CHAPTER 2

The Laws of Motion Part 2

In the previous chapter, we saw how things move from place to place and encountered energy, an important conserved quantity. But motion doesn't always involve a change of position, and energy isn't nature's only conserved quantity. In this chapter, we'll take a look at a second type of motion—rotation—and at two other conserved quantities—momentum and angular momentum. Spinning objects are quite common, and we'll do well to explore their laws of motion before proceeding much further. With those additional concepts under our belts, we'll be ready to explore the physics behind a broad assortment of mechanical objects.

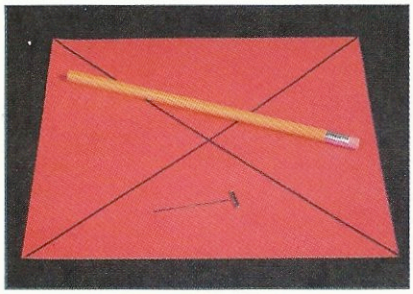
EXPERIMENT: A Spinning Pinwheel

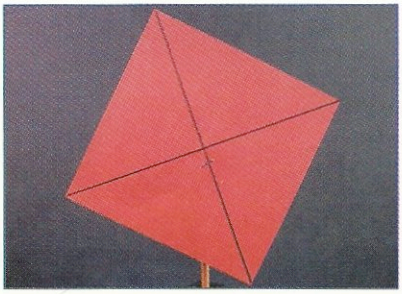
Spinning a toy pinwheel seems like a simple activity. But don't let its uncomplicated appearance deceive you: there are lots of physics involved in balancing the pinwheel, starting it spinning, and keeping it going.

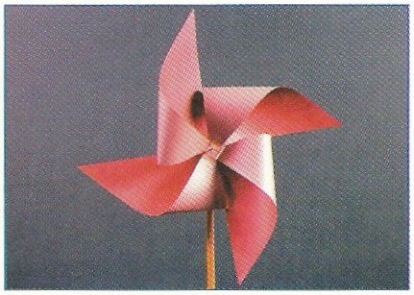
To experiment with a pinwheel, cut a square from a sheet of sturdy paper and draw two straight lines diagonally from corner to corner. Stick a pin through the center of the square, where the two lines cross. Enlarge the pinhole just enough that the square can turn easily on the pin. Now stick the pin into the eraser at the end of a pencil.

When you hold the pencil upright and release the paper square so that it can rotate, what happens to the square? If it begins rotating, does it eventually settle in a particular orientation? If it doesn't begin rotating, does altering the orientation of the square before you release it affect the square's behavior? Why might the square settle into a particular orientation? What must you do so that it can remain stationary at any orientation?

If necessary, reposition the pinhole until the square does not begin rotating when you release it from rest, regardless of its initial orientation. You have balanced the square. What does it mean to be balanced?







Courtesy Lou Bloomfield

Now spin the paper square. What must you do to start the square spinning, and what keeps it spinning once you let go? Why does it gradually slow down and stop? Is it still balanced, even when it is spinning?

To transform this square into a pinwheel, unpin it and cut it along the diagonal lines you drew. Start at each corner and cut along the line, stopping when you are about ½ inch from the center. Each of the four corners now has a pair of narrow sections. As you look down on the square, each pair of sections has a section that is further clockwise than the other section. Flip the four clockwise-located sections up and point them toward the center of the square. Carefully glue the tips of these four sections onto the center of the square, one on top of the other to form a stack. Stick the pin through the stack of tips and through the center of the square, and then into the pencil eraser. You now have a pinwheel.

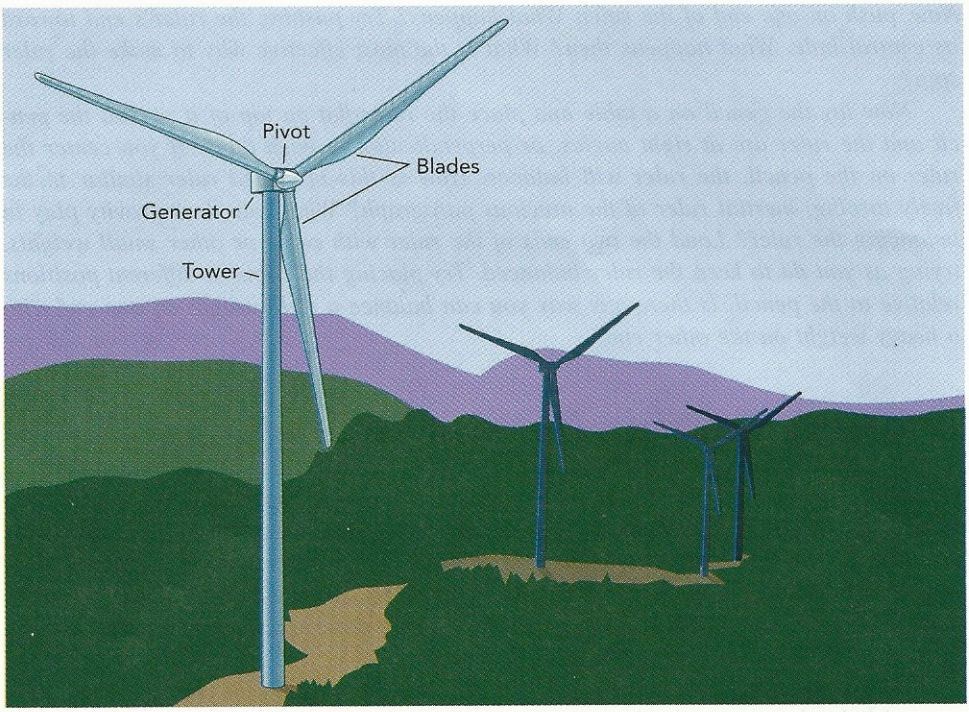
Hold the pencil vertically and make sure that the pinwheel turns easily on the pin. Check that it is balanced—that it doesn't spontaneously settle into a particular orientation—and adjust it if necessary.

Now blow gently toward the pinwheel. It should start to spin. Watch how it starts—does it immediately reach full speed or does its speed gradually increase? What do we mean by speed, since the pinwheel isn't going anywhere? When you stop blowing, does the pinwheel stop immediately or does it slow down gradually?

As you blow on the pinwheel, notice in which direction the pinwheel spins. Why does it turn in that direction? What could you do to reverse the direction? Feel the air as it moves out of your mouth toward the pinwheel. Now feel the air's motion after it passes the pinwheel. Did the pinwheel affect the velocity of the air? If the pinwheel affects the air, how does the air affect the pinwheel?

Chapter Itinerary

We're going to explore the laws of rotational motion and two new conserved quantities in the context of three everyday objects: (1) *wind turbines*, (2) *wheels*, and (3) *bumper cars*. In *wind turbines*, we'll look at twists and turns, and see how the wind manages to spin the turbine and do work on an electric generator. In *wheels*, we'll examine how friction affects motion and learn how wheels make a vehicle more mobile. In *bumper cars*, we'll learn the physics behind collisions and uncover some of the simple rules that govern what initially appear to be complicated motions. For a more complete preview of what we'll examine in this chapter, flip ahead to the Chapter Summary at the end of the chapter.



SECTION 2.1 Wind Turbines

Our need for renewable energy sources is growing rapidly and so is the number of wind turbines dotting the landscape. Each wind turbine consists of several blades that are pushed around in a circle by the wind. Although these turbines are moving devices that provide mechanical energy to electric generators, their motion differs from that of the objects in Chapter 1. Wind turbines don't go anywhere; they rotate. In this section, we'll revisit many of the laws of motion that we encountered previously, but this time we'll see these laws in a new context: rotational motion.

Questions to Think About: *What must the wind do to start a wind turbine spinning? To keep it spinning? How does the wind supply energy to the wind turbine? How does the turbine transfer that energy to the electric generator? A wind turbine only balances when its blades are arranged symmetrically about its center. Why does it matter just how the blades are arranged? What do we mean by a balanced wind turbine?*

Experiments to Do: *To get a feel for how rotating objects work, find a rigid ruler with a hole in its center—the kind that can be clipped into a three-ring binder. If you support the ruler by putting the tip of an upright pencil into the central hole, you'll find that the ruler exhibits rotational inertia; that is, it either remains stationary, at whatever orientation you choose, or rotates steadily about the central hole. (Eventually, the ruler comes to rest because of friction, a detail that we'll continue to ignore for now.)*

Now, push on one end of the ruler. What happens? Try pushing the ruler's end toward its central hole. What happens then? What is the most effective way to make the ruler spin?

Now lay the pencil on a table and place the ruler flat on top of it so that the pencil and the ruler are at right angles, or perpendicular, to each other. If you center the ruler on the pencil, the ruler will balance. How is this balanced ruler similar to the freely turning, inertial ruler of the previous paragraph? What role does gravity play in balancing the ruler? Load the two ends of the ruler with coins or other small weights, trying as you do to keep the ruler balanced. Try placing the coins at different positions relative to the pencil. Is there any way you can balance a light weight on one end with a heavy weight on the other end?

A Wind Turbine Rotates

Because fossil fuels are becoming ever scarcer and more expensive and because burning them is altering our climate in unpleasant ways, we need to expand our use of renewable energy sources. That's why the wind is generating an increasingly important fraction of our electric power. With wind turbines in all 50 states and most countries around the world, their time has come. They are also simple mechanical wonders, and we can learn a great deal of physics by watching them turn.

Whether you find the graceful arc of their blades delightful or consider them eyesores, it's hard not to appreciate the simplicity of their shapes and motions. With rare exception, a wind turbine consists of two or more identical blades spaced evenly around a central pivot and it spins about that pivot when blown by the wind. That symmetrical, balanced arrangement allows the turbine to spin smoothly and easily, and to extract energy efficiently from the wind. Before we can look at how the rotating turbine obtains energy from the wind, however, we need to understand rotation itself. That's where we'll start.

Always eager to do your part for the environment, you decide to harness the wind to power your home and neighborhood. After some investigation on the Web, you order a twin-bladed wind turbine and a tower to support it high in your backyard. While you're waiting for the turbine to arrive, let's begin to think about how it is going to work.

On a windless day, there are only three forces acting on the twin-blade wind turbine shown in Fig. 2.1.1: two downward forces (the weights of the two blades) and one upward force (the support force of the central pivot). Seeing those three forces, you might think about net force and begin to look for some overall acceleration of this device. We know, however, that the wind turbine remains where it is on its tower and isn't likely to head off for Kalamazoo or the center of the earth anytime soon. Because the turbine's fixed pivot always provides just enough upward and sideways force to keep the turbine from accelerating as a whole, the turbine always experiences zero net force and never leaves the tower. Overall movement of an object from one place to another is called **translational motion**. Although the turbine never experiences translational motion, it can turn around the pivot and thus it experiences a different kind of motion. Motion around a fixed point is called **rotational motion**.

Rotational motion is crucial to a wind turbine—the wind spins it and thereby powers an electric generator. But what causes the wind turbine to rotate, and what observations can we make about the process of rotation?

To answer those questions, we'll need to examine several new physical quantities associated with rotation and explore the laws of rotational motion that relate them to one

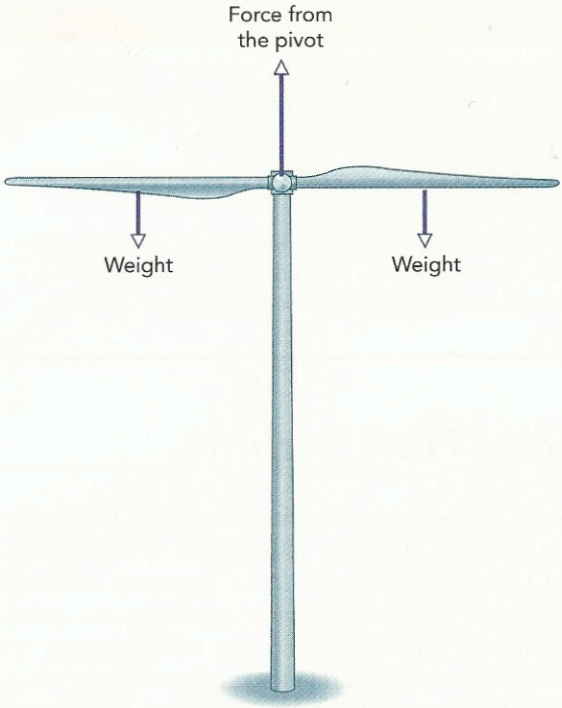


Fig. 2.1.1 The wind turbine experiences three forces: the downward weight of each blade and the upward support of the pivot. The net force on the turbine is zero, so it doesn't accelerate.

another. We'll do these things both by studying the workings of wind turbines and other rotating objects and by looking for analogies between translational motion and rotational motion.

Imagine that the wind turbine in Fig. 2.1.1 is motionless when you climb up the tower and start twisting it clockwise as hard as you can. The turbine begins to rotate clockwise. The turbine's motion is fairly slow at first, but it moves more and more quickly until eventually your hands are no longer able to keep up with it. We could describe the wind turbine being twisted from rest in the following way:

> "The turbine starts out not rotating at all. When you start twisting the turbine clockwise, the turbine begins to rotate clockwise. The turbine's rate of rotation increases continuously in the clockwise direction until the turbine is rotating too fast for you to continue twisting."

This description sounds a lot like the description of a skater being pushed from rest:

> "The skater starts out not moving at all. When you start pushing the skater northward, the skater begins to move northward. The skater's rate of translation increases continuously in the northward direction until the skater is translating too fast for you to continue pushing."

The statement about the wind turbine involves rotational motion, while the statement about the skater involves translational motion. Their similarity isn't a coincidence; the concepts and laws of rotational motion have many analogies in the concepts and laws of translational motion. The familiarity that we've acquired with translational motion will help us examine rotational motion.

Check Your Understanding #1: Wheel of Fortune Cookies

The guests at a large table in a Chinese restaurant use a revolving tray, a lazy Susan, to share the food dishes. How does the lazy Susan's motion differ from that of the passing dessert cart?

Answer: The lazy Susan undergoes rotational motion, while the dessert cart undergoes translational motion.

Why: The lazy Susan has a fixed pivot at its center. This pivot never goes anywhere, no matter how you rotate the lazy Susan. In contrast, the dessert cart moves about the room and has no fixed point. The server can rotate the dessert cart when necessary, but its principal motion is translational.

The Free Motion of a Wind Turbine

In the previous chapter we looked at the concept of translational inertia, which holds that a body in motion tends to stay in motion and a body at rest tends to stay at rest. This concept led us to Newton's first law of translational motion. Inserting the word "translational" here is a useful revision because we're about to encounter the corresponding concepts associated with rotational motion. We'll begin that encounter by observing a balanced wind turbine that's free of outside rotational influences: no wind, no air resistance, and no generator. We'll then examine how the turbine responds to such outside influences. Because of the similarities between rotational and translational motions, this section will closely parallel our earlier examination of skating and falling balls.

Your new twin-blade wind turbine is being installed on its tower and it is presently dangling overhead at the end of a cable (Fig. 2.1.2). The cable is attached to the middle of the turbine, where it supports the turbine's weight but exerts no other influences on the turbine. Most important, the dangling turbine can spin and pivot with complete freedom—nothing pushes on it or twists it (for simplicity, let's ignore wind effects or the possibility that the cable will get in the turbine's way as it moves). This dangling turbine is free to rotate in any direction. You, the observer, are standing motionless on the ground. When you look up at the turbine, what does it do?

If the turbine is stationary, then it will remain stationary. However, if it's rotating, it will continue rotating at a steady pace, about a fixed line in space. What keeps the turbine rotating? Its **rotational inertia**. A body that's rotating tends to remain

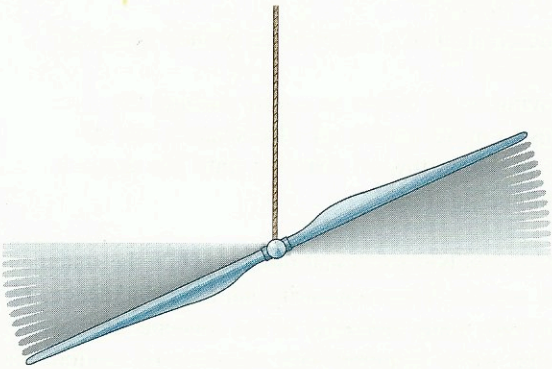


Fig. 2.1.2 A wind turbine that's dangling from a rope at its middle. Since nothing twists it, the wind turbine rotates steadily about a fixed line in space.

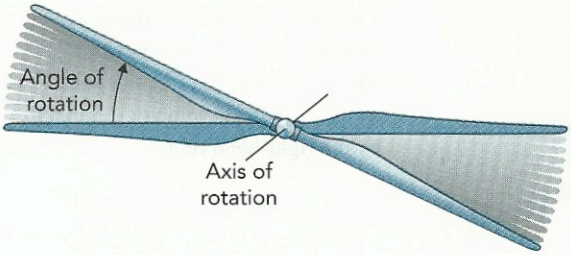


Fig. 2.1.3 You can specify this wind turbine's angular position, relative to its horizontal reference orientation, as the axis about which it was rotated to reach its new orientation and the angle through which it was rotated.

rotating; a body that's not rotating tends to remain not rotating. That's how our universe works.

To describe the turbine's rotational inertia and rotational motion more accurately, we'll need to identify several physical quantities associated with rotational motion. The first is the turbine's orientation. At any particular moment, the turbine is oriented in a certain way—that is, it has an **angular position**. Angular position describes the turbine's orientation relative to some reference orientation; it can be specified by determining how far the turbine has rotated away from its reference orientation and the axis or line about which that rotation has occurred. The turbine's angular position is a vector quantity of relatively minor importance, pointing along the rotation axis with a magnitude equal to the rotation angle (Fig. 2.1.3).

