

$$\text{In[1]:= } \mathbf{x1[t] = L1 Sin[ang1[t]]}$$

$$\mathbf{y1[t] = -L1 Cos[ang1[t]]}$$

$$\mathbf{x2[t] = x1[t] + L2 Sin[ang2[t]]}$$

$$\mathbf{y2[t] = y1[t] - L2 Cos[ang2[t]]}$$

$$\text{Out[1]= } L1 Sin[ang1[t]]$$

$$\text{Out[2]= } -L1 Cos[ang1[t]]$$

$$\text{Out[3]= } L1 Sin[ang1[t]] + L2 Sin[ang2[t]]$$

$$\text{Out[4]= } -L1 Cos[ang1[t]] - L2 Cos[ang2[t]]$$

$$\text{In[7]:= } \mathbf{T1 = 1/2 M1 (D[x1[t], t]^2 + D[y1[t], t]^2)}$$

$$\text{Out[7]= } \frac{1}{2} M1 (L1^2 Cos[ang1[t]]^2 ang1'[t]^2 + L1^2 Sin[ang1[t]]^2 ang1'[t]^2)$$

$$\text{In[8]:= } \mathbf{U1 = M1 g (y1[t] + L1)}$$

$$\text{Out[8]= } g M1 (L1 - L1 Cos[ang1[t]])$$

$$\text{In[9]:= } \mathbf{T2 = 1/2 M2 (D[x2[t], t]^2 + D[y2[t], t]^2)}$$

$$\text{Out[9]= } \frac{1}{2} M2 ((L1 Cos[ang1[t]] ang1'[t] + L2 Cos[ang2[t]] ang2'[t])^2 + (L1 Sin[ang1[t]] ang1'[t] + L2 Sin[ang2[t]] ang2'[t])^2)$$

$$\text{In[10]:= } \mathbf{U2 = M2 g (L1 + L2 + y2[t])}$$

$$\text{Out[10]= } g M2 (L1 + L2 - L1 Cos[ang1[t]] - L2 Cos[ang2[t]])$$

$$\text{In[11]:= } \mathbf{L = T1 + T2 - (U1 + U2)}$$

$$\begin{aligned} \text{Out[11]= } & -g M1 (L1 - L1 Cos[ang1[t]]) - g M2 (L1 + L2 - L1 Cos[ang1[t]] - L2 Cos[ang2[t]]) + \\ & \frac{1}{2} M1 (L1^2 Cos[ang1[t]]^2 ang1'[t]^2 + L1^2 Sin[ang1[t]]^2 ang1'[t]^2) + \\ & \frac{1}{2} M2 ((L1 Cos[ang1[t]] ang1'[t] + L2 Cos[ang2[t]] ang2'[t])^2 + \\ & (L1 Sin[ang1[t]] ang1'[t] + L2 Sin[ang2[t]] ang2'[t])^2) \end{aligned}$$

In[12]:= **E1 = D[L, ang1[t]] - D[D[L, ang1'[t]], t]**

Out[12]= 
$$\begin{aligned} & -g L1 M1 \sin[\text{ang1}[t]] - g L1 M2 \sin[\text{ang1}[t]] + \\ & \frac{1}{2} M2 (-2 L1 \sin[\text{ang1}[t]] \text{ang1}'[t] (L1 \cos[\text{ang1}[t]] \text{ang1}'[t] + L2 \cos[\text{ang2}[t]] \text{ang2}'[t]) + \\ & \quad 2 L1 \cos[\text{ang1}[t]] \text{ang1}'[t] (L1 \sin[\text{ang1}[t]] \text{ang1}'[t] + L2 \sin[\text{ang2}[t]] \text{ang2}'[t])) - \\ & \frac{1}{2} M1 (2 L1^2 \cos[\text{ang1}[t]]^2 \text{ang1}''[t] + 2 L1^2 \sin[\text{ang1}[t]]^2 \text{ang1}''[t]) - \\ & \frac{1}{2} M2 (-2 L1 \sin[\text{ang1}[t]] \text{ang1}'[t] (L1 \cos[\text{ang1}[t]] \text{ang1}'[t] + L2 \cos[\text{ang2}[t]] \text{ang2}'[t]) + \\ & \quad 2 L1 \cos[\text{ang1}[t]] \text{ang1}'[t] (L1 \sin[\text{ang1}[t]] \text{ang1}'[t] + L2 \sin[\text{ang2}[t]] \text{ang2}'[t]) + \\ & \quad 2 L1 \cos[\text{ang1}[t]] (-L1 \sin[\text{ang1}[t]] \text{ang1}'[t]^2 - L2 \sin[\text{ang2}[t]] \text{ang2}'[t]^2 + \\ & \quad L1 \cos[\text{ang1}[t]] \text{ang1}''[t] + L2 \cos[\text{ang2}[t]] \text{ang2}''[t]) + \\ & \quad 2 L1 \sin[\text{ang1}[t]] (L1 \cos[\text{ang1}[t]] \text{ang1}'[t]^2 + L2 \cos[\text{ang2}[t]] \text{ang2}'[t]^2 + \\ & \quad L1 \sin[\text{ang1}[t]] \text{ang1}''[t] + L2 \sin[\text{ang2}[t]] \text{ang2}''[t])) \end{aligned}$$

