CHAPTER 4

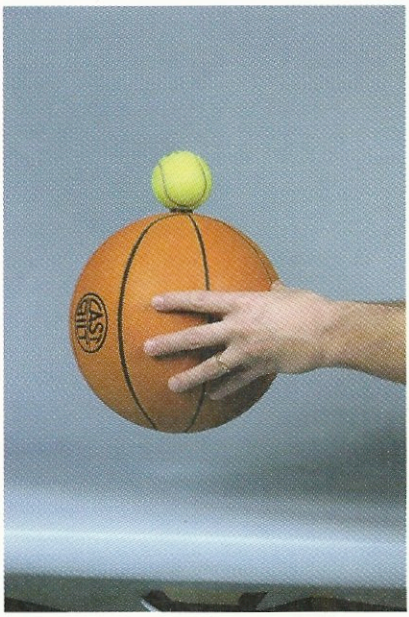
Mechanical Objects Part 2

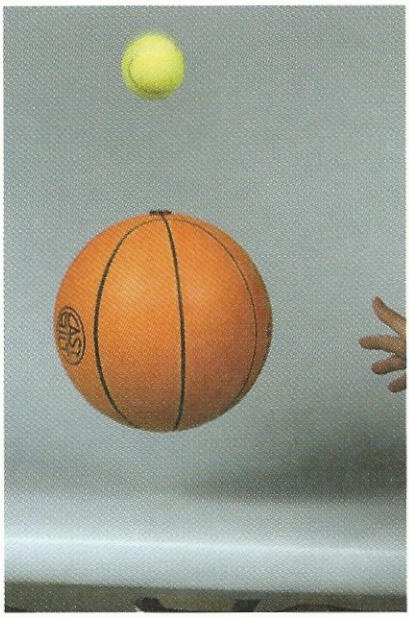
No matter how sophisticated the machines around us appear, most are based in large part on the simple principles that we have already encountered. In this chapter, we'll take a look at two more fascinating machines and see what makes them tick. As we do, we'll find ourselves revisiting familiar issues and exploring a few new ones all the way to the frontiers of science and the cosmos.

EXPERIMENT: High-Flying Balls

Among the physical issues that we'll discuss in this chapter are the reaction effects that push rockets forward. These reaction effects appear in an interesting way in a simple experiment that involves two different-sized balls, a basketball and a tennis ball.

If you can't find a basketball and a tennis ball, any two lively balls of very different masses will do. Drop the balls separately and see how high they bounce. You'll immediately notice that an individual ball can't bounce higher than the point from which you dropped it. Such a rebound would give it more energy than it had originally. In fact, some of the ball's energy will be lost to thermal energy so that it will not even reach its original height.





Courtesy Lou Bloomfield

But what will happen when you stack the smaller ball on top of the larger ball and drop the two balls together? Think about the sequence of events that will occur, then give it a try. Make sure that the small ball remains directly above the large ball as the two balls fall to the floor.

Why does the smaller ball rebound as it does? How does your choice of balls influence this effect? If you put the smaller ball on the bottom, will that change the outcome? What about dropping the pair of balls from a different height? Try the same experiment with balls of various sizes to gather more insight into what is happening.

Chapter Itinerary

The objects in this chapter combine many of the concepts we have discussed and add a few new ones of their own. These objects are (1) *bicycles* and (2) *rockets*. In *bicycles*, we'll see how motion and clever design can make an apparently unstable vehicle stable enough to ride easily, even without hands on the handlebars. In *rockets*, we'll look at the principle of action and reaction, and see how this basic idea makes it possible for spacecraft to leave the earth's surface and even travel toward the stars. For a more complete preview of this chapter, turn ahead to the Chapter Summary at the end of the chapter.

This chapter revisits many ideas from the previous chapters, but it also introduces a variety of new concepts we can use. In *bicycles*, we'll learn about dynamic stability and explore issues that waiters, water-skiers, and warehouse clerks must deal with all the time. We'll also encounter gyroscopes and their peculiar responses to torques. In *rockets*, we'll see how gravity causes objects to orbit one another, giving rise to the intricate motions of the planets, moons, and comets. We'll also look at new physics that appears near the extremes of speed and gravity.

SECTION 4.1 Bicycles

A bicycle is a wonderfully energy efficient, human-powered vehicle. Its wheels allow its rider to coast forward easily on level surfaces and accelerate effortlessly down hills. Compare the easy motion of a bicycle to that of walking, which requires effort every step of the way. Bicycles are very simple machines, and most of their moving parts are quite visible: the pedals, sprockets, brakes, and steering mechanisms, to name a few. Their simplicity and visibility make bicycles relatively easy to fix, even for a novice.

Questions to Think About: *Why is a two-wheeled bicycle preferable to the apparently more stable three-wheeled tricycle? Why do you lean a bicycle in the direction that you are turning? How is it possible to ride a bicycle without hands on the handlebar? How does pedaling a bicycle make it move forward? Why does a bicycle have such a complicated drive system between its pedals and its rear wheel?*

Experiments to Do: *If you know how to ride a bicycle, pay attention to its stability as you ride it. Notice how you lean the bicycle in the direction that you are turning and how the bicycle naturally steers into that turn as you begin to lean. This automatic steering is part of the bicycle's self-stabilizing behavior. As you ride, observe how fast the pedals are turning and how hard you must push on the pedals to keep them turning at that rate. Go up and down some hills in various gears and see how each gear choice affects the pedaling rate and the forces you must exert on the pedals. Why do you choose a certain gear for a certain situation? What pedaling rate do you find most comfortable? What pedaling force feels best?*

Tricycles and Static Stability

Fig. 4.1.1 A tricycle is very stable when it's standing still. However, it tips over easily during a high-speed turn because the rider can't lean in the direction that the tricycle is turning. The rider must also pedal furiously to move at a reasonable speed.

Bicycles have a stability problem. With only two wheels to support them, stationary bicycles tip over easily. Why then do we use two-wheeled bicycles for transportation?

We can begin to answer that question by looking at **static stability**, an object's stability at rest. To have static stability, an object needs a stable equilibrium. Equilibrium always means zero net force or torque, but near a *stable* equilibrium an object experiences restoring influences—forces and/or torques—which push it back toward that equilibrium. Aided by such influences, a statically stable object returns to its stable equilibrium after being disturbed slightly. A stool exhibits such static stability, but a balancing bicycle most definitely does not. If you want a pedal-powered vehicle that's statically stable, try a tricycle (Fig. 4.1.1).

An upright tricycle has static stability because it experiences restoring influences following a tip (Fig. 4.1.2*a*). Actually, the tricycle has only static rotational stability—it experiences restoring torques that act to return it to upright, a particular angular position. In contrast, a marble in a bowl has only static translational stability—it experiences restoring forces that act to return it to the bottom of the bowl, a particular (translational) position. The fact that a tricycle doesn't have static translational stability is a good thing; a tricyclist doesn't want to stay near one place, she only wants to stay near one orientation: upright!

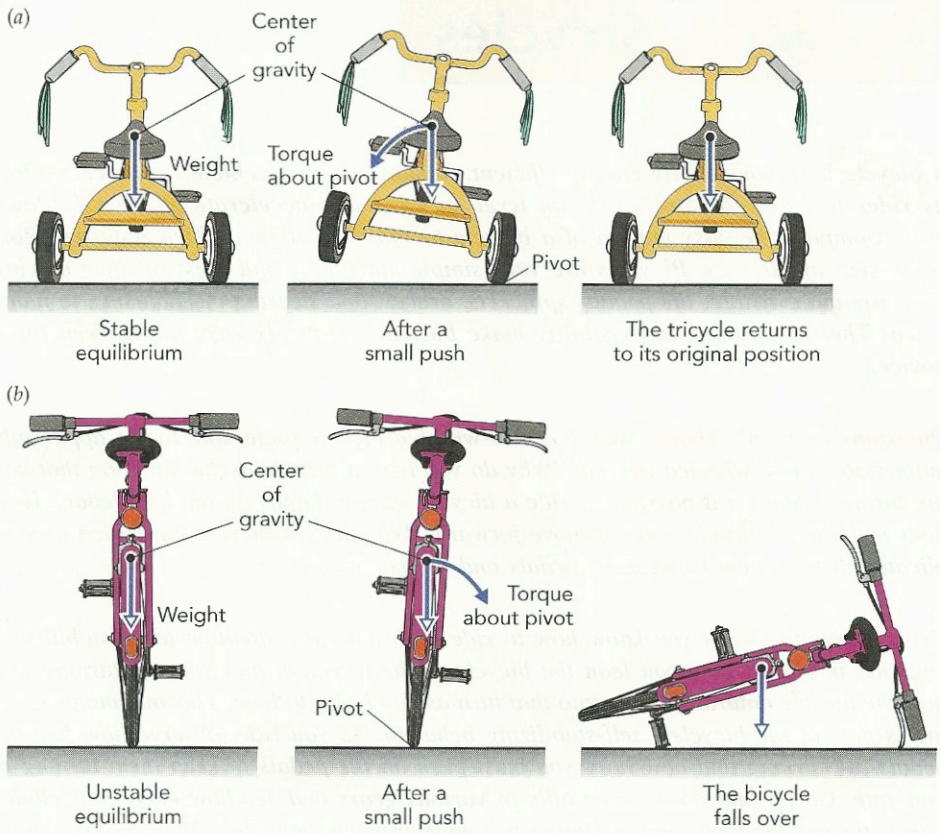


Fig. 4.1.2 (*a*) A tricycle is in a stable equilibrium. When it's disturbed, its center of gravity and gravitational potential energy rise, and it experiences restoring forces that return it to upright. (*b*) A bicycle is in an unstable equilibrium. Any disturbance causes it to fall.

Why does the tipped tricycle experience restoring torques? Rather than look for those torques directly, let's take a much more general approach. Part of my training as a physicist was to always look for the big-picture concept hiding behind every simple example and sometimes I just have to do it.

Recall that an object accelerates in whichever direction reduces its total potential energy as quickly as possible. In this case, the tricycle undergoes angular acceleration in whichever rotational direction reduces its total potential energy as quickly as possible. Since small tips always raise the tricycle's center of gravity and gravitational potential energy, the tipped tricycle can certainly reduce its total potential energy by rotating back to upright. The upright tricycle is thus in a stable **rotational equilibrium**—it is experiencing zero net torque and will return spontaneously to that situation after being tipped slightly. No wonder children love tricycles!