The SI unit of angular position is the **radian**, the natural unit for angles. It's a natural unit because it follows directly from geometry, not from an arbitrary human choice or convention the way most units do. Geometry tells us that a circle of radius 1 has a circumference of 2π. By letting arc lengths around that circle's circumference specify angles, we are using radians. For example, there are 2π radians (or 360°) in a full circle and $\pi/2$ radians (or 90°) in a right angle. Since the radian is a natural unit, it is often omitted from calculations and derived units.

If the turbine is rotating, then its angular position is changing; in other words, it has an angular velocity. **Angular velocity** is our first important vector quantity of rotational motion and measures how quickly the turbine's angular position changes; it consists of the angular speed at which the turbine is rotating and the axis about which that rotation proceeds. The turbine's **angular speed** is its change in angle divided by the time required for that change:

$$\text{angular speed} = \frac{\text{change in angle}}{\text{time}}.$$

The SI unit of angular velocity is the **radian-per-second** (abbreviated 1/s).

The turbine's **axis of rotation** is the line in space about which the turbine is rotating. But just knowing that line isn't quite enough: is the turbine rotating clockwise or counterclockwise? To resolve this ambiguity, we take advantage of the fact that any line has two directions to it. Once we have identified the line about which the turbine is rotating, we can look down that line at the turbine from both directions. From one direction, the turbine appears to be rotating clockwise; from the other direction, counterclockwise.

By convention, we choose to view the turbine from the direction in which it appears to be rotating clockwise and say that the turbine's rotation axis points away from our eye toward the turbine. This convention is called the **right-hand rule** because if the fingers of your right hand are curling around the axis in the way the wind turbine is rotating,

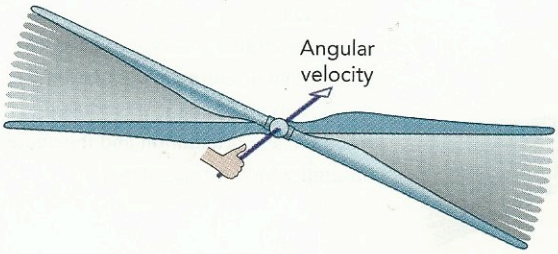


Fig. 2.1.4 This wind turbine is spinning about the rotation axis shown. The direction of the wind turbine's angular velocity is defined by the right-hand rule.

then your thumb is pointing along the wind turbine's rotation axis (Fig. 2.1.4). (While experimenting with the right hand rule, be sure to hold this book in your left hand, Fig. 2.1.5.)

Remembering this convention isn't as important as understanding why you must specify the direction about which rotation occurs when describing a rotating object's angular velocity. Just as translational velocity consists of a translational speed and a direction in which the translational motion occurs, so angular velocity consists of a rotational speed and a direction about which the rotational motion occurs.

We're now prepared to describe the rotational motion of the dangling turbine. Because of its freedom from outside influences and its rotational inertia, its angular velocity is constant. The dangling turbine just keeps on turning and turning, always at the same angular speed, always about the same axis of rotation.

As you might suspect, this observation isn't unique to wind turbines. It is **Newton's first law of rotational motion**, which states that a rigid object that is not wobbling and is not subject to any outside influences rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation. The outside influences referred to in this law are called **torques**—a technical term for twists and spins. When you twist off the lid of a jar or spin a top with your fingers, you're exerting a torque.

This law excludes objects that wobble or can change shape as they rotate because those objects have more complicated motions. They are covered instead by a more general principle—the conservation of angular momentum—that we'll meet up with in Section 2.3.

Courtesy Lou Bloomfield

Fig. 2.1.5 Right-hand rules were created to help us remember several physics conventions, such as the agreed-upon direction of angular velocity. These rules start with a convention that we all find extremely familiar, specifically which hand is your right hand, and use it to specify the conventions associated with physical laws. Even if you are left-handed, you still know which hand to call your right hand and therefore understand this convention.

Newton's First Law of Rotational Motion

A rigid object that is not wobbling and is not subject to any outside torques rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation.

Check Your Understanding #2: Going for a Spin

A rubber basketball floats in a swimming pool. It experiences zero torque, no matter which end of it is up. If you spin the basketball and then let go, how will it move?

Answer: It will continue to spin at a steady pace about a fixed rotational axis (although friction with the water will gradually slow the ball's rotation).

Why: Because the basketball is free of torques, the outside influences that affect rotational motion, it has a constant angular velocity. If you spin the basketball, it will continue to spin about whatever axis you chose. If you don't spin the basketball, its angular velocity will be zero and it will remain stationary.

The Wind Turbine's Center of Mass

Even without looking for wind turbines, you can find many objects that are nearly free from torques: a baton thrown overhead by a baton twirler, for example, or a juggler's club whirling through the air between two clowns. These motions, however, are complicated because those freely moving objects rotate and translate at the same time. The spinning baton travels up and down, the turning club arcs through the air, and, if the cable breaks, your wind turbine will fall as it spins. How can we distinguish their translational motions from their rotational motions?

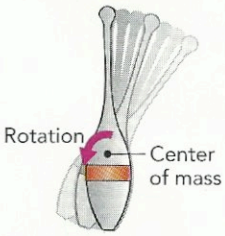


Fig. 2.1.6 This club spins about its center of mass, which remains stationary.

Once again, we can make use of a wonderful simplification of physics. There's a special point in or near a free object about which all of its mass is evenly distributed and about which it naturally spins—its **center of mass**. The axis of rotation passes through this point so that, as the free object rotates, the center of mass doesn't move unless the object has an overall translational velocity. The center of mass of a typical ball is at its geometrical center, while the center of mass of a less symmetrical object depends on how the mass of that object is distributed. You can begin to find a small object's center of mass by spinning it on a smooth table and looking for the fixed point about which it spins (Fig. 2.1.6).

Center of mass allows us to separate an object's translational motion from its rotational motion. As a juggler's club arcs through space, its center of mass follows the simple path we discussed in Section 1.2 on falling balls (Fig. 2.1.7). At the same time, the club's rotational motion about its center of mass is that of an object that's free of outside torques: if it's not wobbling, it rotates with a constant angular velocity.

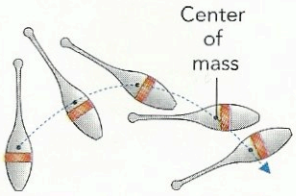


Fig. 2.1.7 A juggler's club that is traveling through space rotates about its center of mass as its center of mass travels in the simple arc associated with a falling object.

Many of the objects we'll examine in this book translate and rotate simultaneously, and it's worth remembering that we can often separate these two motions by paying attention to an object's center of mass. For example, your wind turbine has been designed with its center of mass located exactly where the pivot will go. As a result, the pivot will prevent any translational motion of the turbine while permitting nearly free rotational motion of the turbine about its center of mass, at least about one axis.

Check Your Understanding #3: Tracking the High Dive

When a diver does a rigid, open somersault off a high diving board, his motion appears quite complicated. Can this motion be described simply? How?

Answer: Yes. His center of mass falls smoothly, obeying the rules governing falling objects. As he falls, his body rotates at constant angular velocity about his center of mass.

Why: Like a thrown football or tossed baton, the diver is a rigid, rotating object. His motion can be separated into translational motion of his center of mass (it falls) and rotational motion about his center of mass (he rotates about it at constant angular velocity). While the diver may never think of his motion in these terms, he is aware intuitively of the need to handle both his rotational and translational motions carefully. Hitting the water with his chest because he mishandled his rotation isn't much more fun than hitting the board because he mishandled his translation.

How the Wind Turbine Responds to Torques

The workers are eating lunch, so your wind turbine is still hanging from the cable. Why can't this dangling turbine change its rotational speed or axis of rotation? Because it has rotational mass . **Rotational mass** is the measure of an object's *rotational* inertia, its

For clarity and simplicity, this book refers to the measure of an object's rotational inertia as "rotational mass." However, this quantity is known more formally as **moment of inertia**.

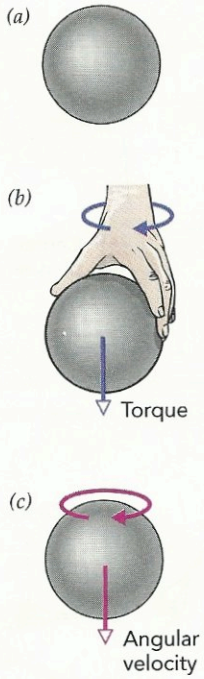


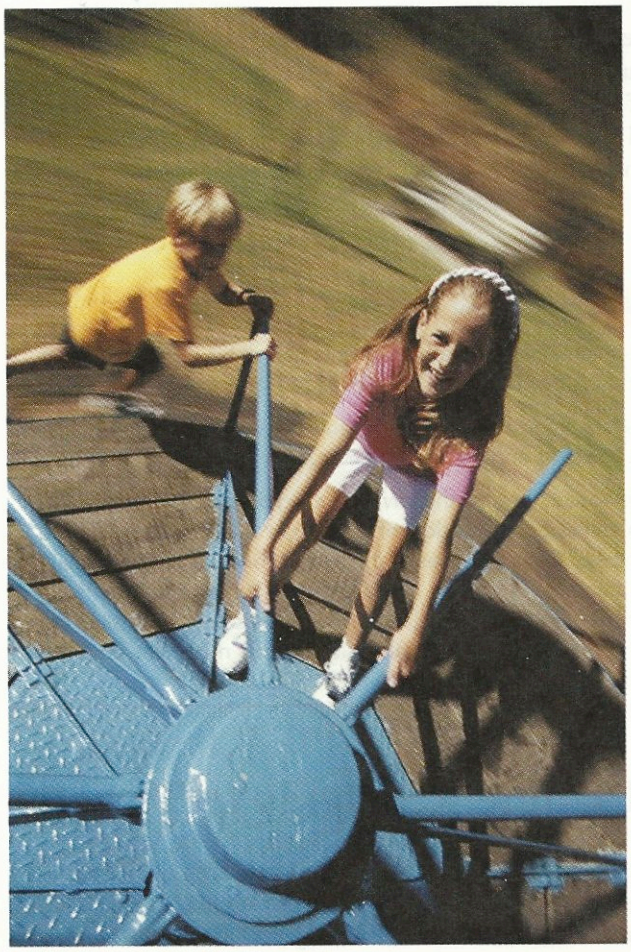
Fig. 2.1.8 If you start with a ball that's not spinning (*a*) and twist it with a torque (*b*), the ball will acquire an angular velocity (*c*) that's in the same direction as that torque.

resistance to changes in its *angular* velocity. An object's rotational mass depends both on its ordinary mass and on how that mass is distributed within the object. The SI unit of rotational mass is the **kilogram-meter^2** (abbreviated kg · m^2). Because the wind turbine has rotational mass, its angular velocity will change only if something twists it or spins it. In other words, it must experience a torque.

Torque—our second important vector quantity of rotational motion—has both a magnitude and a direction. The more torque you exert on the wind turbine, the more rapidly its angular velocity changes. Depending on the direction of the torque, you can make the wind turbine turn more rapidly or less rapidly or even rotate about a different axis. How do you determine the direction of a particular torque? One way is to imagine exerting this torque on a stationary ball floating in water (Figs. 2.1.8*a, b*). The ball will begin to rotate, acquiring a nonzero angular velocity (Fig. 2.1.8*c*). The direction of this angular velocity is that of the torque. The SI unit of torque is the **newton-meter** (abbreviated N · m).

The larger an object's rotational mass, the more slowly its angular velocity changes in response to a specific torque (Fig. 2.1.9). You can easily spin a basketball with the tips of your fingers, but it's much harder to spin a bowling ball. The bowling ball's larger rotational mass comes about primarily because it has a greater ordinary mass than the basketball.

However, rotational mass also depends on an object's shape, particularly on how far each portion of its ordinary mass is from the axis of rotation. The farther a portion of



Chris Harvey/Stone/Getty Images

Fig. 2.1.9 Spinning a merry-go-round is difficult because of its large rotational mass. Despite the large torque exerted by this boy, the merry-go-round's angular velocity increases slowly.

mass is from that axis, the more rapidly it must accelerate as the entire object undergoes angular acceleration and the more leverage it has with which to oppose that acceleration. We'll examine levers shortly, but the consequence of these two effects of distance from the rotation axis is that each portion of mass contributes to the object's rotational mass in proportion to the square of its distance from that axis. That's why an object that has most of its mass located near the axis of rotation will have a much smaller rotational mass than an object of the same mass that has most of its mass located far from that axis. Thus a ball of pizza dough has a smaller rotational mass than the finished pizza. Also, the bigger the pizza gets, the harder it is to start or stop spinning.

Because an object's rotational mass depends on how far its mass is from the axis of rotation, changes in the axis of rotation are likely to change its rotational mass. For example, less torque is required to spin a tennis racket about its handle (Fig. 2.1.10*a*) than to flip the racket head-over-handle (Fig. 2.1.10*b*). When you spin the tennis racket about its handle, the axis of rotation runs right through the handle so that most of the racket's mass is fairly close to the axis and the rotational mass is small. When you flip the tennis racket head-over-handle, the axis of rotation runs across the handle so that both the head and the handle are far away from the axis and the rotational mass is large. The tennis racket's rotational mass becomes even larger when you hold it in your hand and make it rotate about your shoulder rather than about its center of mass (Fig. 2.1.10*c*).

When something exerts a torque on the dangling wind turbine, its angular velocity changes; in other words, it undergoes angular acceleration, our third important vector quantity of rotational motion. **Angular acceleration** measures how quickly the turbine's *angular* velocity changes. It's analogous to acceleration, which measures how quickly an object's *translational* velocity changes. Just as with acceleration, angular acceleration involves both a magnitude and a direction. An object undergoes angular acceleration when its angular speed increases or decreases or when its angular velocity changes directions. The SI unit of angular acceleration is the **radian-per-second^2** (abbreviated $1/s^2$).

If the wind turbine is experiencing several torques at once, it can't respond to them individually. Instead, it undergoes angular acceleration in response to the **net torque** it experiences—the sum of all the individual torques being exerted on it.

There is a simple relationship between the net torque exerted on the wind turbine, its rotational mass, and its angular acceleration. The wind turbine's angular acceleration is equal to the net torque exerted on it divided by its rotational mass or, as a word equation,

$$\text{angular acceleration} = \frac{\text{net torque}}{\text{rotational mass}}. \qquad (2.1.1)$$

The turbine's angular acceleration, as we've seen, is in the same direction as the net torque exerted on it.

This relationship is **Newton's second law of rotational motion**. Structuring the relationship this way distinguishes the causes (net torque and rotational mass) from their effect (angular acceleration). Nonetheless, it has become customary to rearrange the relationship to eliminate the division. In its traditional form, the relationship can be written in a word equation:

$$\text{net torque} = \text{rotational mass} \cdot \text{angular acceleration}, \qquad (2.1.2)$$

in symbols:

$$\tau_{\text{net}} = I \cdot \alpha,$$

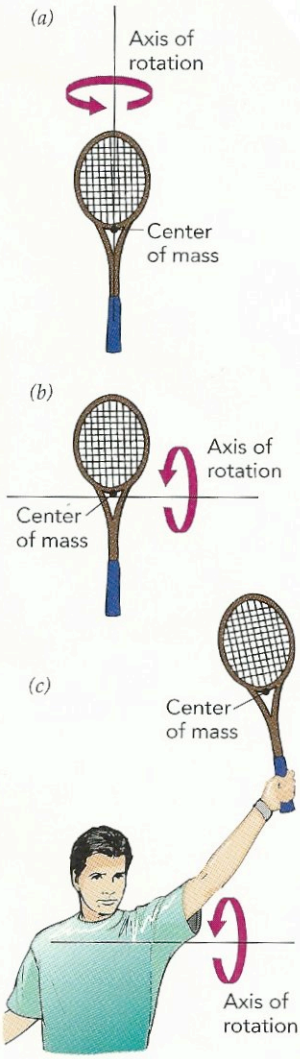


Fig. 2.1.10 A tennis racket's rotational mass depends on the axis about which it rotates. Its rotational mass is small (*a*) when it rotates about its handle and large (*b*) when it rotates head-over-handle. (*c*) If you make it rotate about your shoulder, its rotational mass becomes even larger.

and in everyday language:

Spinning a marble is much easier than spinning a merry-go-round.

It's like Newton's second law of translational motion (net force = mass · acceleration), except that net torque has replaced net force, rotational mass has replaced mass, and angular acceleration has replaced acceleration. This new law doesn't apply to wobbling objects, however, because they're being affected by more than one rotational mass simultaneously (see the discussion of tennis rackets above) and follow a much more complicated law.

Newton's Second Law of Rotational Motion

The net torque exerted on an object that is not wobbling is equal to the product of that object's rotational mass times its angular acceleration. The angular acceleration points in the same direction as the net torque.

Because it's an equation, the two sides of Eq. 2.1.1 are equal. Any change in the net torque you exert on the wind turbine must be accompanied by a proportional change in its angular acceleration. As a result, the harder you twist the wind turbine, the more rapidly its angular velocity changes.

We can also compare the effects of a specific torque on two different rotational masses. Equation 2.1.1 indicates that a decrease in rotational mass must be accompanied by a corresponding increase in angular acceleration. If we replace the wind turbine with a toy pinwheel, the rotational mass will decrease and the angular acceleration will increase. The angular velocity of a pinwheel thus changes more rapidly than the angular velocity of a wind turbine when the two experience identical net torques.

In summary:

1. Your angular position indicates exactly how you're oriented.
2. Your angular velocity measures how quickly your angular position changes.
3. Your angular acceleration measures how quickly your angular velocity changes.
4. In order for you to undergo angular acceleration, you must experience a net torque.
5. The more rotational mass you have, the less angular acceleration you experience for a given net torque.

This summary of the physical quantities of rotational motion is analogous to the summary for translational motion in Section 1.1. Take a moment to compare the two.

