The Result: A Perfect Integrator Chain

Under the feedback law $u(x, v)$, the system's behavior from the new input v to the output y y is now equivalent to a chain of r integrators in Controllable Canonical Form.

New State Definition

Define a new state vector z of dimension r:

$$z = [y, \dot{y}, \dots, y^{(r-1)}]^T$$

Linear Dynamics

$$\dot{z} = Az + Bv$$
$$y = Cz$$

Next Step

Standard linear control techniques (e.g., pole placement) can now be used to design a controller for v to place the poles of this new, simplified linear system.

Matrix A — Companion Matrix

0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

Matrix B

0
0
0
0
1

Matrix C

1	0	0	0	0	0	0	0	0	0