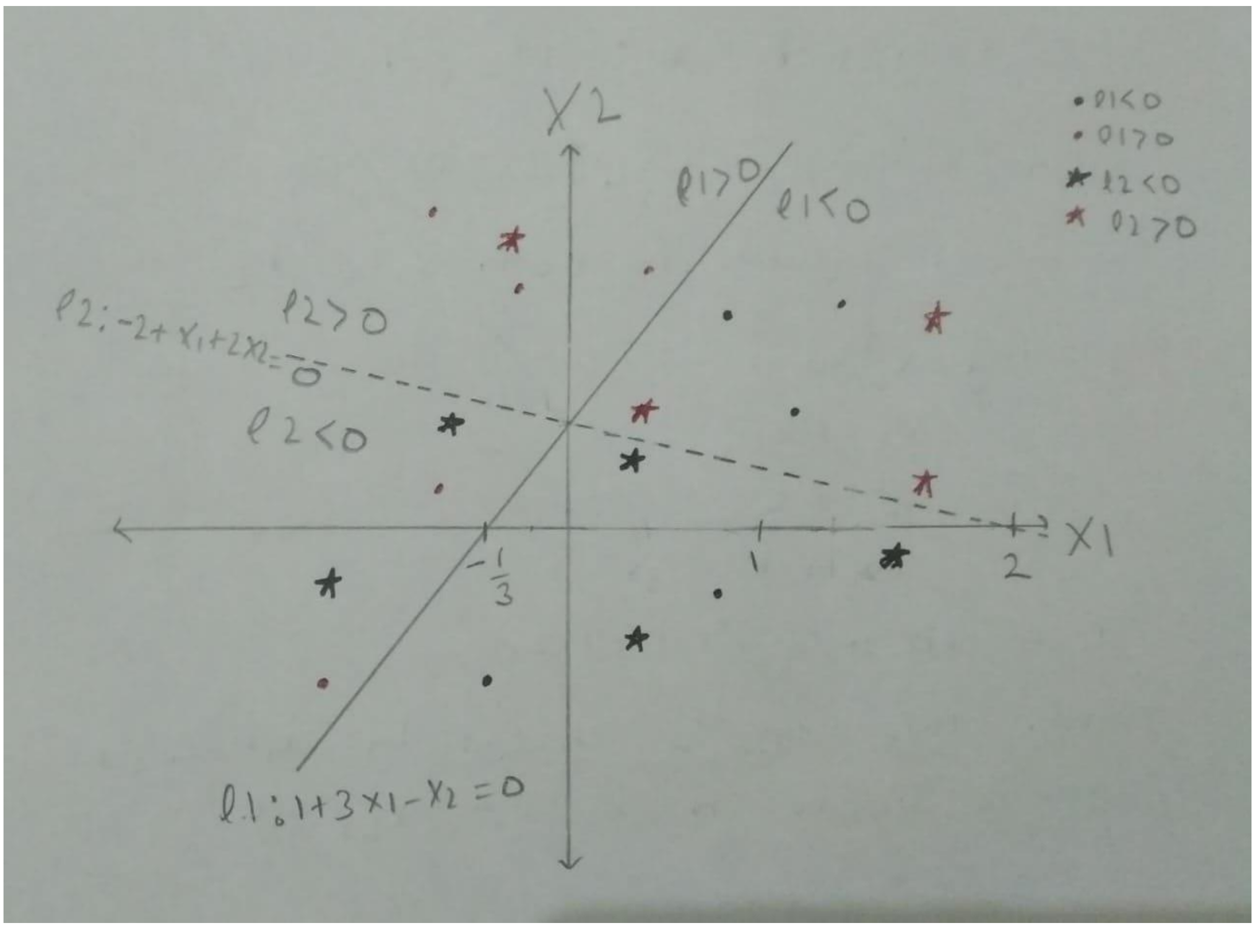
9.7 EXERCISES
Conceptual Questions

Chapter – 9

1. This problem involves hyperplanes in two dimensions.

(a) Sketch the hyperplane $1 + 3X1 - X2 = 0$. Indicate the set of points for which $1 + 3X1 - X2 > 0$, as well as the set of points for which $1 + 3X1 - X2 < 0$.