Quantity	*SI Unit*	*English Unit*	*SI → English*	*English → SI*
Angular position	radian (1)	radian (1)		
Angular velocity	radian-per-second (1/s)	radian-per-second (1/s)		
Angular acceleration	radian-per-second^2 (1/s^2)	radian-per-second^2 (1/s^2)		
Torque	newton-meter (N · m)	foot-pound (ft · lbf)	1 N · m = 0.73757 ft · lbf	1 ft · lbf = 1.3558 N · m
Rotational mass	kilogram-meter^2 (kg · m^2)	pound-foot^2 (lbm · ft^2)	1 kg · m^2 = 23.730 lbm · ft^2	1 lbm · ft^2 = 0.042140 kg · m^2

Check Your Understanding #4: **The Merry-Go-Round**

The merry-go-round is a popular playground toy (see Fig. 2.1.9). Already challenging to spin empty, a merry-go-round is even harder to start or stop when there are lots of children on it. Why is it so difficult to change a full merry-go-round's angular velocity?

Answer: The full merry-go-round has a huge rotational mass.

Why: Starting or stopping a merry-go-round involves angular acceleration. As the pusher, you exert a torque on the merry-go-round and it undergoes angular acceleration. This angular acceleration depends on the merry-go-round's rotational mass, which in turn depends on how much mass it has and how far that mass is from the axis of rotation. With many children adding to the merry-go-round's rotational mass, its angular acceleration tends to be small.

Check Your Figures #1: **Hard to Turn**

Automobile tires are normally hollow and filled with air. If they were made of solid rubber, their rotational masses would be about 10 times as large. With the wheel lifted off the ground, how much more torque would an automobile have to exert on a solid tire to make it undergo the same angular acceleration as a hollow tire?

Answer: If would need about 10 times as much torque.

Why: To keep the angular acceleration in Eq. 2.1.1 unchanged while increasing the rotational mass by a factor of 10, the torque must also increase by a factor of 10. Solid tires are extremely difficult to spin or to stop from spinning, which is why automobiles use hollow tires.

Forces and Torques: Spinning the Turbine

The workers have finally installed your wind turbine on its tower. Because the shaft connecting the wind turbine to the tower passes directly through the turbine's center of mass, the turbine pivots about its own natural rotation axis. The shaft supports the turbine's weight while leaving it free to obey Newton's first law of rotational motion. That is, when not influenced by the air or anything else, the turbine is either motionless or rotates with constant angular velocity about its pivot.

To change the turbine's angular velocity, you have to exert a torque on it. But how do you actually exert a torque? To find out, you climb part way up the tower and push the nearby blade to the left (Fig. 2.1.11*a*). If the turbine was motionless, it now starts turning. If it was already turning, its rate of rotation changes. You have indeed exerted a torque on the turbine.

You started by exerting a *force* on the turbine—you pushed on it—so forces and torques must be related somehow. Sure enough, a force can produce a torque and a torque can produce a force. To help us explore their relationship, let's think of all the ways not to exert a torque on the wind turbine.

What happens if you climb farther up the tower and push leftward on the turbine's central pivot (Fig. 2.1.11*b*)? Nothing happens—there is no angular acceleration. If you move down a little and push the blade to the left again, you can get the turbine rotating but you have to push hard. You do much better if you push on the end of the blade, where even a small force can start the turbine rotating. The distance from the pivot to the place where you push on the turbine is called the **lever arm**; in general, the longer the lever arm, the less force it takes to cause a particular angular acceleration. Our first

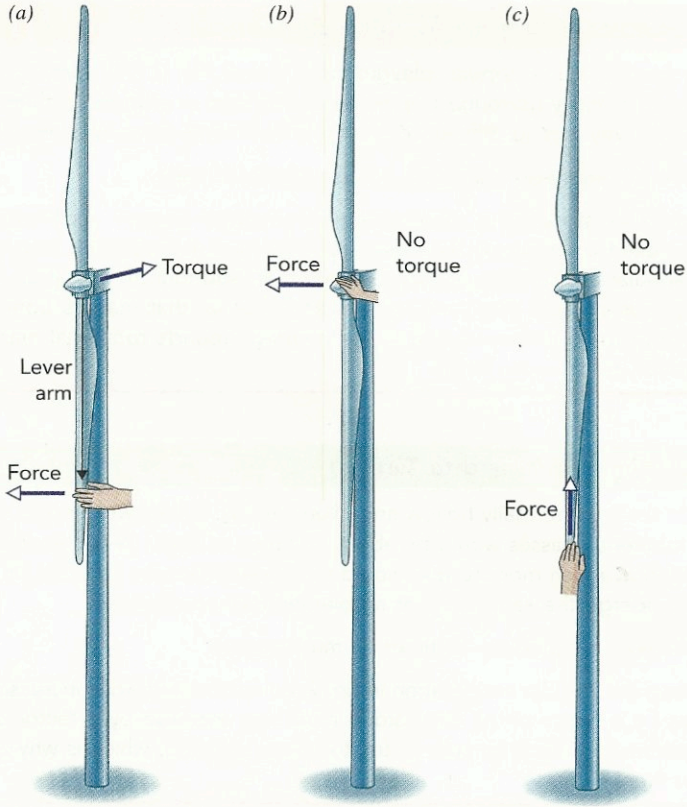


Fig. 2.1.11 (*a*) When you push leftward on the wind turbine, far below its pivot, you exert a torque on it. But when you (*b*) exert your force at the pivot or (*c*) exert your force toward the pivot, you exert no torque.

observation about producing a torque with a force is this: you obtain more torque by exerting that force farther from the pivot or axis of rotation. In other words, the torque is proportional to the lever arm.

Another ineffective way to start the turbine rotating is to push the end of a blade directly toward or away from the pivot (Fig. 2.1.11*c*). A force exerted toward or away from the axis of rotation doesn't produce any torque about that axis. At least a component of the force you exert must be perpendicular to the lever arm, which is actually a vector pointing along the wind turbine's blade from the pivot to the place where you push on the blade. Our second observation about producing a torque with a force is that you must exert at least a component of that force perpendicular to the lever arm. Only that component of force contributes to the torque. To produce the most torque, push perpendicular to the lever arm.

We can summarize these two observations as follows: the torque produced by a force is equal to the product of the lever arm times that force, where we include only the component of the force that is perpendicular to the lever arm. This relationship can be written as a word equation:

$$\text{torque} = \text{lever arm} \cdot \text{force perpendicular to lever arm}, \tag{2.1.3}$$

in symbols:

$$\tau = r \cdot F_{\perp},$$

and in everyday language:

When twisting an unyielding object, it helps to use a long wrench.

The directions of the force and lever arm also determine the direction of the torque. The three directions follow another right-hand rule (Fig. 2.1.12). If you point your right index finger in the direction of the lever arm and your bent middle finger in the direction of the force, then your thumb will point in the direction of the torque. Thus in Fig. 2.1.12, the lever arm points downward, the force points leftward, and the resulting torque points into the page so that the wind turbine undergoes angular acceleration in the clockwise direction.

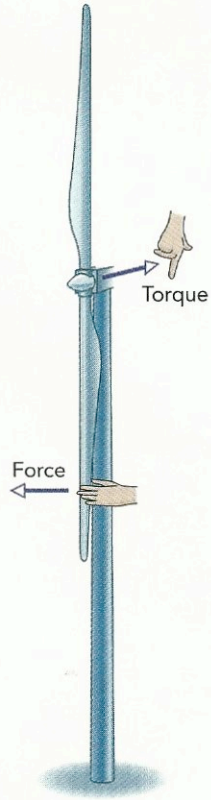


Fig. 2.1.12 The torque on a wind turbine obeys a right-hand rule: if your index finger points downward along the lever arm and your middle finger points leftward along the force, your thumb points into the paper along the torque.

Check Your Understanding #5: Cutting Up Cardboard

When you cut cardboard with a pair of scissors, it's best to move the cardboard as close as possible to the scissors' pivot. Explain.

Answer: The closer the cardboard is to the pivot, the more force it must exert on the scissors to produce enough torque to keep the scissors from rotating closed. When the cardboard is unable to produce enough torque, the scissors cut through it.

Why: When you place paper close to the pivot of a pair of scissors, you are requiring that paper to exert enormous forces on the scissors to keep them from rotating closed. Rotations are started and stopped by torques, and forces exerted close to the pivot exert relatively small torques.

Check Your Figures #2: A Few Loose Screws

You're trying to remove some rusty screws from your refrigerator, using an adjustable wrench with a 0.2-meter (20-centimeter) handle. Although you push as hard as you can on the handle, you can't produce enough torque to loosen one of the screws. You have a 1.0-meter (100-cm) pipe that you can slip over the handle of the wrench to make the wrench effectively 1 meter long. How much more torque will you then be able to exert on the screw?

Answer: You'll be able to exert five times as much torque as before.

Why: The pipe increases the wrench's lever arm by a factor of 5, from 0.2 meter to 1.0 meter. According to Eq. 2.1.3, the same force exerted five times as far from the pivot will produce five times as much torque about that pivot. Extending the handle of a lever-like tool is a common technique to increase the available torque, although it can be hazardous for both the tool and its user. Some tools that are designed for such extreme use come with removable handle extensions.

And Now for the Wind!

As you climb down the tower, the previously calm air begins to move and soon a brisk wind is blowing past the turbine's blades. Your turbine is about to become a *wind* turbine. Although we'll save the details of how moving air pushes on surfaces for Chapter 6, it's time for a preview.

Each turbine blade is essentially a wing, a structure that deflects a passing airstream. When moving air flows around a wing, the wing bends that airstream by pushing the air perpendicular to its path (Fig. 2.1.13). Though air may seem insubstantial, deflecting it

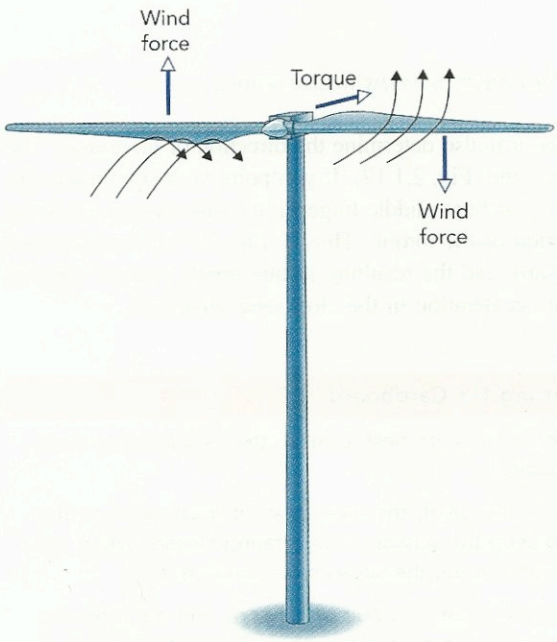


Fig. 2.1.13 As wind flows past the wind turbine, it is deflected downward by the left blade and upward by the right blade. Since the blades are pushing on the air, the air pushes back on the blades. The upward wind force on the left blade and the downward wind force on the right blade both exert torques on the wind turbine that point into the paper.

does require a force and, in accordance with Newton's third law, the air pushes back on the wing with an equal but oppositely directed force. When a wing is deflecting lots of fast-moving air, the forces that the air and wing exert on one another can be enormous.

As we'll see in Section 6.3, an airplane manages to support its entire weight by pushing the passing air downward with both of its wings. That air pushes upward on those wings and keeps the plane from falling. However, a wind turbine doesn't need to be supported, it needs to be spun. Thus its two blades are arranged so that they deflect the wind in *opposite* directions. For example, when the blades are horizontal (Fig. 2.1.13), the blade on the left deflects the wind downward while the blade on the right deflects the wind upward. The wind responds by pushing up on the left blade and down on the right blade. The wind is twisting the wind turbine!

Each of the turbine's blades is experiencing a torque from the wind, but how much torque and in which direction? We know the wind force on each blade, but we also need to know the lever arm—the vector from the turbine's central pivot to the point at which that wind force acts. Although the wind force on each blade is actually spread out along its length, that force effectively acts at a single point near the middle of the blade.

The lever arm for the left blade therefore points toward the left and is approximately half the length of the blade. Since this leftward lever arm is perpendicular to the upward wind force, the resulting torque is simply the product of the lever arm times the force and it points, according to the right-hand rule, into the page. In simple terms, the wind twists this blade clockwise. Similarly, the lever arm for the right blade points toward the right and is approximately half the length of the blade. Since the wind force on the right blade is downward and the lever arm is rightward, the resulting torque also points into the page—clockwise. Both blades are experiencing wind torques in the same direction.

The wind turbine can't respond to each torque individually; it responds to the net torque it experiences. Since the torques from its blades both point into the page, the net torque on the wind turbine also points into the page and it undergoes angular acceleration in that direction. The turbine begins to rotate clockwise.

Although this rotation tips the wind turbine's blades away from horizontal, the torques on those blades don't change. That's because the lever arm associated with a

and in everyday language:

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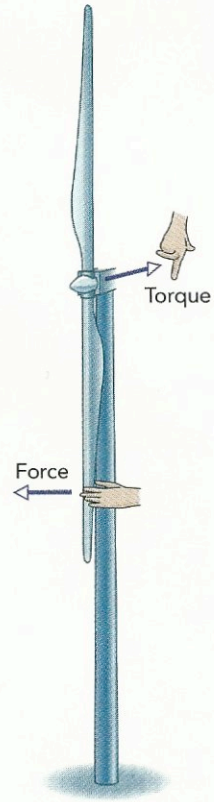
The directions of the force and lever arm also determine the direction of the torque. The three directions follow another right-hand rule (Fig. 2.1.12). If you point your right index finger in the direction of the lever arm and your bent middle finger in the direction of the force, then your thumb will point in the direction of the torque. Thus in Fig. 2.1.12, the lever arm points downward, the force points leftward, and the resulting torque points into the page so that the wind turbine undergoes angular acceleration in the clockwise direction.



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And Now for the Wind!

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Each turbine blade is essentially a wing, a structure that deflects a passing airstream. When moving air flows around a wing, the wing bends that airstream by pushing the air perpendicular to its path (Fig. 2.1.13). Though air may seem insubstantial, deflecting it

blade and the wind force that acts on it tip together and they remain perpendicular. The deflected wind always twists both blades counterclockwise. Since the net torque on the wind turbine remains constant as it rotates, it experiences steady angular acceleration and spins clockwise faster and faster.

When the wind turbine is spinning quickly enough, a sensor activates the electric generator. Now the wind turbine is turning the generator and producing electric power. The fact that the wind turbine is exerting a torque on the generator also has important consequences for the wind turbine itself. Just as there is a Newton's third law for translational motion, so there is a **Newton's third law of rotational motion**: if one object exerts a torque on a second object, then the second object will exert an equal but oppositely directed torque on the first object.

Newton's Third Law of Rotational Motion

For every torque that one object exerts on a second object, there is an equal but oppositely directed torque that the second object exerts on the first object.

To keep the generator turning along with it, the wind turbine must exert a torque on the generator that points into the page (clockwise). The generator responds by exerting an opposite torque on the wind turbine: out of the page (counterclockwise). The net torque on the wind turbine now includes three torques, with the generator's torque opposing the wind's torques. When the generator is properly adjusted, with the total torque the wind exerts on it equal to the torque it exerts on the generator, the three torques on the wind turbine will cancel exactly and the turbine will coast at constant angular velocity. That is actually the normal operating condition for a wind turbine: steady rotation with all of the wind torque being conveyed to the generator.

Summary of Newton's Laws of Rotational Motion

1. A rigid object that is not wobbling and is not subject to any outside torques rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation.
2. The net torque exerted on an object that is not wobbling is equal to the product of that object's rotational mass times its angular acceleration. The angular acceleration points in the same direction as the torque.
3. For every torque that one object exerts on a second object, there is an equal but oppositely directed torque that the second object exerts on the first object.

Note: These laws are the rotational analogs of the translational laws in Section 1.3.

Check Your Understanding #6: Turning on Ice

A light at your skating rink needs replacing, so you stand on the slippery ice and reach up overhead to unscrew the light. As you twist the light, you begin to rotate. What caused you to rotate?

Answer: The light exerted a torque on you.

Why: To unscrew the light, you must exert a torque on it. In accordance with Newton's third law of rotational motion, that light exerts an equal but opposite torque on you. The ice is too slippery to exert a torque on you, so you undergo angular acceleration and begin to rotate.

Wind Energy: Torque, Rotation, and Work

Since your wind turbine is now generating electric energy and energy is a conserved quantity, energy must be going from the wind to the wind turbine to the generator. Let's use our current understanding of work to follow the first step in that process: the transfer of energy from the wind to the wind turbine.

The wind is pushing each blade as the blade moves in the direction of that push, so the wind is certainly doing work on the turbine. The amount of work done on each blade during a full revolution of the turbine is equal to the product of force times distance—in this case, the wind force on the blade times the circumference of the circle swept out by the lever arm as the blade pivots, or

$$\begin{aligned}\text{work (for a full revolution)} &= \text{force} \cdot \text{distance (around circle)}\\ &= \text{force} \cdot (\text{lever arm} \cdot 2\pi).\end{aligned}$$

Recalling that the product of force times lever arm is torque, we can rewrite that same work as

$$\begin{aligned}&= (\text{force} \cdot \text{lever arm}) \cdot 2\pi\\ &= \text{torque} \cdot 2\pi.\end{aligned}$$

Since 2π is the angle of a full revolution, as measured in radians, the work done on the turbine during a full rotation is simply the product of the torque times the angle of the full rotation. Since the work is proportional to that angle, the work done during any amount of rotation is the product of torque times the angle of rotation. This relationship can be written as a word equation:

$$\text{work} = \text{torque} \cdot \text{angle (in radians)} \tag{2.1.4}$$

in symbols:

$$W = \boldsymbol{\tau} \cdot \boldsymbol{\theta},$$

and in everyday language:

If you're not twisting or it's not turning, then you're not working.

This simple relationship assumes that the wind's torque is constant while it's doing the work. If that torque varies, the calculation of work will have to recognize that variation and may require the use of calculus.

