 N (0.22 lbf).

Why: Air exerts a buoyant force on him equal to the weight of the air he displaces. The density of air near sea level is about 1.25 kg/m^3, so 0.08 m^3 of air has a mass of about 0.1 kg (1.25 kg/m^3 times 0.08 m^3) and a weight of about 1 N. So the upward buoyant force on him is about 1 N. This buoyant force due to the air is real and reduces the weight you read when you stand on a scale by about 0.125%.

Hot-Air Balloons

Since air is very light, with a density of only 1.25 kg/m^3 (0.078 lbm/ft^3), few objects float in it. One of these rare objects is a balloon with a vacuum inside it. Assuming that the balloon has a very thin outer shell or *envelope*, it will weigh almost nothing and have an average density near zero. Because its negligible weight is less than the upward buoyant force it experiences, the empty balloon will float upward nicely.

Unfortunately, this empty balloon won't last long. Because it's surrounded by atmospheric pressure air, each square meter of its envelope will experience an inward force of 100,000 N. With nothing inside the balloon to support its envelope against this crushing force, it will smash flat. A thick, rigid envelope might be able to withstand the pressure of the surrounding air, but then the balloon's average density would be large and it would sink. So an empty balloon won't work.

What will work is a balloon filled with something that exerts an outward pressure on the envelope equal to the inward pressure of the surrounding air. Then each portion of the envelope will experience zero net force and the balloon will not be crushed. We could fill the balloon with outside air, but that would make its average density too high. Instead, we need a gas that has the same pressure as the surrounding air but a lower density.

One gas that has a lower density at atmospheric pressure is hot air. Filling our balloon with hot air takes fewer particles than filling it with cold air, since each hot-air particle is moving faster and contributes more to the overall pressure than does a cold-air particle. A hot-air balloon contains fewer particles, has less mass, and weighs less than it would if it contained cold air. Now we have a practical balloon with an average density less than that of the surrounding air. The buoyant force it experiences is larger than its weight, and up it goes (Fig. 5.1.4).

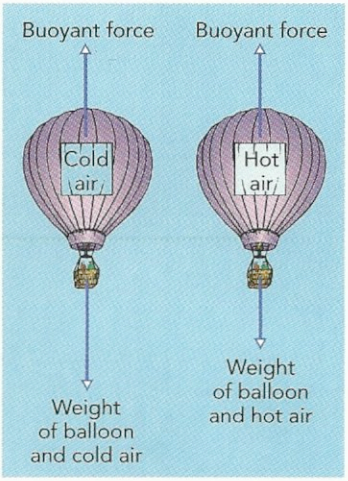


Fig. 5.1.4 A balloon filled with hot air contains fewer air particles and weighs less than a balloon filled with cold air. If the balloon's weight is small enough, the net force on the balloon will be in the upward direction and the balloon will accelerate upward.

Because the air pressure inside a hot-air balloon is the same as the air pressure outside the balloon, the air has no tendency to move in or out (an issue we will cover in the next section), and the balloon doesn't need to be sealed (Fig. 5.1.5). A large propane burner, located beneath the balloon's open end, heats the air that fills the envelope. The hotter the air in the envelope, the lower its density and the less the balloon weighs. The balloon's pilot controls the flame so that the balloon's weight is very nearly equal to the buoyant force on the balloon. If the pilot raises the air's temperature, particles leave the envelope, the balloon's weight decreases, and the balloon rises. If the pilot allows the air to cool, particles enter the envelope, the balloon's weight increases, and the balloon descends.

Courtesy Lou Bloomfield

Fig. 5.1.5 The bottom of a hot-air balloon is open so that heated air can flow in and cold air can flow out. The heated air displaces more than its weight in cold air and makes the balloon lighter.

Even if the pilot heats the air to be very hot, the balloon won't rise upward forever. As the balloon ascends, the air becomes thinner and the pressure decreases both inside and outside the envelope. Although the balloon's weight decreases as the air thins out, the buoyant force on it decreases even more rapidly, and it becomes less effective at lifting its cargo. When the air becomes too thin to lift the balloon any higher, the balloon reaches a *flight ceiling* above which it can't rise, even if the pilot turns the flame on full blast. For each hot-air temperature, then, there is a cruising altitude at which the balloon will hover. When the balloon reaches that altitude, it's in a stable equilibrium. If the balloon shifts downward for some reason, the net force on it will be upward; if it shifts upward, the net force on it will be downward.

Check Your Understanding #5: **Ballooning Weather**

Can a hot-air balloon lift more on a hot day or a cold day?

Answer: It can lift more on a cold day.

Why: On a cold day, the outside air is relatively dense and the buoyant force on a balloon is larger than it would be on a hot day. The hot air in the balloon will cool off more quickly on a cold day, but the balloon will be able to carry a heavier load. Airplanes also fly better on cold days.

Helium Balloons

Although the particles in hot and cold air are similar, there are fewer of them in each cubic meter of hot air than in each cubic meter of cold air. We call the number of particles per unit of volume **particle density**,

$$\text{particle density} = \frac{\text{particles}}{\text{volume}},$$

and hot air has a smaller particle density than cold air (Fig. 5.1.6*a, b*). Because they contain similar particles, hot air also has a smaller density than cold air and is lifted upward by the buoyant force.

There's another way to make one gas float in another: use a gas consisting of very light particles. Helium atoms, for example, are much lighter than air particles. When they have equal pressures and temperatures, helium gas and air also have equal particle densities. Since each helium atom weighs 14% as much as the average air particle, 1 m^3 of helium weighs only 14% as much as 1 m^3 of air. Thus a helium-filled balloon has only a fraction of the weight of the air it displaces, and the buoyant force carries it upward easily.

Why should air and helium have the same particle densities whenever their pressures and temperatures are equal? Because a gas particle's contribution to the pressure doesn't

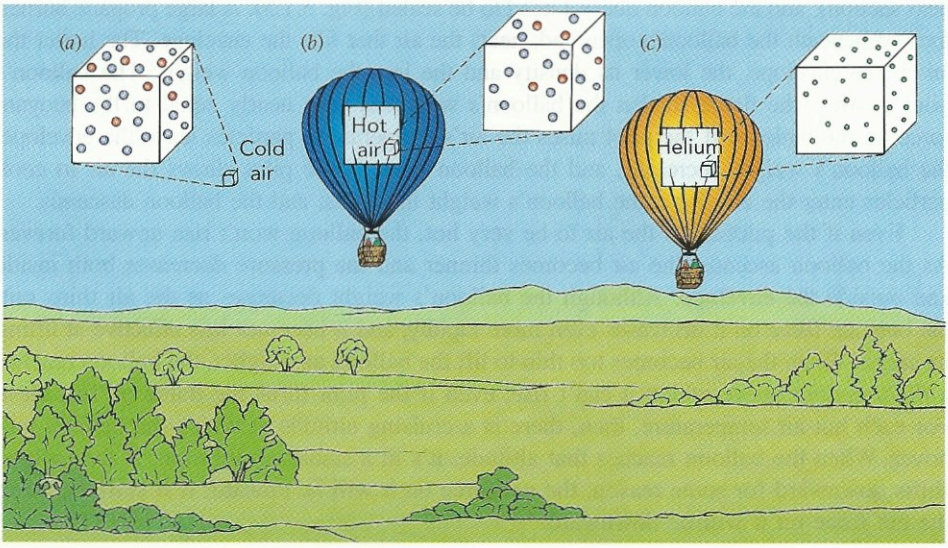


Fig. 5.1.6 A cube of cold air (*a*) contains more air particles than an equivalent cube of hot air (*b*). Since it weighs less than the cold air it displaces, the hot air inside the balloon experiences an upward buoyant force that is greater than its weight. Although an equivalent cube of helium (*c*) contains the same number of particles as the cube of cold air (*a*), each helium particle weighs less than the average air particle. The helium inside the balloon therefore experiences an upward buoyant force that is greater than its weight.

depend on its mass (or weight). At a particular temperature, each particle in a gas has the same average kinetic energy in its translational motion, regardless of its mass. Although a helium atom is much less massive than a typical air particle, the average helium atom moves much faster and bounces more often. As a result, lighter but faster-moving helium atoms are just as effective at creating pressure as heavier but slower-moving air particles.

