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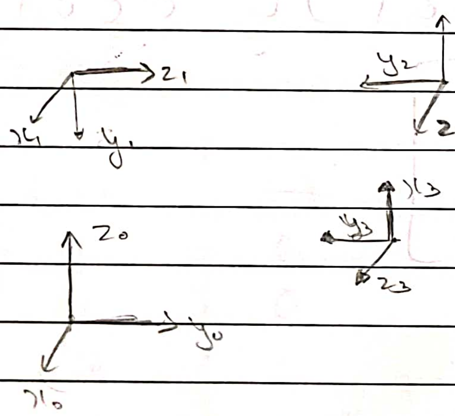
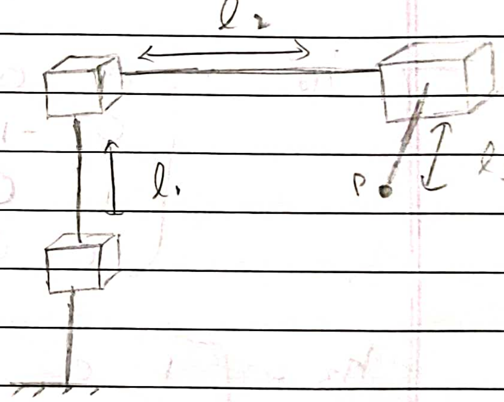
Assignment 3&4

1. Configuration at which a parallel robot loses it rigidity in which the end-effector degree of freedom become uncontrollable.
Singular configurations can be determined by the rank of matrix J^{-1} (Jacobian matrix inverse).

Yes, we can determine if a particular configuration is close to a singular configuration using Manipulator Jacobian by calculating the rank of jacobian matrix. If the rank is less than the degree of freedom than it indicates that the manipulator is close to ~~the~~ a singularity.

5.





Link	d	θ	a	α
1	l_1	0	0	$-90°$
2	l_2	$-90°$	0	$-90°$
3	l_3	0	0	0

$$H_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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PAGE NO.:

DATE: / /

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore H_0^2 = H_1 H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_2 \\ 1 & 0 & 0 & l_1 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