This relationship between static stability and total potential energy is universal. There is no need to look directly for the restoring influences, which may be complicated anyway. When an object is situated so that its total potential energy rises whenever it shifts, it's in a stable equilibrium and will tend to remain there. That useful rule applies not only to static rotational stability and tricycles, but also to static translational stability and everything from canoes to bridge spans to mobiles. If you want an object to be stable at rest, ensure that any small shift increases its total potential energy.

Stable Equilibrium and Potential Energy

An object is in a stable equilibrium when any small shift increases its total potential energy.

That general observation underlies a simple rule of thumb for the behavior of an independent object like a tricycle resting on a surface: that object will be in a stable rotational equilibrium as long as its center of gravity is positioned over its **base of support**—the polygon defined by its contact points with the ground. This rule follows from geometry: if the object's center of gravity is positioned over its base of support, then tipping it raises its center of gravity and increases its gravitational potential energy. Assuming no other potential energies are significant, the tipped object will undergo angular acceleration back toward that rotational equilibrium and the equilibrium will be stable.

Geometry also sets the limits of the object's static rotational stability: if its center of gravity moves outside the original base of support, it can reduce its total potential energy by tipping over and will promptly do so. That's why leaning back too far while on a tricycle is a recipe for disaster! The tricycle's three wheels define a triangular base of support, so it will be statically stable only as long as its rider keeps the overall center of gravity above that triangle.

With just two wheels touching the ground, a bicycle has no base of support (Fig. 4.1.2*b*) and therefore no static rotational stability. Although it's at rotational equilibrium when perfectly upright, this equilibrium isn't stable. If the bicycle tips to one side, its center of gravity will descend and its gravitational potential energy will decrease. The bicycle will actually undergo angular acceleration *away* from equilibrium! The upright bicycle is thus in an **unstable equilibrium**; instead of experiencing restoring forces or torques, the disturbed bicycle will tip farther and faster until it hits the ground.

Unstable Equilibrium and Potential Energy
An object's equilibrium is unstable when a small shift can decrease its total potential energy.

Check Your Understanding #1: Keeping Coffee in Its Place

Some travel mugs taper outward at the bottom so that they have very wide bases. Why does this shape make such a mug particularly stable and keep it from flipping over during the morning commute?

Answer: The mug's wide base ensures that its total potential energy will continue to increase as it tips, even if that tip becomes quite severe, so it will naturally return to its stable equilibrium.

Why: A mug with a narrow base is stable, but only if it never tips very far. As soon as its center of gravity moves beyond its original base of support, over it will go. However, a mug with a wide base has a broad base of support and will recover even from severe tips.

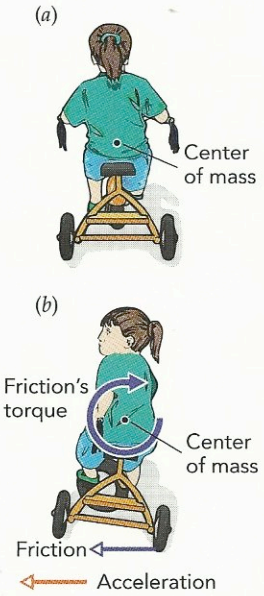


Fig. 4.1.3 (*a*) A tricycle that is heading straight is stable because any tip causes its center of gravity to move upward. (*b*) During a fast left turn, however, the tricycle accelerates left and friction exerts a large leftward force on the wheels. This frictional force produces a torque about the tricycle and rider's combined center of mass and can cause the tricycle to tip over.

Bicycles and Dynamic Stability

Static rotational stability matters most to people who have difficulty balancing. That's why children learn to ride tricycles first (Fig. 4.1.1). But when a tricycle is moving, static rotational stability doesn't guarantee safety. If a child rolls down a steep hill and then makes a sudden sharp turn, he or she will probably flip over. What has gone wrong?

A moving tricycle stays upright only if the girl riding it avoids sudden accelerations, such as a left turn at high speed. To make such a turn, she steers the tricycle's wheels so that friction with the pavement pushes the tricycle to the left (Fig. 4.1.3). Frictional forces on the wheel accelerate the tricycle leftward, redirecting its speed so that it turns. Of course, the girl needs to turn, too, so the tricycle pushes her along with it. As long as the turn is slow, a gentle push is all that's required, and the tricycle and girl turn together safely.

If the turn is too abrupt, the girl doesn't complete the turn along with the tricycle. Instead, her body goes straight as the tricycle drives out from under her. Crash. As you can see, a tricycle has good static stability but poor **dynamic stability**—stability in motion.

The tricycle flips because it can't handle the enormous torque that friction exerts on it during a sharp turn. Because the horizontal frictional force that turns the tricycle is exerted well below the tricycle and rider's combined center of mass (Fig. 4.1.3), it produces a torque about that center of mass. If this torque is small, the tricycle's static rotational stability will provide a restoring torque in the opposite direction that prevents any angular acceleration. If the turn is too sharp, the huge frictional torque will overwhelm the limited restoring torque, and the tricycle and rider will flip over. During high-speed turns, the tricycle is dynamically *unstable*.

Since the goal of a wheeled vehicle is to go somewhere, dynamic rotational stability is ultimately more important than static rotational stability. And while a bicycle lacks static stability, a moving bicycle is remarkably stable. Its dynamic stability is so good that it's almost hard to tip over and can even be ridden without any hands on the handlebars. This feat is a popular daredevil stunt among children who haven't yet realized how easy it is.

Courtesy Dr. David Jones

Fig. 4.1.4 In 1970, British physicist David Jones investigated the origins of a bicycle's dynamic stability. He built a series of "unridable" bicycles, including one that had a front wheel so small that it often skidded and became almost red hot as the bicycle was ridden. Jones found that gyroscopic effects alone did not account for a bicycle's stability. He discovered that this stability also comes from the shape of the front fork.

As British physicist David Jones discovered (Fig. 4.1.4), the bicycle's incredible dynamic stability results from its tendency to steer automatically in whatever direction it's leaning. For example, if the bicycle begins leaning to the left, the front wheel will automatically steer toward the left so as to return the bicycle to an upright position. Although a stationary bicycle falls over when it's disturbed from its unstable equilibrium, a forward-moving bicycle naturally drives under the combined center of mass and returns to that unstable equilibrium.

There are two physical mechanisms acting together to produce this automatic steering effect: one involving rotation and one involving potential energy. The first is based on the wheels alone: they behave as *gyroscopes*. Because each wheel is spinning, it has angular momentum and tends to continue spinning at a constant angular speed about a fixed axis in space. Since a wheel's angular momentum can be changed only by a torque, it naturally tends to keep its upright orientation.

But angular momentum alone doesn't prevent the bicycle from tipping over, any more than it prevents a tricycle from doing so. Instead, it prompts the bicycle to steer automatically by way of gyroscopic **precession**—the pivoting of a gyroscope's rotational axis caused by a torque exerted perpendicular to its angular momentum. When a bicycle is upright, the pavement exerts no perpendicular torque on the front wheel. But when the bicycle leans to the left, the pavement's upward support force no longer points at the wheel's center of mass, and it produces a perpendicular torque on the wheel. This torque is what makes that wheel precess; its axis of rotation pivots toward the left and thereby steers the bicycle to safety!

Assisting gyroscopic precession in this automatic steering process is a second effect due to potential energy. Because of the shape and angle of the fork supporting its front wheel, a leaning bicycle can lower its center of gravity and total potential energy by steering its front wheel in the direction that the bicycle is leaning. When the bicycle leans to the left, its front wheel accelerates toward the left—the direction that lowers the bicycle's total potential energy as quickly as possible. Once again the bicycle automatically steers in the direction that it's leaning and avoids falling over. These self-correcting effects explain why a riderless bicycle stays up so long when you roll it forward or down a hill.

There is little a bicycle designer can do to change the gyroscopic effect, but the potential energy effect depends on fork shape and angle. To be stable, the front wheel must touch the ground behind the steering axis (Fig. 4.1.5). If the fork is flawed, so that the wheel touches the ground ahead of the steering axis, the bicycle will steer the wrong way when it leans and be virtually unridable.

Fig. 4.1.5 A bicycle is stable when moving forward in part because its front wheel touches the ground behind the steering axis. As a result, the front wheel naturally steers in the direction that the bicycle is leaning and returns the bicycle to an upright position.

The front fork of a typical adult's bicycle arcs forward so that the wheel touches the ground just behind the steering axis. This situation leaves the bicycle dynamically stable enough to ride yet highly maneuverable. In contrast, the front fork of a typical child's bicycle is relatively straight so that the wheel touches the ground far behind the steering axis. The child's bicycle is therefore more dynamically stable than the adult's bicycle but also less easy to turn. That trade-off between dynamic stability and maneuverability is universal, appearing not only in pedal-powered vehicles, but in cars, boats, and aircraft as well.

Check Your Understanding #2: Like a Rolling Coin

Standing a coin on edge is difficult, but a rolling coin stays upright easily. What effect is stabilizing the moving coin?

Answer: The effect is gyroscopic precession.

Why: On edge, the coin has little or no static stability and tips over easily. But when it's rolling forward quickly, the same gyroscopic precession effect that causes a bicycle to steer under its center of gravity also dynamically stabilizes the coin's otherwise unstable equilibrium.

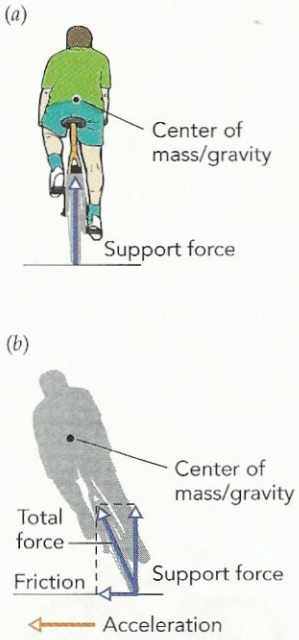


Fig. 4.1.6 (*a*) A bicycle that is heading straight is in rotational equilibrium when it's perfectly upright. The support force from the road produces no torque about its center of mass. (*b*) A bicycle that is turning left is in rotational equilibrium when it's tilted to the left. Together, the support and frictional forces from the road produce no torque about its center of mass.