Thus, if you allow the helium atoms inside a balloon to spread out until the pressures and temperatures inside and outside the balloon are equal, the particle densities inside and outside the balloon will also be equal (Fig. 5.1.6*a, c*). Since the helium atoms inside the balloon are lighter than the air particles outside it, the balloon weighs less than the air it displaces, and it will be lifted upward by the buoyant force.

The pressure of a gas is proportional to the product of its particle density and its absolute temperature, as the following formula indicates:

$$\text{pressure} \propto \text{particle density} \cdot \text{absolute temperature}. \qquad (5.1.2)$$

This proportionality holds regardless of the gas's chemical composition. Our previous proportionality, Eq. 5.1.1, worked only as long as the gas's composition didn't change, so that density and particle density remained proportional to one another. Now, however, we have a relationship with a wider applicability.

Equation 5.1.2, with an associated constant of proportionality, is called the **ideal gas law**. This law relates pressure, particle density, and absolute temperature for a gas in which the particles are perfectly independent. It's also fairly accurate for real gases in which the particles do interact somewhat. The constant of proportionality is the **Boltzmann constant**, with a measured value of 1.381×10^{-23} Pa · m^3/(particle · K). Using the Boltzmann constant, the ideal gas law can be written as a word equation:

$$\text{pressure} = \text{Boltzmann constant} \cdot \text{particle density} \cdot \text{absolute temperature}, \qquad (5.1.3)$$

in symbols:

$$p = k \cdot \rho_{\text{particle}} \cdot T,$$

and in everyday language:

Don't incinerate a spray can. A hot, dense gas tends to burst its container.

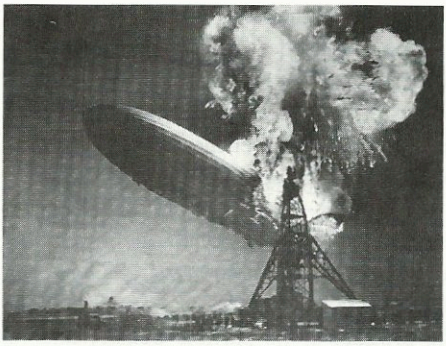
The Ideal Gas Law

The pressure of a gas is equal to the product of the Boltzmann constant times the particle density times the absolute temperature.

Helium isn't the only "lighter-than-air" gas. Hydrogen gas, which is half as dense as helium, is also used to make balloons float. However, don't expect hydrogen to lift twice as much weight as helium. A balloon's lifting capacity is the difference between the upward buoyant force it experiences and its downward weight. Although the gas in a hydrogen balloon weighs half that in a similar helium balloon, the balloons experience the same buoyant force. Thus the hydrogen balloon's lifting capacity is only slightly more than that of the helium balloon. Hydrogen's main advantage is that it's cheap and plentiful, while helium is scarce (see ■). Hydrogen is also dangerously flammable, so it's avoided in situations where safety is important (Fig. 5.1.7). However, even helium-filled airships can have problems (see ■).

■ Helium gas is obtained as a by-product of natural gas production from underground reservoirs in the United States, where it formed through the gradual radioactive decay of uranium and other unstable elements. While some of this gas is saved for industrial and commercial use, much is simply released into the atmosphere. The only other source of helium is the atmosphere, where helium is present at a level of 5 parts per million. Once the underground stores are consumed, helium will become a relatively rare and expensive gas.

■ Even helium-filled airships were easily destroyed by bad weather. The *Shenandoah*, one of two U.S. airships based on German designs, was destroyed by air turbulence on September 3, 1925, near Ava, Ohio. Crowds from a local fair immediately poured over the wreckage, collecting souvenirs.



Corbis Images

Fig. 5.1.7 Rigid airships were appropriately named because they were truly ships that floated through the air. Unfortunately, the hydrogen gas that filled most airships, making them light enough that air could lift them, is very flammable. The *Hindenburg* burned on May 6, 1937, while trying to land at Lakehurst, New Jersey. Because hydrogen is so buoyant in air, most of the combustion occurred above the airship and many passengers survived.

Check Your Understanding #6: **What Not to Put in a Balloon**

A carbon dioxide molecule is heavier than an average air particle. If you pour carbon dioxide gas from a cup, which way does it flow in air, up or down?

Answer: It flows down.

Why: Carbon dioxide, found in carbonated beverages, dry ice, and fire extinguishers, is heavier than air because its molecules are heavier than air particles. The carbon dioxide you pour from the cup has the same pressure and temperature as the air around it and thus the same particle density. However, each carbon dioxide molecule weighs more, so the carbon dioxide is the denser gas (it has higher mass density) and flows down in air. This tendency to flow along the floor makes carbon dioxide very good at extinguishing low-lying flames by depriving them of oxygen.

Check Your Figures #2: **Popped Out of the Refrigerator**

When you take an air-filled plastic container out of the refrigerator, it warms from 2 °C to 25 °C. How much does the pressure of the air inside it change?

Answer: It increases by 8.4%.

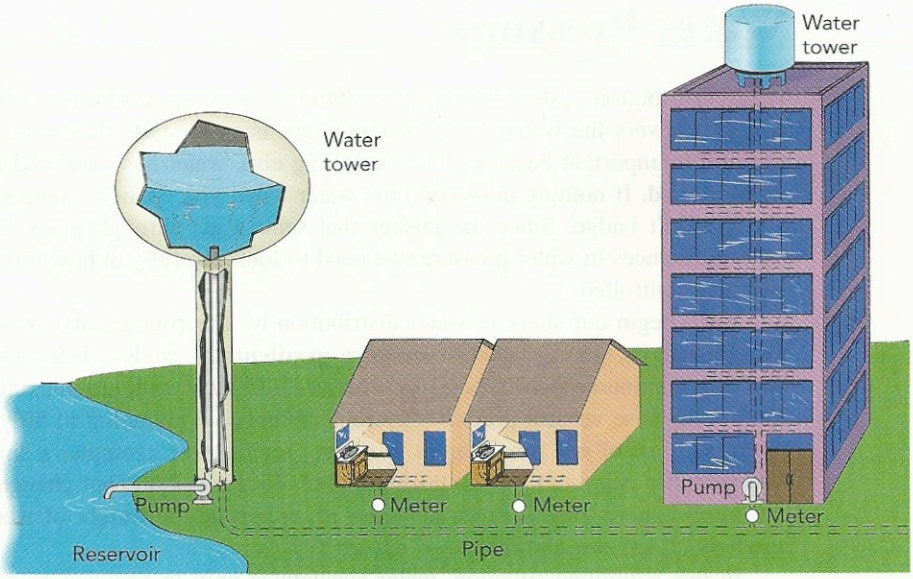
Why: To use Eq. 5.1.3 to determine the pressure change, we need temperatures measured on an absolute scale, such as the Kelvin scale. Since 0 °C is about 273 K, 2 °C is about 275 K and 25 °C is about 298 K. We can write Eq. 5.1.3 twice, once for each temperature:

$$\text{pressure}_{298\,\text{K}} = \text{Boltzmann constant} \cdot \text{particle density} \cdot 298\,\text{K},$$
$$\text{pressure}_{275\,\text{K}} = \text{Boltzmann constant} \cdot \text{particle density} \cdot 275\,\text{K}.$$