That relationship between torque, angle, and work is the rotational equivalent of Eq. 1.3.1 and it allows us to calculate rotational work without resorting to forces and distances. For example, it tells us that the total work done on the turbine by the wind is the product of the total wind torque times the turbine's rotation angle, measured in radians.

We can also use Eq. 2.1.4 to follow the transfer of energy from the wind turbine to the generator. As the turbine twists the generator around and around, the work the turbine does on the generator is simply the product of the torque it exerts on the generator times the angle of rotation, measured in radians. When the wind turbine is exerting just as much torque on the generator as the wind exerts on it, all of the work that the wind does on the turbine is transferred to the generator, which converts it to electric energy. The wind turbine then spins at a steady angular velocity, turning wind energy into electric energy.

Check Your Understanding #7: **Going Nowhere One Foot at a Time**

You're pedaling a stationary bicycle when your shoelace comes undone. While you retie the lace, you can only pedal with one foot and exert half as much torque on the pedals as before. The pedals are still turning just as quickly, so are you still exercising as hard?

Answer: No.

Why: By halving the torque you exert on the pedals, you halve the work that you do on those pedals each time they complete a rotation.

Check Your Figures #3: **We All Knead Energy**

When your kitchen mixer is kneading a loaf of sourdough bread, it exerts a torque of 20 newton-meters on its mixing blade and that blade completes 500 rotations. How much energy does the mixer supply to its blade and the dough?

Answer: It supplies about 63,000 joules.

Why: According to Eq. 2.1.4, the energy that the mixer transfers to its blade is the product of the torque it exerts on the blade times the angle through which the blade turns. Since there are 2π radians per complete rotation, the blade turns approximately 3140 radians. The work that the mixer does is 20 N · m times 3140, or about 63,000 J.

Mechanical Advantage

A 50%-off coupon arrives by email and you decide to upgrade your wind turbine by doubling the lengths of its blades. With twice the length to deflect air, each blade experiences twice the wind force. But the torque on that blade doesn't just double, it quadruples!

That's because the lever arm, the vector from the pivot to the effective point at which the wind force acts, also doubles. Since torque is the product of lever arm times force, doubling both the lever arm and the force quadruples the torque. This effect, whereby the torque produced by a given force increases in proportion to the lever arm or "leverage," is another case of mechanical advantage, one of the great benefits of levers. You can increase the torque produced by a lever by either pushing on it harder or increasing its length. Here, we have done both!

With the rotating wind turbine experiencing four times as much torque, it does four times as much work on the generator per revolution and provides four times as much electric energy. It obtains that increased energy by sweeping its longer blades through a cross section of the wind that has four times the area. Not surprisingly, it takes four times as much wind to produce four times as much electric energy.

It turns out, however, that your twin-blade wind turbine has a serious shortcoming: the wind pushes unevenly on its two blades when those blades are vertical. Since the wind is strongest high above the ground, the higher blade always experiences a stronger wind force than the lower blade. That is not a problem for power generation, because the two blades act together to turn the generator, but it is a problem for the central pivot. The net force on the wind turbine must be zero, so if there is an overall wind force on the turbine, the pivot must compensate by pushing the turbine in the opposite direction.

When the two blades are both horizontal, the wind forces they experience are nearly equal in amount but opposite in direction, so they cancel. The pivot doesn't have to fight an overall wind force. When the blades are vertical, however, the wind pushes the upper

blade to the right much harder than it pushes the lower blade to the left. The turbine experiences an overall wind force to the right, and the pivot must compensate by pushing the turbine toward the left. The pivot's leftward force on the wind turbine increases and decreases twice with each revolution of the wind turbine, and that cyclic stress gradually wears out the entire structure.

Rather than allowing your wind turbine to age prematurely, you order a third blade. With an odd number of blades, your improved wind turbine never has two of its blades in a vertical line and thus avoids extreme overall wind forces. Since the pivot doesn't have to fight so hard to keep the wind turbine in place, it doesn't wear out as quickly and you'll be doing much less maintenance.

Check Your Understanding #8: **Pulling Nails**

Some hammers have a special claw designed to remove nails from wood. When you slide the claw under the nail's head and rotate the hammer by pulling on its handle, the claw pulls the nail out of the wood. The hammer's head contacts the wood to form a pivot that's about 10 times closer to the nail than to the handle. The torque you exert on the hammer twists it in one direction, while the torque that the nail exerts on the hammer twists it in the opposite direction. The hammer isn't undergoing any significant angular acceleration, so the torques must nearly balance. If you're exerting a force of 100 N (22 lbf) on the hammer's handle, how much force is the nail exerting on the hammer's claw?

Answer: The nail is exerting about 1000 N (220 lbf).

Why: Since the nail is 10 times closer to the pivot, the nail must exert 10 times the force on the hammer to create the same magnitude of torque as you do pulling on the handle. As the nail pulls on the hammer, the hammer pulls on the nail. Although the wood exerts frictional forces on the nail to keep it from moving, the extracting force overwhelms this friction and the nail slides slowly out of the wood.

Balancing Your Wind Turbine

Adding that third blade brings up a topic that I alluded to early in this section: balance. What is balance and how do you balance your wind turbine? Fortunately, we can now use our understanding of torque to answer those questions. A **balanced** wind turbine is one that is experiencing no overall torque due to gravity. While gravity normally exerts a torque on each individual blade, those torques cancel one another perfectly and the complete wind turbine is free of gravitational influence.

It's easy to see that the twin-bladed turbine is balanced when it's horizontal (Fig. 2.1.14*a*). We need to show only that the torque due to the right blade's weight exactly cancels the torque due to the left blade's weight. To determine those two torques, however, we need to know the appropriate lever arms—the vectors from the pivot to each blade's weight. But where exactly is a blade's weight located?

A blade's weight is actually spread out along the blade's entire length, but we can consider it as acting at a single point near the blade's center. Known as the blade's **center of gravity**, this point is the effective location of the blade's weight. Center of gravity (a gravitational concept) is different from center of mass (an inertial concept). However, the two conveniently coincide: a blade's center of gravity and center of mass are located at the same point. That coincidence stems from the proportionality between an object's weight and its mass near the earth's surface (see Section 1.2).

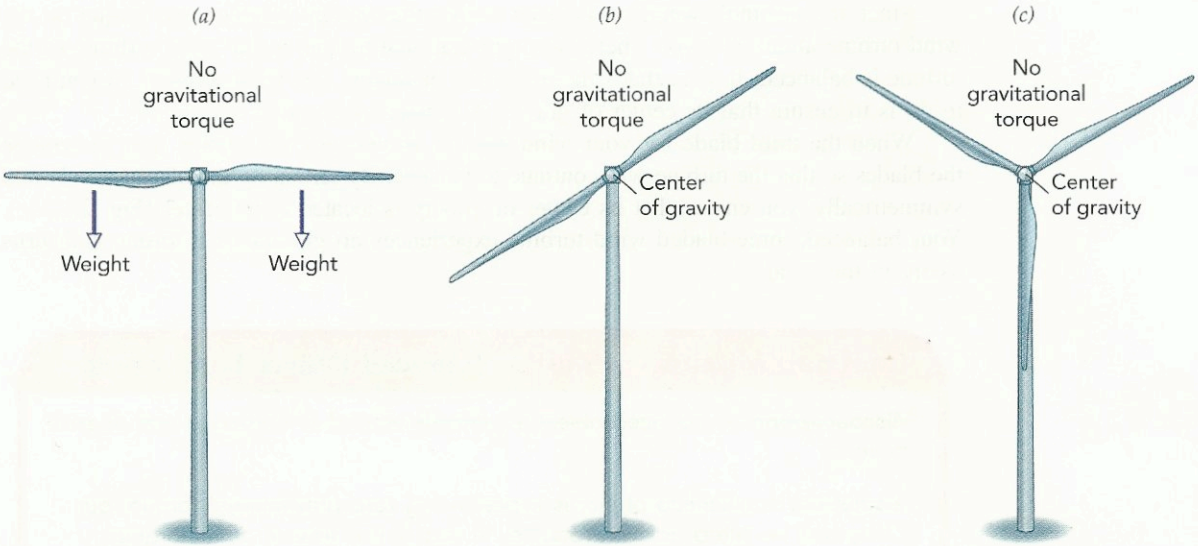


Fig. 2.1.14 (*a*) It's easy to see that a two-bladed wind turbine is balanced when its blades are horizontal. They experience opposite torques due to gravity, so the turbine experiences no overall gravitational torque. (*b*) When the blades are tilted, the easiest way to show that the turbine is balanced is to recognize that its center of gravity is at its pivot, so its weight produces no torque. (*c*) Because its center of gravity is at the pivot, this three-bladed wind turbine also balances.

Common Misconceptions: Center of Mass Is Center of Gravity

Misconception: Center of mass and center of gravity are the same idea and can be used interchangeably.

Resolution: Center of mass is an inertial concept: the effective location of an object's mass as it translates and the natural pivot as it rotates. Center of gravity is a gravitational concept: the effective location of the object's weight. Although these two centers coincide for objects near the earth's surface, interchanging the terms is as incorrect as confusing mass and weight.

The lever arm on each blade is a vector pointing from the pivot to that blade's center of gravity. Since the right blade's lever arm is horizontal and its weight is vertical, the torque on the right blade is simply the product of its lever arm times its weight and points, according to the right-hand rule, into the page. Similarly, the torque on the left blade is the product of its lever arm times its weight and points out of the page. Since the blades are identical, they have equal weights and opposite lever arms, so these two torques are equal but opposite and cancel one another. The turbine is truly balanced.

When the wind turbine isn't horizontal (Fig. 2.1.14*b*), however, a blade's lever arm and its weight are no longer perpendicular and determining the gravitational torque on it isn't so easy. To show that the wind turbine is still balanced, let's use a different approach: let's locate the center of gravity for the entire wind turbine. Since the turbine is symmetrical about its center, its center of gravity can't be in either one of its blades. In fact, there is only one logical place for the turbine's center of gravity: at its geometrical center!

Since the turbine's weight acts at its center, that weight produces no torque on the wind turbine about its pivot. There is no gravitational torque on the wind turbine, so the turbine is balanced. In fact, that's the recipe for balancing *any* wind turbine: all you need to do is to ensure that its center of gravity is located at its pivot.

When the third blade for your wind turbine arrives, you now know how to arrange the blades so that the turbine will continue to balance. By arranging the blades evenly and symmetrically, you ensure that its center of gravity is located at its center (Fig. 2.1.14*c*). Your balanced, three-bladed wind turbine experiences no gravitational torque and turns easily in the wind.

Common Misconceptions: A Balanced Object Is Motionless

Misconception: A balanced object is perfectly vertical or horizontal and doesn't move.

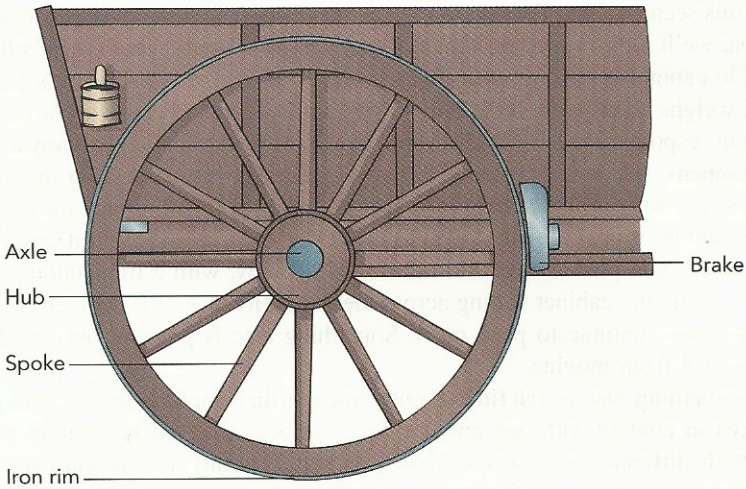
Resolution: A balanced object is experiencing zero gravitational torque, but it may well be tilted or even rotating. In fact, if it is free of other torques, Newton's first law of rotational motion may apply to it. Only when an object's balance is easily disturbed (e.g., a pencil standing upright on its point or a pendulum hanging directly below its pivot) are orientation and motionlessness important and then only because any small rotation will give rise to a gravitational torque and consequently to the loss of balance.

Check Your Understanding #9: Rocking the Boat

Loading a large container ship requires some care in balancing the cargo and fastening it down firmly. The effective pivot about which the ship can rotate in the water is located roughly along the centerline of the ship, from its bow to its stern. Why is improperly fastened-down cargo so dangerous on such a ship, possibly causing it to capsize during a storm?

Answer: A substantial shift in the cargo's position during a storm can unbalance the ship, giving rise to a gravitational torque that may cause the ship to rotate about the effective pivot. The ship may then capsize.

Why: Although most boats can compensate for some amount of cargo imbalance, shifting cargo can easily flip even a fairly stable boat. It happens frequently in real life, often with fatal consequences. Some boats, particularly canoes and racing shells, are notoriously sensitive to unbalanced loading and are easily flipped by careless or moving occupants.



SECTION 2.2 Wheels

Like ramps and levers, wheels are simple tools that make our lives easier. But a wheel's main purpose isn't mechanical advantage, it's overcoming friction. Up until now, we've ignored friction, looking at the laws of motion as they apply only in idealized situations. However, our real world does have friction, and most objects in motion tend to slow down and stop because of it. One of our first tasks in this section will therefore be to understand friction—though, for the time being, we'll continue to neglect air resistance.

Questions to Think About: *If objects in motion tend to stay in motion, why is it so hard to drag a heavy box across the floor? If objects should accelerate downhill on a ramp, why won't a plate slide off a slightly tilted table? What makes the wheels of a cart turn as you pull the cart forward? How does spinning its wheels propel a car forward?*

Experiments to Do: *To observe the importance of wheels in eliminating friction, try sliding a book along a flat table. Give the book a push and see how quickly it slows down and stops. Which way is friction pushing on the book? Does the force that friction exerts on the book depend on how fast the book is moving? Let the book come to a stop. Is friction still pushing on the book when it isn't moving? If you push gently on the stationary book, what force does friction exert on it?*

Lay down three or four round pencils, parallel to one another and a few inches apart. Rest the book on top of the pencils and give the book a push in the direction that the pencils can roll. Describe how the book now moves. What do you think has caused the difference?

Moving a File Cabinet: Friction

When we imagined moving your friend's piano into a new apartment back in Section 1.3, we neglected a familiar force—friction. Luckily for us, your friend's piano had wheels on its legs, and wheels facilitate motion by reducing the effects of friction. We'll focus on

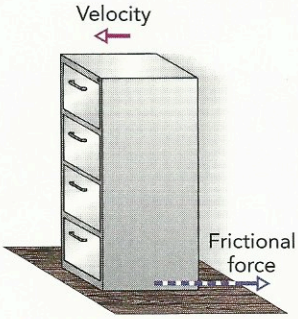


Fig. 2.2.1 A file cabinet sliding to the left across the floor. The file cabinet experiences a frictional force toward the right that gradually brings it to a stop.

wheels in this section. First, though, to help us understand the relationship between wheels and friction, we'll look at another item that needs to be moved—your friend's file cabinet.

The file cabinet is resting on a smooth and level hardwood floor; it's full of sheet music and weighs about 1000 N (225 lbf). Despite its large mass, you know that it should accelerate in response to a horizontal force, so you give it a gentle push toward the door. Nothing happens. Of course, the file cabinet accelerates in response to the net force it experiences, not to each individual force acting on it. Something else must be pushing on the file cabinet in just the right way to cancel your force and keep it from accelerating. Undaunted, you push harder and harder until finally, with a tremendous shove, you manage to get the file cabinet sliding across the floor. However, the cabinet moves slowly even though you continue to push on it. Something else is pushing on the file cabinet, trying to stop it from moving.

That something else is **friction**, a phenomenon that opposes the relative motion of two surfaces in contact with one another. Two surfaces that are in **relative motion** are traveling with different velocities so that a person standing still on one surface would observe the other surface as moving. In opposing relative motion, friction exerts forces on both surfaces in directions that tend to bring them to a single velocity.

For example, when the file cabinet slides by itself toward the left, the floor exerts a rightward frictional force on it (Fig. 2.2.1). The frictional force exerted on the file cabinet, *toward the right*, is in the direction opposite the file cabinet's velocity, *toward the left*. Since the file cabinet's acceleration is in the direction opposite its velocity, the file cabinet slows down and eventually comes to a stop.

According to Newton's third law of motion, an equal but oppositely directed force must be exerted by the file cabinet on the floor. Sure enough, the file cabinet does exert a leftward frictional force on the floor. However, the floor is rigidly attached to the earth, so it accelerates very little. The file cabinet does almost all the accelerating, and soon the two objects are traveling at the same velocity.

Frictional forces always oppose relative motion, but they vary in strength according to (1) how tightly the two surfaces are pressed against one another, (2) how slippery the surfaces are, and (3) whether or not the surfaces are actually moving relative to one another. First, the harder you press two surfaces together, the larger the frictional forces they experience. For example, an empty file cabinet slides more easily than a full one. Second, roughening the surfaces generally increases friction, while smoothing or lubricating them generally reduces it. Riding a toboggan down the driveway is much more interesting when the driveway is covered with snow or ice than when the driveway is bare asphalt. We'll examine the third issue later on.

Check Your Understanding #1: The One That Got Away

Your table at the restaurant isn't level, and your water glass begins to slide slowly downhill toward the edge. Which way is friction exerting a force on it?

Answer: Friction is pushing the glass uphill.

Why: The glass is sliding downhill across the top of the stationary table. Since friction always opposes relative motion, it pushes the glass uphill, in the direction opposite its motion.

A Microscopic View of Friction

As the file cabinet slides by itself across the floor, it experiences a horizontal frictional force that gradually brings it to a stop. From where does this frictional force come? The obvious forces on the file cabinet are both vertical, not horizontal: the cabinet's weight

is downward and the support force from the floor is upward. How can the floor exert a horizontal force on the file cabinet?