Leaning While Turning

Why does a bicycle rider lean during a turn? The answer is that leaning can balance out the torque that friction exerts on him during the turn, the same frictional torque that flipped our unfortunate tricycle rider. With the proper lean, the bicyclist can safely complete even the sharpest turn.

As he rides along, the bicyclist tries to keep himself and the bicycle in rotational equilibrium. Since they experience a frictional torque about their combined center of mass each time they turn, the bicyclist balances that torque by leaning himself and/or the bicycle toward the inside of the turn. The pavement's upward support force on the wheels then produces a torque on them about their combined center of mass and that new torque opposes the frictional torque. When those two torques sum to zero, the bicyclist and bicycle are safely in rotational equilibrium.

Since these opposing torques are both produced by pavement forces on the wheels, we can combine them into a single torque; we can first add the friction and support forces and then see how much torque that overall force produces on the bicyclist. What we'll find in doing this addition is that the torque drops to zero when the pavement pushes the wheels directly toward the bicycle and rider's combined center of mass. As we saw in Section 2.1, a force exerted directly toward a pivot produces zero torque about that pivot. To stay in rotational equilibrium as he rides, the bicyclist always places the combined center of mass directly in line with the pavement's force on the wheels.

For example, when the bicycle is heading straight, the rider can avoid a net torque by keeping the bicycle upright. The pavement pushes the wheels straight upward, directly toward the combined center of mass (Fig. 4.1.6*a*). But when the bicycle is turning left, the rider must lean himself and/or the bicycle to the left to stay in rotational equilibrium. That's because each wheel of the turning bicycle is experiencing not only an upward support force, but also the leftward frictional force it needs to accelerate through the turn (Fig. 4.1.6*b*). The road's total force on the turning bicycle therefore points up and to the left. By leaning leftward just the right amount, the rider ensures

that this total force points directly toward the combined center of mass and exerts no torque about it (Fig. 4.1.7).

After a while, leaning the bike while turning becomes so automatic, so habitual, that you don't even think about it. You simply can't ride a bicycle or motorcycle without leaning as you turn. Even if you turn one of these vehicles so sharply that it skids, leaning can still keep it safely in rotational equilibrium.

All this discussion of leaning during turns begs the question: how do you make the bicycle lean prior to a turn? Actually, you cause that lean unconsciously by steering the bicycle briefly in the wrong direction—the direction opposite the turn itself! The bicycle then drives out from under you and the two of you begin to lean in the desired manner. You then steer in the correct direction and remain in rotational equilibrium throughout the turn. When you're ready to stop turning, you steer extra hard briefly in the direction of the turn. The bicycle then drives under you and the two of you return to upright. The turn is over.

Since only a statically unstable vehicle can lean, a statically stable one must rely on restoring torques to keep it near its rotational equilibrium. As we saw with tricycles, restoring torques are limited and a vehicle can tip out of its rotational equilibrium during rapid accelerations. A car, truck, or SUV will flip if it turns too sharply, and some are particularly prone to such catastrophes. The higher a vehicle's center of mass and the narrower its base of support, the more limited its restoring torques and the more easily it flips during turns. SUVs are surprisingly vulnerable to such rollover accidents, and small trucks that have been boosted up to resemble tractor-trailer cabs are truly hazardous. Even some ordinary cars have been found unsafe in this regard.

Wesley Hitt

Fig. 4.1.7 As they round a turn during a race, these cyclists lean toward the inside of the turn. This leaning prevents the pavement from producing torques on them and tipping them over.

Check Your Understanding #3: Cutting Corners

Why does a skier lean in the direction of a turn during a downhill run?

Answer: She leans into the turn to make sure that the force on her skis is directed toward her center of mass.

Why: In many sports and situations a person must lean in the direction of a turn. Since a turn always involves horizontal acceleration and horizontal force, the person must shift her center of mass toward the inside of the turn. When she leans just the right amount, the net force on her feet points directly toward her center of mass and exerts no torque on her about her center of mass. She remains in rotational equilibrium and doesn't tip over.

Pedaling Bicycles

So far, we have only discussed stability. Once we've settled on a bicycle as the most likely configuration for a useful person-powered vehicle, we need to figure out how to person-power it. The rider could push his feet on the ground, but that would be pretty inconvenient and even dangerous at high speeds. Instead, we use foot pedals to produce a torque on one of the wheels. But which wheel, and how do we produce that torque?

The original answer was to power the front wheel, using cranks attached directly to its axle. A crank is simply a lever that projects from the axle and produces a torque on that axle as you push its free end around in a circle. Each bicycle crank has a pedal installed on its free end so that you can use your foot to push on it. With its axle suspended in bearings, the front wheel turns as you pedal it. Friction between the turning wheel and the ground then pushes the bicycle forward.

Of course, you can't get something for nothing: ground friction also pushes back on the wheel, opposing its rotation. That's why you have to keep pedaling: your pedaling twists the wheel forward, while ground friction twists it backward. When those two torques balance, you'll move along at a steady pace.

This pedaling method is still used in children's tricycles, but it has three drawbacks. First, pedaling the front wheel interferes with its other responsibility: steering. Second, you can't take a break from pedaling; if the vehicle is moving, so are the pedals. Third, you can produce more than enough torque on the front wheel, but you often have trouble moving your legs quickly enough to keep up with the pedals. When you ride on level ground, you find yourself pedaling furiously and feeling very little resistance from the pedals.

The frantic pedaling problem goes to the heart of all pedal-powered vehicles, vehicles that draw power from you, the rider, to overcome inertia, air resistance, and perhaps an uphill slope. You provide that power by doing work on the pedals—a certain amount of work each second. Since work is the product of force times distance, you can do the same work on the pedals each second—that is, you can provide the same power to the vehicle—by pushing the pedals down hard as they turn slowly or by pushing them down gently as they turn quickly. However, for reasons having to do with physiology more than physics, your legs are best at providing power when you are pushing the pedals down medium hard as they turn medium fast.

Unfortunately, the pedals of an ordinary tricycle turn too quickly and too easily to make good use of your pedaling power. When the tricycle is moving fast on level ground, you can barely keep up with its pedals, let alone do work on them. The only time a tricycle makes good use of your capacity to supply power is when it's going uphill at moderate speed; only then do the pedals move medium fast and require that you push on them medium hard.

Fig. 4.1.8 The pennyfarthing used a large, directly driven front wheel to permit the rider to travel at a reasonable speed without having to pedal very rapidly. Its name came from its resemblance to two coins, the large English penny and the smaller farthing.

An early solution to the frantic pedaling problem was to use a gigantic front wheel. In such a configuration, one turn of the wheel would take you a considerable distance, so that you no longer had to pedal furiously while traveling at high speed along a level road. At the same time, ground friction exerted a larger backward torque on the huge wheel while causing the same forward acceleration of the vehicle, so that you had to push harder on the pedals to keep the wheel turning steadily. At last you could reach your peak performance on level ground, pushing medium hard on the pedals as they turned medium fast. The pennyfarthing of the mid-nineteenth century was this sort of bicycle (Fig. 4.1.8). But pedaling still interfered with steering, and you couldn't stop pedaling while the bicycle was moving. Furthermore, this bicycle had a new problem: you couldn't push hard enough with your feet to keep its front wheel turning steadily on uphill stretches.

These problems were solved by removing the cranks from the front wheel's axle and using an indirect drive scheme to convey power to the rear wheel (Fig. 4.1.9).

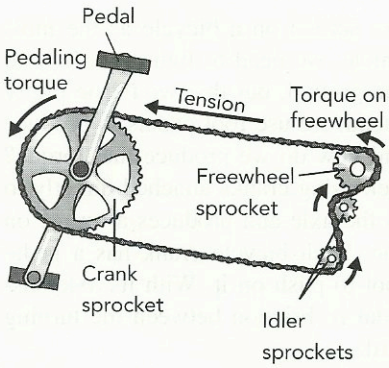


Fig. 4.1.9 The drive system for a modern bicycle. Pedaling produces torque on the crank sprocket, which in turn produces tension in the top segment of the chain. That segment of chain produces torque on the freewheel sprocket, which conveys that torque to the bicycle's rear wheel (not shown). The idler sprockets handle the extra chain.

Employing toothed sprockets and a chain loop, that indirect drive allowed the pedals and the wheels to turn at different rates. This change lets you use mechanical advantage to choose how you supply power to the bicycle: whether you exert large forces on slowly moving pedals or small forces on rapidly moving pedals or, ideally, medium forces on medium-fast-moving pedals. Whether you're zooming along a level road or grinding slowly up a steep hill, you can always find a drive setting or "gear" that lets you comfortably supply your maximum power.

Finally, the nonstop pedaling problem is solved by incorporating a one-way drive or freewheel in the hub of the rear wheel. This freewheel (Fig. 4.1.10) allows the rear wheel to turn freely in one direction so that you can stop pedaling as you coast forward. A modern bicycle with these improvements appears in Fig. 4.1.11.

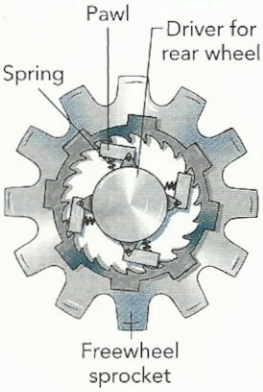


Fig. 4.1.10 The ratchet in a bicycle freewheel. If the relative rotation of the inner and outer parts is in the correct direction, the pawls transmit torque from the outer part to the inner part. If the relative rotation direction is reversed, the pawls compress the springs and skip along the teeth on the inside of the outer part. No torque is transmitted.

