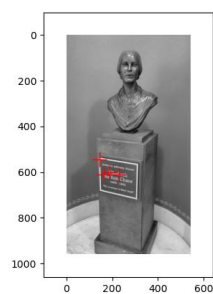


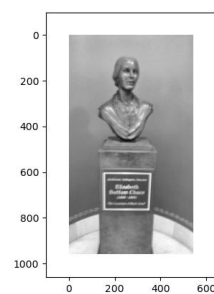
1.1

(a)

结果如下:

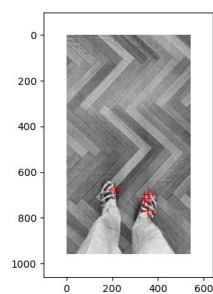


Chase1

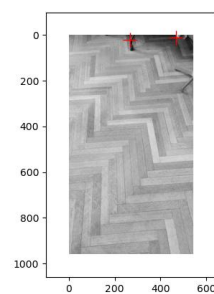


Chase2

图 1: Chase 组

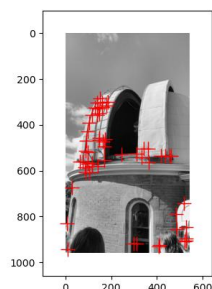


RISHLibrary1

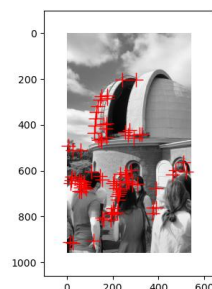


RISHLibrary2

图 2: RISHLibrary



LaddObservatory1



LaddObservatory2

图 3: LaddObservatory 组

(b)

对于 Chase 组, Chase1 找到了比较明显的几个 corners, Chase2 则没有找到, 从原图中可以看到 Chase2 比 Chase1 模糊了很多。

对于 RISHLibrary 组, RISHLibrary1 找到的是鞋子的 corners, RISHLibrary2 找到的是桌角的 corners, 他们都是原图中比较显眼的地方。同时两张原图的光线存在差异。

对于 LaddObservatory 组, 在建筑物上两个结果相差不是特别大, 对于 LaddObservatory2 中的人更多识别到的关于人的 corners 也更多。两张原图应该是不同视角拍摄下来的图片。

从三个对照组来看, 我认为匹配图像特征存在的挑战有:

1. 视角和尺度变化
2. 遮挡
3. 光线变化
4. 噪声问题

1.2

(a)

1. **欧氏距离**是指两个向量之间的直线距离，即它们在空间中的几何距离。在二维平面上，欧氏距离可以被视为连接两个点的直线的长度。在更高维度的空间中，欧氏距离可以看作是两个向量之间的“直线”距离。它主要衡量了向量之间的“接近程度”，距离越短表示向量越相似。当关注向量的绝对值大小和空间位置时，欧氏距离是更合适的选择。例如，在图像处理中，如果需要比较两幅图像的像素值之间的相似性，可以使用欧氏距离来衡量它们之间的距离。
2. **余弦相似度**是指两个向量之间的夹角的余弦值，它衡量了向量之间的方向相似度而不考虑它们的大小。如果两个向量的方向越接近，余弦相似度就越接近于 1；如果它们的方向正交或者相反，余弦相似度就越接近于 0。余弦相似度衡量了向量之间的“方向相似度”，而不受向量大小的影响。当关注向量之间的方向相似度而不考虑它们的绝对大小时，余弦相似度更适合。例如，在自然语言处理中，可以使用余弦相似度来比较文档之间的相似性，因为文档的长度可能不同，但它们的主题方向可能是相似的。

(b)

我认为给定距离度量方式后，一个比较好的特征描述符匹配方法是 nearest neighbor 算法。因为他比较简单，直观且易于理解，在欧氏距离和余弦相似度两种距离度量上都比较适用，还可以通过 KD-Tree 和 Ball-Tree 等数据结构来加速大规模数据集的匹配。

1.3

$$(a) y = \lambda n + x$$

$$x = (u, v) \Rightarrow (u, v, 1)$$

$$n = (a, b)$$

$$y = \lambda \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \lambda a + u \\ \lambda b + v \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda a + u \\ \lambda b + v \\ 1 \end{pmatrix}$$

$$y' = Hy = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} \lambda a + u \\ \lambda b + v \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} h_{11}(\lambda a + u) + h_{12}(\lambda b + v) + h_{13} \\ h_{21}(\lambda a + u) + h_{22}(\lambda b + v) + h_{23} \\ h_{31}(\lambda a + u) + h_{32}(\lambda b + v) + h_{33} \end{pmatrix}$$