In[13]:= **E2 = D[L, ang2[t]] - D[D[L, ang2'[t]], t]**

Out[13]= 
$$\begin{aligned} & -g L2 M2 \sin[\text{ang2}[t]] + \\ & \frac{1}{2} M2 (-2 L2 \sin[\text{ang2}[t]] \text{ang2}'[t] (L1 \cos[\text{ang1}[t]] \text{ang1}'[t] + L2 \cos[\text{ang2}[t]] \text{ang2}'[t]) + \\ & \quad 2 L2 \cos[\text{ang2}[t]] \text{ang2}'[t] (L1 \sin[\text{ang1}[t]] \text{ang1}'[t] + L2 \sin[\text{ang2}[t]] \text{ang2}'[t])) - \\ & \frac{1}{2} M2 (-2 L2 \sin[\text{ang2}[t]] \text{ang2}'[t] (L1 \cos[\text{ang1}[t]] \text{ang1}'[t] + L2 \cos[\text{ang2}[t]] \text{ang2}'[t]) + \\ & \quad 2 L2 \cos[\text{ang2}[t]] \text{ang2}'[t] (L1 \sin[\text{ang1}[t]] \text{ang1}'[t] + L2 \sin[\text{ang2}[t]] \text{ang2}'[t]) + \\ & \quad 2 L2 \cos[\text{ang2}[t]] (-L1 \sin[\text{ang1}[t]] \text{ang1}'[t]^2 - L2 \sin[\text{ang2}[t]] \text{ang2}'[t]^2 + \\ & \quad L1 \cos[\text{ang1}[t]] \text{ang1}''[t] + L2 \cos[\text{ang2}[t]] \text{ang2}''[t]) + \\ & \quad 2 L2 \sin[\text{ang2}[t]] (L1 \cos[\text{ang1}[t]] \text{ang1}'[t]^2 + L2 \cos[\text{ang2}[t]] \text{ang2}'[t]^2 + \\ & \quad L1 \sin[\text{ang1}[t]] \text{ang1}''[t] + L2 \sin[\text{ang2}[t]] \text{ang2}''[t])) \end{aligned}$$

In[14]:= **Solve**[{E1 == 0, E2 == 0}, {ang1''[t], ang2''[t]}]