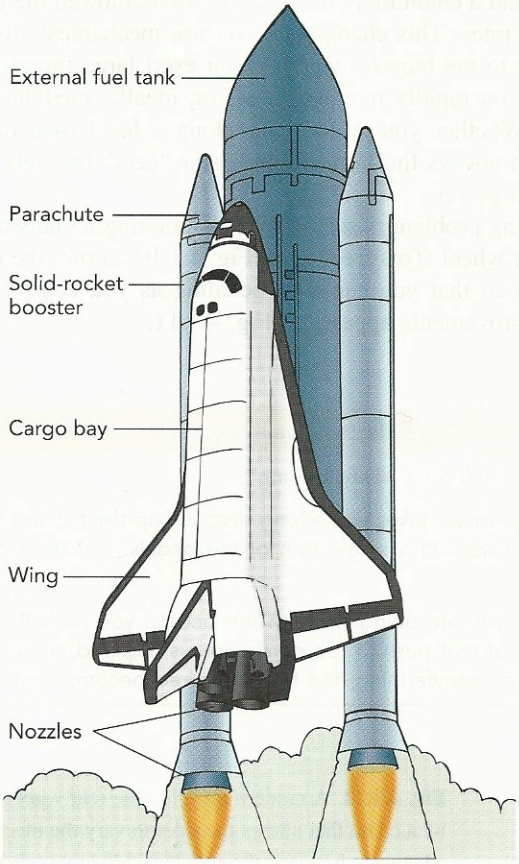
Check Your Understanding #4: Bigger Isn't Always Better

Why is it so difficult to climb a hill on a pennyfarthing?

Answer: Each turn of the huge wheel takes you a long distance up the hill and requires lots of work. To do this much work in a single turn of the pedals, you must push on the pedals very hard.

Why: As you pedal a bicycle, you are doing work on the pedals; you push the pedals as they move in the direction of that push. If too much work is required, as is the case when you're going uphill on a pennyfarthing, the force required becomes unbearable.

Fig. 4.1.11 A modern bicycle. The rear wheel is driven by a chain that allows the rider to vary the mechanical advantage between the pedals and the rear wheel. A freewheel in the hub of the rear wheel lets that wheel turn freely in one direction. This free motion allows the bicycle to coast forward, even when the pedals are stationary.



SECTION 4.2 Rockets and Space Travel

Despite the complexity of modern spacecraft, the rocket is one of the simplest of all machines. It's based on the principle that every action has a reaction. A rocket is propelled forward by pushing material out of its tail. Despite that simplicity, people have been developing better rockets for over 700 years. They're used for such pursuits as space exploration, weaponry, rescue operations, and amusement.

Questions to Think About: *What pushes a rocket forward? How can a rocket work in space, where there seems to be nothing to push against? Why do modern rockets have such fancy exhaust nozzles? What is the fastest speed that a rocket can reach? Why do satellites travel endlessly around the earth? What sort of path does a rocket take as it travels from planet to planet?*

Experiments to Do: *While an afternoon spent launching model rockets (available at hobby and toy stores) would be the best introduction to this section, a toy balloon will do just fine. Blow up the toy balloon and let it go. It will sail around the room. What*

pushes the balloon forward? Are the room's air and walls involved in the propulsion or is the balloon propelled by the very act of ejecting gas through its opening? How could you verify your answer to the previous question?

Rocket Propulsion

Among a rocket's most impressive features are its ability to propel itself forward even in the complete isolation of space and its capacity to reach astonishing speeds using that propulsion. It somehow manages to push itself forward without any outside help and to use that forward push to accelerate seemingly without limits.

Of course, a rocket can't really push itself forward, any more than you can lift yourself up by your boots, and it can't accelerate forever. In reality, it obtains a forward force, a **thrust** force, by pushing against its own limited store of fuel, and when that fuel runs out, it stops accelerating. To understand how a rocket obtains thrust from its fuel supply, let's look at how Newton's third law, the one describing action and reaction, applies to rockets.

Imagine that you're sitting in the middle of a frozen pond with zero velocity and no momentum. It's a warm day and the wet ice is remarkably slippery. Try as you may, you can't seem to get moving at all. How can you get off the ice?

Because of your inertia, the only way you can start moving is if something pushes on you. Sure, you could order a pizza and then push against the delivery truck when it arrives. Instead, you follow the ideas we discussed in Section 2.3. You remove a shoe and throw it as hard as you can toward the east side of the pond (Fig. 4.2.1). As you throw the shoe, you exert a force on it with your hand. The shoe accelerates and heads off across the ice.

What happens to you? You head off toward the west side of the pond! You're moving because when you pushed the shoe toward the east side of the pond, it pushed you equally hard toward the west side of the pond. In the process, you transferred momentum to the shoe and it transferred momentum in the opposite direction to you. Momentum isn't being created or destroyed, it's only being redistributed. Even after you let go of the shoe, your combined momentum remains at zero. The shoe has as much momentum in one direction as you have in the other.

Of course, you are much more massive than the shoe, so you end up traveling slower than it does. Momentum is the product of mass times velocity, so the more massive the object, the less velocity it needs for the same amount of momentum. Still, you've achieved what you set out to do: you're sliding slowly toward the west side of the pond.

Your final speed is limited because you only managed to transfer a small amount of momentum to the shoe and thus received only a small amount of opposite momentum in return. If you'd been able to throw the shoe faster or if you'd thrown a whole box of shoes, you'd have transferred more momentum and would be going faster.

Instead of throwing shoes, you'd have done better to throw very fast-moving gas molecules. Even at room temperature, the molecules in air are traveling about 1800 km/h (1100 mph). When gas molecules are heated to roughly 2800 °C (5000 °F), as they are in a liquid-fuel rocket engine, they move about three times that fast. If you throw something in one direction at that kind of speed, you receive quite a lot of momentum in the other direction.

That's what a conventional rocket engine does (Fig. 4.2.2). It uses a chemical reaction to create very hot exhaust gas from fuels contained entirely within the rocket itself. What started as potential energy in the stored chemical fuels becomes thermal energy in the hot, burned exhaust gas. This thermal energy is mostly kinetic energy, hidden in the

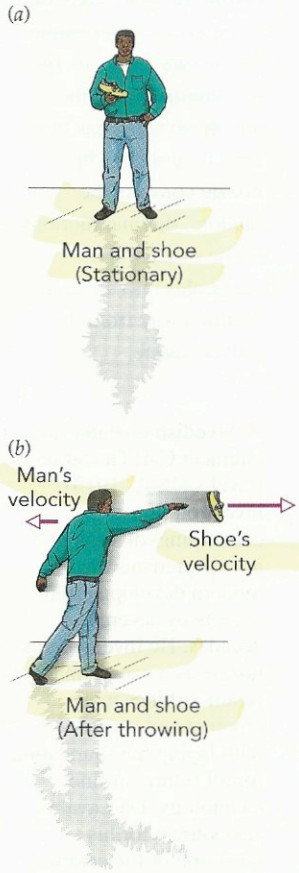


Fig. 4.2.1 A man who is holding a shoe while standing still on ice has zero momentum. Once he has thrown the shoe to the right, the shoe has a momentum to the right and the man has a momentum to the left. The total momentum of the man and shoe is still zero. Because the man is much more massive than the shoe, the shoe moves much faster than the man.

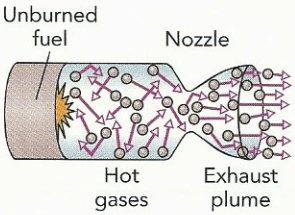


Fig. 4.2.2 A molecular picture of what happens in a chemical rocket engine. The engine burns its fuel in a confined chamber, and the exhaust gas flows out of a nozzle. The nozzle converts the random, thermal motions of the exhaust gas molecules into directed motion away from the rocket engine.

random motion of the tiny molecules themselves. The rocket engine's nozzle steers most of this random motion in one direction, and the engine obtains thrust in the opposite direction.

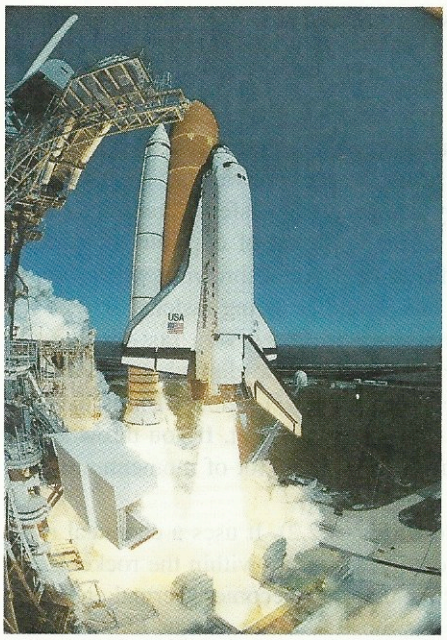
If you've ever watched the launch of a large rocket, you've probably noticed the bell-shaped nozzles through which the exhaust flows (Fig. 4.2.3). Each nozzle allows the rocket to obtain as much forward momentum as possible from its exhaust by directing that exhaust backward and accelerating it to the greatest possible speed. As we'll see in Chapter 6, nozzles allow gases to convert their various internal energies into kinetic energy and are ideally suited for directing and accelerating gases. In the case of rocket exhaust, the most effective nozzle shape is a converging–diverging one, called a de Laval nozzle after its Swedish inventor, Carl Gustaf de Laval ■.

■ Swedish inventor and engineer Carl Gustaf de Laval's (1845–1913) invention of the converging–diverging nozzle predates the modern development of rockets by several decades. He invented this nozzle as a way to make steam turbines more efficient and is credited with laying the foundation for all future turbine technology. De Laval is also known for his invention of the cream separator for milk.

To understand fully why this complicated nozzle structure works so well for rocket engines, we'd need to examine the physics of flowing gases up to and beyond the speed of sound. We'll encounter some of these issues later in this book, but for now a brief summary will have to suffice.

Inside the rocket and before the de Laval nozzle, the hot exhaust gas is tightly packed and its pressure is enormous. Like the gas in a spray bottle, this exhaust gas accelerates rapidly through the nozzle toward the lower pressure environment outside. The narrowing throat of that nozzle aids its acceleration, up to a point. When the gas reaches the narrowest part of the nozzle, it's traveling at the speed of sound and its characteristics begin to change dramatically. To coax the supersonic exhaust gas to accelerate still further, the nozzle stops narrowing and begins to widen. The tightly packed exhaust gas expands in volume as it flows through that widening bell and thereby prepares to enter the more open environment outside the nozzle.

Just how wide the diverging end of the de Laval nozzle must be to obtain the maximum thrust from its exhaust gas depends on the nozzle's surroundings. Near sea level, the exhaust gas flows into ordinary air outside the nozzle and a relatively narrow de Laval nozzle works best. At high altitude or in space, the exhaust gas enters thinner air or nothing at all, so a wider de Laval nozzle is more ideal. Rockets typically make a compromise in their nozzle shapes so as to operate reasonably well in all environments.