The answer lies in the fact that neither the bottom of the file cabinet nor the top of the floor is perfectly smooth. They both have microscopic hills and valleys of various sizes. The file cabinet is actually supported by thousands of tiny contact points, where the file cabinet directly touches the floor (Fig. 2.2.2). As the file cabinet slides, the microscopic projections on the bottom of the file cabinet pass through similar projections on the top of the floor. Each time two projections collide, they experience horizontal forces. These tiny forces oppose the relative motion and give rise to the overall frictional forces experienced by the file cabinet and floor. Because even an apparently smooth surface still has some microscopic surface structure, all surfaces experience friction as they rub across one another.

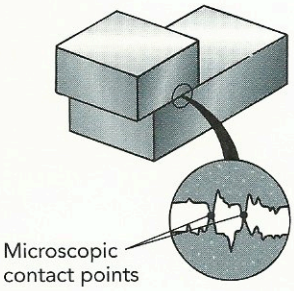


Fig. 2.2.2 Two surfaces that are pressed against one another actually touch only at specific contact points. When the surfaces slide across one another, these contact points collide, producing sliding friction and wear.

Increasing the size or number of these microscopic projections by roughening the surfaces generally leads to more friction. If you put sandpaper on the bottom of the file cabinet, it will experience larger frictional forces as it slides across the floor. On the other hand, a microscopically smoother "nonstick" surface, like that used in modern cookware, would let the file cabinet slide more easily.

Increasing the number of contact points by squeezing the two surfaces more tightly together also leads to more friction. The microscopic projections simply collide more often. That's why adding more sheet music to the file cabinet would make it harder to slide. Doubling the file cabinet's weight would roughly double the number of contact points and make it about twice as hard to move across the floor. A useful rule of thumb is that the frictional forces between two surfaces are proportional to the forces pressing those two surfaces together.

Friction also causes wear when the colliding contact points break one another off. With time, this wear can remove large amounts of material so that even seemingly indestructible stone steps are gradually worn away by foot traffic. The best way to reduce wear between two surfaces (other than to insert a lubricant between them) is to polish them so that they are extremely smooth. The smooth surfaces will still touch at contact points and experience friction as they slide across one another, but their contact points will be broad and round and will rarely break one another off during a collision.

Check Your Understanding #2: Weight and Friction

How much harder is it to slide a stack of two identical books across a table than it is to slide just one of those books?

Answer: It is about twice as hard.

Why: The frictional forces between the table and books are roughly proportional to the forces pressing them together. The book's weight is what pushes them together. When you effectively double the book's weight by stacking a second book on top of it, you double the frictional forces between the table and the books.

Static Friction, Sliding Friction, and Traction

There are really two kinds of friction—sliding and static. When two surfaces are moving across one another, **sliding friction** acts to stop them from sliding. But even when those surfaces have the same velocity, **static friction** may act to keep them from starting to slide across one another in the first place.

You find it particularly hard to start the file cabinet sliding across the floor. Contact points between the cabinet and floor have settled into one another, so a small push does nothing. Static friction always exerts a frictional force that exactly balances your push. Since the net force on the file cabinet is zero, it doesn't accelerate.

However, the force that static friction can exert is limited. To get the file cabinet moving, you need to give it a mighty shove and thereby exert more horizontal force on it than static friction can exert in the other direction. The net force on the file cabinet is then no longer zero and it accelerates.

Once the file cabinet is moving, static friction is replaced by sliding friction. Because sliding friction acts to bring the file cabinet back to rest, you must push on the cabinet to keep it moving. With the file cabinet sliding across the floor, however, the contact points between the surfaces no longer have time to settle into one another, and they consequently experience weaker horizontal forces. That's why the force of sliding friction is generally weaker than that of static friction and why it's easier to keep the file cabinet moving than it is to get it started.

Both forms of friction are incorporated in the concept of **traction**—the largest amount of frictional force that the file cabinet can obtain from the floor at any given moment. When the cabinet is stationary, its traction is equal to the maximum amount of force that static friction can exert on it. Once it begins to slide across the floor, however, its traction reduces to the amount of force that sliding friction exerts.

While the file cabinet's traction is a nuisance that you must overcome, the traction of your shoes on the floor is crucial. Unless you can push against the wall, your shoes are going to need enough traction to provide the horizontal force required to move the file cabinet. Let's hope you're wearing your Dr. Martens!

Check Your Understanding #3: **Skidding to a Stop**

Antilock brakes keep an automobile's wheels from locking and skidding during a sudden stop. Apart from issues of steering, what is the advantage of preventing the wheels from skidding (sliding) on the pavement?

Answer: If the wheels continue to turn, they experience static friction with the pavement. If they lock and begin to skid, they experience sliding friction. Since the traction provided by static friction is greater than that provided by sliding friction, the car will decelerate faster if the wheels don't skid.

Why: For a rapid stop, the car needs the maximum possible force in the direction opposite its velocity. The most effective way to obtain that stopping force from the road is with static friction between the turning wheels and the pavement. Sliding friction, the result of skidding tires, is much less effective at stopping the car, wears out the tires, and diminishes the driver's ability to steer the vehicle.

Work, Energy, and Power

There is another difference between static and sliding friction: sliding friction wastes energy. It can't make that energy disappear altogether because energy, as we've seen, is a conserved quantity: it can't be created or destroyed. However, energy can be transferred between objects or converted from one form to another. What sliding friction does is convert useful, **ordered energy**—energy that can easily be used to do work—into relatively useless, disordered energy. This disordered energy is called **thermal energy** or

internal energy and is the energy we associate with temperature. Sliding friction makes things hotter by turning work into thermal energy.

As we saw in Section 1.3, energy is the capacity to do work and is transferred between objects by doing that work. Energy can also change forms, appearing as either kinetic energy in the motions of objects or as potential energy in the forces between or within those objects. With practice, you can "watch" energy flow through a system just as an accountant watches money flow through a company.

The most obvious form of energy is kinetic energy, the energy of motion. It's easy to see when kinetic energy is transferred into or out of an object. As kinetic energy leaves an object, the object slows down—moving water slows down as it turns a gristmill, and a bowling ball slows down as it knocks over bowling pins. Conversely, as kinetic energy enters an object, the object speeds up. A baseball moves faster as you do work on it during a pitch; you're transferring energy from your body into the baseball, where the energy becomes kinetic energy in the baseball's motion.

Potential energy is stored in the forces between or within objects, and usually isn't as visible as kinetic energy. It can take many different forms, some of which appear in Table 2.2.1. In each case nothing is moving; but because the objects still have a great potential to do work, they contain potential energy.

We measure energy in many common units: joules (J), calories, food Calories (also called kilocalories), and kilowatt-hours, to name only a few. All of these units measure the same thing, and they differ from one another only by numerical conversion factors, some of which can be found in Appendix B. For example, 1 food Calorie is equal to 1000 calories or 4187 J. For example, a jelly donut with about 250 food Calories contains about 1,000,000 J of energy. Since a joule is the same as a newton-meter, 1,000,000 J is the energy you'd use to lift your friend's file cabinet into the second-floor apartment 200 times (1000 N times 5 m upward is 5000 J of work per trip). No wonder eating donuts is hard on your physique!

Of course, you can eventually use up the energy in a jelly donut; it just takes time. You can only do so much work each second. The measure of how quickly you do work is **power**—the amount of work you do in a certain amount of time, or

$$\text{power} = \frac{\text{work}}{\text{time}}.$$

The SI unit of power is the **joule-per-second**, also called the **watt** (abbreviated W). Other units of power include Calories-per-hour and horsepower; like the units for energy, these units differ only by numerical factors, which are again listed in Appendix B. For example, 1 horsepower is equal to 745.7 W. Since a 1-horsepower motor does 745.7 J of work each second, and since it takes 5000 J of work to move the file cabinet to the second floor, that motor has enough power to do the job in about 6.7 s.

Like work itself, power can be supplied by translational motion or rotational motion. You're using translational motion to supply power to the file cabinet when you push it

Table 2.2.1 Several Forms and Examples of Potential Energy

FORM OF POTENTIAL ENERGY	EXAMPLE
Gravitational potential energy	A bowling ball at the top of a hill
Elastic potential energy	A wound clock spring
Electrostatic potential energy	A cloud in a thunderstorm
Chemical potential energy	A firecracker
Nuclear potential energy	Uranium

$W = F \cdot d$

$P = \frac{W}{t}$

$P = \frac{F \cdot d}{t}$ translational

$P = F \cdot V$

across the floor. Since translational work is force times distance (Eq. 1.3.1) and power is work divided by time, translational power is force times distance divided by time, or

$$\text{power} = \text{force} \cdot \text{velocity}.$$

You can supply more power to the file cabinet by pushing it harder or by having it move faster in the direction of your push.

A motor uses rotational motion to supply power to an elevator when it turns the elevator's lifting mechanism and raises the file cabinet to the second floor. Since rotational work is torque times angle (Eq. 2.1.4), rotational power is torque times angle divided by time, or

$$\text{power} = \text{torque} \cdot \text{angular velocity}.$$

ω

Check Your Understanding #4: Apple Overtures

Trace the flow of energy as an archer shoots an apple off the head of her assistant with an arrow.

Answer: The archer does work on the string and bow as she draws the arrow back. (Chemical energy from her body is transferred to the bow, where it is stored as elastic potential energy.) As she releases the arrow, the string and bow do work on the arrow. (Elastic potential energy is transferred to the arrow, where it becomes kinetic energy.) Finally, the arrow does work on the apple, knocking it off the head of the assistant. (Kinetic energy is transferred from the arrow to the apple.)

Why: Because energy is conserved, we could in principle follow it back to the origins of the universe. Whatever energy we see around us now was somewhere in our universe yesterday, last week, and a million years ago, although its form may have changed. It will still be in our universe next year, too, but maybe not in so useful a form.

Friction and Thermal Energy

What about the *thermal* energy produced by sliding friction? Is thermal energy a new kind of potential energy or an alternative to kinetic energy?

In truth, it's neither. Thermal energy is actually a mixture of ordinary kinetic and potential energies. But unlike the kinetic energy in a moving ball or the potential energy in an elevated piano, the kinetic and potential energies in thermal energy are disordered at the atomic and molecular level. Thermal energy makes every microscopic particle in an object jiggle independently; at any moment, each particle has its own tiny supply of potential and kinetic energies, and this dispersed energy is collectively referred to as thermal energy.

As you push the file cabinet across the floor, you do work on it, but it doesn't pick up speed. Instead, sliding friction converts your work into thermal energy, so that the cabinet becomes hotter as the energy you transfer to it disperses among its particles. But although sliding friction easily turns work into thermal energy, there's no easy way to turn thermal energy back into work. Disorder not only makes things harder to use, but it is also difficult to undo. When you drop your favorite coffee mug on the floor and it shatters into a thousand pieces, the cup is still all there, but it's disordered and thus much less useful. Just as dropping the pieces on the floor a second time isn't likely to reassemble your cup, energy converted into thermal energy can't easily be reassembled into useful, ordered energy.

Sliding friction always converts at least some work into thermal energy. Since two surfaces sliding across one another experience frictional forces that oppose their relative motion, sliding friction does negative work on them; it extracts energy from a sliding object and converts that energy into thermal energy. Thus while you do work on the file cabinet by pushing it across the floor, sliding friction does negative work on it. The file cabinet's kinetic energy doesn't change very much, but its thermal energy continues to increase.

In contrast, static friction doesn't convert work into thermal energy. Since two surfaces experiencing static friction don't move relative to one another, there is no distance traveled and thus no work done. You can push against the stationary file cabinet all day without doing any work on it. Even if you lift the file cabinet upward with your hands (no easy task), static friction between your hands and the file cabinet's sides merely assists you in doing work on the file cabinet itself. As you lift the file cabinet upward, all of your work goes into increasing the file cabinet's gravitational potential energy.

Check Your Understanding #5: Burning Rubber

If you push too hard on your car's accelerator pedal when the traffic light turns green, your wheels will slip and you'll leave a black trail of rubber behind. Such a jackrabbit start can cause as much wear on your tires as 50 km (31 miles) of normal driving. Why is skidding so much more damaging to the tires than normal driving?

Answer: Normal driving involves mostly static friction because the surfaces of the tires don't slide across the pavement. Skidding involves sliding friction as the tire surfaces move independently of the pavement. Because it involves sliding friction, skidding creates thermal energy and damages the tires.

Why: The expression "burning rubber" is an appropriate name for skidding during a "jackrabbit start." Substantial thermal energy is produced, and a trail of hot rubber is left on the pavement behind the car. At drag races, the frictional heating that results from skidding at the start can be so severe that the tires actually catch on fire.

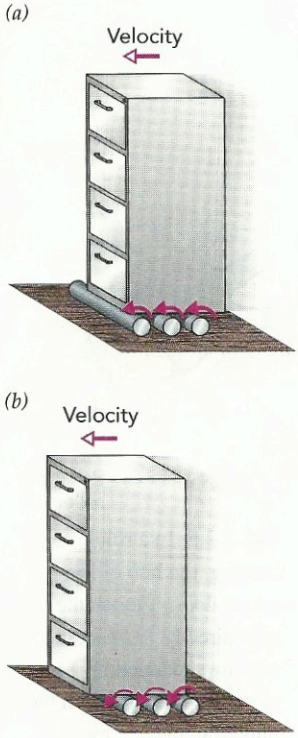


Fig. 2.2.3 (*a*) A file cabinet that's supported on turning rollers experiences only static friction. (*b*) Since the top surface of a roller moves forward with the file cabinet while its bottom surface stays behind with the sidewalk, the roller's center of mass moves only half as fast as the file cabinet. As a result, the rollers are soon left behind.

Wheels

You've wrestled your friend's file cabinet out the door of the old apartment and are now dragging it along the sidewalk. You're doing work against sliding friction the whole way, producing large amounts of thermal energy in both the bottom of the cabinet and the surface of the sidewalk. You're also damaging both objects, since sliding friction is wearing out their surfaces. The four-drawer file cabinet may be down to three drawers by the time you arrive at the new apartment.

Fortunately, there are mechanical systems that can help you move one object across another without sliding or sliding friction. The classic example is a roller (Fig. 2.2.3). If you place the file cabinet on rollers, those rollers will rotate as the file cabinet moves so that their surfaces never slide across the bottom of the cabinet or the top of the sidewalk. To see how the rollers work, make a fist with one hand and roll it across the palm of your other hand. The skin of one hand doesn't slide across the skin of the other hand; since this silent motion doesn't convert work into thermal energy, your skin remains cool. Now slide your two open palms across one another; this time, sliding friction warms your skin.

Although the rollers don't experience sliding friction, they do experience static friction. The top of each roller is touching the bottom of the cabinet, and the two surfaces

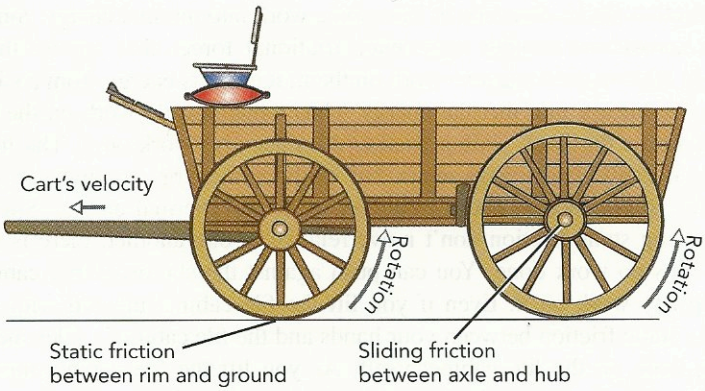


Fig. 2.2.4 As this cart moves toward the left, its wheels rotate counterclockwise. Although the wheel rims experience only static friction with the ground, the wheel hubs slide around the axles and convert the cart's kinetic energy into thermal energy. To reduce this wasted energy, the cart has narrow axles that are lubricated with axle grease.

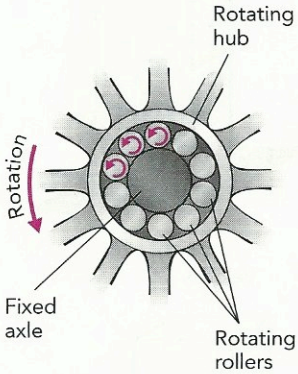


Fig. 2.2.5 In a roller bearing, the hub of the wheel doesn't touch the axle directly. Instead, the two are separated by a set of rollers that turn with the hub. The bottom few rollers bear most of the load since the hub pushes up on them and they push up on the axle. As the wheel turns, the rollers recirculate, traveling up to the right and over the top of the axle before returning down to the left to bear the load once again. The rollers, wheel, and axle experience only static friction, not sliding friction. In a ball bearing, the cylindrical rollers are replaced by spherical balls.

move along together because of static friction; they grip one another tightly until the roller's rotation pulls them apart. A similar process takes place between the rollers and the top of the sidewalk; static friction exerts torques on the rollers and hence is what makes them rotate in the first place. Again, you can illustrate this behavior with your hands. Try to drag your fist across your open palm. Just before your fist begins to slide, you'll feel a torque on it. Static friction between the skins of your two hands, acting to prevent sliding, causes your fist to begin rotating just like a roller.

Once you get the file cabinet moving on rollers, you can keep it rolling along the level sidewalk indefinitely. Without any sliding friction, the cabinet doesn't lose kinetic energy, so it continues at constant velocity without your having to push it. However, the rollers move out from under the file cabinet as it travels, and you frequently have to move a roller from the back of the cabinet to the front. In fact, you need at least three rollers to ensure that the file cabinet never falls to the ground when a roller pops out the back. Although the rollers have eliminated sliding friction, they've created another headache—one that makes the prospect of a long trip unappealing. Is there another device that can reduce sliding friction without requiring constant attention?