(b) On the same plot, sketch the hyperplane $-2 + X1 + 2X2 = 0$. Indicate the set of points for which $-2 + X1 + 2X2 > 0$, as well as the set of points for which $-2 + X1 + 2X2 < 0$



2. We have seen that in p = 2 dimensions, a linear decision boundary takes the form β0+β1X1+β2X2 = 0. We now investigate a non-linear decision boundary.

(a) Sketch the curve

$(1 + X1)\ 2 + (2 - X2)\ 2 = 4.$

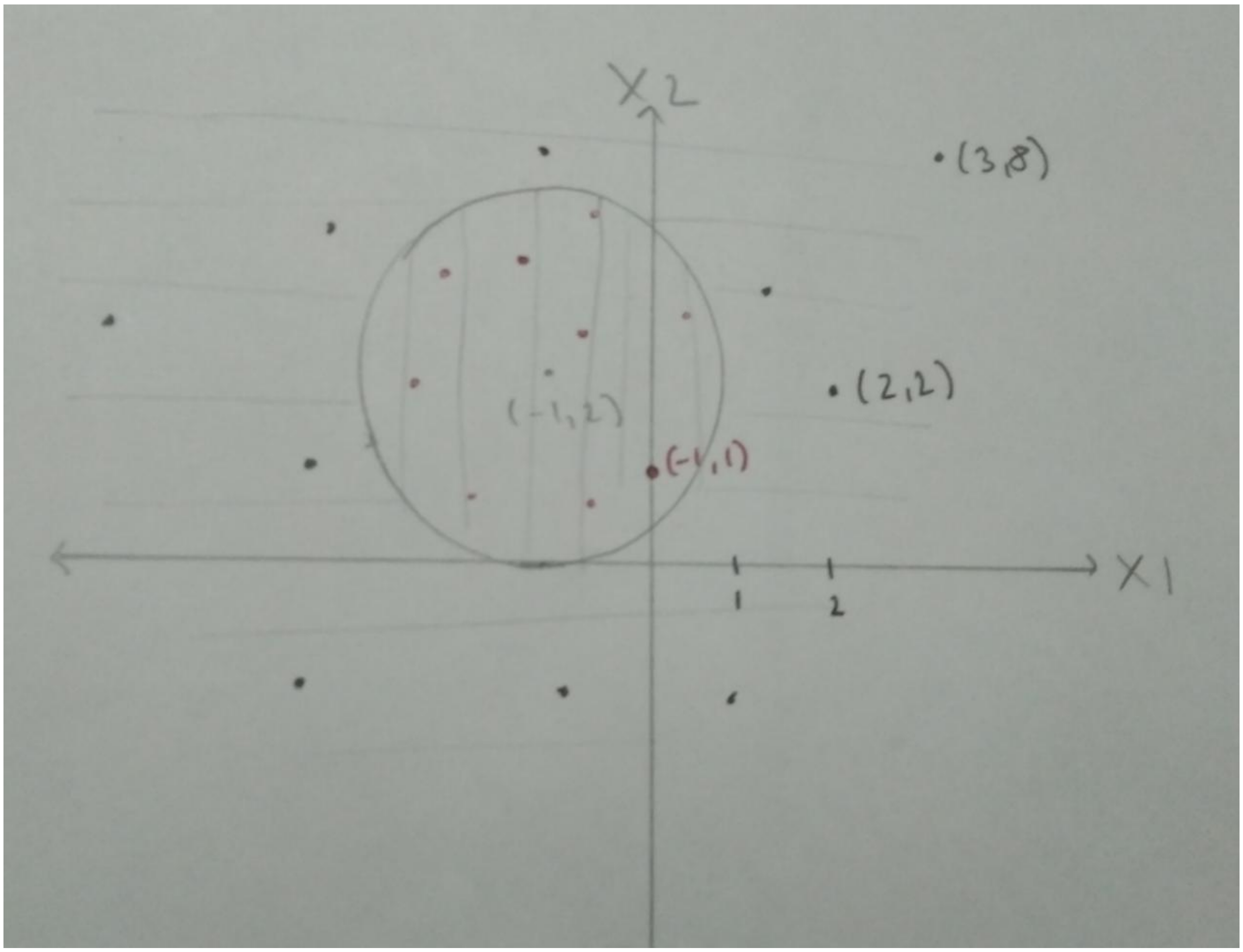
(b) On your sketch, indicate the set of points for which