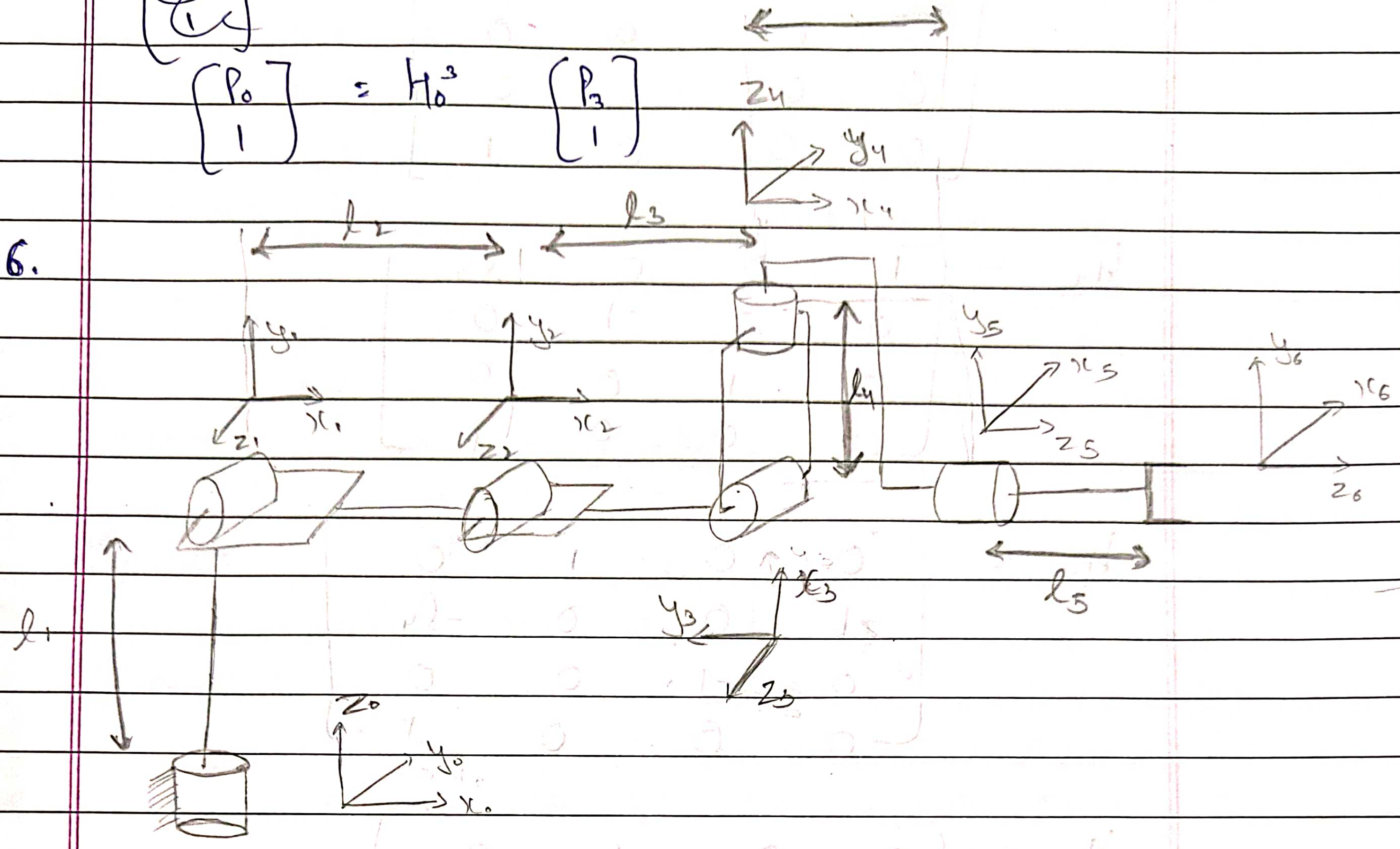
$$H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = H_0^2 H_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_2 \\ 1 & 0 & 0 & l_1 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} 0 & 0 & 1 & l_3 \\ 0 & -1 & 0 & l_2 \\ 1 & 0 & 0 & l_1 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

6.



Link	d	θ	a	α
1	l_1	0	0	90°
2	0	0	l_2	0
3	0	0	l_3	90°
4	~~l_4~~ 0	-90°	~~0~~ l_4	-90°
5	$-l_4$	90°	a_1	+90°
6	l_5	0	0	0

$$\cancel{A_4} = H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PAGE NO.:
DATE : / /

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -l_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^6 = H_1\, H_2\, H_3\, H_4\, H_5\, H_6$$

$$H_1 H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1H_2 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1H_2H_3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1H_2H_3 = \begin{bmatrix} 1 & 0 & 0 & l_2+l_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1H_2H_3H_4 = \begin{bmatrix} 1 & 0 & 0 & l_2+l_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -l_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1H_2H_3H_4 = \begin{bmatrix} 0 & 0 & 1 & l_2+l_3 \\ 1 & 0 & 0 & l_4 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1H_2H_3H_4H_5 = \begin{bmatrix} 0 & 0 & 1 & l_2+l_3 \\ 1 & 0 & 0 & l_4 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1H_2H_3H_4H_5 = \begin{bmatrix} 0 & 1 & 0 & l_2+l_3-l_4 \\ 0 & 0 & 1 & l_4 \\ 1 & 0 & 0 & a_1+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PAGE NO.:
DATE: / /

$$H_1H_2H_3H_4H_5H_6 = \begin{bmatrix} 0 & 1 & 0 & l_2+l_3-l_4 \\ 0 & 0 & 1 & l_4 \\ 1 & 0 & 0 & a_1+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^6 = \begin{bmatrix} 0 & 1 & 0 & l_2+l_3-l_4 \\ 0 & 0 & 1 & l_4+l_5 \\ 1 & 0 & 0 & a_1+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^6 \begin{bmatrix} P_6 \\ 1 \end{bmatrix}$$

Ans

Q10.