Courtesy NASA

Fig. 4.2.3 The space shuttle's nozzles are designed to push its rocket exhaust downward as long, glowing plumes. The gas pushes back, lifting the shuttle upward into space.

By the time the gas reaches the end of the de Laval nozzle, it has converted most of its original energy into kinetic energy, with its velocity directed away from the nozzle. In fact, because the gas actually continues to burn even as it flows through the nozzle, its kinetic energy and speed keep rising until they reach fantastic levels. With the help of the de Laval nozzle, exhaust gas leaves the rocket's engine at an **exhaust velocity** or backward-directed flow speed of between 10,000 and 16,000 km/h (6000 and 10,000 mph).

As it creates this plume of exhaust, the rocket pushes the gas backward and gives it backward momentum. The gas completes the momentum transfer by pushing the rocket forward. The very act of ejecting the exhaust is all that's required to obtain forward thrust; the rocket doesn't need anything external to push and will operate perfectly well in empty space (see ■). When it pushes hard enough on its exhaust, the rocket not only can support its own weight, it can even accelerate upward. The space shuttle weighs about 20,000,000 N (4,500,000 lbf) at launch, but its thrust is about 30,000,000 N (6,750,000 lbf). That means that the space shuttle can accelerate upward at about half the acceleration due to gravity! As the shuttle consumes its fuel, so that its weight and mass diminish, it can accelerate upward even more rapidly.

■ On January 13, 1920, *The New York Times* ran an editorial attacking Robert Goddard for proposing that rockets could be used for travel in space. With modest financial support from the Smithsonian Institution, Goddard was pioneering the development of liquid-fuel rockets. The editorial began: "That Professor Goddard, with his 'chair' in Clark College and the countenancing of the Smithsonian Institution, does not know the relation of action to reaction, and of the need to have something better than a vacuum against which to react—to say that would be absurd. Of course he only seems to lack the knowledge ladled out daily in high schools."

Common Misconceptions: Action and Reaction in Rockets

Misconception: A rocket needs some external object to react against in order to push itself forward.

Resolution: While rocket propulsion does involve a pair of equal but opposite forces, action and reaction, the rocket is pushing its exhaust backward (action) and the exhaust is pushing the rocket forward (reaction). What this exhaust plume hits, if anything, makes no difference to the propulsion effect.

Check Your Understanding #1: A Rocket with a Head Start

When a missile is launched from beneath the wing of a fighter aircraft, what does it push against in order to accelerate forward?

Answer: It pushes against its own exhaust, as do all rockets.

Why: While it may appear that the plume of exhaust beneath a ground-launched rocket is what lifts that rocket upward, the ground itself doesn't contribute to the thrust. The very act of pushing the gas out the nozzle propels the rocket forward.

The Ultimate Speed of a Spacecraft

At rest on the launchpad, a rocket consists principally of a spacecraft and a supply of fuel. Once the rocket's engine begins to fire, exhaust from the burned fuel accelerates backward and the spacecraft accelerates forward. The fuel is gradually consumed until eventually it runs out and the spacecraft coasts along on its own. Although weight and air resistance influence this story, let's neglect both for now to see what determines the spacecraft's eventual speed.

It might seem that the ultimate speed of the spacecraft is limited to the rocket's exhaust speed. Remarkably, there is no such limit. As long as the rocket keeps pushing exhaust backward, it will continue to accelerate forward. However, for the spacecraft to

reach extremely high speed, the rocket must push the vast majority of its initial mass backward as exhaust. For example, if the rocket's initial mass is 90% fuel (i.e., it starts with 9 parts fuel and 1 part spacecraft), then we might expect the spacecraft to end up heading forward at about 9 times the speed of the exhaust gas. After all, that arrangement seems to satisfy the conservation of momentum.

Unfortunately, that analysis overestimates the spacecraft's speed. Because the rocket still has fuel on board as it accelerates forward, some of the forward momentum it obtains from its exhaust goes into its remaining fuel rather than into the spacecraft. Since that fuel will soon be ejected from the rocket as exhaust, giving it precious forward momentum is wasteful and the spacecraft therefore ends up with less forward momentum. Although the total momentum of the spacecraft and fuel still always sum to zero, this inopportune momentum transfer from the exhaust to the fuel reduces both the spacecraft's speed forward and the exhaust's average speed backward.

Despite this problem, a spacecraft can still travel faster than the speed of its rocket exhaust; it just needs more fuel. Neglecting air resistance and weight, the spacecraft's final speed is given by the rocket equation:

$$\text{spacecraft speed} = \text{exhaust speed} \cdot \log_e\left(\frac{\text{mass}_{\text{spacecraft}} + \text{mass}_{\text{fuel}}}{\text{mass}_{\text{spacecraft}}}\right). \qquad (4.2.1)$$

For a rocket that is 90% fuel at launch, its spacecraft can reach 2.3 times the speed of its exhaust gas. If it can eject more than 90% of its initial mass as exhaust, it can go even faster.

There's a problem, however, with trying to burn up and eject a huge fraction of the rocket's original mass as exhaust. It's difficult to construct a rocket that is 99.99% fuel and 0.01% spacecraft. Instead, space-bound rockets use several separate stages, each stage much smaller than the previous stage. Once the first stage has used up all of its fuel, the whole stage is discarded and a new, smaller rocket begins to operate. In this manner, the rocket behaves as though it's ejecting almost all of its mass as rocket exhaust. With the help of stages and lots of fuel, rockets can travel substantially faster than their exhaust velocities and reach earth orbit or the solar system beyond. (For ongoing developments in single-stage rockets, see ■.)

■ In 1989, the U.S. government began a program to develop a reusable rocket vehicle that could achieve earth orbit with only a single stage. Nothing but fuel would be jettisoned during launch so that the vehicle could travel to and from orbit repeatedly with only refueling and minimal maintenance between flights. The challenges facing this program are formidable. Even with liquid hydrogen and oxygen as its fuels, almost 90% of this vehicle's launch weight must be fuel. Nonetheless, efforts to produce such Single Stage To Orbit (SSTO) vehicles are proceeding.

Check Your Understanding #2: Not Everything Is Disposable

The space shuttle doesn't have the staged look of expendable rockets. How does it manage to eject most of its launch mass as exhaust?

Answer: It's actually staged subtly. Its two solid-fuel boosters are effectively the first stage, the external liquid-fuel tank is the second stage, and the orbiter itself is the third stage.

Why: Although the space shuttle is not stacked one stage on the next like a Saturn or Delta rocket, it doesn't travel from ground to space as a single object. It discards empty fuel containers as it accelerates upward. First to go are the two boosters, followed by the external fuel tank. The final mass of the orbiter itself is much less than what left the launchpad.

Orbiting the Earth

If the spacecraft were heading straight up when it ran out of fuel, it would either fall back to the ground or leave the earth forever (more on that later). But if it were heading primarily horizontally when its engine turned off, it might find itself circling the earth

endlessly. With no atmosphere to affect it, the spacecraft follows a path determined only by inertia and gravity, and since the spacecraft's weight causes it to accelerate toward the center of the earth, its trajectory can bend into a huge elliptical loop around the earth.

The spacecraft is following an orbit around the earth. An **orbit** is the path an object takes as it falls freely around a celestial object. Although the spacecraft accelerates directly toward the earth's center at every moment, its huge horizontal speed prevents it from actually hitting the earth's surface. In effect, the spacecraft perpetually misses the earth as it falls (Fig. 4.2.4). To orbit the earth just above the atmosphere, a spacecraft must travel at the enormous speed of 7.9 km/s (about 17,800 mph) and will circle the earth once every 84 minutes.

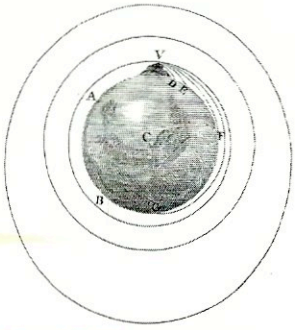


Fig. 4.2.4 Newton's drawing of a cannonball fired horizontally from the top of a tall mountain. As the cannonball's speed increases, it travels farther from the mountain before hitting the earth. Eventually, the cannonball moves so quickly that the curved earth drops away beneath it and it never hits the earth at all. The cannonball then orbits the earth.

However, the farther the spacecraft's orbit is from the earth's surface, the longer its **orbital period**—the time it takes to complete one orbit. First, since the spacecraft must travel farther to complete the larger orbit, the trip takes longer. Second, the spacecraft must travel slower in order to follow a circular path around the earth because the earth's gravity becomes weaker with distance.

Chapter 1 noted that gravity attracts every object in the universe toward every other object in the universe (see ■). In particular, objects are attracted toward the earth. Near the earth's surface, an object's weight is simply its mass times 9.8 N/kg, the acceleration due to gravity. However, as the object's distance from the center of the earth increases, the acceleration due to gravity diminishes. Equation 1.2.1 is only an approximation, valid for objects near the earth's surface and for situations where the altitude doesn't change significantly.

A more general formula relates the gravitational forces between two objects to their masses and the distance separating them. These forces are equal to the gravitational constant times the product of the two masses, divided by the square of the distance separating them. This relationship, discovered by Newton and called the **law of universal gravitation**, can be written as a word equation:

$$\text{force} = \frac{\text{gravitational constant} \cdot \text{mass}_1 \cdot \text{mass}_2}{(\text{distance between masses})^2}, \tag{4.2.2}$$

in symbols:

$$F = \frac{G \cdot m_1 \cdot m_2}{r^2},$$

and in common language:

The pull of gravity is strongest between massive objects but diminishes rapidly with distance.

Note that the force on mass_1 is directed toward mass_2 and the force on mass_2 is directed toward mass_1. Those two forces are equal in magnitude but oppositely directed. The **gravitational constant** is a fundamental constant of nature, with a measured value of $6.6720 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$.

■ English physicist Henry Cavendish (1731–1810) proved that terrestrial objects do exert gravitational forces on one another. His experiment, performed in 1798, measured the tiny forces that two metal spheres exert on one another, using a very sensitive torsion balance. Comparing the forces between the two spheres with those between the earth and those same spheres (their weights), Cavendish was able to deduce the mass of the earth. In effect, Cavendish's tabletop experiment allowed him to weigh the earth.