$$(1 + X1)2 + (2 - X2)2 > 4$$

as well as the set of points for which

$$(1 + X1)2 + (2 - X2)2 \leq 4.$$

(c) Suppose that a classifier assigns an observation to the blue class if (1 + X1) 2 + (2 − X2) 2 > 4, and to the red class otherwise. To what class is the observation (0, 0) classified? (−1, 1)? (2, 2)? (3, 8)?



(d) Argue that while the decision boundary in (c) is not linear in terms of X1 and X2, it is linear in terms of X1, X2 1 , X2, and X2 2 .

d.) $(1+X_1)^2 + (2-X_2)^2 = 4$ - (eq of decision boundary)

$1 + X_1^2 + 2X_1 + 4 + X_2^2 - 4X_2 = 4$

$X_1^2 + 2X_1 + X_2^2 - 4X_2 + 1 = 0$ - ②

eq ② is quad. in both X_1 & X_2

Replacing $X_1^2 \to a$

$X_1 \to b$

$X_2^2 \to c$

$X_2 \to d$

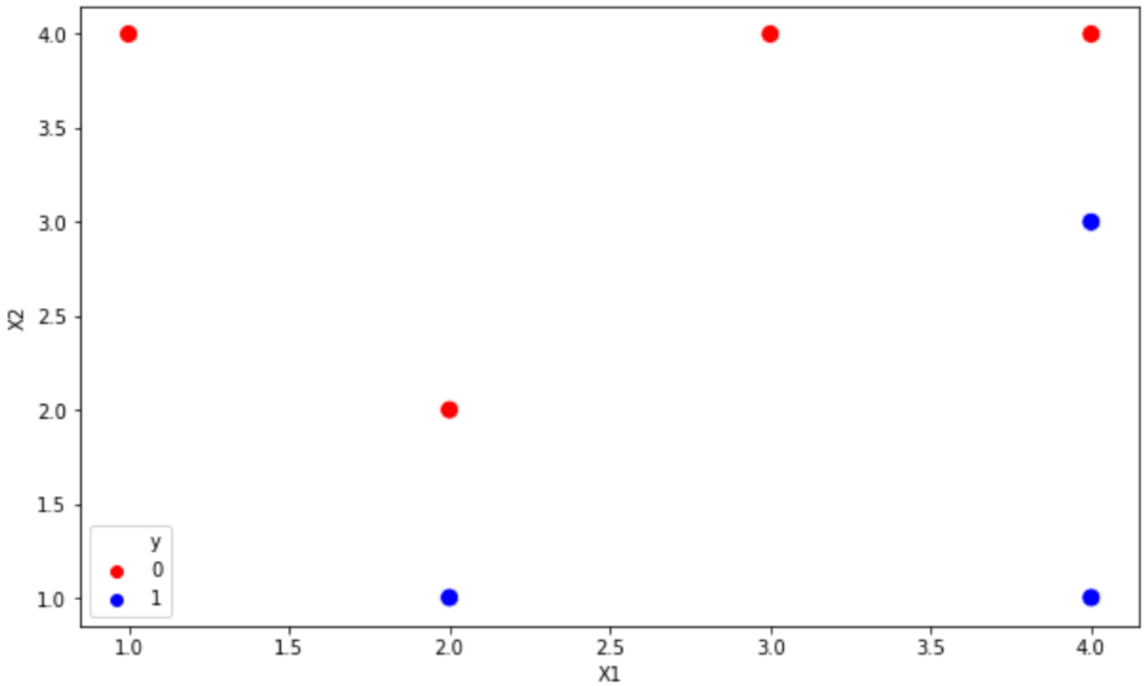
$a + 2b + c - 4d + 1 = 0$

hence this eq is linear in a, b, c, d

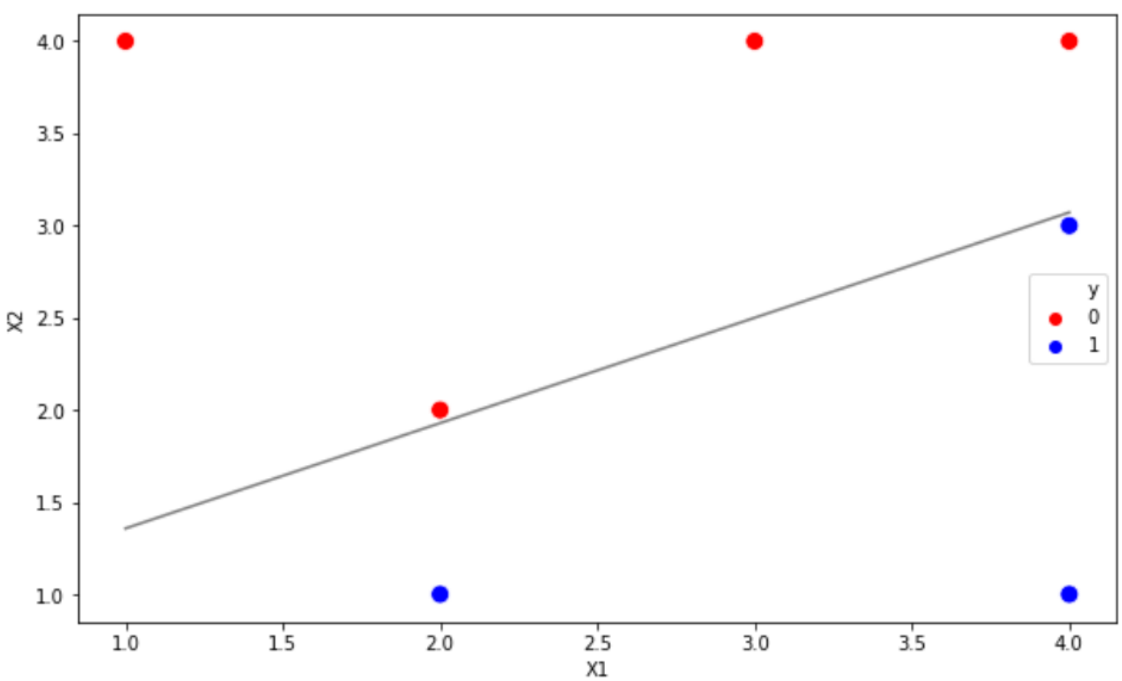
$\therefore$ it is linear in X_1, X_1^2, X_2, X_2^2

3. Here we explore the maximal margin classifier on a toy data set.

(a) We are given n = 7 observations in p = 2 dimensions. For each observation, there is an associated class label.
(table in book)
Sketch the observations



(b) Sketch the optimal separating hyperplane, and provide the equation for this hyperplane (of the form (9.1)).



