

Assignment-2

1. Let $a = [a \ 0 \ 0]^T$, $R = R_{z,a} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$S(a) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -a \\ a & 0 & 0 \end{bmatrix}$$

$$RS(a) = \begin{bmatrix} 0 & 0 & a\sin \theta \\ 0 & 0 & -a\cos \theta \\ 0 & a & 0 \end{bmatrix}, \quad S(R^T) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RS(a)R^T = \begin{bmatrix} 0 & 0 & a\sin \theta \\ 0 & 0 & -a\cos \theta \\ -a\sin \theta & a\cos \theta & 0 \end{bmatrix} \quad \text{---(1)}$$

$$Ra = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$$Ra = \begin{bmatrix} a\cos \theta \\ a\sin \theta \\ 0 \end{bmatrix} = \begin{bmatrix} a\cos \theta & a\sin \theta & 0 \end{bmatrix}^T$$

$$S(Ra) = \begin{bmatrix} 0 & 0 & a\sin \theta \\ 0 & 0 & -a\cos \theta \\ -a\sin \theta & a\cos \theta & 0 \end{bmatrix} \quad \text{---(11)}$$

$$\therefore \text{---(1)} = \text{---(11)}$$

$$\therefore S R S(a) R^T = S(Ra)$$

Hence Proved.

2. $P_3 = \begin{bmatrix} 0 \\ -l_3+d \\ 0 \end{bmatrix}$

$$R_0^1 = R_{2, q_1}$$

$$R_1^2 = [R_{11}, \pi_{12}] [R_{21}, q_{12}]$$

$$R_2^3 = R_{2, 0} = [I] J_{3 \times 3}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

~~$d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$~~

~~$d_2^3 =$~~

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 K_2^3 H_1^2 H_2^3 B_3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} C_{01} - S_{01} & 0 & 0 & 0 \\ S_{01} & C_{01} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} -C_{02}, 0, S_{02}, 0 \\ S_{02}, 0, -C_{02}, 0 \\ 0, 1, 0, l_1 \\ 0, 0, 0, 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 1 & 1 & l_2 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -l_3+d \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} H_2^3 \\ P_3 \end{bmatrix} = \begin{bmatrix} l_2, -l_3+d \\ -l_3+d \\ -l_3+d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ k_1 \\ f \end{bmatrix} =$$

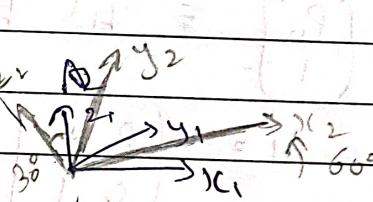
$$H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} -c_{01}(l_2 - l_3 + d) + s_{01}(-l_3 + d) \\ s_{01}(l_2 - l_3 + d) + c_{01}(l_3 - d) \\ l_1 - l_3 + d \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} a(c_{01} - b s_{01}) \\ a s_{01} + b(c_{01}) \\ c \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_{01}[-(c_{02}(l_2 - l_3 + d) + s_{02}(-l_3 + d))] - s_{01}[s_{02}(l_2 - l_3 + d) + c_{02}(l_3 - d)] \\ s_{01}[-(c_{02}(l_2 - l_3 + d) + s_{02}(-l_3 + d))] + c_{01}[s_{02}(l_2 - l_3 + d) + c_{02}(l_3 - d)] \\ l_1 - l_3 + d \end{bmatrix}$$

$$5. R_1^1 = R_{2,0} = I_3$$

$$d_0^1 = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$$



$$R_1^2 = [R_{1x}, \pi_{1x}] [R_{2, \pi_{1x}}]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 - 1/2 & \sqrt{3}/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1/2 - \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 \\ 0 & 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 \end{bmatrix}$$

$$d_1^2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$d_2^3 =$$

$$\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

$$R_2^3 = R_{2,0} = I_3$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^T H_1^{-2} H_2^{-3} \begin{bmatrix} P_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & 0 \\ 3/\sqrt{4} & \sqrt{3}/4 - 1/2 & 0 \\ \sqrt{3}/4 & Y_4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} H_0^T H_1^{-2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Setting off last P-Step (3)

$$= H_0^T \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$

$$P_0 = [0, -1.5, 12.598]^T$$

$$7. R_0^1 = \begin{bmatrix} C_{01}, -S_{01}, 0 \\ S_{01}, C_{01}, 0 \\ 0, 0, 1 \end{bmatrix} \quad R_1^2 = \begin{bmatrix} C_{02}, -S_{02}, 0 \\ S_{02}, C_{02}, 0 \\ 0, 0, 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ d - l_3 \end{bmatrix}$$

These all are already calculated
in question 2.