Out[14]=  $\left\{ \left\{ \text{ang1}''[t] \rightarrow - \left( \left( g M1 \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]] + g M2 \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]] - \right. \right. \right.$   
 $g M2 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]] \sin[\text{ang2}[t]] + g M1 \sin[\text{ang1}[t]] \sin[\text{ang2}[t]]^2 +$   
 $L1 M2 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]] \text{ang1}'[t]^2 -$   
 $L1 M2 \cos[\text{ang1}[t]]^2 \cos[\text{ang2}[t]] \sin[\text{ang2}[t]] \text{ang1}'[t]^2 +$   
 $L1 M2 \cos[\text{ang2}[t]] \sin[\text{ang1}[t]]^2 \sin[\text{ang2}[t]] \text{ang1}'[t]^2 -$   
 $L1 M2 \cos[\text{ang1}[t]] \sin[\text{ang1}[t]] \sin[\text{ang2}[t]]^2 \text{ang1}'[t]^2 +$   
 $L2 M2 \cos[\text{ang2}[t]]^3 \sin[\text{ang1}[t]] \text{ang2}'[t]^2 -$   
 $L2 M2 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]]^2 \sin[\text{ang2}[t]] \text{ang2}'[t]^2 +$   
 $L2 M2 \cos[\text{ang2}[t]] \sin[\text{ang1}[t]] \sin[\text{ang2}[t]]^2 \text{ang2}'[t]^2 -$   
 $L2 M2 \cos[\text{ang1}[t]] \sin[\text{ang2}[t]]^3 \text{ang2}'[t]^2 \Big) /$   
 $\left( L1 \left( M1 \cos[\text{ang1}[t]]^2 \cos[\text{ang2}[t]]^2 + M1 \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]]^2 + \right. \right.$   
 $M2 \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]]^2 - 2 M2 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]]$   
 $\sin[\text{ang1}[t]] \sin[\text{ang2}[t]] + M1 \cos[\text{ang1}[t]]^2 \sin[\text{ang2}[t]]^2 +$   
 $M2 \cos[\text{ang1}[t]]^2 \sin[\text{ang2}[t]]^2 + M1 \sin[\text{ang1}[t]]^2 \sin[\text{ang2}[t]]^2 \Big) \Big),$   
 $\text{ang2}''[t] \rightarrow - \left( \left( -g M1 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]] \sin[\text{ang1}[t]] - \right. \right.$   
 $g M2 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]] \sin[\text{ang1}[t]] +$   
 $g M1 \cos[\text{ang1}[t]]^2 \sin[\text{ang2}[t]] + g M2 \cos[\text{ang1}[t]]^2 \sin[\text{ang2}[t]] -$   
 $L1 M1 \cos[\text{ang1}[t]]^2 \cos[\text{ang2}[t]] \sin[\text{ang1}[t]] \text{ang1}'[t]^2 -$   
 $L1 M2 \cos[\text{ang1}[t]]^2 \cos[\text{ang2}[t]] \sin[\text{ang1}[t]] \text{ang1}'[t]^2 -$   
 $L1 M1 \cos[\text{ang2}[t]] \sin[\text{ang1}[t]]^3 \text{ang1}'[t]^2 -$   
 $L1 M2 \cos[\text{ang2}[t]] \sin[\text{ang1}[t]]^3 \text{ang1}'[t]^2 + L1 M1 \cos[\text{ang1}[t]]^3$   
 $\sin[\text{ang2}[t]] \text{ang1}'[t]^2 + L1 M2 \cos[\text{ang1}[t]]^3 \sin[\text{ang2}[t]] \text{ang1}'[t]^2 +$   
 $L1 M1 \cos[\text{ang1}[t]] \sin[\text{ang1}[t]]^2 \sin[\text{ang2}[t]] \text{ang1}'[t]^2 +$   
 $L1 M2 \cos[\text{ang1}[t]] \sin[\text{ang1}[t]]^2 \sin[\text{ang2}[t]] \text{ang1}'[t]^2 -$   
 $L2 M2 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]] \text{ang2}'[t]^2 +$   
 $L2 M2 \cos[\text{ang1}[t]]^2 \cos[\text{ang2}[t]] \sin[\text{ang2}[t]] \text{ang2}'[t]^2 -$   
 $L2 M2 \cos[\text{ang2}[t]] \sin[\text{ang1}[t]]^2 \sin[\text{ang2}[t]] \text{ang2}'[t]^2 +$   
 $L2 M2 \cos[\text{ang1}[t]] \sin[\text{ang1}[t]] \sin[\text{ang2}[t]]^2 \text{ang2}'[t]^2 \Big) /$   
 $\left( L2 \left( M1 \cos[\text{ang1}[t]]^2 \cos[\text{ang2}[t]]^2 + M1 \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]]^2 + \right. \right.$   
 $M2 \cos[\text{ang2}[t]]^2 \sin[\text{ang1}[t]]^2 - 2 M2 \cos[\text{ang1}[t]] \cos[\text{ang2}[t]]$   
 $\sin[\text{ang1}[t]] \sin[\text{ang2}[t]] + M1 \cos[\text{ang1}[t]]^2 \sin[\text{ang2}[t]]^2 +$   
 $M2 \cos[\text{ang1}[t]]^2 \sin[\text{ang2}[t]]^2 + M1 \sin[\text{ang1}[t]]^2 \sin[\text{ang2}[t]]^2 \Big) \Big) \Big\}$