One alternative would be a four-wheeled cart. The simplest cart rests on fixed poles or axles that pass through central holes or hubs in the four wheels (Fig. 2.2.4). The ground exerts upward support forces on the wheels, the wheels exert upward support forces on the axles, and the axles support the cart and its contents. As the cart moves forward, its wheels turn so that their bottom surfaces don't slide or skid across the ground; instead, each wheel lowers a portion of its surface onto the sidewalk, leaves it there briefly to experience static friction, and then raises it back off the sidewalk, with a new portion of wheel surface taking its place. Because of that touch-and-release behavior, there is only static friction between the cart's wheels and the ground.

Unfortunately, as each wheel rotates, its hub slides across the stationary axle at its center. This sliding friction wastes energy and causes wear to both hub and axle. However, the narrow hub moves relatively slowly across the axle so that the work and wear done each second are small. Still, this sliding friction is undesirable and can be reduced significantly by lubricating the hub and axle with "axle grease."

A better solution is to insert rollers between the hub and axle (Fig. 2.2.5). The result is a roller bearing—a mechanical device that minimizes sliding friction between a hub and an axle. A complete bearing consists of two rings separated by rollers that keep those rings from rubbing against one another. In this case, the bearing's inner ring is attached to the

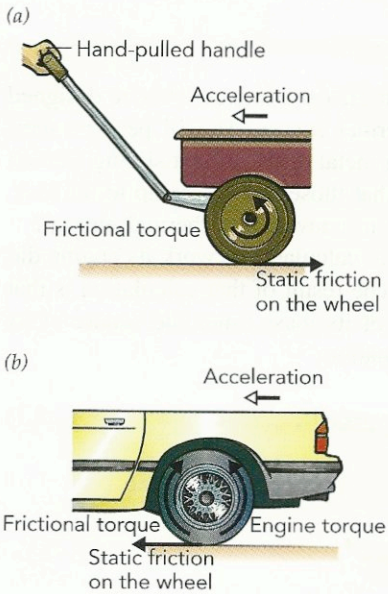


Fig. 2.2.6 (*a*) When a wagon accelerates forward, static friction from the ground exerts the torques that spin its wheels. (*b*) When a car's engine spins its wheels, static friction from the ground pushes the car forward and produces torques that oppose the wheels' rotations.

stationary axle while its outer ring is attached to the spinning wheel hub. The nondriven wheels of an automobile are supported by such bearings on essentially stationary axles. When the vehicle starts forward, static friction from the ground exerts torques on its free wheels and they begin to turn (Fig. 2.2.6*a*). A nondriven bicycle wheel is similarly supported on a stationary axle, but its bearings use balls instead of rollers—ball bearings.

A car's driven wheels are also supported by roller bearings, but these bearings act somewhat differently. Because the engine must be able to exert a torque on each driven wheel, those wheels are rigidly connected to their axles. As the engine spins these axles, the axles spin their wheels (Fig. 2.2.6*b*). A bearing prevents each spinning axle from rubbing against the car's frame. This bearing's outer ring is attached to the stationary car frame while its inner ring is attached to the spinning axle.

As a driven wheel begins to spin, it experiences static friction with the ground and the ground pushes horizontally on the wheel's bottom to keep it from skidding. Since that is the only horizontal force on the automobile, the automobile accelerates forward.

Recognizing a good idea when you think of it, you load the file cabinet into the passenger seat of your red convertible sports car and start the engine. The car isn't quite as responsive as usual because of the added mass, but it's still able to accelerate respectably. In a few seconds, you're cruising down the road toward the new apartment and a very grateful friend.

Check Your Understanding #6: Jewel Movements

Many antique mechanical watches and clocks proudly proclaim that they have "jewel movements." Gears in these timepieces turn on axles that are pointed at either end and are supported at those ends by very hard, polished gemstones. What is the advantage of having needlelike ends on an axle and supporting those needles with smooth, hard jewels?

Answer: Because all the supporting forces are very close to the axis of rotation, the jewels exert almost zero torque on the axle. The axle turns remarkably freely.

Why: Mechanical timepieces need almost ideal motion to keep accurate time. One of the best ways to allow a rotating object free movement is to support it exactly on the axis of rotation, where the support can't exert torque on the object.

Kinetic Energy

As you near your destination, you begin thinking about the car's brakes. They're designed to stop the car by turning its kinetic energy into thermal energy. They'll perform their task by rubbing stationary brake pads against spinning metal discs, so that sliding friction can transform the energy. Although you're confident that those brakes are up to the task, just how much kinetic energy are they going to have to convert into thermal energy?

One way to determine the car's kinetic energy is to calculate the work its engine did on it while bringing it from rest to its current speed. The result of that calculation is that the moving car's kinetic energy is equal to one-half of its mass times the square of its speed. This relationship can be written as a word equation:

$$\text{kinetic energy} = \tfrac{1}{2} \cdot \text{mass} \cdot \text{speed}^2, \tag{2.2.1}$$

in symbols:

$$K = \tfrac{1}{2} \cdot m \cdot v^2,$$

and in everyday language:

Racing around at twice the speed takes four times the energy.

With you and the file cabinet on board, the sports car has a mass of about 1500 kg (3300 lbm). At a speed of 100 km/h (62 mph), it has over 575,000 J of kinetic energy. That enormous energy is four times what it would be at 50 km/h (31 mph), so put down your cell phone and drive carefully. The dramatic increase in kinetic energy that results from a modest increase in speed explains why high-speed crashes are far deadlier than those at lower speeds and why that police officer is checking out your car with a radar gun. Red cars get all the attention.

You're traveling safely within the speed limit and exchange a polite wave with the officer. However, you soon pass another car that has been stopped for a ticket. The light on the nearby police car spins round and round, and rotating objects have kinetic energy, too. Like the kinetic energy of translational motion, the kinetic energy of rotational motion depends on the light's inertia and speed. For a spinning light, however, it's the rotational inertia and rotational speed that matter. The light's kinetic energy is equal to one-half of its rotational mass times the square of its angular speed. This relationship can be written as a word equation:

$$\text{kinetic energy} = \tfrac{1}{2} \cdot \text{rotational mass} \cdot \text{angular speed}^2, \tag{2.2.2}$$

in symbols:

$$K = \tfrac{1}{2} \cdot I \cdot \omega^2,$$

and in everyday language:

It takes a very energetic person to spin his wheels twice as fast.

With the ticket complete, the police car pulls out into traffic with its light still spinning. The light's total kinetic energy is now the sum of two parts: translational kinetic energy and rotational kinetic energy. Its translational kinetic energy depends on the speed

of the light's center of mass, which is equal to the police car's speed through traffic. Its rotational kinetic energy depends on the angular speed at which the light turns about its center of mass.

As the police car disappears in the distance, it occurs to you that the spinning wheels of your car also have rotational kinetic energy that adds to the car's substantial translational kinetic energy. Still, you trust your brakes. In a few minutes, you arrive at your destination and brake to a stop. Although you're aware of the added mass as the car decelerates less quickly than usual, the brakes successfully transform the car's kinetic energy into thermal energy. You've reached your goal safely and are now a hero.

Check Your Understanding #7: **Throwing a Fastball**

A typical grade-school pitcher can throw a baseball at 80 km/h (50 mph), but only a few professional athletes have the extraordinary strength needed to throw a baseball at twice that speed. Why is it so much harder to throw the baseball only twice as fast?

Answer: Doubling the speed of the baseball requires quadrupling the energy transferred to it by the pitcher.

Why: To throw a 160-km/h (100-mph) fastball, a major league pitcher must put four times as much kinetic energy into both the ball and his arm as when pitching an 80-km/h slow ball. He also pitches the fastball in half the time needed to pitch the slow ball. Overall, he must do four times as much work throwing a fastball and he must do that work in one-half the time. That means the pitcher produces eight times as much power while throwing a fastball as when throwing a slow ball. No wonder amateurs have trouble duplicating that feat.

Check Your Figures #1: **Blowing in the Wind**

The air in a hurricane travels at 200 km/h (124 mph). How much more kinetic energy does 1 kg of this air have than 1 kg of air moving at only 20 km/h?

Answer: It has 100 times as much kinetic energy.

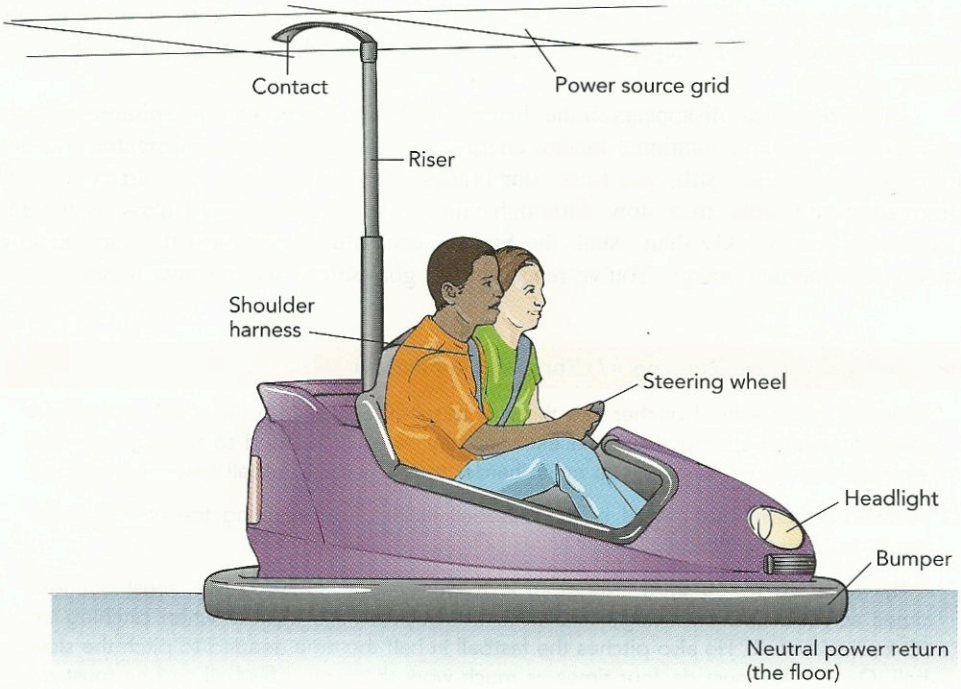
Why: Because kinetic energy is proportional to speed squared, the kilogram of air in the hurricane moves 10 times as fast but has 100 times as much kinetic energy as the slower moving air. This enormous increase in energy is what makes a hurricane's wind dangerous. The air's terrific speed also brings large quantities of it to you quickly, so that the wind power arriving each second is overwhelming.

Check Your Figures #2: **Playing Around at the Playground**

When children climb onto a playground merry-go-round, they increase its rotational inertia. If the children triple the merry-go-round's rotational mass, how will they alter the kinetic energy it has when it spins at a certain angular speed?

Answer: The children will triple the merry-go-round's kinetic energy.

Why: The kinetic energy of a spinning object is proportional to its rotational mass. Since the children triple the merry-go-round's rotational mass, they triple its kinetic energy.



SECTION 2.3 Bumper Cars

While car crashes normally aren't much fun outside of movies or television, there is one delightful exception: bumper cars. For a few minutes, drivers in this amusement park ride race madly about an oval track, deliberately crashing their vehicles into one another and laughing hysterically at the violent impacts. Jolts, jerks, and spins are half the fun, and it's a wonder that no one gets whiplash. Hidden in the excitement, though, are several important physics concepts that influence everything from tennis to billiards.

Questions to Think About: *Why does a stationary car begin rolling forward after being struck by a moving car? What aspects of motion are passed between cars as they collide? Why does your car jolt more when the car that hits it contains two big adults rather than one small child? What would happen if the bumper cars had hard steel bumpers rather than soft rubber ones? Why is your car often set spinning by collisions, and what keeps it spinning?*

Experiments to Do: *Place a coin on a smooth table and flick a second, identical coin so that it slides along the table and strikes the stationary coin squarely. What happens? Try this experiment again, but now use two coins with different masses. How is the collision different? Does it matter which coin you crash into the other?*

Now line up several identical coins so that they touch, and slide another coin into one end of this line. How does the collision affect the coin that was originally moving? How does it affect the line of coins? What was transferred among the coins by the collision?

Now stand a coin on its edge and flick it so that it spins rapidly. Did you give it something that keeps it spinning? Why does the coin eventually stop spinning?

Coasting Forward: Linear Momentum

Bumper cars are small, electrically powered vehicles that can turn on a dime and are protected on all sides by rubber bumpers. Each car has only two controls: a pedal that activates its motor and a steering wheel that controls the direction in which the motor pushes the car. Since the car itself is so small, its occupants account for much of the car's total mass and rotational mass.

Imagine that you have just sat down in one of these cars and put on your safety strap. The other people also climb into their cars, usually one person per car, and the ride begins.

With your car free to move or turn, you quickly become aware of its translational and rotational inertias. The car's translational inertia makes it hard to start or stop, and its rotational inertia makes it difficult to spin or stop from spinning. While we've seen these two types of inertia before, let's take another look at them and at how they affect your bumper car. This time, we'll see that they're associated with two new conserved quantities—linear momentum and angular momentum. As promised, energy isn't the only conserved quantity in nature!

When fast-moving bumper cars crash into one another, they exchange more than just energy. Energy is directionless—it's not a vector quantity—yet these cars seem to be exchanging some quantity of motion that has a direction associated with it. For example, if your car is hit squarely by a rightward moving car, then your car's motion shifts somewhat rightward in response. Your car is receiving a vector quantity of motion from the other car, a conserved vector quantity known as linear momentum.

Linear momentum, usually just called **momentum**, is the measure of an object's translational motion—its tendency to continue moving in a particular direction. Roughly speaking, your car's momentum indicates which way it's heading and just how difficult it was to get the car moving with its current velocity.

The car's momentum is its mass times its velocity and can be written as a word equation:

$$\text{momentum} = \text{mass} \cdot \text{velocity}, \tag{2.3.1}$$

in symbols:

$$\mathbf{p} = m \cdot \mathbf{v},$$

and in everyday language:

It's hard to stop a fast-moving truck.

Note that momentum is a vector quantity and that it has the same direction as the velocity. As we might expect, the faster your car is moving or the more mass it has, the more momentum it has in the direction of its motion. The SI unit of momentum is the **kilogram-meter-per-second** (abbreviated kg · m/s).

To physicists, conserved quantities are rare treasures that make it easier to understand otherwise complicated motions. Like all conserved quantities, momentum can't be created or destroyed. It can only be transferred between objects. Momentum plays a very basic role in bumper cars: the whole point of crashing them into one another is to enjoy

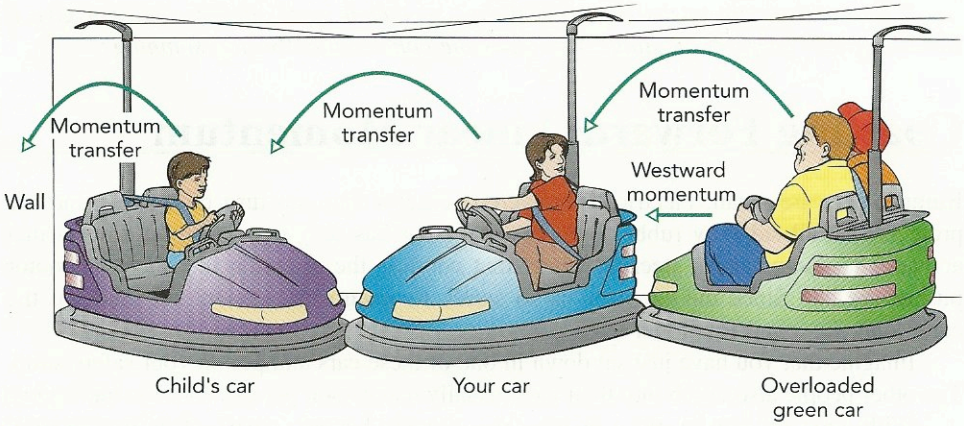


Fig. 2.3.1 Your car is hit by a fast-moving, massive car with westward momentum. Much of that westward momentum is transferred to your car. You crash into a child's car, transferring the westward momentum to it. It then crashes into the wall, transferring the westward momentum to the wall. Although that wall doesn't appear to move, it and the earth to which it's attached actually accelerate westward a tiny amount as they receive the westward momentum.

the momentum transfers. During each collision, momentum shifts from one car to the other so that they abruptly change their speeds or directions or both. As long as these momentum transfers aren't too jarring, everyone has a good time.

You've stopped your car, so it has zero velocity and zero momentum. To begin moving again, something must transfer momentum to your car. While you could press the pedal and let the motor gradually transfer momentum from the ground to your car, that's not much fun. Instead, you let two grinning couch potatoes in an overloaded green car slam into you at breakneck speed (Fig. 2.3.1).

The green car was heading westward, and in a few moments your car is moving westward, too, while the green car has slowed significantly. Before you recover from the jolt, your car pounds a child's car westward and your car slows down abruptly. Finally, its impact with a wall stops the child's car. Despite disapproving looks from the child's parents, there's no harm done. Overall, westward momentum has flowed from the spudmobile to your car, to the child's car, and into the wall. No momentum has been created or destroyed; you've all simply enjoyed passing it along from car to car.

Check Your Understanding #1: **Stuck on the Ice**

Suppose you're stuck in the middle of a frozen lake, with a surface so slippery that you can't get any traction. You take off a shoe and throw it toward the southern shore. You find yourself coasting toward the northern shore and soon escape from the lake. Why did this scheme work?

Answer: By transferring southward momentum to the shoe, you obtained northward momentum.

Why: Initially, both you and your shoe have zero momentum. But when you throw the shoe southward, you give it southward momentum. Since the only source of that southward momentum is you, you must have lost southward momentum. A negative amount of southward momentum is actually northward momentum, and thus you coast northward. Interestingly enough, the total momentum of you and the shoe hasn't changed. It's still zero, as it must be because momentum is conserved. It has simply been redistributed.

Check Your Figures #1: Follow That Train!