The particle density of the air in the container can't change as it warms up because its volume is fixed. Therefore, we can divide the upper equation by the lower one and cancel the Boltzmann constant and particle density on the right-hand side:

$$\frac{\text{pressure}_{298\,\text{K}}}{\text{pressure}_{275\,\text{K}}} = \frac{298\,\text{K}}{275\,\text{K}} = 1.084.$$

The pressure in the container thus increases by a factor of almost 1.084, or about 8.4%. This elevated pressure will cause the container to emit a "pop" sound when you open it.



SECTION 5.2 Water Distribution

Now that we've explored the behavior of objects in fluids, let's turn to the behavior of fluids in objects. In this section, as we examine how plumbing distributes water, we'll see that pressure, density, and weight are just as important in plumbing as they are in ballooning. To keep things simple, we'll focus on the causes of water's motion through plumbing, leaving most of the complications associated with the motion itself for the next chapter. For example, we'll temporarily ignore drag and viscosity and the fascinating pressure changes that accompany fluid motion.

Questions to Think About: *Why is the water pressure higher in the basement than it is in the attic? Why does a deep water well require a pump at the bottom? If a water tower is only a storage device, why is it so tall? Why do skyscrapers have complicated plumbing systems that include reservoirs at various levels in the building? What causes water to flow up through a drinking straw to your mouth?*

Experiments to Do: *To see the effects of pressure and weight on water, try these simple experiments with a drinking straw. First, "suck" water up the straw from a glass to your mouth. Are you exerting an attractive force on the water, or is some other force pushing it upward toward your mouth? With the straw full of water, seal the top with your finger; keeping the seal tight, remove the straw from the glass. What happens to the water inside? What happens to the water when you release the seal? Now blow into one end of a straw full of water while sealing the far end with your finger. When you release the seal on the far end, what happens to the water? What forces are responsible for this effect?*

Water Pressure

Water distribution systems require two things: plumbing and water pressure. Plumbing is what delivers the water, and water pressure is what starts that water flowing. Water pressure is important because, like everything else, water has mass and accelerates only when pushed. If nothing pushed on the water when you opened a faucet, the water simply wouldn't budge. Since the pushes that send water through pipes come principally from differences in water pressure, we need to look carefully at how this pressure is created and controlled.

We'll begin our study of water distribution by ignoring gravity. As we've seen with the atmosphere, gravity creates **pressure gradients** in fluids—distributions of pressure that vary continuously with position. Pressure decreases with altitude and increases with depth, and these vertical pressure gradients complicate plumbing in hilly cities and skyscrapers. If all of our plumbing is in a level region—for example, a single-story house in a very flat city—our job is much simpler. With no significant changes in height, we can safely ignore gravity, since no water is supporting the weight of water above it and gravity's effects are minimal.

In this simplified situation, water accelerates only in response to unbalanced pressures. Just as unbalanced forces make a solid object accelerate, so too do unbalanced pressures make a fluid accelerate. If the water pressure inside a pipe is uniform, then each portion of water feels no net force and doesn't accelerate; it either remains stationary or coasts at constant velocity (Fig. 5.2.1*a*). If the pressure is out of balance, however, then each portion of water experiences a net force and accelerates toward the region of lowest pressure (Figs. 5.2.1*b*, *c*).

This acceleration doesn't mean that the water will instantly begin moving toward the lowest pressure. Because of its inertia, water changes velocity gradually: it speeds up, slows down, or turns to the side, depending on where the lowest pressure is located. A complicated arrangement of high and low pressures can steer water through an intricate maze of pipes, and that is exactly how water reaches your home from the city pumping station. Every change in its velocity during its trip through the plumbing is caused by a pressure imbalance.

You can create a pressure imbalance in water simply by squeezing parts of it. The pressure in a squeezed portion will rise and it will accelerate toward lower pressures elsewhere. Since this sort of pressure change isn't caused by the water's motion, it's a **static variation** in pressure. The water's motion can also affect its pressure, and such **dynamic variations** in pressure can be complicated and fascinating. As we'll see in the next chapter, they contribute to such diverse effects as the spray of a garden hose nozzle, the lift on an airplane's wing, and the curve of a curve ball.

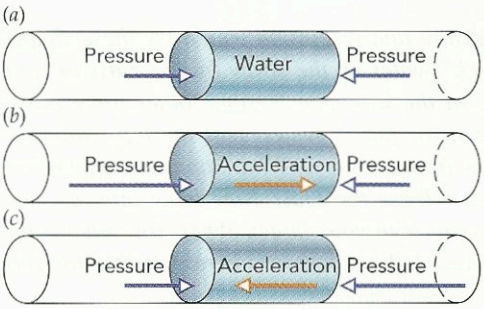


Fig. 5.2.1 (*a*) If the water in a horizontal pipe is exposed to a uniform pressure, it will not accelerate. (*b*, *c*) If the pressure along the pipe is not uniform, however, the imbalance will create a net force on each portion of the water and the water will accelerate toward the side with the lower pressure.

Check Your Understanding #1: Under Pressure in the Garden

With the water faucet open and the nozzle at the end of your garden hose shut tightly, the hose is full of high-pressure water. Why doesn't this water accelerate?

Answer: The water pressure (force on a unit of surface area) inside the hose is uniform throughout, so the water experiences no net force and doesn't accelerate.

Why: In the absence of gravity, fluids accelerate only when they experience pressure imbalances. Since water throughout the hose is at the same pressure, there is no pressure imbalance and no acceleration. When you open the nozzle, the pressure at that end of the hose drops and the water accelerates toward it.

Creating Water Pressure with Water Pumps

To start water flowing through the plumbing in a level house or city, you need a water pump—a device that uses mechanical work to deliver pressurized water through a pipe. At its most basic level, a water pump squeezes a portion of water to raise its local pressure so that it accelerates toward regions of lower pressure. The pump continues to squeeze that water as it flows out through the plumbing.

To understand how a pump works, picture a sealed plastic soda bottle full of water. When you don't squeeze the bottle, the pressure inside it is atmospheric and uniform (remember that we're neglecting gravity). But when you squeeze the sides of the bottle and push inward on the water, the water responds by pushing outward on you—Newton's third law—and it does this by increasing its pressure. The harder you push, the harder the water pushes back and the greater its pressure becomes.

Because water, like all liquids, is **incompressible** and experiences almost no change in volume as its pressure increases, the bottle won't get smaller. But the rise in water pressure inside it can be substantial. It doesn't take much force, exerted with your thumb on a small area of the bottle, to increase the pressure in the bottle from atmospheric to twice that value or more.