(b) 当 $h_{31}(\lambda a + u) + h_{32}(\lambda b + v) + h_{33}$ 无穷大,

即 l 与投影平面平行时,

当 l 平行于投影平面或沿投影方向时, l' 收敛于一个消失点.

$$y'_v = \lim_{\lambda \rightarrow \infty} \left(\frac{h_{11}\lambda a + h_{12}\lambda b + h_{13}}{h_{31}\lambda a + h_{32}\lambda b + h_{33}}, \frac{h_{21}\lambda a + h_{22}\lambda b + h_{23}}{h_{31}\lambda a + h_{32}\lambda b + h_{33}} \right)$$

$$= \left(\frac{h_{11}a + h_{12}b}{h_{31}a + h_{32}b}, \frac{h_{21}a + h_{22}b}{h_{31}a + h_{32}b} \right).$$

(c) 当 H 不是有效的变换矩阵,

或 l 与图像平面的平行线相交时.

2.3

(a)

实现如下:

```
1     def get_interest_points(image, feature_width):
2
3     # 设置参数
4     sigma = 1.0
5     k = 0.06
6     threshold = 0.1
7
8     # 从main.py中可以知道传入的图片都是灰度图，可以直接
      计算Ix, Iy,求得海森矩阵
9     Ix = filters.sobel_v(image=image)
10    Iy = filters.sobel_h(image=image)
11
12    Ixx = Ix**2
13    Iyy = Iy**2
14    Ixy = Ix * Iy
15
16    # 将高斯滤波器用于上面算子
17    Ixx = filters.gaussian(Ixx, sigma=sigma)
18    Iyy = filters.gaussian(Iyy, sigma=sigma)
19    Ixy = filters.gaussian(Ixy, sigma=sigma)
20
21    # 计算每个像素的  $R = \det(H) - k(\text{trace}(H))^2$ 。 $\det(H)$ 
      表示矩阵H的行列式， $\text{trace}$ 表示矩阵H的迹。通常k的
      取值范围为[0.04,0.16]。
22    detH = (Ixx * Iyy) - (Ixy ** 2)
23    traceH = Ixx + Iyy
24    R = detH - k * ((traceH) ** 2)
25
26    # 满足  $R \geq \max_{\text{R}} * \text{th}$  的像素点即为角点。th常取0.1，
```

```

    不知道peak_local_max的threshold_rel是不是就是在
    做这个
27  # 参数中的feature_width好像是没用的，又或者说他与
    min_distance有关？我感觉它只是在get_features中
    会用到
28  points = feature.peak_local_max(R, min_distance=
    feature_width, threshold_rel=threshold)
29
30  xs = points[:, 1]
31  ys = points[:, 0]
32
33  return xs, ys

```

(b)

实现如下:

normalized patches:

```

1  def get_features(image, x, y, feature_width):
2  features = []
3
4  # Iterate over interest points
5  for i in range(len(x)):
6      # Get the coordinates of the current interest
        point
7      x_coord = int(x[i])
8      y_coord = int(y[i])
9
10     # Calculate the half-width of the feature
        patch
11     half_width = feature_width // 2
12
13     # Extract the patch around the interest point
14     patch = image[y_coord - half_width:y_coord +

```

```

15         half_width,
16         x_coord - half_width:x_coord +
17         half_width]
18
19     # Normalize the patch to range [0, 1] for each
20     channel
21     patch_normalized = patch / 255.0
22
23     # Flatten the patch and append it to the
24     features list
25     features.append(patch_normalized.flatten())
26
27 # Convert features list to numpy array
28 features = np.array(features)
29
30 return features

```

SIFT:

```

1  def get_features(image, x, y, feature_width):
2      h = image.shape[0]
3      w = image.shape[1]
4
5      offset = feature_width // 2
6      num_points = x.shape[0]
7
8      # 计算索引范围
9      x_start = x - offset
10     x_stop = x + offset
11     y_start = y - offset
12     y_stop = y + offset
13
14     # Compute padding parameters
15     x_min = x_start.min()