The Law of Universal Gravitation

Every object in the universe attracts every other object in the universe with a force equal to the gravitational constant times the product of the two masses, divided by the square of the distance separating the two objects.

This relationship describes any gravitational attraction, whether it's between two planets or between the earth and you. The effective location of an object's mass is its center of mass, so the distance used in Eq. 4.2.2 is the distance separating the two centers of mass. For a spacecraft orbiting the earth just above its atmosphere, that distance is roughly the earth's radius of 6378 km (3964 miles). For a spacecraft far above the atmosphere, however, the distance is larger and the force of gravity is weaker. That spacecraft experiences a smaller acceleration due to gravity. To give it the additional time it needs for its path to bend around in a circle, the high-altitude spacecraft must travel more slowly than the low-altitude spacecraft. This reduced speed explains the long orbital periods of high-altitude spacecraft.

At 35,900 km (22,300 miles) above the earth's surface, the orbital period reaches 24 hours. A satellite traveling eastward in such an orbit turns with the earth and is said to be *geosynchronous*. If a geosynchronous satellite orbits the earth around the equator, it's also *geostationary*—it always remains over the same spot on the earth's equator. Such a fixed orientation is useful for communications and weather satellites.

Not all orbits are circular. The orbits of some spacecraft are elliptical, so that their altitudes vary up and down once per orbit. At *apogee*, its greatest distance from the earth's center, a spacecraft travels relatively slowly because it has converted some of its kinetic energy into gravitational potential energy. At *perigee*, its smallest distance from the earth's center, the spacecraft travels relatively rapidly because it has converted some of its gravitational potential energy into kinetic energy. Of course, the perigee should not bring the spacecraft into the earth's atmosphere or it will crash.

The orbit of a spacecraft can also be hyperbolic. If the spacecraft is traveling too fast, the earth's gravity will be unable to bend its path into a closed loop and the spacecraft will coast off into interplanetary space. The spacecraft's path near the earth is then a hyperbola. The spacecraft only follows this hyperbolic path once and then drifts away from the earth forever.

A spacecraft usually enters a hyperbolic orbit by firing its rocket engine. It starts in an elliptical orbit around the earth and uses its rocket engine to increase its kinetic energy. The spacecraft then arcs away from the earth and its kinetic energy gradually transforms into gravitational potential energy. But the earth's gravity becomes weaker with distance and the spacecraft's gravitational potential energy slowly approaches a maximum value even as its distance from the earth becomes infinite. If the spacecraft has more than enough kinetic energy to reach this maximum gravitational potential energy, it will be able to escape completely from earth's gravity.

The speed that a spacecraft needs in order to escape from the earth's gravity is called the **escape velocity**. This escape velocity depends on the spacecraft's altitude and is about 11.2 km/s (25,000 mph) near the earth's surface. A spacecraft traveling at more than the escape velocity follows a hyperbolic orbital path and heads off toward the other planets or beyond.

Common Misconceptions: Astronauts and "Weightlessness"

Misconception: An astronaut orbiting the earth is too far from the earth to experience gravity and is truly weightless.

Resolution: The astronaut is still so near the earth's surface that he experiences almost his full earth weight. He only feels weightless because he is in free fall.

Check Your Understanding #3: Speeding Up the Lunar Month

The moon orbits the earth every 27.3 days, at a distance of 384,400 km from the earth's center of mass. For the moon to orbit in less time, how would its distance from the earth have to change?

Answer: The distance between the earth and moon would have to decrease.

Why: The moon behaves just like a spacecraft orbiting the earth at a distance of 384,400 km. Such a spacecraft would also have an orbital period of 27.3 days. To reduce this orbital period, the moon would have to move closer to the earth so that the earth's gravity could bend its path more rapidly.

Check Your Figures #1: Attractive Cars

How much gravitational force does a 1000-kg automobile exert on an identical car located 10 m away?

Answer: It exerts about 6.73×10^{-7} N.

Why: We use Eq. 4.2.2 to obtain the force:

$$\text{force} = \frac{6.6720 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2 \cdot 1000\,\text{kg} \cdot 1000\,\text{kg}}{(10\,\text{m})^2}$$
$$= 6.6720 \times 10^{-7}\,\text{N}.$$

This force is roughly equal to the weight of a grain of sand. No wonder it's hard to feel gravity from anything but the entire earth.

Orbiting the Sun: Kepler's Laws

Once it escapes from the earth's gravity and again turns off its rocket engine, the spacecraft behaves like a tiny planet and orbits the sun. If you watch it patiently as it travels and compare its orbital motion with the motions of the planets themselves, you may begin to notice three universal features of all these solar orbits. First recognized by German astronomer Johannes Kepler (1571–1630) through his careful analysis of the extensive observational data collected by Danish astronomer Tycho Brahe, those three orbital behaviors are known as Kepler's laws.

Kepler's first law is already rather familiar to us from our examination of earth orbits. This law describes the shape of the spacecraft's looping orbit around the sun: it's an ellipse, with the sun at one focus of that ellipse (Fig. 4.2.5). An ellipse isn't an arbitrary oval; it's a planar curve with two foci and a rule stating that each point on the curve has the same sum of distances to the two foci. In this case, the sun occupies one focus and the other focus is empty. If you add the distance from the spacecraft to the sun plus the distance from the spacecraft to the empty focus, that sum will remain constant as the spacecraft orbits the sun. A circular orbit around the sun is a particularly simple elliptical one; its two foci coincide and the sun occupies them both.

Kepler recognized that every object orbiting the sun follows such an elliptical path. The planets move along nearly circular ellipses while the comets travel in highly elongated ones. Our spacecraft's orbit may be circular or elongated, depending on its position and velocity at the time its engine stopped firing. To reach another planet, the spacecraft's solar orbit and that of its destination planet must overlap and the two objects

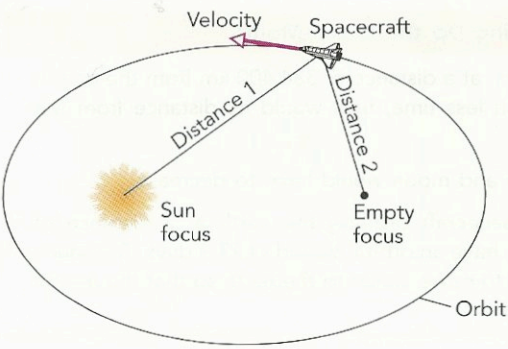


Fig. 4.2.5 A spacecraft's orbit around the sun is an ellipse, with the sun occupying one focus and the other focus empty. The sum of distance_1 and distance_2 is the same for all points on this ellipse.

must reach that overlapping point at the same time. Traveling from planet to planet is clearly a tricky business.

Newton later recognized that these elliptical orbits are a direct consequence of the law of universal gravitation (Eq. 4.2.2) and its inverse square relationship between force and distance (force $\propto$ 1/distance^2). Any other relationship between force and distance would yield curving paths that don't close on themselves at all, let alone form ellipses. As you follow the spacecraft's elliptical orbit around the sun, you are witnessing an elegant exhibition of the law of universal gravitation.

Kepler's First Law: Orbits
All planets move in elliptical orbits, with the sun at one focus of the ellipse.

Kepler's second law describes the area swept out by a line stretching from the sun to the spacecraft: that line sweeps out equal areas in equal times (Fig. 4.2.6). Regardless of how circular or elongated the spacecraft's orbit is, or where the spacecraft is along that orbit, the area marked off each second by that moving line is always the same.

This observation demonstrates another physical law: conservation of angular momentum. Since the sun's gravity pulls the spacecraft directly toward the sun, it exerts no torque on the spacecraft and the spacecraft's angular momentum about the sun is constant. Remarkably, the rate at which this line sweeps out area is proportional to the spacecraft's angular momentum, so the steadiness of that sweep demonstrates the constancy of the spacecraft's angular momentum.

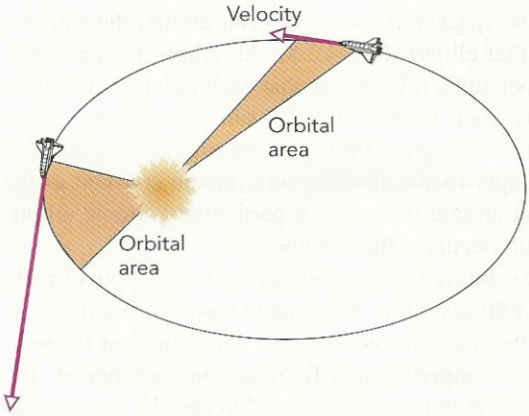


Fig. 4.2.6 The orbiting spacecraft sweeps out the same orbital area each second, despite variations in its distance from the sun. This steady sweep is a result of the spacecraft's constant angular momentum about the sun.

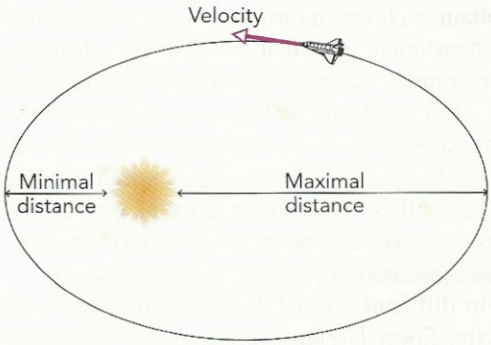


Fig. 4.2.7 The square of the spacecraft's orbital period is proportional to the cube of its mean distance from the sun—the average of its maximal and minimal distances from the sun.

Kepler's Second Law: Areas
A line stretching from the sun to a planet sweeps out equal areas in equal times.

Kepler's third law describes the spacecraft's orbital period around the sun: the square of its orbital period is proportional to the cube of its mean distance from the sun—that is, the average of its maximal and minimal distances from the sun (Fig. 4.2.7). This relationship can be derived from the law of universal gravitation (Eq. 4.2.2), the equations describing centripetal acceleration (e.g., Eq. 3.3.1), and Newton's second law (Eq. 1.1.2).

Kepler's Third Law: Periods
The square of a planet's orbital period is proportional to the cube of that planet's mean distance from the sun.