The pressure increases uniformly throughout the water bottle, an observation known as **Pascal's principle**—a change in the pressure of an enclosed incompressible fluid is conveyed undiminished to every part of the fluid and to the surfaces of its container. This uniform pressure rise leads to a large upward force on the bottle's cap. If the cap were wider and had more surface area, the upward force on it might be large enough to blow it off the bottle. That effect is the basis for hydraulic systems and lifts, where pressure produced in an incompressible fluid by a small force exerted on a small area of the fluid's container results in a large force exerted on a large area of the fluid's container (Fig. 5.2.2). It also explains why plastic drinking bottles usually have small caps and why wide-mouth plastic containers are better suited for candies, cookies, and nuts.

Pascal's Principle

A change in the pressure of an enclosed incompressible fluid is conveyed undiminished to every part of the fluid and to the surfaces of its container.

It's time to remove the cap from the water bottle. Now when you squeeze the bottle and increase the local water pressure, the water can move. As its pressure rises inside the bottle, the water begins to accelerate toward the lower pressure above the bottle's open top and the result is a fountain. You are pumping water!

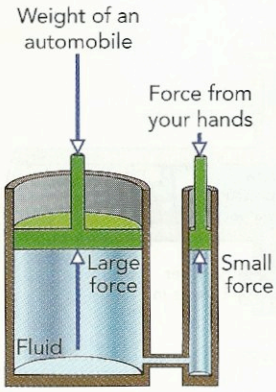


Fig. 5.2.2 The force a pressurized fluid exerts on a piston is proportional to the surface area of that piston. This fact underlies hydraulic systems, in which a small force exerted on a small piston pressurizes a trapped liquid so that it exerts a large force on a large piston. In this figure, your gentle downward push on the small piston is balanced by a gentle upward force exerted on that small piston by the pressurized fluid. At the same time, the strong downward push of an automobile on the large piston is balanced by a strong upward force exerted on that large piston by the pressurized fluid. If the small piston descends quickly at constant velocity, the large piston will rise slowly at constant velocity and you'll be doing the work of lifting the heavy car a short distance by pushing the small piston down gently for a large distance. The hydraulic system is providing you with mechanical advantage and allowing you to raise something that's too heavy for you to lift unaided.

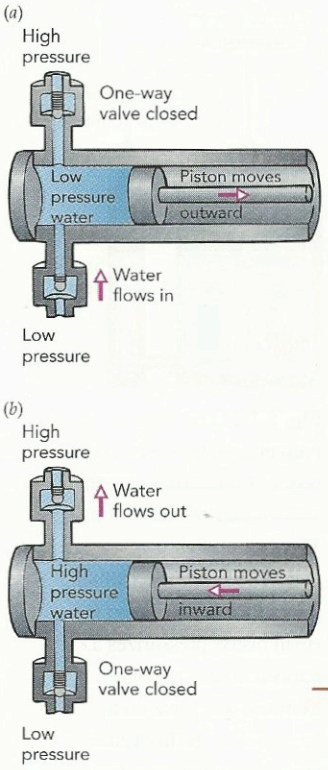


Fig. 5.2.3 Water is pumped from a region of low pressure to a region of high pressure by a reciprocating piston pump. (*a*) As the piston is drawn outward, water flows into the cylinder from the low-pressure region. (*b*) As the piston is pushed inward, the inlet one-way valve closes and water is driven out of the cylinder and into the high-pressure region.

You are also doing work: as the water flows out of the bottle, your hands move inward. Since you are pushing inward on the water and the water is moving inward, you are doing work on the water. Pumps do work when they deliver pressurized water, and pressurized water carries with it the energy associated with that work.

Although a water bottle can act as a pump briefly, it soon runs out of water. A more practical pump appears in Fig. 5.2.3. In this pump, a piston slides back and forth in the open end of a hollow cylinder, making a watertight seal. Pushing inward on that piston squeezes any water in the cylinder and raises the local water pressure. Water begins to flow.

In contrast to our simple bottle, this pump's cylinder is easy to refill. The cylinder actually has two openings, each with a valve that permits water to flow in only one direction. Water can leave the cylinder only through the top opening and can enter only through the bottom opening. As you push the pump's piston into the water-filled cylinder, the water pressure in the cylinder rises and water accelerates and flows out through the top valve. As you pull the pump's piston out of the water-filled cylinder, the water pressure inside the cylinder drops and water accelerates and flows in through the bottom valve. In fact, as you withdraw the piston, the pressure inside the cylinder drops below atmospheric pressure, so that even water in an open reservoir nearby will accelerate toward the partial vacuum in the cylinder and refill it.

Check Your Understanding #2: Working a Water Pump

Which normally requires more work: pulling the piston of a water pump out of the cylinder or pushing it back in?

Answer: Pushing it back in usually requires more work.

Why: When you pull the piston out of the cylinder, you are moving air out of the way and creating a partial vacuum inside the cylinder. This action requires a modest amount of work because the air doesn't push terribly hard on the back of the piston, and pressure from the water flowing into the cylinder assists you. But as you push the piston back into the cylinder, you are pressurizing the water. Depending on the pressure of water in the outlet hose, the water in the cylinder may exert a very large force on the piston. In that case, you must do a great deal of work on the water as you push the piston inward and drive the water out of the cylinder.

Moving Water: Pressure and Energy

The pump of Fig. 5.2.3 can draw low-pressure water from a pond and fill a hose with high-pressure water. If the other end of the hose is open, the water will accelerate toward lower pressure at that end and will have considerable kinetic energy as it sprays out of the hose. From where does this kinetic energy come?

The energy comes from you and the pump. As you push inward on the piston, pressurizing the water and squeezing it out through the top valve, you're doing work on the water because you're exerting an inward force on the water's surface and the water is moving inward. The amount of work you do is equal to the product of the water pressure times the volume of water you pump. This simple relationship between work, pressure, and volume comes about because the inward *force* you exert on the water with the piston is equal to the water pressure times the surface area of the piston, or

$$\text{force} = \text{pressure} \cdot \text{surface area},$$

and because the *distance* the water moves in the direction of that force is equal to the volume of water pumped divided by the surface area of the piston, or

$$\text{distance} = \frac{\text{volume}}{\text{surface area}}.$$

Since work equals *force* times *distance*, the work you do pumping water is

$$\text{work} = \text{pressure} \cdot \text{volume}.$$

As you pump the water, it sprays out of the open hose. The energy that makes the water accelerate out of the hose actually travels through the water directly from the pump to the end of the hose. Since water is incompressible, each time a liter of water leaves the pump, a liter of water also leaves the hose. Although the water never really stores any energy, the pump gives each liter of water a certain amount of energy as it leaves the hose so we can imagine that this energy is associated with the water and not with the pump. We create a useful fiction: **pressure potential energy**. Water that's under pressure has a pressure potential energy equal to the product of the water's volume times its pressure.

Because pressure potential energy actually comes from the pump, it vanishes as soon as you break the link between the water and the pump: you can't save a bottle of high-pressure water and expect it to retain this potential energy. The concept of pressure potential energy is only meaningful if the water is flowing freely so that water leaving the plumbing is immediately replaced; then whatever energy leaves the plumbing as kinetic energy in the water is put back into the plumbing by the pump. Actually, the details of the pump aren't as important as the idea that any water moving through the plumbing is immediately replaced by more water with the same pressure. As long as the water is flowing steadily, you can safely use the concept of pressure potential energy, even if you don't know where the pump is or whether there actually is one.