The bad guys are getting away in a four-car train and you're trying to catch them. The train has a mass of 20,000 kg and it's rolling forward at 22 m/s (80 km/h or 50 mph). What is the train's momentum?

Answer: The train's momentum is 440,000 kg · m/s, in the forward direction.

Why: You can use Eq. 2.3.1 to calculate the train's momentum from its mass and velocity:

$$\text{linear momentum} = 20{,}000\,\text{kg} \cdot 22\,\text{m/s}$$
$$= 440{,}000\,\text{kg} \cdot \text{m/s}$$

That momentum is in the same direction the train is moving: the forward direction.

Exchanging Momentum in a Collision: Impulses

Momentum is transferred to a car by giving it an **impulse**—that is, a force exerted on it for a certain amount of time. When the motor and floor push your bumper car forward for a few seconds, they give your car an impulse and transfer momentum to it. This impulse is the change in your car's momentum and is equal to the product of the force exerted on the car times the duration of that force. This relationship can be written as a word equation:

$$\text{impulse} = \text{force} \cdot \text{time}, \tag{2.3.2}$$

in symbols:

$$\Delta \mathbf{p} = \mathbf{F} \cdot t,$$

and in everyday language:

The harder and longer you push a bobsled forward at the start of a race, the more momentum it will have when it starts down the hill.

The more force or the longer that force is exerted, the larger the impulse and the more your bumper car's momentum changes. Remember that an impulse, like momentum itself, is a vector quantity and has a direction. If your aim is off and the misdirected impulse you obtain from the floor sends you crashing into the wall, don't say you hadn't been warned!

Different forces exerted for different amounts of time can transfer the same momentum to a car:

$$\text{impulse} = \text{large force} \cdot \text{short time}$$
$$= \text{small force} \cdot \text{long time}. \tag{2.3.3}$$

Thus you can get your car moving with a certain forward momentum either by letting the motor and floor push it with a small forward force of long duration or by letting the colliding green car push on it with a large forward force of short duration.

We can now explain why bumper cars have soft rubber bumpers. If the bumpers were hard steel, the collision between the green car and your car would last only an instant and would involve an enormous forward force. You'd be in need of a neck brace

and the services of a personal injury lawyer. However, amusement parks don't like lawsuits and sensibly limit the car impact forces. To do this, they use rubber bumpers and rather slow-moving cars.

Nonetheless, you can get a pretty good jolt when you collide head-on with another car. Your two cars then start with oppositely directed momenta, and the collision roughly exchanges those momenta between cars. In almost no time, you go from heading forward to heading backward. The impulse that causes this reversal of motion is especially large because it not only stops your forward motion, it also causes you to begin heading backward.

Why should momentum be a conserved quantity? It's conserved because of Newton's third law of motion. When one car exerts a force on a second car for a certain amount of time, the second car exerts an equal but oppositely directed force on the first car for exactly the same time. Because of the equal but oppositely directed nature of the two forces, cars that push on one another receive impulses that are equal in amount but opposite in direction. Since the momentum gained by one car is exactly equal to the momentum lost by the other car, we say that momentum is transferred from one car to the other.

The more mass a car has, the less its velocity changes as a consequence of a momentum transfer. That's why the green car doesn't stop completely when it crashes into your car, whereas your car speeds up dramatically. Just a fraction of the green car's forward momentum causes a large change in your car's velocity. Like a bug being hit by an automobile windshield, your bumper car does most of the accelerating.

Conserved Quantity: Momentum **Transferred By: Impulse**

Momentum: The measure of an object's translational motion—its tendency to continue moving in a particular direction. Momentum is a vector quantity, meaning that it has a direction. It has no potential form and therefore cannot be hidden; momentum = mass · velocity.

Impulse: The mechanical means for transferring momentum; impulse = force · time.

Common Misconceptions: Momentum and Force

Misconception: A massive, moving object carries a force with it—the "force of its momentum."

Resolution: Although the impulses that transfer momentum involve forces, momentum itself does not. A moving object carries only momentum, not force. Most important, a coasting object is free of any net force. However, when that object hits an obstacle, the two exchange momentum via impulses and those impulses do involve forces.

Check Your Understanding #2: Bowling Them Over

When a beanbag hits the wall, it transfers all of its forward momentum to the wall and comes to a stop. When a rubber ball hits the wall, it transfers all of its forward momentum, comes to a stop, and then rebounds. During the rebound it transfers still more forward momentum to the wall. If you wanted to knock over a weighted bowling pin at the county fair, which would be the more effective projectile: the rubber ball or the beanbag, assuming they have identical masses and you throw them with identical velocities?

Answer: The bouncy rubber ball would be more effective.

Why: Either projectile will transfer all of its original momentum to the bowling pin while coming to a stop. However, the bouncy rubber ball will bounce back and continue to exert a force on the bowling pin. The impulse (force · time) delivered by the rubber ball will be greater than that delivered by the beanbag because the ball will exert its forward force for a longer time (during stopping *and* rebounding). The ball will rebound with its momentum reversed, having transferred roughly twice its original momentum to the pin.

Check Your Figures #2: **Stop That Train!**

The engine of the train you're chasing (see Check Your Figures #1) has broken down, but it's still rolling forward. To stop it, you grab onto the last car and begin to drag your boot heels on the ground. The backward force on the train is 200 N. How long will it take you to stop the train?

Answer: It will take 2200 s, so kiss your boot heels goodbye!

Why: To stop the train, you must give it a backward impulse that completely cancels its forward momentum. Since its forward momentum is 440,000 kg · m/s, the backward impulse must be 440,000 kg · m/s. Since 200 N can also be written as 200 kg · m/s^2, we can use Eq. 2.3.2 to find the time:

$$\text{time} = \frac{440{,}000 \text{ kg} \cdot \text{m/s}}{200 \text{ kg} \cdot \text{m/s}^2} = 2200 \text{ s.}$$

Spinning in Circles: Angular Momentum

When bumper cars are set spinning during crashes, they are exchanging yet another conserved quantity. Like momentum, it's a conserved vector quantity, but now it's associated with the angular speed and direction of rotational motion around a specific pivot. For example, when your car receives a glancing blow from a car that is circling you clockwise, your car's rotational motion or spin shifts somewhat clockwise in response. Your car is receiving a vector quantity of motion from the other car, a conserved vector quantity known as angular momentum.

Angular momentum is the measure of an object's rotational motion—its tendency to continue spinning about a particular axis. Simply put, your car's angular momentum indicates the direction of its rotation and just how difficult it was to get it spinning with its current angular velocity. The car's angular momentum is its rotational mass times its angular velocity and can be written as a word equation:

$$\text{angular momentum} = \text{rotational mass} \cdot \text{angular velocity}, \qquad (2.3.4)$$

in symbols:

$$\mathbf{L} = I \cdot \omega,$$

and in everyday language:

It's hard to stop a spinning carousel.

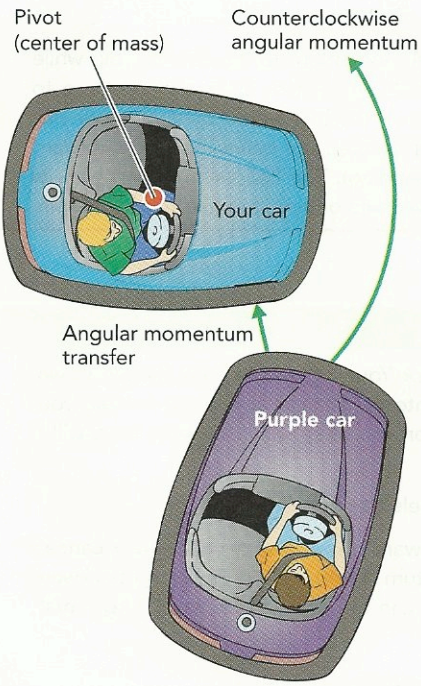


Fig. 2.3.2 Since the purple car is circling your car counterclockwise, it has counterclockwise angular momentum. When it hits your car, it transfers some of that angular momentum to your car. Because of this transfer, the purple car stops circling quickly as your car begins to spin counterclockwise.

Note that angular momentum is a vector quantity and that it has the same direction as the angular velocity. The faster your car is spinning or the larger its rotational mass, the more angular momentum it has in the direction of its angular velocity. The SI unit of angular momentum is the **kilogram-meter^2-per-second** (abbreviated kg · m^2/s).

Angular momentum is another conserved quantity; it can't be created or destroyed, only transferred between objects. For your car to begin spinning, something must transfer angular momentum to it, and your car will then continue to spin until it transfers this angular momentum elsewhere. To study angular momentum properly, however, we must pick the pivot about which all the spinning will occur. In the present situation, a good choice for this pivot is your car's initial center of mass.

Your bumper car is stationary again, so it has zero angular velocity and zero angular momentum. Suddenly, a purple car sweeps by and strikes your car a glancing blow (Fig. 2.3.2). Because the purple car was circling your car counterclockwise, it had counterclockwise angular momentum about the pivot. Its impact transfers some of this angular momentum to your car, which begins spinning counterclockwise itself. Since it has given up some of its angular momentum, the purple car circles your car less rapidly. Your car gradually stops spinning as its wheels and friction transfer the angular momentum to the ground and earth. Overall, no angular momentum was created or destroyed during the collision. Instead, it was transferred from the purple car to your car to the earth.

Check Your Understanding #3: Many Happy Re-Turns

Satellites are often set spinning during launch in order to give them added stability. When astronauts visit these satellites years later, they find them still spinning. Why don't the satellites stop spinning?

Answer: The satellites are unable to get rid of their angular momentum.

Why: Because of their extreme isolation, orbiting satellites have nothing with which to exchange angular momentum. The angular momentum given to them at launch stays with them indefinitely, so they continue to spin for decades.

Check Your Figures #3: Want to Go for a Spin?

Spinning satellites are particularly stable. Suppose that the astronauts launching a particular satellite decide to increase its angular velocity by a factor of 5. How will that change affect the satellite's angular momentum?

Answer: The angular momentum will increase by a factor of 5.

Why: Because the satellite's angular momentum is proportional to its angular velocity, spinning it five times faster will increase its angular momentum by that same factor.

Glancing Blows: Angular Impulses

Angular momentum is transferred to a bumper car by giving it an **angular impulse**—that is, a torque exerted on it for a certain amount of time. When the purple car hits your car and exerts a torque on it briefly, it gives your car an angular impulse and transfers angular momentum to it. This angular impulse is the change in your car's angular momentum and is equal to the product of the torque exerted on your car times the duration of that torque. This relationship can be written as a word equation:

$$\text{angular impulse} = \text{torque} \cdot \text{time}, \tag{2.3.5}$$

in symbols:

$$\Delta L = \tau \cdot t,$$

and in everyday language:

To get a merry-go-round spinning rapidly, you must twist it hard and for a long time.

The more torque or the longer that torque is exerted, the larger the angular impulse and the more your car's angular momentum changes. Once again, an angular impulse is a vector quantity and has a direction. Had the purple car been circling your car clockwise when it struck the glancing blow, its angular impulse would have been in the opposite direction and you'd be spinning the other way.

Different torques exerted for different amounts of time can transfer the same angular momentum to a car:

$$\begin{aligned}\text{angular impulse} &= \text{large torque} \cdot \text{short time}\\ &= \text{small torque} \cdot \text{long time}.\end{aligned} \tag{2.3.6}$$

Thus you can get your car spinning with a certain angular momentum either by letting the motor and floor twist it with a small torque of long duration or by letting the colliding purple car twist it with a large torque of short duration. As with linear momentum, sudden transfers of angular momentum can break things, so the cars are designed to limit their impact torques to reasonable levels. Even so, you may find yourself reaching for the motion sickness bag after a few spinning collisions.

Why should angular momentum be a conserved quantity? Like linear momentum, angular momentum is conserved because of Newton's third law of rotational motion. When one car exerts a torque on a second car for a certain amount of time, the second car exerts an equal but oppositely directed torque on the first car for exactly the same

Steven E. Sutton/Duomo Photography, Inc.

Fig. 2.3.3 When Kristi Yamaguchi pulls in her arms, she reduces her rotational mass. Since she is experiencing zero net torque, her angular momentum must remain constant and she begins to spin more rapidly.

amount of time. Because of the equal but oppositely directed nature of the two torques, cars that exert torques on one another receive angular impulses that are equal in amount but opposite in direction. Since the angular momentum gained by one car is exactly equal to the angular momentum lost by the other car, we say that angular momentum is transferred from one car to the other.

Because a car's angular momentum depends on its rotational mass, two different cars may end up rotating at different angular velocities even though they have identical angular momenta. For example, when the purple car hits the overloaded green car and transfers angular momentum to it, the green car's enormous rotational mass makes it spin relatively slowly. The same sort of behavior occurs with linear momentum, where a car's mass affects how fast it travels when it's given a certain amount of linear momentum. But while a bumper car can't change its mass, it can change its rotational mass. If it does so while it's spinning, its angular *momentum* won't change, but its angular *velocity* will!

To see this change in angular velocity, consider the overloaded green car. Its two large occupants are disappointed with the ride because their huge mass and rotational mass prevent them from experiencing the intense jolts and spins that you've been enjoying. Suddenly they get a wonderful idea. As their car slowly spins, one of them climbs into the other's lap and the two sit very close to the car's center of mass. By rearranging the car's mass this way, they have reduced the car's overall rotational mass and the car actually begins to spin faster than before.

As the green car's mass is being redistributed, it's not a freely turning rigid object covered by Newton's first law of rotational motion. However, it is freely turning and thus covered by a more general and equally useful rule: an object that is not subject to any outside torques has constant angular momentum. As the car's rotational mass becomes smaller, its angular velocity must increase in order to keep its angular momentum constant. That's just what happens. This effect of changing one's rotational mass explains how an ice skater can achieve an enormous angular velocity by pulling herself into a thin, spinning object on ice (Fig. 2.3.3).

Conserved Quantity: Angular Momentum **Transferred By: Angular Impulse**

Angular Momentum: The measure of an object's rotational motion—its tendency to continue spinning about a particular axis. Angular momentum is a vector quantity, meaning that it has a direction. It has no potential form and therefore cannot be hidden; angular momentum = rotational mass · angular velocity.

Angular Impulse: The mechanical means for transferring angular momentum; angular impulse = torque · time.

Check Your Understanding #4: Spinning the Merry-Go-Round

A person who is initially motionless starts a merry-go-round spinning and then returns to being motionless. If angular momentum is truly conserved, what is the source of the angular momentum that the spinning merry-go-round now has?

Answer: It came from the entire earth.

Why: Because the person stood on the earth as he started the merry-go-round spinning, he transferred angular momentum from the earth to the merry-go-round. The merry-go-round spins in one direction, and the earth's rotation changes ever so slightly in the other direction. Because the earth is so huge and has such an enormous rotational mass, its slight change in rotation is undetectable.

Check Your Figures #4: Spin Away!

How much longer will it take the astronauts launching the satellite in Check Your Figures #3 to bring it to the faster angular velocity, if they use the initially planned torque?

Answer: It will take them five times as long.

Why: To reach the new, faster angular velocity, the astronauts will need an angular impulse that's five times as large as originally planned. Since they will be using the same torque, they will have to exert that torque for five times as long.

The Three Conserved Quantities

As you drive your bumper car around the oval track, its motion is governed in large part by three conserved quantities: energy, linear momentum, and angular momentum (Table 2.3.1). While you can exchange those quantities with the earth and the power company by steering your car or switching on its motor, most of the interesting exchanges involve collisions.

Each time your car shoves another car forward, your car does work on that other car and transfers energy to it. Each time your car pushes another car northward briefly, your car gives a northward impulse to that other car and transfers northward momentum to it. And each time your car twists another car clockwise about its center of mass, your car gives a clockwise angular impulse to the other car and transfers clockwise angular momentum to it. These exchanges of energy, momentum, and angular momentum are fast and furious and make for an exciting ride.

Quantity	*SI Unit*	*English Unit*	*SI → English*	*English → SI*
Momentum	kilogram-meter-per-second (kg · m/s)	pound-foot-per-second (lbm · ft/s)	1 kg·m/s = 7.2329 lbm · ft/s	1 lbm·ft/s = 0.13826 kg · m/s
Angular momentum	kilogram-meter^2-per-second (kg · m^2/s)	pound-foot^2-per-second (lbm · ft^2/s)	1 kg·m^2/s = 23.730 lbm · ft^2/s	1 lbm · ft^2/s = 0.042140 kg · m^2/s

Check Your Understanding #5: Hitting the Wall

You're backing out of a parking space and accidentally hit a concrete wall. The wall doesn't move and your car sustains some damage. Did your car transfer any energy or momentum to the wall?

Answer: Your car transferred momentum but no energy.

Why: To transfer momentum to the wall, your car must give it an impulse: it must push on the wall for an amount of time. It did so and thus transferred all its backward momentum to the wall. But to transfer energy to the wall, your car must do work on it: it must push on the wall as the wall moves in the direction of that push. The wall is immobile, however, so your car couldn't do work on it. Instead, the car's energy stayed in the car, where it caused damage.

Table 2.3.1 The Three Conserved Quantities of Motion and Their Transfer Mechanisms

CONSERVED QUANTITY	TRANSFER MECHANISM
Energy	Work
Linear momentum	Impulse
Angular momentum	Angular impulse

Potential Energy, Acceleration, and Force

Shortly before the bumper car ride stops, you notice that there is a low point in the floor. After years of use, its metal surface has dented into a bowl-shaped depression, and you observe that cars naturally tend to roll into this bowl and accelerate toward its bottom. As each car tries to coast through the depression, it accelerates downhill and its velocity changes as a result. While some cars pass directly through the bowl's bottom, accelerating forward during their descents and backward during their subsequent rises, most cars encounter the bowl at an angle and have their paths bent by the accelerations they experience. Each deflected car emerges from the bowl traveling in a somewhat altered direction.

Common Misconceptions: Acceleration and Velocity

Misconception: An object's acceleration and velocity always point along the same line.