Equation = $0.615 * X1 - 1.0769 * X2 + 0.84 = 0$

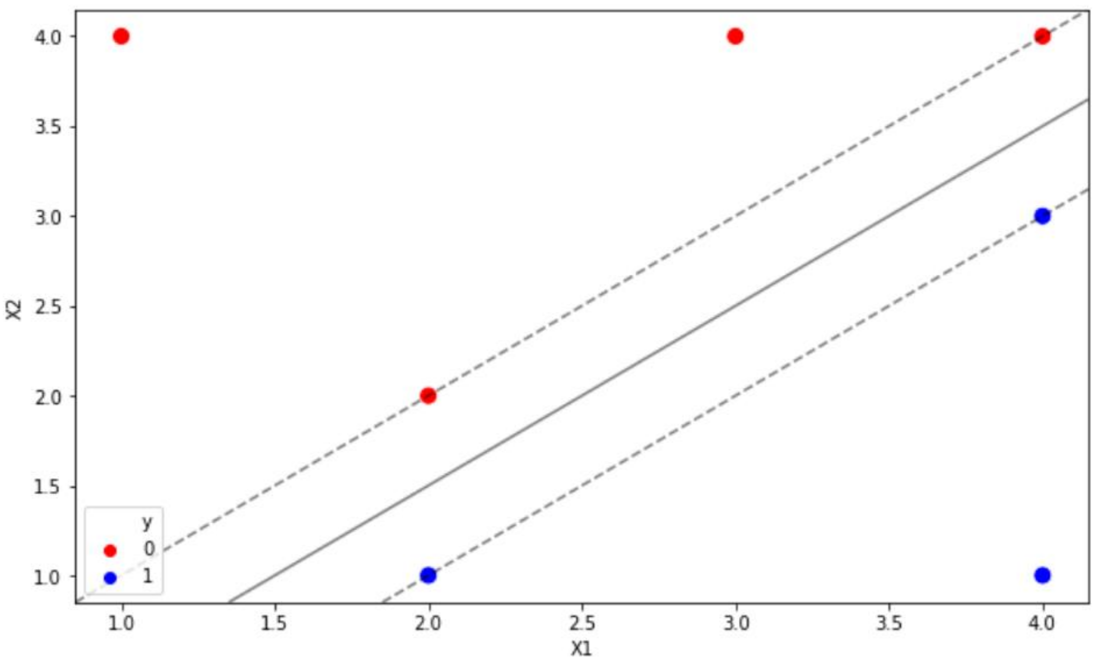
(c) Describe the classification rule for the maximal margin classifier. It should be something along the lines of "Classify to Red if β0 + β1X1 + β2X2 > 0, and classify to Blue otherwise." Provide the values for β0, β1, and β2.