Pressure potential energy is most meaningful in **steady-state flow**—a situation in which fluid flows continuously and steadily through a stationary environment, without starting or stopping or otherwise changing its characteristics anywhere. You can tell you're watching steady-state flow when you can't detect the passage of time in the fluid or its environment; if a video of the situation would look the same as a still photograph, then it's steady-state flow. With small allowances for the inevitable imperfections of real life, water spraying steadily out of a hose, wind blowing smoothly across your motionless face, and a gentle current flowing in a quiet river are all cases of steady-state flow in fluids.

Without gravity, the energy in a certain volume of water in steady-state flow is equal to the sum of its pressure potential energy and its kinetic energy. We've already seen that the pressure potential energy is the product of the water's volume times its pressure. The water's kinetic energy is given by Eq. 2.2.1 as one-half the product of its mass times the square of its speed. Since water's mass is its density times its volume, this sum is

$$\begin{aligned}\text{energy} &= \text{pressure potential energy} + \text{kinetic energy}\\ &= \text{pressure} \cdot \text{volume} + \tfrac{1}{2} \cdot \text{density} \cdot \text{volume} \cdot \text{speed}^2,\end{aligned} \tag{5.2.1}$$

If we divide both sides of this expression by the volume involved, we can obtain another useful form of this relationship:

$$\begin{aligned}\frac{\text{energy}}{\text{volume}} &= \frac{\text{pressure potential energy}}{\text{volume}} + \frac{\text{kinetic energy}}{\text{volume}}\\ &= \text{pressure} + \tfrac{1}{2} \cdot \text{density} \cdot \text{speed}^2.\end{aligned} \tag{5.2.2}$$

As a professor in Basel, Daniel Bernoulli (Swiss mathematician, 1700–1782) taught not only physics, but also botany, anatomy, and physiology. He correctly proposed that the pressure a gas exerts on the walls of its container results from the countless impacts of tiny particles that make up the gas. He also derived an important relationship between the pressure, motion, and height of a fluid—Bernoulli's equation.

As each volume of water moves along with the flow, it is soon replaced by a new volume of water. Because the flow is steady-state, the energy in the new volume of water must be exactly the same as in the volume that preceded it; thus the energy in each volume of water that flows along a particular path must be identical. The particular path that a volume of water takes is called a **streamline**, and the energy-per-volume of fluid along a streamline is constant:

$$\begin{aligned}\frac{\text{energy}}{\text{volume}} &= \frac{\text{pressure potential energy}}{\text{volume}} + \frac{\text{kinetic energy}}{\text{volume}} \\ &= \text{pressure} + \tfrac{1}{2}\cdot\text{density}\cdot\text{speed}^2 \\ &= \text{constant}\,(\textit{along a streamline}).\end{aligned} \tag{5.2.3}$$

Equation 5.2.3 is called **Bernoulli's equation**, after Swiss mathematician Daniel Bernoulli (see ■) whose work led to its development, although Swiss mathematician Leonhard Euler (1707–1783) actually completed it.

Because energy is conserved, an incompressible fluid such as water that's in steady-state flow can exchange pressure for speed or speed for pressure as it flows along a streamline. As water accelerates out of a hose with a nozzle, for example, its pressure drops but its speed increases because it's converting pressure potential energy into kinetic energy. And as that moving water sprays against the car you're washing, it slows down but its pressure increases because it's converting kinetic energy back into pressure potential energy. In both cases, the water's total energy is constant.

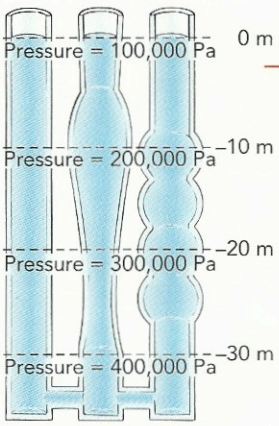


Fig. 5.2.4 The pressure of stationary water in pipes increases with depth by about 10,000 Pa per meter of depth. *The shape of the pipes doesn't matter.* For plumbing that's open on top and connected near the bottom, as shown here, water will tend to flow until its height is uniform throughout the plumbing.

Check Your Understanding #3: How Does Your Garden Grow?

Water in your garden hose has considerable pressure and arcs several meters through the air as you water your plants. What is the water pressure in the falling water once it leaves the end of the hose?

Answer: It is atmospheric pressure.

Why: As the water accelerates out of the hose, its pressure drops. It is converting pressure potential energy into kinetic energy. The water pressure drops until it reaches the pressure of the surrounding air, which is atmospheric pressure.

Gravity and Water Pressure

Gravity creates a pressure gradient in water: the deeper the water, the more weight there is overhead and the greater the pressure. Since water is much denser than air, water pressure increases rapidly with depth. In a vertical pipe that's open on top, the water's surface is at atmospheric pressure (about 100,000 Pa), but only 10 m (33 feet) below the water's surface, the pressure has already doubled to 200,000 Pa. At that modest depth, the water overhead weighs as much as the air overhead, even though the atmosphere is several kilometers thick.

The shape of the pipe doesn't affect the relationship between pressure and depth. No matter how complicated the plumbing, the pressure of stationary water inside it increases with depth by 10,000 Pa per meter or 10,000 Pa/m (Fig 5.2.4). This uniform

pressure gradient creates an upward buoyant force on anything immersed in the water. In fact, that buoyant force is what supports the water itself (Fig. 5.2.5).

The dependence of water pressure on depth has a number of important implications for water distribution. First, water pressure at the bottom of a tall pipe is substantially higher than at the top of that same pipe. Consequently, if only a single pipe supplies water to a skyscraper, then the water pressure on the ground floor will be dangerously high, while the pressure in the penthouse will be barely enough for a decent shower. Tall buildings must therefore handle water pressure very carefully; they can't simply supply water to every floor directly from the same pipe.

Second, pressure in a city water main does more than just accelerate water out of a showerhead; it also supports water in the pipes of multistory buildings. Lifting water to the third floor against the downward force of gravity requires a large upward force, and that force is provided by water pressure. The higher you want to lift the water, the more water pressure you need at the bottom of the plumbing. Lifting the water also requires energy, which is often provided by a water pump.

Third, as water travels up and down the streets of a hilly city, its pressure varies with height. In the valleys the pressure can be very large, and at the tops of hills the pressure can be very small. Water mains in valleys must therefore be particularly strong to keep from bursting. The large pressure in a valley is quite useful because it helps push the water back uphill on the other side of the valley (Fig. 5.2.6). Nonetheless, a hilly city must have pumping stations and other water pressure control systems located throughout in order to provide reasonable water pressures to all the buildings, regardless of their altitudes.

Apart from those pressure-control systems, even a hilly city's plumbing often involves steady-state flow and can be explained using a version of Bernoulli's equation that includes gravity. Before we explore that topic, though, let's take a look at the non-steady-state flow that occurs when water in an isolated section of plumbing has free surfaces that can move up or down so that you see things change with time.