Resolution: Although acceleration measures how quickly velocity is changing, those two quantities are independent at any given moment and may or may not point along the same line. When they are not aligned, the direction of the object's velocity will shift over time toward the direction of its acceleration and the object's path will bend.

We've seen this tendency to accelerate downhill before with ramps, but now let's look at it in terms of energy: a car always accelerates in the direction that reduces its total potential energy as quickly as possible. Since a lone car's only potential energy is gravitational potential energy, it accelerates so as to reduce its gravitational potential energy as quickly as possible: down the steepest route to the bottom of the bowl.

This behavior of accelerating in the direction that reduces total potential energy as quickly as possible is universal. Potential energy and forces are intimately related, and the direction of quickest total potential energy decrease *is* the direction of the net force and therefore of acceleration.

We can explore the origins of this relationship by considering what happens when you push your bumper car steadily up a smooth, frictionless ramp. To keep that car moving at a constant velocity, the force you exert on it must point directly uphill in the direction of the steepest slope. If you accidentally push the car to the side, for example, it will accelerate sideways.

You are doing work on the car and, according to Eq. 1.3.1, your work is the product of the force you exert on the car times the distance the car travels in the direction of your force:

$$\text{work you do on car} = \text{force you exert on car} \cdot \text{distance traveled by car.}$$

Since your force is uphill along the line of steepest slope, the work you do depends only on the car's motion along that direction. We can divide both sides of the equation by the distance the car travels along that direction and obtain:

$$\frac{\text{work you do on car}}{\text{distance traveled by car up steepest slope}} = \text{force you exert on car,}$$

where your force on the car points *uphill* along the ramp's steepest slope.

Since the car moves steadily and without friction, all of your work becomes gravitational potential energy of the car. And because the car isn't accelerating as you push it uphill, it must be experiencing a second force of equal magnitude in the downhill direction—a force due to its potential energy. Incorporating those two observations into the previous equation, it becomes:

$$-\frac{\text{increase in car's potential energy}}{\text{distance traveled by car up steepest slope}} = \text{force on car due to its potential energy,}$$

where the force on the car due to its potential energy points *downhill* along the ramp's steepest slope (hence the minus sign).

If you stop pushing on the car, this force due to its potential energy will cause the car to accelerate downhill along the steepest slope. Moreover, the car's acceleration will be proportional to how quickly its potential energy would increase if it were to travel up that slope—a measure of the ramp's steepness. We saw exactly this behavior in Section 1.3 on ramps, but this time we're attributing the downhill force on the car to its gravitational potential energy. The story hasn't changed; we're just interpreting it in a different way.

This new interpretation applies to all objects and all forms of potential energy. Whenever an object can reduce its total potential energy by moving, it will experience a force in the direction that reduces its total potential energy as quickly as possible. The force is equal but opposite to how quickly its potential energy would increase if it were to move toward increasing potential energy along the steepest slope—a quantity known as the **potential gradient**. We can write this relationship as a word equation,

$$\text{force} = -\text{potential gradient} = -\frac{\text{increase in potential energy}}{\text{distance up steepest slope}}, \tag{2.3.7}$$

in symbols:

$$\mathbf{F} = -\mathbf{Gradient}(U),$$

and in everyday language:

An object accelerates in the direction that reduces its total potential energy as quickly as possible

where the force points in the direction of most rapid potential energy decrease. A similar rule applies for rotational motion and torques. These rules are a useful way to determine how motion will proceed: which way a spring will leap, a chair will tip, or a bumper car will roll. We'll use them frequently in this book.

Potential Energy and Acceleration
An object accelerates in the direction that reduces its total potential energy as quickly as possible.

Check Your Understanding #6: Heading Down

When you pull a child back on a playground swing and let go, which way does that child accelerate?

Answer: The child accelerates forward, in the direction that will reduce the child's potential energy as quickly as possible.

Why: The child has only one form of potential energy—gravitational potential energy. This gravitational potential energy is lowest when the child is directly below the swing's supporting bar. The child accelerates forward because that will put the child below the support as quickly as possible.

Check Your Figures #5: Pumpkin Chucking

You've constructed a catapult for a pumpkin chucking contest. When you let go of the basket holding the pumpkin, the catapult will swing that basket forward and release 10,000 J of potential energy as it travels 1 m forward. What force must you exert on the basket to hold it in place prior to letting go?

Answer: You must exert 10,000 N on the basket in the backward direction.

Why: When you let go, the basket will accelerate forward—the direction that reduces its potential energy as quickly as possible. According to Eq. 2.3.7, since the basket acquires 10,000 J in traveling backward 1 m, the forward force acting on the basket is:

$$-\frac{10{,}000\ \text{J}}{-1\ \text{m}} = 10{,}000\ \text{N}.$$

To keep the basket stationary, you must exert a force of 10,000 N in the backward direction.

Epilogue for Chapter 2

In this chapter we looked at rotating and colliding objects and studied the physical laws that describe their motions. In *wind turbines*, we examined rotational inertia and saw how torques cause angular accelerations. We also noticed how useful it can be to separate an object's rotational motion from its translational motion. In *wheels*, we discussed another important type of force, friction, as well as a new type of energy, thermal energy—the energy associated with heat and temperature. In *bumper cars*, we introduced two more conserved physical quantities: momentum and angular momentum. As we'll see, following the flows of energy, momentum, and angular momentum between objects often helps in understanding how those objects work.

Explanation: A Spinning Pinwheel

The paper square balances when the pin passes directly through its center of gravity. With the square's weight acting exactly at the pivot, gravity exerts no torque on the square and the torque-free paper square obeys Newton's first law of rotational motion. It can remain stationary at any orientation and it can spin at a constant angular velocity.

Because the balanced paper square has rotational inertia, torques are required both to start it spinning and to stop it from doing so. When you twist the square with your hand, the torque you exert gives the square an angular impulse and sets it spinning with a certain amount of angular momentum. If the pivot were truly frictionless and there were no air resistance, the square would spin indefinitely because it would be unable to get rid of its angular momentum. However, air resistance and friction in the pivot exert small but significant torques that oppose the square's motion and gradually slow it down. As these opposing torques transfer angular momentum out of the square and into the air and the pencil, the square turns more and more slowly until it finally comes to a stop.

When the square has become a pinwheel, it can interact with a passing wind. Its paper blades deflect the wind to one side by exerting forces on the moving air. The air pushes back on the blades. Because the air's force on each blade is exerted perpendicular to a lever arm, it produces a torque on the blade and the pinwheel. All four blades experience torques in the same direction and the pinwheel undergoes angular acceleration in that direction.

The turning pinwheel experiences significant air resistance, so it reaches a maximum rotational speed. At that speed, the torque produced by your breath acting on the blades exactly cancelled by an opposing torque produced by air resistance. When you stop blowing, that air resistance torque quickly slows the pinwheel to a stop.

Chapter Summary

How Wind Turbines Work: A wind turbine is a rotating device that obtains mechanical power from the wind and conveys that power to an electric generator. The turbine's blades are actually wings, and each blade deflects the passing wind by pushing it to the side. The wind pushes back on the blade, producing a torque on the blade about its pivot. The wind torques on the individual blades act together, twisting the turbine and doing work on it as it rotates. The turbine is motionless at first, and this overall torque causes it to begin spinning faster and faster. Once the turbine has reached operating speed, however, it starts to power the generator and its speed stops increasing. The turbine is twisting the generator and doing work on it as it rotates and the generator converts that work into electric energy.

A wind turbine functions best when it's perfectly balanced, meaning that it experiences zero gravitational torque. To ensure that it's balanced, the wind turbine is built with its center of gravity located at its pivot. Since the turbine's weight then acts at the pivot, it produces no torque on the turbine and the turbine can rotate freely.

How Wheels Work: Wheels facilitate motion by eliminating or reducing sliding friction between an object and a surface. The wheels convey the support forces needed to hold

the object up but allow the object to move without sliding. As a cart with freely turning wheels moves along the surface, static friction between each wheel and the surface exerts a torque on that wheel and causes it to turn. However, rubbing may occur between the wheel's hub and the axle, where sliding friction can waste energy and cause wear. To eliminate this sliding friction, roller or ball bearings are often used.

The torque that causes a powered wheel on a vehicle to turn comes from an engine by way of an axle. In this case, static friction between the outside of the wheel and the ground exerts a torque on the wheel that opposes the torque from the engine. This static frictional force also contributes to the net force on the vehicle and causes it to accelerate.

Once supported on wheels and bearings, objects can move freely and can retain linear momentum, angular momentum, and energy for long periods of time. By eliminating sliding friction, wheels can also keep objects from converting ordered energy into thermal energy. Wheels allow vehicles to hold onto these conserved quantities for extended periods and make transportation far more practical.

How Bumper Cars Work: Since they start from rest, bumper cars must obtain their initial momenta and angular momenta from the ground and their initial kinetic energies from the power company. They do this with the help of motors and wheels, which gradually transfer energy, momentum, and angular momentum into the cars.

Once the cars are moving, they can begin to exchange those conserved quantities by way of collisions. Each impact usually changes the cars' speeds and directions of travel in a manner that may seem rather complicated. However, following the exchanges of momentum, angular momentum, and energy often makes it easier to understand these collisions.

Cars containing massive riders respond weakly when collisions transfer momentum and angular momentum to them. That's because their large masses and rotational masses minimize their changes in velocity and angular velocity. Because of their small masses, children experience the wildest rides.

Important Laws and Equations

1. Newton's First Law of Rotational Motion: A rigid object that is not wobbling and is not subject to any outside torques rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation.

2. Newton's Second Law of Rotational Motion: The net torque exerted on an object is equal to the product of that object's rotational mass times its angular acceleration, or

$$\text{net torque} = \text{rotational mass} \cdot \text{angular acceleration}. \quad (2.1.2)$$

The angular acceleration points in the same direction as the torque. This law doesn't apply to objects that are wobbling.

3. Relationship between Force and Torque: The torque produced by a force is equal to the product of the lever arm times the component of that force perpendicular to the lever arm, or

$$\text{torque} = \text{lever arm} \cdot \text{force perpendicular to lever arm}. \quad (2.1.3)$$

4. The Rotational Definition of Work: The work done on an object is equal to the product of the torque exerted on that object times the angle (in radians) through which that object rotates in the direction of the torque, or

$$\text{work} = \text{torque} \cdot \text{angle (in radians)} \quad (2.1.4)$$

5. Kinetic Energy: An object's translational kinetic energy is one-half of its mass times the square of its speed, or

$$\text{kinetic energy} = \tfrac{1}{2} \cdot \text{mass} \cdot \text{speed}^2. \quad (2.2.1)$$

An object's rotational kinetic energy is one-half of its rotational mass times the square of its angular speed, or

$$\text{kinetic energy} = \tfrac{1}{2} \cdot \text{rotational mass} \cdot \text{angular speed}^2. \quad (2.2.2)$$

6. Linear Momentum: An object's linear momentum is its mass times its velocity, or

$$\text{linear momentum} = \text{mass} \cdot \text{velocity}. \quad (2.3.1)$$

7. The Definition of Impulse: The impulse given to an object is equal to the product of the force exerted on that object times the length of time that force is exerted, or

$$\text{impulse} = \text{force} \cdot \text{time}. \tag{2.3.2}$$

8. Angular Momentum: An object's angular momentum is its rotational mass times its angular velocity, or

$$\text{angular momentum} = \text{rotational mass} \cdot \text{angular velocity}. \tag{2.3.4}$$

9. The Definition of Angular Impulse: The angular impulse given to an object is equal to the product of the torque exerted on that object times the length of time that torque is exerted, or

$$\text{angular impulse} = \text{torque} \cdot \text{time}. \tag{2.3.5}$$

10. Newton's Third Law of Rotational Motion: For every torque that one object exerts on a second object, there is an equal but oppositely directed torque that the second object exerts on the first object.

11. Potential Energy, Acceleration, and Force: An object experiences a force and accelerates in the direction that reduces its total potential energy as quickly as possible. That force is equal but opposite to how quickly its potential energy would increase if it were to move toward increasing potential energy along the steepest slope, or

$$\text{force} = -\text{potential gradient} = -\frac{\text{increase in potential energy}}{\text{distance up steepest slope}}. \tag{2.3.7}$$

Exercises

1. The chairs in an auditorium aren't all facing the same direction. How could you describe their angular positions in terms of a reference orientation and a rotation?

2. When an airplane starts its propellers, they spin slowly at first and gradually pick up speed. Why does it take so long for them to reach their full rotational speed?

3. A mechanic balances the wheels of your car to make sure that their centers of mass are located exactly at their geometrical centers. Neglecting friction and air resistance, how would an improperly balanced wheel behave if it were rotating all by itself?

4. An object's center of mass isn't always inside the object, as you can see by spinning it. Where is the center of mass of a boomerang or a horseshoe?

5. Why is it hard to start the wheel of a roulette table spinning, and what keeps it spinning once it's started?

6. Why can't you open a door by pushing its doorknob directly toward or away from its hinges?

7. Why can't you open a door by pushing on its hinged side?

8. It's much easier to carry a weight in your hand when your arm is at your side than it is when your arm is pointing straight out in front of you. Use the concept of torque to explain this effect.

9. A gristmill is powered by falling water, which pours into buckets on the outer edge of a giant wheel. The weight of the water turns the wheel. Why is it important that those buckets be on the wheel's outer edge?

10. How does the string of a yo-yo get the yo-yo spinning?

11. One way to crack open a walnut is to put it in the hinged side of a door and then begin to close the door. Why does a small force on the door produce a large force on the shell?

12. A common pair of pliers has a place for cutting wires, bolts, or nails. Why is it so important that this cutter be located very near the pliers' pivot?

13. You can do push-ups with either your toes or your knees acting as the pivot about which your body rotates. When you pivot about your knees, your feet actually help you to lift your head and chest. Explain.

14. Tightrope walkers often use long poles for balance. Although the poles don't weigh much, they can exert substantial torques on the walkers to keep them from tipping and falling off the ropes. Why are the poles so long?

15. Some racing cars are designed so that their massive engines are near their geometrical centers. Why does this design make it easier for these cars to turn quickly?

16. How does a bottle opener use mechanical advantage to pry the top off a soda bottle?

17. A jar-opening tool grabs onto a jar's lid and then provides a long handle for you to turn. Why does this handle's length help you to open the jar?

18. When you climb out on a thin tree limb, there's a chance that the limb will break off near the trunk. Why is this disaster most likely to occur when you're as far out on the limb as possible?

19. How does a crowbar make it easier to lift the edge of a heavy box a few centimeters off the ground?

20. The basket of a wheelbarrow is located in between its wheel and its handles. How does this arrangement make it relatively easy for you to lift a heavy load in the basket?

21. Skiers often stop by turning their skis sideways and skidding them across the snow. How does this trick remove energy from a skier, and what happens to that energy?

22. A horse does work on a cart it's pulling along a straight, level road at a constant speed. The horse is transferring energy to the cart, so why doesn't the cart go faster and faster? Where is the energy going?

23. Explain why a rolling pin flattens a piecrust without encountering very much sliding friction as it moves.

24. Professional sprinters wear spikes on their shoes to prevent them from sliding on the track at the start of a race. Why is energy wasted whenever a sprinter's foot slides backward along the track?

25. A yo-yo is a spool-shaped toy that spins on a string. In a sophisticated yo-yo, the end of the string forms a loop around the yo-yo's central rod so that the yo-yo can spin almost freely at the end of the string. Why does the yo-yo spin longest if the central rod is very thin and very slippery?

26. As you begin pedaling your bicycle and it accelerates forward, what is exerting the forward force that the bicycle needs to accelerate?

27. When you begin to walk forward, what exerts the force that allows you to accelerate?

28. If you are pulling a sled along a level field at constant velocity, how does the force you are exerting on the sled compare to the force of sliding friction on its runners?

29. Why does putting sand in the trunk of a car help to keep the rear wheels from skidding on an icy road?

30. When you're driving on a level road and there's ice on the pavement, you hardly notice that ice while you're heading straight at a constant speed. Why is it that you only notice how slippery the road is when you try to turn left or right, or to speed up or slow down?

31. Describe the process of writing with chalk on a blackboard in terms of friction and wear.

32. Falling into a leaf pile is much more comfortable than falling onto the bare ground. In both cases you come to a complete stop, so why does the leaf pile feel so much better?

33. In countless movie and television scenes, the hero punches a brawny villain who doesn't even flinch at the impact. Why is the immovable villain a Hollywood fantasy?

34. Why can't an acrobat stop himself from spinning while he is in midair?

35. While a gymnast is in the air during a leap, which of the following quantities must remain constant for her: velocity, momentum, angular velocity, or angular momentum?

36. If you sit in a good swivel chair with your feet off the floor, the chair will turn slightly as you move about but will immediately stop moving when you do. Why can't you make the chair spin without touching something?

37. When a star runs out of nuclear fuel, gravity may crush it into a neutron star about 20 km (12 miles) in diameter. Although the star may have taken a year or so to rotate once before its collapse, the neutron star rotates several times a second. Explain this dramatic increase in angular velocity.

38. A toy top spins for a very long time on its sharp point. Why does it take so long for friction to slow the top's rotation?

39. It's easier to injure your knees and legs while hiking downhill than while hiking uphill. Use the concept of energy to explain this observation.

40. When you first let go of a bowling ball, it's not rotating. But as it slides down the alley, it begins to rotate. Use the concept of energy to explain why the ball's forward speed decreases as it begins to spin.

41. Firefighters slide down a pole to get to their trucks quickly. What happens to their gravitational potential energy, and how does it depend on the slipperiness of the pole?

Problems

1. When you ride a bicycle, your foot pushes down on a pedal that's 17.5 cm (0.175 m) from the axis of rotation. Your force produces a torque on the crank attached to the pedal. Suppose that you weigh 700 N. If you put all your weight on the pedal while it's directly in front of the crank's axis of rotation, what torque do you exert on the crank?