- First we find the kinetic energy using D(q).
- Now L = K - V, put the value of K(kinetic energy calculated in the first step) and V(q).
- Find the value of $\frac{\partial L}{\partial \dot{q}_k}$ $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k}$, and $\frac{\partial L}{\partial q_k}$
- Apply the Euler-Lagrange equations to derive the equations of motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i$$

- By interchanging the order of summation and using symmetry,we get the Euler-Lagrange equation in the form:

$$\sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j \quad = \quad \sum_{i,j}\frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j$$

- Here comes a notation that reduces our efforts of involving the symbols in computing which is the Christoffel symbol(c_{ijk}).

$$c_{ijk} \quad = \quad \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}$$

- In the end, we determine a function Φ_k as:

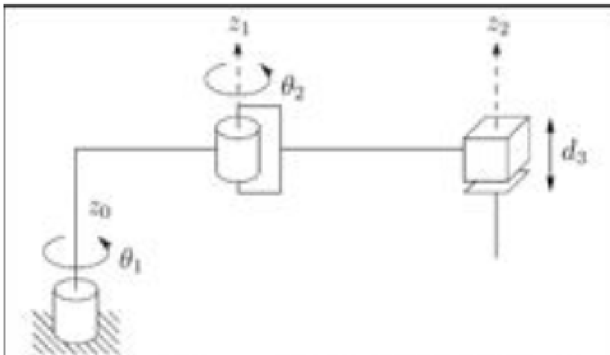
$$\phi_k \quad = \quad \frac{\partial P}{\partial q_k}$$

- Now we get the Equation of motion:

$$\sum_i d_{kj}(q)\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k, \qquad k = 1, \ldots, n$$

Q4.

I have taken a simple RRP SCARA configuration. In this, most simple DH parameters are taken.



Schematics of SCARA Robot

	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	180°	0	θ_2^*
3	0	0	d_3^*	0

DH Parameters

I have taken a1 = a2 = 0, d3* = 1, θ1* = 0, and θ2* = 0.

```
Your DH Matrix is as below:
0 0 0 0
0 0 0 180
1 0 0 0
```

```
The end effector position is [0.] i + [0.80115264] j + [-0.59846007] k
```

Q7.

Direct-Drive Configuration:

Each joint is directly coupled to the actuator in a direct drive 2R manipulator. This indicates a direct connection between the motor and the link's joint shaft.

<u>Advantages:</u>

- It has an easy design.
- High torque and accuracy are provided.
- Backlash is decreased.

<u>Disadvantages:</u>

- It offers only a small range of motion.
- It is unable to deliver fast speed.

Remotely-Driven Configuration:

Each joint in a remotely driven 2R manipulator is operated remotely by a linkage or other mechanism rather than being physically attached to the actuator.

<u>Advantages:</u>

- Increased range of motion.
- The robot arm may be made smaller so that it can fit in small locations.
- The trade-off between speed and torque may be managed with the help of the remote linkage.

<u>Disadvantages:</u>

- There is an extra mechanism in this arrangement, which makes it more complicated.
- The extra links may result in a loss of accuracy.

5-Bar Parallelogram Arrangement:
The 2R manipulator is created in this design as a closed-loop parallelogram with both joints coupled in a parallelogram shape.
<u>Advantages:</u>

- Throughout the work, it keeps the original orientation.
- The likelihood of achieving singularity is decreased.
- It enhances the stability of the structure.

<u>Disadvantages:</u>

- It has a limited range of motion.
- Additionally, the structure's flexibility is decreased.