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Assignment-2

1. Let $a = [a\ 0\ 0]^T$, $R = R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$S(a) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -a \\ 0 & a & 0 \end{bmatrix}$$

$$RS(a) = \begin{bmatrix} 0 & 0 & a\sin\theta \\ 0 & 0 & -a\cos\theta \\ 0 & a & 0 \end{bmatrix} \quad \& \quad R^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RS(a)R^T = \begin{bmatrix} 0 & 0 & a\sin\theta \\ 0 & 0 & -a\cos\theta \\ -a\sin\theta & a\cos\theta & 0 \end{bmatrix} \quad -(1)$$

$$Ra = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$$Ra = \begin{bmatrix} a\cos\theta \\ a\sin\theta \\ 0 \end{bmatrix} = [a\cos\theta \quad a\sin\theta \quad 0]^T$$

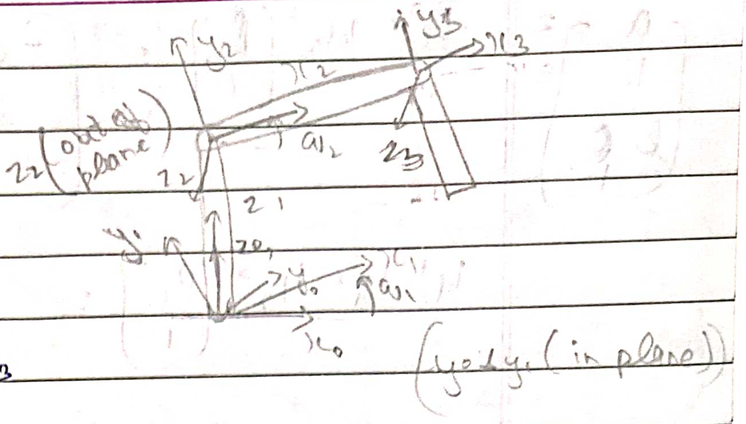
$$S(Ra) = \begin{bmatrix} 0 & 0 & a\sin\theta \\ 0 & 0 & -a\cos\theta \\ -a\sin\theta & a\cos\theta & 0 \end{bmatrix} \quad -(11)$$

$\therefore$ (11) = (1)

$\therefore$ $RS(a)R^T = S(Ra)$

Hence Proved.

2. $P_3 = \begin{bmatrix} 0 \\ -l_3+d \\ 0 \end{bmatrix}$



$R_0^1 = R_{z,q_1}$

$R_1^2 = [R_{x_1,\pi/2}][R_{z,q_2}]$

$R_2^3 = R_{z,0} = [I]_{3\times3}$

$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\cancel{d_1^2 = \begin{bmatrix} l_2 \\ d \\ 0 \end{bmatrix}}$ $d_2^3 =$

$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$ $d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \cancel{H_2^3} H_1^2 H_2^3 \cancel{R_3} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$

$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$ $H_0^1 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 & 0 \\ Sq_1 & Cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H_1^2 = \begin{bmatrix} Cq_2 & 0 & Sq_2 & 0 \\ Sq_2 & 0 & -Cq_2 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $H_2^3 = \begin{bmatrix} 1 & 1 & 1 & l_2 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

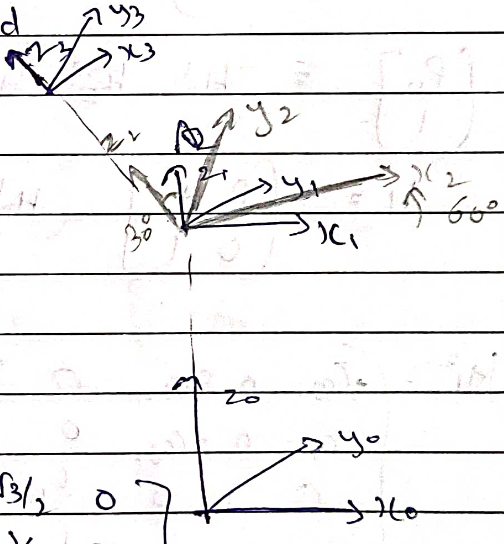
$\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -l_3+d \\ 0 \\ 1 \end{bmatrix}$