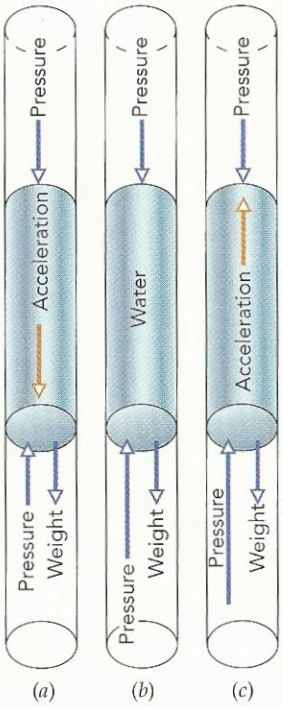
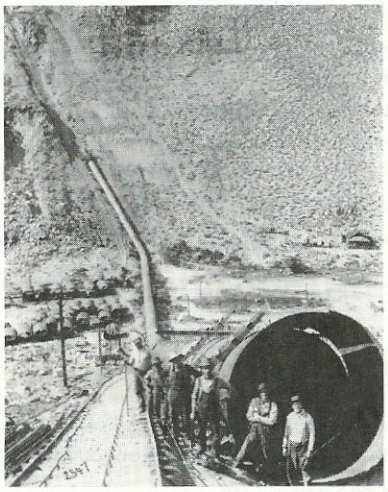


Fig. 5.2.5 When a pipe is oriented vertically, gravity affects the motion of water in the pipe. (*a*) If the water's pressure doesn't change with depth, the water will accelerate downward (fall) because of its weight. (*b*) If the water's pressure increases with depth by 10,000 Pa/m, the water won't accelerate. (*c*) If the water's pressure increases with depth by more than that amount, the water will accelerate upward.



Courtesy Department of Water and Power of the City of Los Angeles

Fig. 5.2.6 Los Angeles receives much of its water from Owens Valley, 300 km to the north. The water negotiates the mountains and valleys in between, driven by gravity alone. Giant pipes allow pressure to build during downhill stretches in order to push the water back uphill later on. Parts of the 1913 aqueduct support so much pressure that the steel pipe used in them has to be more than an inch thick.

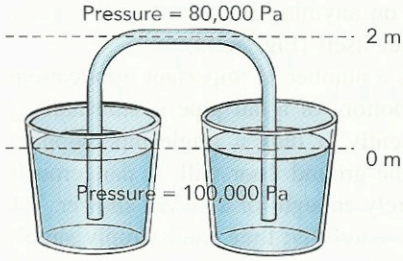


Fig. 5.2.7 The two open containers of water are connected by a siphon. This U-shaped tube permits water to flow until its level is equal in both containers. The sturdy tube permits the water pressure in the siphon to drop below atmospheric pressure.

The simplest case is plumbing that's open on top, so that all the water's free surfaces are at atmospheric pressure. Like any object, water accelerates in the direction that lowers its total potential energy as quickly as possible. With neither the immovable plumbing beneath it nor the uniform air pressure above it contributing any potential energy, water's only potential energy is gravitational and it accelerates in the direction that lowers that gravitational potential as quickly as possible.

If the water's free surface is higher in one place than another, it can reduce its average height and therefore its gravitational potential energy by letting its highest water fill in its lowest valley. After some sloshing about, the water settles down in stable equilibrium with all of its free surfaces smooth and level at a single, uniform height. No matter how complicated the plumbing, water that is open to the air on top always "seeks its level." The natural flow associated with this leveling effect is often used in water delivery (see ■).

■ The Romans used gravity to convey water to Rome from sources up to 90 km away. A very gradual slope in the aqueducts kept the water moving in spite of frictional effects that opposed the water's progress. Poisoning from the lead pipes used in some of the aqueducts is blamed in part for the decay of the Roman Empire.

If you seal off part of the isolated plumbing and reduce the pressure above one of the water's free surfaces, that surface will rise higher than all of the others. It will rise until the added pressure produced by the taller column of water replaces the missing pressure above the water's free surface. The less pressure there is above that surface, the higher the water must rise to make up for the missing pressure. This effect lifts water in a drinking straw and allows it to travel between two open containers in order to "seek its level" through an elevated pipe known as a siphon (Fig. 5.2.7).

However, removing all of the air pressure above the water's free surface inside a long straw or siphon will raise its height only about 10 m (33 feet) above the level of the water elsewhere in an open container. Even with no pressure above it, this 10-m column of elevated water completely replaces the absent air pressure and thus prevents water in the rest of the plumbing from lifting it any higher. It's therefore impossible to draw water from a deep well simply by lowering a pipe into that well and reducing the air pressure in the pipe: the water will rise no more than 10 m upward. Instead, a pump must be attached to the bottom of the pipe to pressurize the water and push it all the way to the top of the pipe.

Check Your Understanding #4: What's the Water Pressure?

If all of the water to a 400-m-tall skyscraper were delivered from a single pipe, how much higher would the water pressure be on the ground floor than on the top floor?

Answer: It would be about 4,000,000 Pa (40 atmospheres) higher.

Why: The weight of water inside the pipe would create an enormous excess pressure near the bottom of the building. Water spraying from an open faucet on the first floor at this enormous pressure could accelerate to 319 km/h (200 mph), as it does in some high-pressure jet washing and cutting machines.

Moving Water Again: Gravity

As we've seen, it takes pressure and energy to lift water to the third floor of a building. We can now expand our statement of energy conservation in fluids undergoing steady-state flow to include gravity and gravitational potential energy.

Water's gravitational potential energy is equal to its weight times its height (the force required to lift it times the distance it has been lifted), and its gravitational potential energy-per-volume is its weight-per-volume times its height. Since its weight-per-volume is its density times the acceleration due to gravity, water's gravitational potential energy-per-volume is its density times the acceleration due to gravity times its height.

If we include gravitational potential energy in Eq. 5.2.2 and recognize that, for fluids in steady-state flow along a streamline, the energy-per-volume is constant, we obtain a relationship that can be written as a word equation:

$$\begin{aligned}\frac{\text{energy}}{\text{volume}} &= \frac{\text{pressure potential energy}}{\text{volume}} + \frac{\text{kinetic energy}}{\text{volume}} + \frac{\text{gravitational potential energy}}{\text{volume}} \\ &= \text{pressure} + \tfrac{1}{2} \cdot \text{density} \cdot \text{speed}^2 + \text{density} \cdot \text{acceleration due to gravity} \cdot \text{height} \\ &= \text{constant}\ (\textit{along a streamline}), \qquad (5.2.4)\end{aligned}$$

in symbols:

$$p + \tfrac{1}{2} \cdot \rho \cdot v^2 + \rho \cdot g \cdot h = \text{constant}\ (\textit{along a streamline}),$$

and in everyday language:

When a stream of water speeds up in a nozzle or flows uphill in a pipe, its pressure drops.

This is a revised version of Bernoulli's equation, one that includes gravity. It correctly describes steady-state flow in streamlines that change height.

Bernoulli's Equation

For an incompressible fluid in steady-state flow, the sum of its pressure potential energy, its kinetic energy, and its gravitational potential energy is constant along a streamline. Equation 5.2.4 expresses this law as a formula.

Because energy is conserved, an incompressible fluid such as water that's in steady-state flow can exchange the energies associated with its speed, pressure, and height for one another. Thus as water flows downward, its speed or pressure or both increase; if it falls from an open faucet, its speed increases; and if it descends steadily inside a uniform pipe, its pressure increases. The reverse happens as water flows upward. Water rising from a fountain loses speed as it ascends, while water rising steadily in a uniform pipe loses pressure on its way up.