$$R_0^2 = R_0^1 R_1^2 = \begin{bmatrix} (a_1, -s_{a_1}, 0) \\ s_{a_1}, (a_1, 0) \\ 0, 0, 1 \end{bmatrix} \begin{bmatrix} (a_2, -s_{a_2}, 0) \\ s_{a_2}, (a_2, 0) \\ 0, 0, 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} (a_1, a_2, -s_{a_1}a_2) & -s_{a_1}s_{a_2} - s_{a_2}a_1 & 0 \\ s_{a_1}(a_2 + s_{a_2}a_1) & -s_{a_1}s_{a_2} + (a_1, a_2) & 0 \\ 0, 0, 1 & 0, 0, 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} (c(a_1+a_2)) & -s(a_1+a_2) & 0 \\ s(a_1+a_2) & (a_1+a_2) & 0 \\ 0, 0, 1 & 0, 0, 1 \end{bmatrix}$$

$$R_0^3 = R_0^2 R_2^3 \quad (R_2^3 \text{ is Identity matrix})$$

$$\therefore R_0^3 = \begin{bmatrix} (a_1+a_2) & -s(a_1+a_2) & 0 \\ s(a_1+a_2) & (a_1+a_2) & 0 \\ 0, 0, 1 & 0, 0, 1 \end{bmatrix}$$

$$H_0^2 = H_0^1 H_1^2 = \begin{bmatrix} (a_1, -s_{a_1}, 0, 0) \\ s_{a_1}, (a_1, 0, 0) \\ 0, 0, 1, 0 \\ 0, 0, 0, 1 \end{bmatrix} \begin{bmatrix} (a_2, -s_{a_2}, 0, 0) \\ s_{a_2}, (a_2, 0, 0) \\ 0, 0, 1, 0 \\ 0, 0, 0, 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} (a_1+a_2) & -s(a_1+a_2) & 0 & (a_1, l_1) \\ s(a_1+a_2) & (a_1+a_2) & 0 & -s_{a_1}l_1 \\ 0, 0, 1, 0 & 0, 0, 1, 0 \\ 0, 0, 0, 1 & 0, 0, 0, 1 \end{bmatrix}$$

$$d_0^2 = \begin{bmatrix} (a_1, l_1) \\ s_{a_1}l_1 \\ 0 \end{bmatrix}$$

$$H_0^3 = H_0^2 \cdot H_1^3 = \begin{bmatrix} (C_{q_1} + a_{12}) & -S_{(q_1 + a_{12})} & 0 & C_{q_1} l_1 \\ S_{(q_1 + a_{12})} & (C_{q_1} + a_{12}) & 0 & S_{q_1} l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} (C_{q_1} + a_{12}) & -S_{(q_1 + a_{12})} & 0 & C_{q_1} l_1 + (C_{q_1} + a_{12}) l_2 \\ S_{(q_1 + a_{12})} & (C_{q_1} + a_{12}) & 0 & S_{q_1} l_1 + S_{(q_1 + a_{12})} l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} C_{q_1} l_1 + (C_{q_1} + a_{12}) l_2 \\ S_{q_1} l_1 + S_{(q_1 + a_{12})} l_2 \\ 0 \end{bmatrix}$$

Linear

Prismatic

Rotational

& Linear

$$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_i^0 - d_{i-1}^0)$$

Rotational

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

In SCARA, RRP there are 3 joints $\Rightarrow (n=3)$

$$R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \times \begin{bmatrix} C_{q_1} l_1 + (C_{q_1} + a_{12}) l_2 \\ S_{q_1} l_1 + S_{(q_1 + a_{12})} l_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -S_{q_1} l_1 - S_{(q_1 + a_{12})} l_2 \\ C_{q_1} l_1 + (C_{q_1} + a_{12}) l_2 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
 R_1^o \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times (d_3^o - d_1^o) &= \begin{pmatrix} C_{11}, -S_{11}, 0 \\ S_{11}, C_{11}, 0 \\ 0, 0, 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times (d_3^o - d_1^o) \\
 &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \left[C_{11}l_1 + (C_{11} + S_{11})l_2, S_{11}l_1 + S(C_{11} + S_{11})l_2 \right] \\
 &= \begin{pmatrix} -S_{11}l_1 - S(C_{11} + S_{11})l_2 \\ C_{11}l_1 + (C_{11} + S_{11})l_2 \\ 0 \end{pmatrix}
 \end{aligned}$$