$[H_2^3]\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} l_2 - l_3+d \\ -l_3+d \\ -l_3+d \\ 1 \end{bmatrix}$

$$\begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} = H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} -c_{q_2}(l_2 - l_3 + d) + s_{q_2}(-l_3 + d) \\ s_{q_2}(l_2 - l_3 + d) + c_{q_2}(l_3 - d) \\ l_1 - l_3 + d \\ 1 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} a c_{q_1} - b s_{q_1} \\ a s_{q_1} + b c_{q_1} \\ c \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_{q_1}\left[-c_{q_2}(l_2 - l_3 + d) + s_{q_2}(-l_3 + d)\right] - s_{q_1}\left[s_{q_2}(l_2 - l_3 + d) + c_{q_2}(l_3 - d)\right] \\ s_{q_1}\left[-c_{q_2}(l_2 - l_3 + d) + s_{q_2}(-l_3 + d)\right] + c_{q_1}\left[s_{q_2}(l_2 - l_3 + d) + c_{q_2}(l_3 - d)\right] \\ l_1 - l_3 + d \\ 1 \end{bmatrix}$$



5. $R_0^1 = R_{z,0} = I_3$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_1^2 = [R_{x,\pi/6}][R_{z,\pi/3}]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \quad R_2^3 = R_{z,0} = I_3$$

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$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 0 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= H_0^1 H_1^2 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= H_0^1 \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 & -1.5 & 12.598 \end{bmatrix}^T$$

Ans

7. $R_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

$P_3 = \begin{bmatrix} 0 \\ 0 \\ d-l_3 \end{bmatrix}$ These all are already calculated in question 2.

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$$R_0^2 = R_0^1 R_1^2 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} Cq_1 Cq_2 - Sq_1 q_2 & -Cq_1 Sq_2 - Sq_1 Cq_2 & 0 \\ Sq_1 Cq_2 + Sq_2 Cq_1 & -Sq_1 Sq_2 + Cq_1 Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} C(q_1+q_2) & -S(q_1+q_2) & 0 \\ S(q_1+q_2) & C(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^3 = R_0^2 R_2^3 \qquad (R_2^3 \text{ is Identity matrix})$$

$$\therefore R_0^3 = \begin{bmatrix} C(q_1+q_2) & -S(q_1+q_2) & 0 \\ S(q_1+q_2) & C(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = H_0^1 H_1^2 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 & 0 \\ Sq_1 & Cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 & l_1 \\ Sq_2 & Cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} C(q_1+q_2) & -S(q_1+q_2) & 0 & Cq_1 l_1 \\ S(q_1+q_2) & C(q_1+q_2) & 0 & Sq_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_0^2 = \begin{bmatrix} Cq_1 l_1 \\ Sq_1 l_1 \\ 0 \end{bmatrix}$$

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$$H_0^3 = H_0^2 H_2^3 = \begin{bmatrix} C(q_1+q_2) & -S(q_1+q_2) & 0 & Cq_1 l_1 \\ S(q_1+q_2) & C(q_1+q_2) & 0 & Sq_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} C(q_1+q_2) & -S(q_1+q_2) & 0 & Cq_1 l_1 + C(q_1+q_2) l_2 \\ S(q_1+q_2) & C(q_1+q_2) & 0 & Sq_1 l_1 + S(q_1+q_2) l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} Cq_1 l_1 + C(q_1+q_2) l_2 \\ Sq_1 l_1 + S(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

	Prismatic	Rotational
Linear	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