This interchangeability of height, pressure, and speed makes it possible to pressurize plumbing by connecting a tall column of water to the pipes. That's why cities, communities, and even individual buildings have water towers (Fig. 5.2.8). A water tower is built at a relatively high site within the region it serves. A pump fills the water tower with

Steve Lewis/The Image Bank/Getty Images

Fig. 5.2.8 Many buildings in New York City have water towers on their roofs. These towers maintain water pressure in the plumbing and help in firefighting.

water, and then gravity maintains a constant high pressure throughout the plumbing that connects to it (Fig. 5.2.9). The water is at atmospheric pressure at the top of the water tower, but the pressure is much higher at the bottom; at the base of a 50-m-high water tower, for example, the pressure is about 600,000 Pa or six times atmospheric pressure.

In addition to providing a fairly steady pressure in the water mains, a water tower stores energy efficiently and can deliver that energy quickly. When water is drawn out of the water tower, its gravitational potential energy at the top becomes pressure potential energy at the bottom. The water tower replaces a pump, supplying a steady flow of water at an almost constant high pressure. Unlike a pump, however, the water tower can supply this high-pressure water at an enormous rate. As long as the water level doesn't drop too far, high-pressure water keeps flowing.

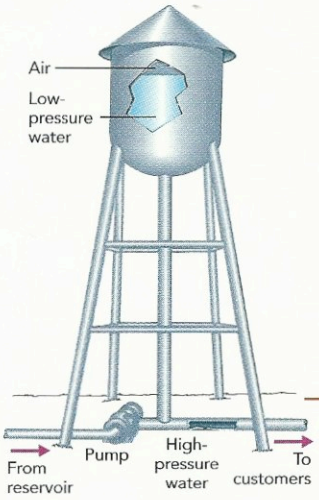


Fig. 5.2.9 A water tower uses the weight of water to create a large water pressure near the ground. The higher the tower, the greater the pressure near the ground. The water tower is able to maintain the water pressure passively and doesn't require constant pumping. Even during periods of peak water consumption, it maintains a fairly steady pressure. When the water level in the water tower drops below a certain set point, a pump refills the tower.

Check Your Understanding #5: Water Power

A hydroelectric power plant extracts energy from water that has descended from an elevated reservoir in a pipe. In the reservoir, this energy takes the form of gravitational potential energy. Just before the power plant, what form does the energy take?

Answer: It takes the form of pressure potential energy (and some kinetic energy).

Why: As the water descends inside the pipe, its gravitational potential energy is converted into pressure potential energy. The water reaching the power plant is under enormous pressure, and it's this pressure that exerts the forces needed to turn the turbines that run the generators. Work is required to turn the turbines, so the water gives up much of its energy in the power plant. This energy leaves the power plant via the electric power lines.

Check Your Figures #1: Up or Out

If the water pressure at the entrance to a building is 1,000,000 Pa, how high can the water rise up inside the building, and how fast will it flow out of a faucet right at the entrance? (A liter of water inside a pipe has a mass of 1 kg.)

Answer: It can rise up about 100 m or emerge from the faucet at about 45 m/s (101 mph).

Why: As the water flows through the pipe (a streamline), its pressure potential energy can become gravitational potential energy or kinetic energy. At the start, the water's energy is all pressure potential energy so, from Eq. 5.2.4, the water's energy-per-volume

is 1,000,000 Pa. If the water flows up the pipe, that energy will become gravitational potential energy. We can rearrange Eq. 5.2.4 to find the height it can reach:

$$\text{height} = \frac{\text{energy}}{\text{volume}} \cdot \frac{1}{\text{density} \cdot \text{acceleration due to gravity}}$$

$$= 1{,}000{,}000\,\text{Pa} \cdot \frac{1}{1000\,\text{kg/m}^3 \cdot 9.8\,\text{m/s}^2} = 102\,\text{m}.$$

If the water flows out of the faucet, that energy will become kinetic energy. We can also rearrange Eq. 5.2.4 to find the speed it will attain:

$$\text{speed} = \sqrt{\frac{\text{energy}}{\text{volume}} \cdot \frac{2}{\text{density}}}$$

$$= \sqrt{\frac{2{,}000{,}000\,\text{Pa}}{1000\,\text{kg/m}^3}} = 45\,\text{m/s}.$$

Epilogue for Chapter 5

In this chapter we investigated some of the basic concepts associated with fluids. In *balloons*, we explored the concept of pressure and the way in which air pressure structures and supports the earth's atmosphere. We saw that increased air pressure beneath an object produces an upward buoyant force on that object, and that this buoyant force can float objects, such as hot-air and helium balloons, that are less dense than the surrounding air.

In *water distribution*, we examined how water pressure causes water to accelerate through plumbing, from high pressure to low pressure. We then focused on ways to produce water pressure, either with pumps or with gravity. By studying the forms of energy in water, we were led to Bernoulli's equation, which describes the interconversion of a fluid's energy between pressure potential energy, kinetic energy, and gravitational potential energy. Although the most dramatic applications of Bernoulli's equation are ahead of us, we've already used it to understand the changes in pressure and speed that accompany water's movement up and down in pipes and in fountains.

Explanation: A Cartesian Diver

The diver floats because its average density—that is, the mass of the vial and its contents divided by the volume of space those two components occupy—is less than the density of water. Since the upward buoyant force on the diver exceeds its downward weight, the diver floats upward toward the top of the bottle. When the diver begins to stick out of the water, it displaces less water and more air and the buoyant force it experiences decreases. Eventually it experiences zero net force and floats without accelerating at the water's surface.

When you squeeze the soda bottle, you increase the pressure inside it. Because water is incompressible, its density doesn't change as the pressure goes up. However, the air bubble inside the vial is compressed and takes up less space inside the vial. Water flows into the vial and increases the average density of the vial and its contents. When the average density of the diver finally exceeds the density of water, the diver sinks.

To keep the diver hovering in the water, you must adjust the water pressure until the diver's average density is exactly that of water. This adjustment is impossible to make without looking at the diver. Even the slightest overpressure or underpressure will cause the diver to drift slowly down or up.

Chapter Summary

How Balloons Work: A hot-air balloon floats because its total weight (basket, envelope, and hot air) is less than that of the cooler air it displaces. By heating the air in the envelope with a flame, the pilot reduces its density. As the air warms up, fewer particles are needed to fill the envelope, the extra particles flow out of the envelope through the opening at its bottom, and the balloon becomes lighter.

A helium balloon also weighs less than the air it displaces. However, its lower weight is due to the lightness of the individual helium atoms, each of which weighs much less than the air particle it replaces. By filling a balloon with helium, the balloon's weight is dramatically reduced. Since the buoyant force exerted on the balloon by the surrounding air exceeds the balloon's weight, the balloon accelerates upward.

How Water Distribution Works: Water distribution begins when a pump transfers low-pressure water from a reservoir to high-pressure plumbing. Along a level path, water accelerates toward regions of lower pressure, such as open hoses or showerheads. Pressure imbalances allow the water to negotiate bends in the pipes en route to its destination. During its travels, the water may rise or fall in height; as it does, its pressure changes, decreasing as the water moves upward and increasing as the water moves downward. In low-lying regions, where the water pressure may be too high to use directly, a pressure regulator may have to be added to the plumbing. In high-lying regions, where the water pressure may be too low to be practical, an additional pump may have to be employed to boost its pressure.