```

```

16     x_max = x_stop.max()
17     y_min = y_start.min()
18     y_max = y_stop.max()
19
20     x_pad = [0, 0]
21     y_pad = [0, 0]
22     if x_min < 0:
23         x_pad[0] = -x_min
24     if y_min < 0:
25         y_pad[0] = -y_min
26     if x_max - w >= 0:
27         x_pad[1] = x_max - w + 1
28     if y_max - h >= 0:
29         y_pad[1] = y_max - h + 1
30
31     x_start += x_pad[0]
32     x_stop += x_pad[0]
33     y_start += y_pad[0]
34     y_stop += y_pad[0]
35
36     # 对图片进行padding
37     image = np.pad(image, [y_pad, x_pad], mode="
        constant")
38
39     # 对每个维度窗口下建立索引
40     cell_size = 4
41     num_blocks = feature_width // cell_size
42
43     x_idx = np.array([np.arange(start, stop) for start
        , stop in zip(x_start, x_stop)])
44     y_idx = np.array([np.arange(start, stop) for start
        , stop in zip(y_start, y_stop)])
45

```



```

46 # 转置之前 : (num_blocks, num_points, cell_size)
47 x_idx = np.array(np.split(x_idx, num_blocks, axis
48                          =1))
49 y_idx = np.array(np.split(y_idx, num_blocks, axis
50                          =1))
51
52 # 转置之后: (num_points, num_blocks, cell_size)
53 x_idx = x_idx.transpose([1, 0, 2])
54 y_idx = y_idx.transpose([1, 0, 2])
55
56 # 对窗口中的每个像素建立索引
57 x_idx = np.tile(np.tile(x_idx, cell_size), [1,
58                          num_blocks, 1]).flatten()
59 y_idx = np.tile(np.repeat(y_idx, cell_size, axis
60                          =2), num_blocks).flatten()
61
62 # 计算偏导数
63 partial_x = filters.sobel_h(image)
64 partial_y = filters.sobel_v(image)
65 # 计算梯度模长
66 magnitude = np.sqrt(partial_x * partial_x +
67                      partial_y * partial_y)
68 # 计算梯度方向
69 orientation = np.arctan2(partial_y, partial_x) +
70                     np.pi
71 # 梯度近似为最近的角度
72 orientation = np.mod(np.round(orientation / (2.0 *
73                     np.pi) * 8.0), 8)
74 orientation = orientation.astype(np.int32)
75 # 对梯度做高斯平滑
76 magnitude = filters.gaussian(magnitude, sigma=
77                             offset)

```

```

71 # 所有的块数据转为1维
72 magnitude_in_pixels = magnitude[y_idx, x_idx]
73 orientation_in_pixels = orientation[y_idx, x_idx]
74
75 # 转为数组 (num_patches, cell_size, cell_size)
76 magnitude_in_cells = magnitude_in_pixels.reshape
77     ((-1, cell_size * cell_size))
78 orientation_in_cells = orientation_in_pixels.
79     reshape((-1, cell_size * cell_size))
80
81 # 对每个cel计算梯度方向加权和
82 features = np.array(list(
83     map(lambda array, weight: np.bincount(array,
84         weight, minlength=8), orientation_in_cells,
85         magnitude_in_cells)))
86
87 # 每一行都是角点对应的一个特征向量
88 features = features.reshape((num_points, -1))
89 features = features / np.linalg.norm(features,
90     axis=-1).reshape((-1, 1))
91 features[features >= 0.2] = 0.2
92 features = features / np.linalg.norm(features,
93     axis=-1).reshape((-1, 1))
94
95 return features

```

(c)

实现如下:

```

1 def match_features(im1_features, im2_features):
2
3     assert im1_features.shape[1] == im2_features.shape
4         [1]

```

```

4      # 设置 confidence
5      confidence = 1
6
7      matches = []
8      confidences = []
9
10     for i in range(im1_features.shape[0]):
11         distances = np.sqrt(
12             ((im1_features[i, :] - im2_features) ** 2)
13             .sum(axis=1))
14         indexes = np.argsort(distances)
15         min_distance = distances[indexes[0]]
16         second_min_distance = distances[indexes[1]]
17
18         ratio = min_distance / second_min_distance if
19             second_min_distance != 0 else 0
20         if ratio < confidence:
21             matches.append([i, indexes[0]])
22             confidences.append(1 - ratio)
23
24     matches = np.asarray(matches)
25     confidences = np.asarray(confidences)
26
27     return matches, confidences