In SCARA, RRP there are 3 joints $\Rightarrow (n=3)$

$$R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \times \begin{bmatrix} Cq_1 l_1 + C(q_1+q_2) l_2 \\ Sq_2 l_2 + S(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -Sq_1 l_1 - S(q_1+q_2) l_2 \\ Cq_1 l_1 + C(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

$$R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_1^0) = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_1^0)$$

$$= \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} Cq_1 l_1 + C(q_1+q_2) l_2 \\ Sq_1 l_1 + S(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -Sq_1 l_1 - S(q_1+q_2) l_2 \\ Cq_1 l_1 + C(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

$$\cancel{R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} =} \begin{bmatrix}0\\0\\1\end{bmatrix}$$

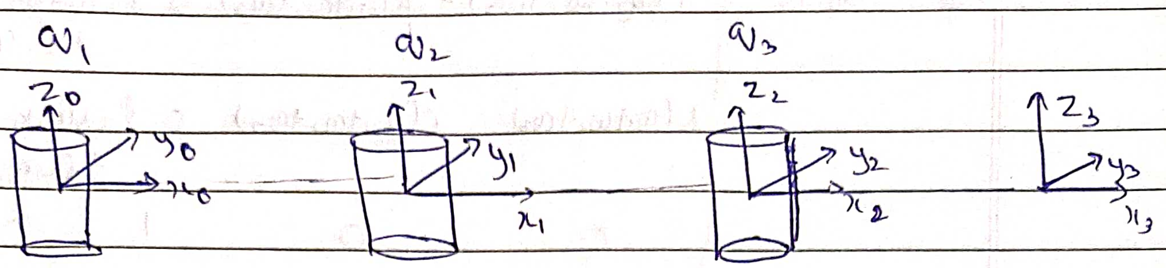
$$\underbrace{R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix}}_{= \begin{bmatrix}0\\0\\1\end{bmatrix}} \times (d_3^0 - d_2^0) = \begin{bmatrix}0\\0\\1\end{bmatrix} \begin{bmatrix} C(q_1+q_2) l_2 \\ S(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -S(q_1+q_2) l_2 \\ C(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

$$R_3^0 \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$J = \begin{bmatrix} -Sq_1 l_1 - S(q_1+q_2) l_2 & -S(q_1+q_2) l_2 & 0 \\ Cq_1 l_1 + C(q_1+q_2) l_2 & C(q_1+q_2) l_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

9.



$$H_0^1 = \begin{pmatrix} Cq_1 & -Sq_1 & 0 & l_1 Cq_1 \\ Sq_1 & Cq_1 & 0 & l_1 Sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_1^2 = \begin{bmatrix} Cq_2 & -Sq_2 & 0 & l_2 Cq_2 \\ Sq_2 & Cq_2 & 0 & l_2 Sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{pmatrix} Cq_3 & -Sq_3 & 0 & l_3 Cq_3 \\ Sq_3 & Cq_3 & 0 & l_3 Sq_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_0^2 = H_0^1 H_1^2 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 & l_1 Cq_1 \\ Sq_1 & Cq_1 & 0 & l_1 Sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 & l_2 Cq_2 \\ Sq_2 & Cq_2 & 0 & l_2 Sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} C(q_1+q_2) & -S(q_1+q_2) & 0 & l_2 C(q_1+q_2) + l_1 Cq_1 \\ S(q_1+q_2) & C(q_1+q_2) & 0 & l_2 S(q_1+q_2) + l_1 Sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = H_0^2 H_2^3 = \begin{pmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 & l_2 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 cq_1 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 & l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$J = \begin{pmatrix} R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_1^0) & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_2^0) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{pmatrix}$$

$$J = \begin{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 cq_1 \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 sq_1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} l_3 c(q_1+q_2+q_3) \\ l_3 s(q_1+q_2+q_3) \\ 0 \end{bmatrix} \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{pmatrix}$$

$$J = \begin{pmatrix} -(l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 sq_1) & -[l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2)] & -l_3 s(q_1+q_2+q_3) \\ l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 cq_1 & l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) & l_3 c(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$