Important Laws and Equations

1. Archimedes' Principle: An object partially or wholly immersed in a fluid is acted on by an upward buoyant force equal to the weight of the fluid it displaces.

2. The Ideal Gas Law: The pressure of a gas is equal to the product of the Boltzmann constant times the particle density times the absolute temperature, or

$$\text{pressure} = \text{Boltzmann constant} \cdot \text{particle density} \cdot \text{absolute temperature}. \qquad (5.1.3)$$

3. Pascal's Principle: A change in the pressure of an enclosed incompressible fluid is conveyed undiminished to every part of the fluid and to the surfaces of its container.

4. Bernoulli's Equation: For an incompressible fluid in steady-state flow, the sum of its pressure potential energy, its kinetic energy, and its gravitational potential energy is constant along a streamline, or

$$\begin{aligned}\frac{\text{energy}}{\text{volume}} &= \frac{\text{pressure potential energy}}{\text{volume}} + \frac{\text{kinetic energy}}{\text{volume}} + \frac{\text{gravitational potential energy}}{\text{volume}} \\ &= \text{pressure} + \tfrac{1}{2} \cdot \text{density} \cdot \text{speed}^2 + \text{density} \cdot \text{acceleration due to gravity} \cdot \text{height} \\ &= \text{constant}\ (\textit{along a streamline}). \qquad (5.2.4)\end{aligned}$$

Exercises

1. A helium-filled balloon floats in air. What will happen to an air-filled balloon in helium? Why?

2. A log is much heavier than a stick, yet both of them float in water. Why doesn't the log's greater weight cause it to sink?

3. An automobile will float on water as long as it doesn't allow water to leak inside. In terms of density, why does admitting water cause the automobile to sink?

4. Many grocery stores display frozen foods in bins that are open at the top. Why doesn't the warm room air enter the bins and melt the food?

5. Some clear toys contain two colored liquids. No matter how you tilt one of those toys, one liquid remains above the other. What keeps the upper liquid above the lower liquid?

6. When the car you are riding in stops suddenly, heavy objects move toward the front of the car. Explain why a helium-filled balloon will move toward the rear of the car.

7. Water settles to the bottom of a tank of gasoline. Which takes up more space: 1 kg of water or 1 kg of gasoline?

8. Oil and vinegar salad dressing settles with the oil floating on top of the vinegar. Explain this phenomenon in terms of density.

9. When a fish is floating motionless below the surface of a lake, what is the amount and direction of the force the water is exerting on it?

10. Some fish move extremely slowly, and it's hard to tell whether they are even alive. However, if a fish is floating at a middle height in your aquarium and not at the top or bottom of the water, you can be pretty certain that it's alive. Why?

11. A barometer, which is often used to monitor the weather, is a device that measures air pressure. How could you use a barometer to measure your altitude as you climbed in the mountains?

12. If you seal a soft plastic bottle or juice container while hiking high in the mountains and then return to the valley, the container will be dented inward. What causes this compression?

13. Many jars have dimples in their lids that pop up when you open the jar. What holds the dimple down while the jar is sealed, and why does it pop up when the jar is opened?

14. If you place a hot, wet cup upside down on a smooth counter for a few seconds, you may find it difficult to lift up again. What is holding that cup down on the counter?

15. You seal a rigid container that is half full of hot food and put it in the refrigerator. Why is the container's lid bowed inward when you look at it later?

16. Why aren't there any thermometers that read temperatures down to −300 °C?

17. You use your breath to inflate a large rubber tube and then ride down a snowy hill on it. After a few minutes in the snow the tube is underinflated. What happened to the air?

18. A marshmallow is filled with air bubbles. Why does a marshmallow puff up when you toast it?

19. Wasp and hornet sprays proudly advertise just how far they can send insecticide. How does the pressure inside the spray can affect that distance, and why is the direction of the spray important (vertical vs. horizontal)?

20. Ice tea is often dispensed from a large jug with a faucet near the bottom. Why does the speed of tea flowing out of the faucet decrease as the jug empties?

21. Why must tall dams be so much thicker at their bases than at their tops?

22. Waterproof watches have a maximum depth to which they can safely be taken while swimming. Why?

23. When you stand in a pool with water up to your neck, you find that it's somewhat more difficult to breathe than when you're out of the water. Why?

24. How does pushing on the plunger of a syringe cause medicine to flow into a patient through a hollow hypodermic needle?

25. Why must the pressure inside a whistle teakettle exceed atmospheric pressure before the whistle can begin to make noise?

26. Each time you breathe in, air accelerates toward your nose and lungs. How does the pressure in your lungs compare with that in the surrounding air as you breathe in?

27. You can inflate a plastic bag by holding it up so that it catches the wind. Use Bernoulli's equation to explain this effect.

28. When someone pulls a fire alarm in a skyscraper, pumps increase the water pressure in the section of the building nearest that alarm box. How does this pressure change assist firefighters who must battle the blaze?

Problems

1. The particle density of standard atmospheric air at 273.15 K (0 °C) is 2.687×10^{25} particles/m^3. Using the ideal gas law, calculate the pressure of this air.

2. How much force is the air exerting on the front surface of this book?

3. If you fill a container with air at room temperature (300 K), seal the container, and then heat the container to 900 K, what will the pressure be inside the container?

4. An air compressor is a device that pumps air particles into a tank. A particular air compressor adds air particles to its tank until the particle density of the inside air is 30 times that of the outside air. If the temperature inside the tank is the same as that outside, how does the pressure inside the tank compare to the pressure outside?

5. If you seal a container of air at room temperature (20 °C) and then put it in the refrigerator (2 °C), how much will the pressure of the air in the container change?

6. If you submerge an 8-kg log in water and it displaces 10 kg of water, what will the net force on the log be the moment you let go of the log?

7. If your boat weighs 1200 N, how much water will it displace when it's floating motionless at the surface of a lake?

8. The density of gold is 19 times that of water. If you take a gold crown weighing 30 N and submerge it in water, what will the buoyant force on the crown be?

9. How much upward force must you exert on the submerged crown in Problem 8 to keep it from accelerating?

10. How could you use your results from Problems 8 and 9 to determine whether the crown was actually gold rather than gold-plated copper? (The density of copper is nine times that of water.)

11. Your town is installing a fountain in the main square. If the water is to rise 25 m (82 feet) above the fountain, how much pressure must the water have as it moves slowly toward the nozzle that sprays it up into the air?

12. Rather than putting a pump in the fountain (see Problem 11), the town engineer puts a water storage tank in one of the nearby high-rise office buildings. How high up in that building should the tank be for its water to rise to 25 m when spraying out of the fountain? (Neglect friction.)

13. To clean the outside of your house you rent a small high-pressure water sprayer. The sprayer's pump delivers slow-moving water at a pressure of 10,000,000 Pa (about 150 atmospheres). How fast can this water move if all of its pressure potential energy becomes kinetic energy as it flows through the nozzle of the sprayer?

14. When the water from the sprayer in Problem 13 hits the side of your house, it slows to a stop. If it hasn't lost any energy since leaving the sprayer, what is the pressure of the water at the moment it stops completely?

15. To dive far below the surface of the water, a submarine must be able to withstand enormous pressures. At a depth of 300 m, what pressure does water exert on the submarine's hull?