In[15]:= **FullSimplify**[%]

Out[15]=  $\left\{ \left\{ \text{ang1}''[t] \rightarrow - \left( \left( g (2 M1 + M2) \sin[\text{ang1}[t]] + g M2 \sin[\text{ang1}[t] - 2 \text{ang2}[t]] + 2 M2 \right. \right. \right.$   
 $\sin[\text{ang1}[t] - \text{ang2}[t]] (L1 \cos[\text{ang1}[t] - \text{ang2}[t]] \text{ang1}'[t]^2 + L2 \text{ang2}'[t]^2) \Big) /$   
 $(L1 (2 M1 + M2 - M2 \cos[2 (\text{ang1}[t] - \text{ang2}[t])])) \Big),$   
 $\text{ang2}''[t] \rightarrow (2 \sin[\text{ang1}[t] - \text{ang2}[t]] ((M1 + M2) (g \cos[\text{ang1}[t]] + L1 \text{ang1}'[t]^2) +$   
 $L2 M2 \cos[\text{ang1}[t] - \text{ang2}[t]] \text{ang2}'[t]^2) /$   
 $(L2 (2 M1 + M2 - M2 \cos[2 (\text{ang1}[t] - \text{ang2}[t])])) \Big) \Big\}$

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In[18]= Experimental`OptimizeExpression[
  { - ( (g (2 M1 + M2) Sin[ang1[t]] + g M2 Sin[ang1[t] - 2 ang2[t]] +
    2 M2 Sin[ang1[t] - ang2[t]] (L1 Cos[ang1[t] - ang2[t]] ang1'[t]^2 + L2 ang2'[t]^2)) /
    (L1 (2 M1 + M2 - M2 Cos[2 (ang1[t] - ang2[t]]))) , (2 Sin[ang1[t] - ang2[t]]
    ((M1 + M2) (g Cos[ang1[t]] + L1 ang1'[t]^2) + L2 M2 Cos[ang1[t] - ang2[t]] ang2'[t]^2)) /
    (L2 (2 M1 + M2 - M2 Cos[2 (ang1[t] - ang2[t]]))) ) , OptimizationLevel -> 2]

Out[18]= Experimental`OptimizedExpression[
  Block[{Compile`$1, Compile`$2, Compile`$3, Compile`$4,
    Compile`$5, Compile`$6, Compile`$7, Compile`$8, Compile`$9,
    Compile`$10, Compile`$11, Compile`$22, Compile`$23, Compile`$25,
    Compile`$26, Compile`$32, Compile`$20, Compile`$21}, Compile`$1 =  $\frac{1}{L1}$ ;

  Compile`$2 = 2 M1;
  Compile`$3 = ang1[t];
  Compile`$4 = ang2[t];
  Compile`$5 = -Compile`$4;
  Compile`$6 = Compile`$3 + Compile`$5;
  Compile`$7 = 2 Compile`$6;
  Compile`$8 = Cos[Compile`$7];
  Compile`$9 = -M2 Compile`$8;
  Compile`$10 = Compile`$2 + M2 + Compile`$9;
  Compile`$11 =  $\frac{1}{\text{Compile`$10}}$ ;
  Compile`$22 = ang1'[t];
  Compile`$23 = Compile`$22^2;
  Compile`$25 = ang2'[t];
  Compile`$26 = Compile`$25^2;
  Compile`$32 =  $\frac{1}{L2}$ ;
  Compile`$20 = Sin[Compile`$6];
  Compile`$21 = Cos[Compile`$6];
  { -Compile`$1 Compile`$11 (g (Compile`$2 + M2) Sin[Compile`$3] +
    g M2 Sin[Compile`$3 - 2 Compile`$4] + 2 M2 Compile`$20 (L1 Compile`$21
    Compile`$23 + L2 Compile`$26)) , 2 Compile`$32 Compile`$11 Compile`$20
    ((M1 + M2) (g Cos[Compile`$3] + L1 Compile`$23) + L2 M2 Compile`$21 Compile`$26)) } ] ]

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