Computer assignment 4
Portfolio selection

Introduction

In this assignment you will backtest several portfolios and compare their performance to the equal weight portfolio and the risk parity portfolio. You will also determine the ”mean-tracking error” efficient portfolios with respect to the S&P 500-index.

Data and parameter estimation:

You will use weekly data for the period 19960101 – 20201231 (in the file Dow_SP500_9620_weekly.csv) consisting of 26 stocks from the Dow Jones index and the S&P 500 index. To estimate the parameters (means and covariances) for the Markowitz model you will use a moving average procedure with an estimation window consisting of 100 weekly gross returns.

Backtesting procedure:

The backtesting procedure is as follows:

1. Starting at the beginning of the 101st week, estimate the parameters using the first 100 weekly gross returns.
2. Determine the optimal allocation.
3. Invest your money according to the allocation above (the initial investment is 100).
4. Calculate the portfolio value at the end of the week.
5. Repeat from step 1 by first updating your means and covariances, by sliding the estimation window one week forward, and then using your new portfolio value when investing for the coming week.

Note: When shorting is not allowed and we have a target return, we need to modify the backtesting procedure in the cases where the optimizer is not able to find a solution. These cases arise if in any time period:

- the target return is less than the mean value of all assets’ returns, or
- the target return is greater than the mean value of all assets’ returns

In the first case, invest everything in the asset with the lowest mean and in the second case, invest everything in the asset with the highest mean.

1. The mean-volatility diagram

At the starting point, using the first 100 weekly returns to estimate the parameters:

a) Plot the individual stocks (assuming 100 invested in each of the stocks) in a mean-volatility diagram with x-limits: [0, 8] and y-limits: [99, 102]
b) Plot the efficient frontiers, with and without short selling allowed, in the same mean-volatility diagram.
c) Plot, in the same mean-volatility diagram, the Sharpe portfolio for the case when shorting is allowed.
d) Plot the Sharpe allocation as a bar plot
e) Plot the gross return correlations as an annotated heatmap

Discussion: Looking at the mean-volatility diagram, did Apple look like a good investment*?*

2. Backtesting the standard Markowitz model

Backtest the Markowitz model using the backtesting procedure outlined above for the following portfolios (by target portfolio we mean a portfolio with weekly gross return target, $\mu_0 = 1{,}00275$):

- the target portfolio, with shorting allowed,
- the target portfolio, shorting not allowed
- the equal weight portfolio (rebalanced every week),

Then:

a) Plot the evolution of the portfolios in the same plot.
b) Include a table with the portfolio values at every 100th week, beginning at week 0.
c) Calculate annualized means, volatilities, Sharpe ratios, maximum drawdown and average ENCs for the portfolios' weekly percentage returns over the backtesting period.
d) Plot the portfolio percentage weights for the no short target portfolio using the area plot.

Discussion: How do these portfolios compare? *What does c) and d) tell you?*

3. Benchmark allocation

Determine the ”mean-tracking error” efficient frontier using S&P 500 as the benchmark index, when V(0) = L(0) = 100. Assume that a risk-free asset is available with $R_0 = 1$ and use the last 100 weekly returns in the data for parameter estimation.

a) Plot the optimal hedge portfolio allocation as a bar plot and include the values in a table
b) Plot the “mean – tracking error” efficient frontier
c) What is the smallest tracking error that can be achieved?
d) What is the expected excess return (in excess of S&P500) of the minimum tracking error portfolio?

4. Backtesting Sharpe portfolios

Repeat the backtest for the following Sharpe portfolios:

- The Sharpe portfolio[1], shorting not allowed
- The Sharpe portfolio, shorting not allowed and applying shrinkage to means and covariances (use shrinkage constant = 0.3 for the covariance matrix)
- The Sharpe portfolio, shorting not allowed applying shrinkage to means and denoising the covariance matrix

Then:

a) Plot the evolution of these portfolios in the same plot as in 2.a)
b) Include their values at every 100th week, beginning at week 0, in the table in 2b)
c) Calculate annualized means and volatilities, Sharpe ratios, maximum drawdown and average ENCs for the portfolios’ weekly percentage returns over the backtesting period.
d) Plot the every portfolios’ percentage weights using the area plot (three separate plots).

Discussion: How do these portfolios compare to the ones in 2? Did shrinkage and denoising have any positive effects?

[1] *To determine the no-short Sharpe portfolio, see: maximizing the sharpe ratio.pdf (sc.edu)*

5. Backtesting equal risk contribution portfolios

Repeat the backtest for

- the MVP, shorting not allowed
- the risk parity portfolio

Then:

a) Plot the evolution of these portfolios (but not in the same plot as problems 2 and 4)
b) Include a table with the portfolio values at every 100th week, beginning at week 0
c) Calculate annualized means and volatilities, Sharpe ratios, maximum drawdown and average ENCs for the portfolios' weekly percentage returns over the backtesting period.
d) Plot the weights of the MVP and the risk parity portfolio over the backtest period using an area plot (two separate plots)

Discussion: How do these portfolios compare?

Grading

These are the requirements for the grades 3, 4 and 5:

Problem	3	4	5
1	All	All	All
2	All	All	All
3	All	All	All
4	-	All	All
5	-	-	All