Observation is classified to Red if

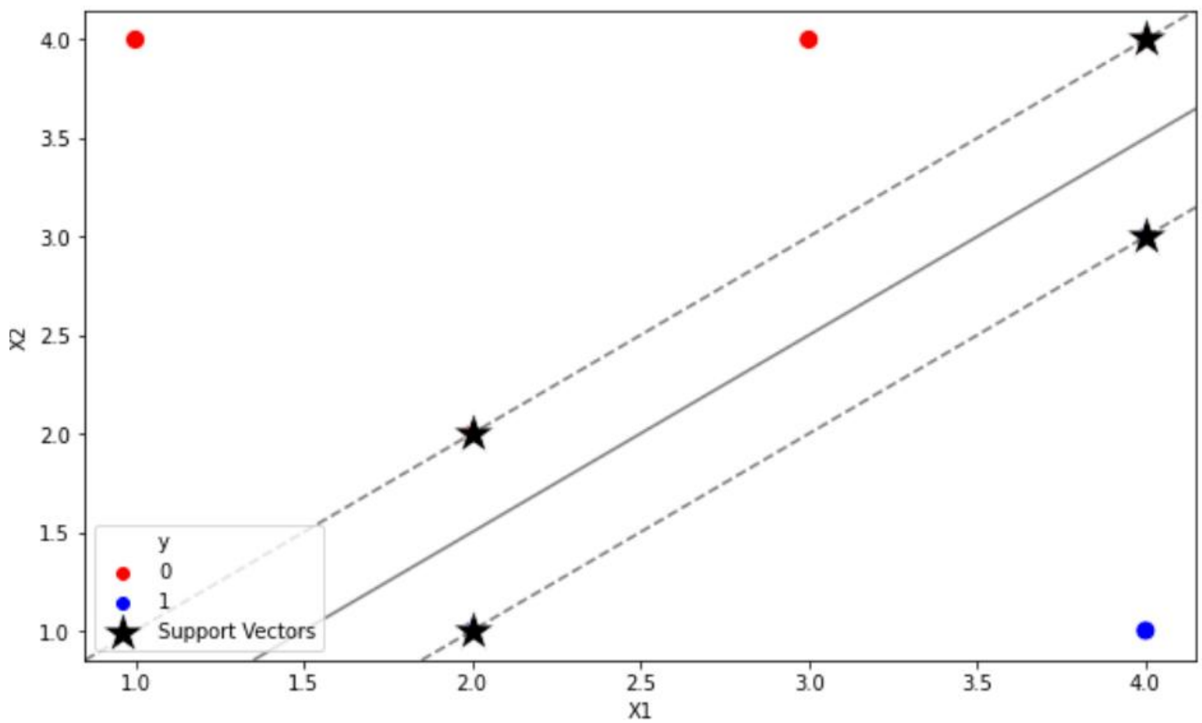
$$0.615 * X1 - 1.0769 * X2 + 0.84 > 0$$

, and classifier to Blue otherwise.

(d) On your sketch, indicate the margin for the maximal margin hyperplane.



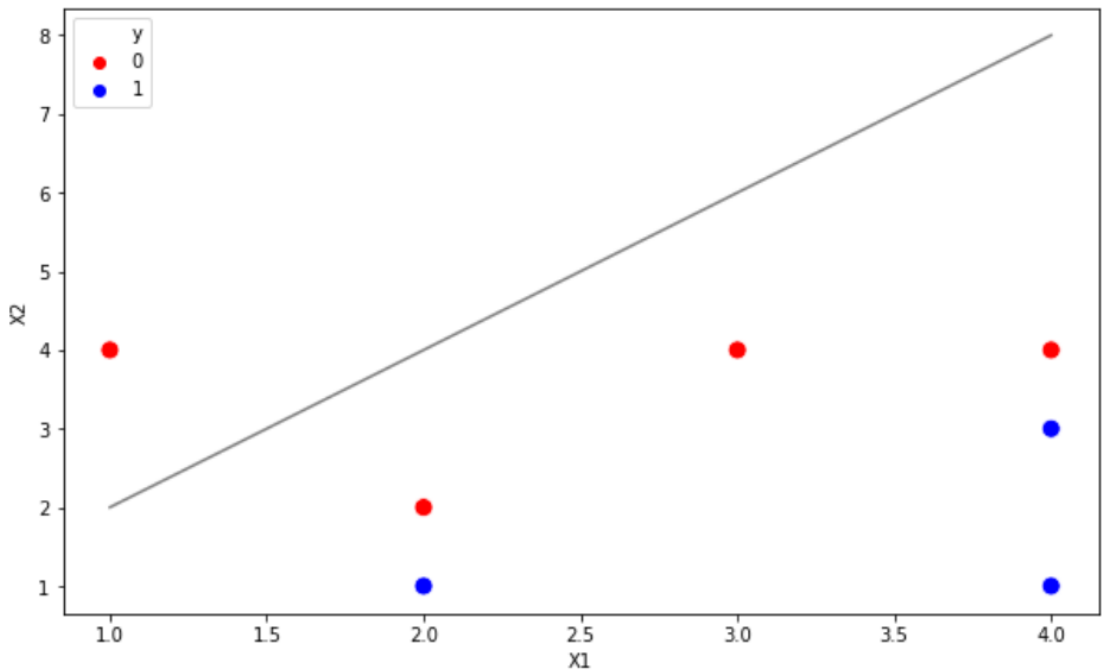
(e) Indicate the support vectors for the maximal margin classifier.



(f) Argue that a slight movement of the seventh observation would not affect the maximal margin hyperplane.

The seventh observation is the one which is bottom right corner, since it does not belong to support vectors, hence the slight movement of the seventh observation would not affect the maximal margin hyperplane.

(g) Sketch a hyperplane that is not the optimal separating hyperplane, and provide the equation for this hyperplane.



Equation of the hyperplace = X1 + 2*X2 = 0

(h) Draw an additional observation on the plot so that the two classes are no longer separable by a hyperplane.

Added a Blue observation at (2,3)