```

(d)

(i)

我在 main.py 中看到传入的图像已经做了转化为灰度图的处理，所以我在实现时没有考虑彩色图。实现时主要就是按照 harris corner, SIFT, Nearest neighbor 算法的描述用 numpy 和 skimage 提供的 api 进行实现。实现 Nearest neighbor 时 comment 提到要设置一个 confidence，我设置为 0.9。

(ii)

使用 `cheat_interest_points` 时, 实现 `get_features` 两种方案的结果:

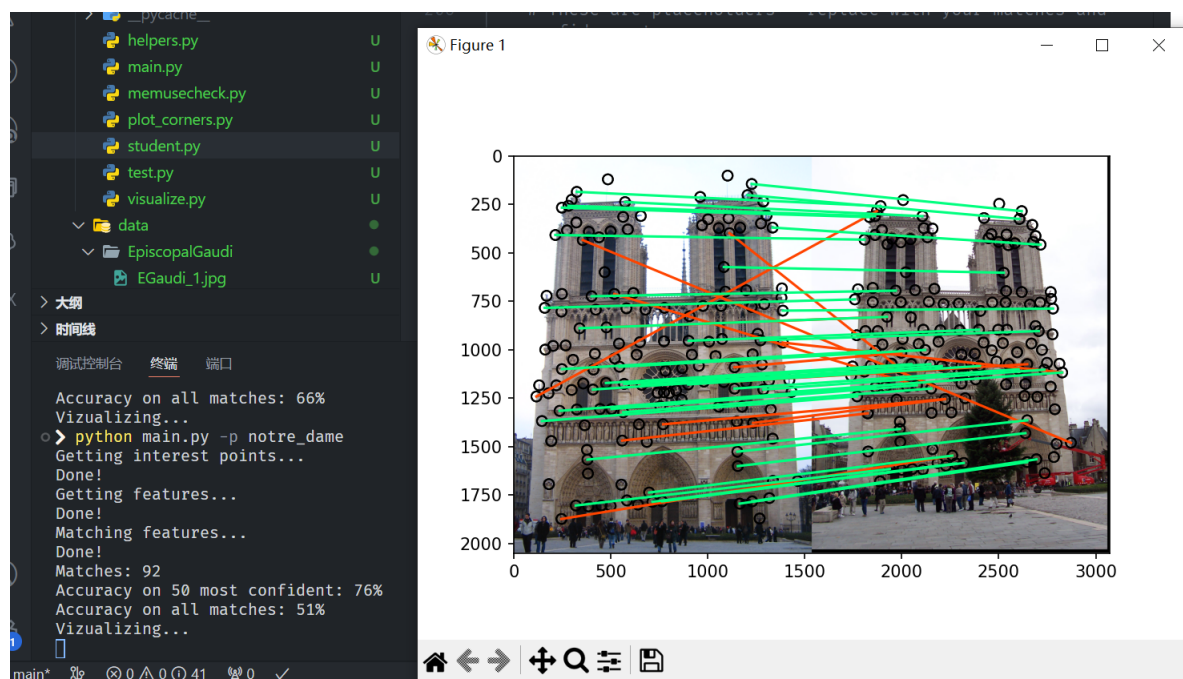


图 5: normalized patches 结果

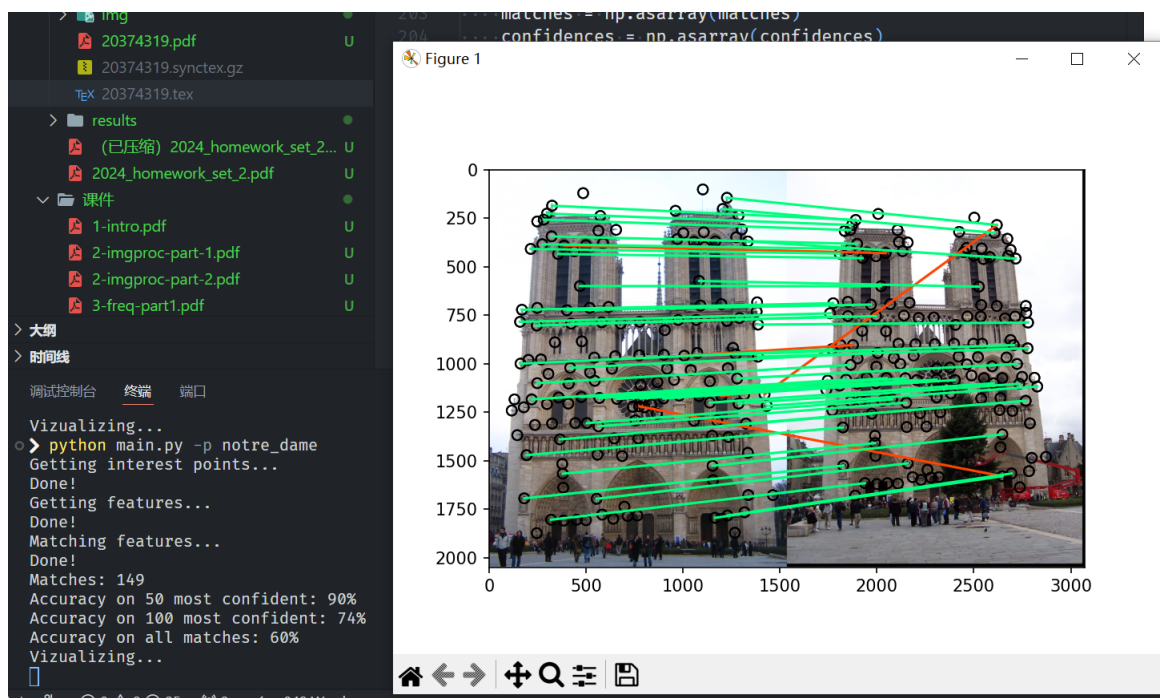


图 6: sift 结果

从图片中可以看到，使用 sift 相比于 normalized patches 大概能提升 10%。

(iii)

结果如下:

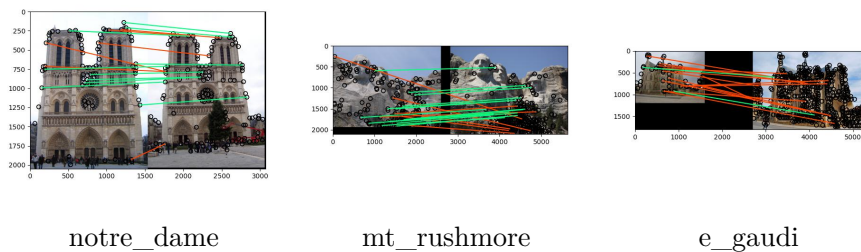
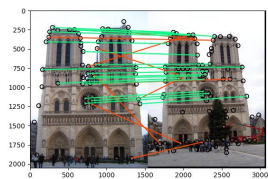


图 7: 结果展示