$$R_2^o \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

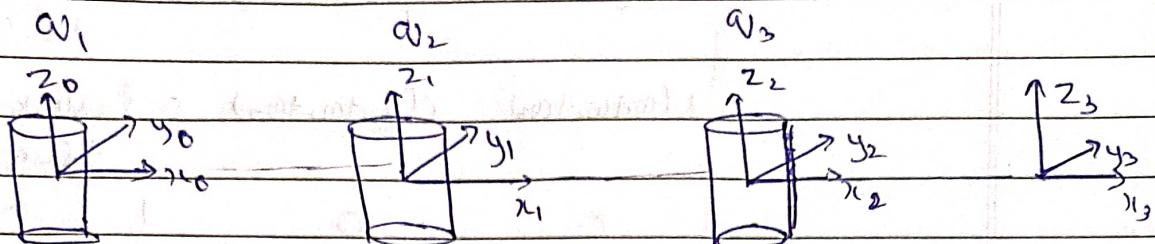
$$\begin{aligned}
 R_2^o \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times (d_3^o - d_2^o) &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} C(C_{11} + C_{22})l_2 \\ S(C_{11} + C_{22})l_2 \\ 0 \end{pmatrix} \\
 &= \begin{pmatrix} -S(C_{11} + C_{22})l_2 \\ C(C_{11} + C_{22})l_2 \\ 0 \end{pmatrix}
 \end{aligned}$$

$$R_3^o \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$I = \begin{pmatrix} -S_{11}l_1 - S(C_{11} + S_{11})l_2 & -S(C_{11} + S_{11})l_2 & 0 \\ C_{11}l_1 + (C_{11} + S_{11})l_2 & (C_{11} + S_{11})l_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

Ans

9.



$$H_0^1 = \begin{bmatrix} (c\omega_1 - s\omega_1, 0, l_1 c\omega_1) \\ s\omega_1, (c\omega_1, 0, l_1 s\omega_1) \\ 0, 0, 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} (c\omega_2 - s\omega_2, 0, l_2 c\omega_2) \\ s\omega_2, (c\omega_2, 0, l_2 s\omega_2) \\ 0, 0, 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} (c\omega_3 - s\omega_3, 0, l_3 c\omega_3) \\ s\omega_3, (c\omega_3, 0, l_3 s\omega_3) \\ 0, 0, 1 \end{bmatrix}$$

$$H_0^2 = H_0^1 \quad H_0^3 = \begin{bmatrix} (c\omega_1 - s\omega_1, 0, l_1 c\omega_1) & (c\omega_2 - s\omega_2, 0, l_2 c\omega_2) \\ s\omega_1, (c\omega_1, 0, l_1 s\omega_1) & s\omega_2, (c\omega_2, 0, l_2 s\omega_2) \\ 0, 0, 1 & 0, 0, 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} (l_1 c\omega_1) & (s\omega_1 + \omega_2) & 0 & l_1(l\omega_1 + \omega_2) + l_1 c\omega_1 \\ s(\omega_1 + \omega_2) & (c\omega_1 + \omega_2) & 0 & l_2(s\omega_1 + \omega_2) + l_2 s\omega_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K_0^3 = K_0^2 K_1^3 = \begin{cases} (C_{q_1}, t_{w_1}, t_{w_2}) - \Delta(C_{q_1}, t_{w_1}, t_{w_2}) & l_1(C_{q_1}, t_{w_1}, t_{w_2}) + l_2(C_{q_1}, t_{w_1}, t_{w_2}) \\ & + l_3(C_{q_1}, t_{w_1}, t_{w_2}) \end{cases}$$

$$\Delta(C_{q_1}, t_{w_1}, t_{w_2}) = (C_{q_1}, t_{w_1}, t_{w_2}) - l_2(C_{q_1}, t_{w_1}, t_{w_2}) + l_3(C_{q_1}, t_{w_1}, t_{w_2})$$

$$\begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$J = \left[\begin{matrix} R_0^0 [0] \times (d_2^0 - d_0^0) & R_0^1 [0] \times (d_2^1 - d_0^1) & R_2^0 [0] \times (d_2^0 - d_1^0) \\ R_0^0 [0] & R_0^1 [0] & R_2^0 [0] \end{matrix} \right]$$

$$J = \left[\begin{matrix} \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \times \begin{matrix} l_3 C_{q_1}, t_{w_1}, t_{w_2} + l_2 C_{q_1}, t_{w_1}, t_{w_2} + l_1 \\ l_3 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_2 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_1 \end{matrix} \end{matrix} \right] \left[\begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \times \begin{matrix} l_3 C_{q_1}, t_{w_1}, t_{w_2} + l_2 C_{q_1}, t_{w_1}, t_{w_2} \\ l_3 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_2 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_1 \end{matrix} \right]$$

$$\begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \times \begin{matrix} l_3 C_{q_1}, t_{w_1}, t_{w_2} \\ l_3 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) \end{matrix}$$

$$R_0^0 [0]$$

$$J = \begin{matrix} -(l_3 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_2 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_1 \Delta(C_{q_1}, t_{w_1}, t_{w_2})) \\ -[l_3 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_2 \Delta(C_{q_1}, t_{w_1}, t_{w_2}) + l_1 \Delta(C_{q_1}, t_{w_1}, t_{w_2})] \end{matrix}$$

$$l_3 C_{q_1}, t_{w_1}, t_{w_2} + l_2 C_{q_1}, t_{w_1}, t_{w_2} + l_1 C_{q_1}, t_{w_1}, t_{w_2}$$

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}$$