Check Your Understanding #4: Out of Round

Your spacecraft has just left the earth behind and is now orbiting the sun independently. Its elongated orbit has the same mean distance from the sun as the earth's orbit, but its maximal distance from the sun is twice its minimal distance. Compare its speeds at the maximal and minimal distances from the sun. Will it ever meet up with the earth again?

Answer: It is moving half as fast at its maximal distance from the sun as at its minimal distance and it will meet the earth again in exactly one year.

Why: In accordance with Kepler's first law, the spacecraft is traveling in an elliptical orbit. Since Kepler's second law requires it to sweep out area at a constant rate, the spacecraft must move half as fast when it is twice as far from the sun. Finally, because the spacecraft's mean distance from the sun is the same as that of the earth, the two have the same orbital periods. They'll both complete their orbits in one year and meet up at that time.

Travel to the Stars: Special Relativity

Despite formidable challenges, it may one day be possible for manned spacecraft to venture away from the solar system and travel to the stars. The distances involved are so vast that the only way to cover them in an astronaut's lifetime would be to move at fantastic speeds, speeds comparable to that of light itself.

■ In 1905, while Albert Einstein (German-born Swiss then American physicist, 1879–1955) was working as a patent examiner in Bern, he published four revolutionary papers in three different areas of physics. For the 26-year-old doctoral student from Germany, it was quite a banner year. Though Einstein is often portrayed as an elderly, wild-haired gentleman, his most important contributions to science were made when he was a vibrant young man who had only just married his first wife two years earlier.

Should spacecraft one day be able to attain such enormous speeds, they'll find that the basic laws of motion, the Galilean and Newtonian laws that we have been learning up to this point, are incomplete. Although extremely accurate at ordinary speeds, those laws falter near the **speed of light** (exactly 299,792,458 m/s). They turn out to be low-speed approximations for the more accurate laws of motion developed by Einstein in 1905 (■). Built on the observation that light always travels at the same speed, regardless of an observer's inertial frame of reference, these **relativistic laws of motion** are accurate at any speed. They are part of Einstein's **special theory of relativity**, the conceptual framework that describes space, time, and motion in the absence of gravity.

We saw in Section 1.1 that observers in different inertial frames of reference can disagree on an object's position and velocity. Special relativity recognizes that those observers can also disagree on the distance and time separating two events. More broadly, two inertial observers who are in relative motion perceive space and time somewhat differently. If they're moving at ordinary speeds, that difference in perceptions is negligible and the Newtonian laws of motion are nearly perfect. But if they're moving relative to one another at a substantial fraction of the speed of light, then they perceive space and time quite differently. In that case, the Newtonian approximations fail and the full laws of relativity are required.

Special relativity has many consequences for high-speed space travel, but we'll concentrate on how relativity alters two familiar conserved quantities: momentum and energy. At low speeds, our spacecraft's momentum takes its usual Newtonian value: mass times velocity (Eq. 2.3.1). But with increasing speed, a new relativistic factor enters the picture: $(1 - \text{speed}^2/\text{light speed}^2)^{-1/2}$. **Relativistic momentum** is the product of the object's mass times its velocity times that factor. This relationship can be written as a word equation:

$$\text{relativistic momentum} = \frac{\text{mass} \cdot \text{velocity}}{\sqrt{1 - \text{speed}^2/\text{light speed}^2}}, \tag{4.2.3}$$

in symbols:

$$\mathbf{p} = \frac{m \cdot \mathbf{v}}{\sqrt{1 - v^2/c^2}},$$

and in common language:

For a spacecraft to reach the speed of light, its momentum would have to be infinite.

At ordinary speeds, the relativistic factor is so nearly equal to 1 that this relativistic relationship is beautifully approximated by the Newtonian one. However, as the spacecraft's speed begins to approach that of light itself, the relativistic factor spoils the simple proportionality between momentum and velocity. Momentum then increases more rapidly than velocity. One result of this change is that it becomes impossible to reach the speed of light, let alone exceed it. Even if the spacecraft's thrust increases its forward momentum at a steady rate, its speed will increase less and less quickly. It will approach but never reach the speed of light.

A similar change happens to the spacecraft's energy as it approaches the speed of light. At low speeds, our isolated spacecraft's kinetic energy takes its usual Newtonian value: half its mass times the square of its speed (Eq. 2.2.1). At high speeds, however, we must begin using **relativistic energy**. Relativistic energy is the product of the object's

mass times the square of the speed of light times the relativistic factor. This relationship can be written as a word equation:

$$\text{relativistic energy} = \frac{\text{mass} \cdot \text{light speed}^2}{\sqrt{1 - \text{velocity}^2/\text{light speed}^2}}, \tag{4.2.4}$$

in symbols:

$$E = \frac{m \cdot c^2}{\sqrt{1 - v^2/c^2}},$$

and in common language:

A spacecraft starts with a rest energy and its energy grows toward infinity as its speed approaches the speed of light.

At ordinary speeds, the spacecraft's relativistic energy can be approximated as:

$$\text{relativistic energy} \simeq \text{mass} \cdot \text{light speed}^2 + \tfrac{1}{2} \cdot \text{mass} \cdot \text{speed}^2.$$

The usual Newtonian kinetic energy appears at the right in this approximation, but to its left is a new energy that we've never seen before. Called the *rest energy*, it's present even when the spacecraft is motionless. Because the rest energy is constant, it doesn't affect low-speed motion and was overlooked in the Newtonian laws. However, this energy associated with mass itself (stated symbolically as $E = mc^2$) does have consequences and is surely the most famous feature of the special theory of relativity.

The relativistic version of energy has two implications for our spacecraft. First, the spacecraft's energy increases so quickly as it nears the speed of light that it can never reach that speed. Second, the spacecraft's initial store of energy before launch is associated with its initial mass. That mass and energy are so closely related is something to which we'll return in Chapter 16.

Check Your Understanding #5: **Stellar Tugboats**

A stellar tugboat is pulling forward on a spacecraft that is already traveling close to the speed of light. Although the tugboat is steadily increasing the spacecraft's forward momentum, the spacecraft is speeding up less and less quickly. Why?

Answer: Although the spacecraft's forward momentum increases steadily, in accordance with the relativistic relationship between momentum and velocity, the spacecraft's velocity increases ever more slowly.

Why: At ordinary speeds, a steady transfer of momentum to an object will yield a steady increase in its speed. Near the speed of light, however, the object's increase in speed no longer keeps pace with its increase in momentum.

Check Your Figures #2: **The Real Speed Limit**

A spacecraft passes your planetary base going half the speed of light. Using special equipment, you transfer exactly enough momentum to that spacecraft to double its forward momentum. How fast is it going now?

Answer: It is going $\sqrt{4/7}$ times the speed of light.

Why: When the spacecraft is traveling at ½ the speed of light, Eq. 4.2.3 gives its forward momentum as $\sqrt{1/3}$ times its mass times the speed of light. Doubling that momentum and solving Eq. 4.2.3 for the velocity yields $\sqrt{4/7}$ times the speed of light.

Check Your Figures #3: It Packs a Wallop

The speed of a 1-kg minispacecraft is $\sqrt{3/4}$ times the speed of light. What is its relativistic energy and what fraction of that energy is rest energy?

Answer: Its total energy is 1.8×10^{17} J, of which half is rest energy.

Why: For an object traveling at $\sqrt{3/4}$ times the speed of light, Eq. 4.2.4 gives a total energy of 2 times its mass times the square of the speed of light. For a 1-kg object, that energy is 1.8×10^{17} J. Its rest energy is simply its mass times the square of the speed of light, or 9.0×10^{16} J. Both values of energy are astonishingly large.

Visiting the Stars: General Relativity

In its travels near stars and other massive celestial objects, our spacecraft is likely to encounter another surprise: the Newtonian view of gravity is also an approximation! Near extremely dense, massive objects, gravity is no longer accurately described by Newton's law of universal gravitation. Instead, understanding gravity requires a new conceptual framework that Einstein first presented in 1916, the **general theory of relativity**.

This new framework is based on the observation that you can't distinguish between downward gravity and upward acceleration. As we saw in Section 3.3, they feel exactly the same. For example, if you feel heavy as you stand inside a closed spacecraft, you can't be sure whether you're experiencing a weight due to downward gravity or a feeling of acceleration due to upward acceleration. In fact, the spacecraft's scientific instruments can't help you because they can't distinguish the effects of gravity from those of acceleration, either. No matter how hard you try, you can't tell the difference.

At the heart of this problem is the concept of mass. Up to this point, we have seen mass play two apparently different roles that we could refer to as gravitational mass and inertial mass. When you're experiencing a weight, your **gravitational mass** is acting together with gravity to make you feel heavy. When you're experiencing a feeling of acceleration, your **inertial mass** is acting together with acceleration to make you feel heavy. However, in spite of their different roles, these two masses seem to be related. Without exception, an object that has a large inertial mass and is therefore difficult to shake also has a large gravitational mass and is therefore hard to support against gravity. In fact, the two masses seem to be the same. That observation led Einstein to propose the **principle of equivalence**: that these two masses, gravitational and inertial, are truly identical and therefore that no experiment you perform inside your spacecraft can distinguish between free fall and the absence of gravity. The general theory of relativity is based on this principle of equivalence.

As long as your spacecraft stays in regions of weak gravity, Newton's law of universal gravitation will adequately describe its motion. At the extremes of gravity, however, the general theory of relativity is necessary. That theory describes a universe in which massive objects distort the structure of nearby space and time, and in which extreme masses produce extreme distortions. One of the most startling predictions of this theory is the existence of objects so radical in their gravitational warping of nearby space

and time that they are **black holes**—spherical or nearly spherical surfaces from which not even light can escape. A number of black holes have been discovered, including an enormous one at the center of our galaxy. You might want to avoid them.

Check Your Understanding #6: Rip van Twinkle

You awaken aboard your small spacecraft only to discover that you have been asleep for almost 20 years and the crew has vanished. The windows are closed, and you have no idea where your spacecraft is located or what it is doing. You realize that you feel pulled toward the floor of the spacecraft. Are you experiencing weight or a feeling of acceleration?

Answer: Trick question! You can't possibly determine the answer!

Why: The central principle of general relativity is that no experiment you make inside your spacecraft can distinguish between the consequences of gravity and the consequences of acceleration. You're simply going to have to look out the window to figure it out.