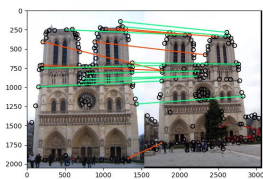
(e)

(i)

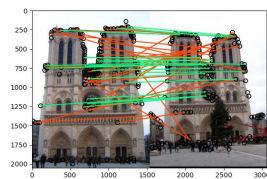
不同 scale 下的结果:



notre_dame_0.3

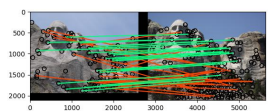


notre_dame_0.5

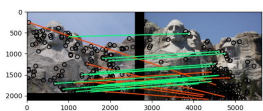


notre_dame_0.8

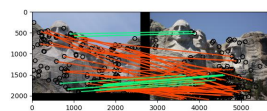
图 8: notre_dame



mt_rushmore_0.3

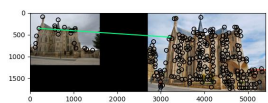


mt_rushmore_0.5

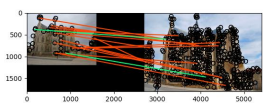


mt_rushmore_0.8

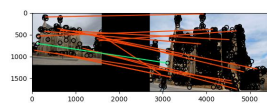
图 9: mt_rushmore



e_gaudi_0.3



e_gaudi_0.5



e_gaudi_0.8

图 10: e_gaudi

(ii)