Epilogue for Chapter 4

This chapter discussed the physical concepts behind two types of machines. In *bicycles*, we investigated the concept of dynamic stability, where an object that falls over when stationary becomes remarkably stable in motion. We also saw how the need for mechanical advantage between the pedals and the wheels led to the development of the modern multispeed bicycle.

In *rockets*, we studied the ways in which reaction forces propel rockets forward. We found that a rocket's ultimate speed is not limited by its exhaust velocity, allowing it to lift itself into orbit around the earth. We also learned that gravity weakens as the distance between two gravitating objects increases, making it possible for a spaceship to break free from earth's gravity and begin orbiting the sun. Finally, we took a brief look at the exotic physics a spacecraft would encounter if it approached the speed of light or passed through the extreme gravity near some celestial objects.

Explanation: High-Flying Balls

The top ball doesn't bounce off the ground; it bounces off the bottom ball. The top ball actually completes its bounce as the bottom ball is heading upward. After colliding, the two balls push off one another in just the same way that a rocket pushes off its exhaust. The balls exchange momentum and energy as they push against one another, and the small top ball—which has less mass and accelerates much more easily than the bottom ball—ends up with a large upward momentum and more than its fair share of energy. It flies upward as though it were hit by a massive upward-moving *bat*. In fact, it has been hit by a massive upward-moving *ball* and it rebounds at high speed.

If the small ball's mass were really negligible in comparison to that of the large ball and the balls bounced without wasting any energy, then the small ball would rebound from the big ball traveling 3 times as fast as when it arrived. Its kinetic energy would be 9 times as great as before, and it would rebound to 9 times its original height. Of

course, real balls aren't perfect, so the tennis ball won't bounce quite so high. Still, the effect is pretty impressive.

Chapter Summary

How Bicycles Work: While a stationary bicycle tips over easily, a moving bicycle is remarkably stable. It remains upright with the help of two stabilizing effects of motion—one due to gyroscopic precession and the other due to the shape and angle of the front fork. These effects work together to steer the bicycle in the direction that it's leaning. Whenever it tips, it automatically drives under its center of gravity and returns to upright. Leaning is also an essential part of turns. By leaning his body and/or the bicycle properly during a turn, the bicyclist ensures that there is no overall torque on the bicycle and that it doesn't tip over.

The rider powers the bicycle by pushing its pedals around in a circle. Since the rider can provide the most pedaling power when the pedals are turning at a moderate rate and he is pushing on them with moderate forces, a modern multispeed bicycle allows him to adjust the relative rotation rates of the pedals and the wheels. By choosing the right gear, the bicycle allows him to provide his peak power comfortably.

How Rockets Work: A rocket obtains its thrust by ejecting gas from an engine in its tail. The rocket pushes on the gas and the gas pushes back. This gas usually comes from burning chemical fuels contained entirely inside the rocket itself and is released from the rocket's engine through a nozzle. A carefully designed nozzle permits the rocket to make efficient use of the energy stored in the fuel by ensuring that gas leaves the rocket at the maximum possible speed. The rocket's thrust is used to lift the rocket against the force of gravity and to accelerate it upward. Once in space, with its engine inactive, the rocket will orbit the earth or sun, or it may travel off into interstellar space.

Important Laws and Equations

1. The Law of Universal Gravitation: Every object in the universe attracts every other object in the universe with a force equal to the gravitational constant times the product of the two masses, divided by the square of the distance separating the two objects, or

$$\text{force} = \frac{\text{gravitational constant} \cdot \text{mass}_1 \cdot \text{mass}_2}{(\text{distance between masses})^2}, \qquad (4.2.2)$$

2. Kepler's First Law: All planets move in elliptical orbits, with the sun at one focus of the ellipse.

3. Kepler's Second Law: A line stretching from the sun to a planet sweeps out equal areas in equal times.

4. Kepler's Third Law: The square of a planet's orbital period is proportional to the cube of that planet's mean distance from the sun.

5. Relativistic Momentum: An object's relativistic momentum is the product of its mass times its velocity times the relativistic factor, or

$$\text{relativistic momentum} = \frac{\text{mass} \cdot \text{velocity}}{\sqrt{1 - \text{speed}^2/\text{light speed}^2}}, \qquad (4.2.3)$$

6. Relativistic Energy: An object's relativistic energy is the product of its mass times the square of the speed of light times the relativistic factor, or

$$\text{relativistic energy} = \frac{\text{mass} \cdot \text{light speed}^2}{\sqrt{1 - \text{velocity}^2/\text{light speed}^2}}, \qquad (4.2.4)$$

Exercises

1. Sprinters start their races from a crouched position with their bodies well forward of their feet. This position allows them to accelerate quickly without tipping over backward. Explain this effect in terms of torque and center of mass.

2. If the bottom of your bicycle's front wheel becomes caught in a storm drain, your bicycle may flip over so that you travel forward over the front wheel. Explain this effect in terms of rotation, torque, and center of mass.

3. When you turn while running, you must lean in the direction of a turn or risk falling over. If you lean left as you turn left, why don't you fall over to the left?

4. If a motorcycle accelerates too rapidly, its front wheel will rise up off the pavement. During this stunt the pavement is exerting a forward frictional force on the rear wheel. How does that frictional force cause the front wheel to rise?

5. As a skateboard rider performs stunts on the inside of a U-shaped surface, he often leans inward toward the middle of the U. Why does leaning keep him from falling over?

6. Most racing cars are built very low to the ground. While this design reduces air resistance, it also gives the cars better dynamic stability on turns. Why are these low cars more stable than taller cars with similar wheel spacings?

7. The crank of a hand-operated kitchen mixer connects to a large gear. This gear meshes with smaller gears attached to the mixing blades. Since each turn of the crank makes the blades spin several times, how is the force you exert on the crank handle related to the forces the mixing blades exert on the batter around them?

8. The starter motor in your car is attached to a small gear. This gear meshes with a large gear that's attached to the engine's crankshaft. The starter motor must turn many times to make the crankshaft turn just once. How does this gearing allow modest forces inside the starter motor to turn the entire crankshaft?

9. A bread-making machine uses gears to reduce the rotational speed of its mixing blade. While its motor spins about 50 times each second, the blade spins only once each second. The motor provides a certain amount of work each second, so why does this arrangement of gears allow the machine to exert enormous forces on the bread dough?

10. The chain of a motorcycle must be quite strong. Since the motorcycle has only one sprocket on its rear wheel, the top of the chain must pull forward hard to keep the rear wheel turning as the motorcycle climbs a hill. Why would replacing the motorcycle's rear sprocket for one with more teeth make it easier for the chain to keep the rear wheel turning?

11. As you clear the sidewalk with a leaf blower, the blower pushes you away from the leaves. What is pushing on the blower so that it can push on you?

12. When a sharpshooter fires a pistol at a target, the gun recoils backward very suddenly, leaping away from the target. Explain this recoil effect in terms of the transfer of momentum.

13. Which action will give you more momentum toward the north: throwing one shoe southward at 10 m/s or two shoes southward at 5 m/s?

14. Do both of the actions in Exercise 13 take the same amount of energy? If not, which one requires more energy?

15. You are propelling yourself across the surface of a frozen lake by hitting tennis balls toward the southern shore. From your perspective, each ball you hit heads southward at 160 km/h. You have a huge bag of balls with you and are already approaching the northern shore at a speed of 160 km/h (100 mph). When you hit the next ball southward, will you still accelerate northward?

16. Can you use the tennis ball scheme of Exercise 15 to propel yourself to any speed, or are you limited to the speed at which you can hit the tennis balls?

17. You and a friend are each wearing roller skates. You stand facing each other on a smooth, level surface and begin tossing a heavy ball back and forth. Why do you drift apart?

18. The time it takes a relatively small object to orbit a much larger object doesn't depend on the mass of the small object. Use an astronaut who is walking in space near the space shuttle to illustrate that point.

19. As the moon orbits the earth, which way is the moon accelerating?

20. Which object is exerting the stronger gravitational force on the other, the earth or the moon, or are the forces equal in magnitude?

21. To free an Apollo spacecraft from the earth's gravity took the efforts of a gigantic Saturn V rocket. Freeing a lunar module from the moon's gravity took only a small rocket in the

lunar module's base. Why was it so much easier to escape from the moon's gravity than from the earth's?

22. Spacecraft in low earth orbit take about 90 minutes to circle the earth. Why can't they be made to orbit the earth in half that amount of time?

23. As a comet approaches the sun, it arcs around the sun more rapidly. Explain.

24. Mars has a larger orbital radius than the earth. Compare the solar years on those two planets.

Problems

1. If an 80-kg baseball pitcher wearing frictionless roller skates picks up a 0.145-kg baseball and pitches it toward the south at 42 m/s (153 km/h or 95 mph), how fast will he begin moving toward the north?

2. How high above the earth's surface would you have to be before your weight would be only half its current value?

3. As you walked on the moon, the earth's gravity would still pull on you weakly and you would still have an earth weight. How large would that earth weight be, compared to your earth weight on the earth's surface? (*Note:* The earth's radius is 6378 km and the distance separating the centers of the earth and moon is 384,400 kilometers.)

4. If you and a friend 10 m away each have masses of 70 kg, how much gravitational force are you exerting on your friend?

5. The gravity of a black hole is so strong that not even light can escape from within its surface or "event horizon." Even outside that surface, enormous energies are needed to escape. Suppose that you are 10 km away from the center of a black hole that has a mass of 10^{31} kg. If your mass is 70 kg, how much do you weigh?

6. In Problem 5, how much work would something have to do on you to lift you 1 m farther away from the black hole?

7. A 1000-kg spacecraft passes you traveling forward at ½ the speed of light. What is its relativistic momentum?

8. A spacecraft passes you traveling forward at ⅓ the speed of light. By what factor would its relativistic momentum increase if its speed doubled?

9. For a rocket traveling at 10,000 m/s, by what factor does its relativistic momentum differ from its ordinary momentum?

10. What is the rest energy of 1 kg of uranium?

11. How much mass has a rest energy of 1000 J?

12. A 1000-kg spacecraft passes you at ½ the speed of light. What is its relativistic energy?

13. A spacecraft passes you at ⅓ the speed of light. By what factor would its relativistic energy increase if its speed doubled?

14. By what factor does the relativistic energy in Problem 13 increase if you subtract out the constant rest energy?