实现一个 Multi-scale descriptor, 我通过在 params 中接受一个 scales 数组实现:

```
1     def get_features(image_origin, x, y, feature_width
2         , scales):
3     features_final = []
4     for scale in scales:
5         image = np.float32(rescale(image_origin, scale
6             ))
7         h = image.shape[0]
8         w = image.shape[1]
9
10        offset = feature_width // 2
11        num_points = x.shape[0]
12
13        # 计算索引范围
14        x_start = x - offset
15        x_stop = x + offset
16        y_start = y - offset
17        y_stop = y + offset
18
19        # Compute padding parameters
20        x_min = x_start.min()
21        x_max = x_stop.max()
22        y_min = y_start.min()
23        y_max = y_stop.max()
24
25        x_pad = [0, 0]
26        y_pad = [0, 0]
27        if x_min < 0:
28            x_pad[0] = -x_min
29        if y_min < 0:
30            y_pad[0] = -y_min
```

```

29         if x_max - w >= 0:
30             x_pad[1] = x_max - w + 1
31         if y_max - h >= 0:
32             y_pad[1] = y_max - h + 1
33
34         x_start += x_pad[0]
35         x_stop += x_pad[0]
36         y_start += y_pad[0]
37         y_stop += y_pad[0]
38
39         # 对图片进行padding
40         image = np.pad(image, [y_pad, x_pad], mode="
            constant")
41
42         # 对每个维度窗口下建立索引
43         cell_size = 4
44         num_blocks = feature_width // cell_size
45
46         x_idx = np.array([np.arange(start, stop) for
            start, stop in zip(x_start, x_stop)])
47         y_idx = np.array([np.arange(start, stop) for
            start, stop in zip(y_start, y_stop)])
48
49         # 转置之前 : (num_blocks, num_points,
            cell_size)
50         x_idx = np.array(np.split(x_idx, num_blocks,
            axis=1))
51         y_idx = np.array(np.split(y_idx, num_blocks,
            axis=1))
52
53         # 转置之后: (num_points, num_blocks, cell_size
            )
54         x_idx = x_idx.transpose([1, 0, 2])

```



```

55     y_idx = y_idx.transpose([1, 0, 2])
56
57     # 对窗口中的每个像素建立索引
58     x_idx = np.tile(np.tile(x_idx, cell_size), [1,
59                               num_blocks, 1]).flatten()
60     y_idx = np.tile(np.repeat(y_idx, cell_size,
61                               axis=2), num_blocks).flatten()
62
63     # 计算偏导数
64     partial_x = filters.sobel_h(image)
65     partial_y = filters.sobel_v(image)
66     # 计算梯度模长
67     magnitude = np.sqrt(partial_x * partial_x +
68                           partial_y * partial_y)
69     # 计算梯度方向
70     orientation = np.arctan2(partial_y, partial_x)
71     + np.pi
72     # 梯度近似为最近的角度
73     orientation = np.mod(np.round(orientation /
74     (2.0 * np.pi) * 8.0), 8)
75     orientation = orientation.astype(np.int32)
76     # 对梯度做高斯平滑
77     magnitude = filters.gaussian(magnitude, sigma=
78     offset)
79
80     # 所有的块数据转为1维
81     magnitude_in_pixels = magnitude[y_idx, x_idx]
82     orientation_in_pixels = orientation[y_idx,
83     x_idx]
84
85     # 转为数组 (num_patches, cell_size, cell_size)
86     magnitude_in_cells = magnitude_in_pixels.
87     reshape((-1, cell_size * cell_size))

```

```

80         orientation_in_cells = orientation_in_pixels.
            reshape((-1, cell_size * cell_size))
81
82     # 对每个cel计算梯度方向加权和
83     features = np.array(list(
84         map(lambda array, weight: np.bincount(
            array, weight, minlength=8),
            orientation_in_cells,
            magnitude_in_cells)))
85
86     # 每一行都是角点对应的一个特征向量
87     features = features.reshape((num_points, -1))
88     features = features / np.linalg.norm(features,
            axis=-1).reshape((-1, 1))
89     features[features >= 0.2] = 0.2
90     features = features / np.linalg.norm(features,
            axis=-1).reshape((-1, 1))
91     features_final.append(features)
92
93     return features_final

```

(iii)

使用 Ball-Tree 来加速匹配过程:

```

1     def match_features(im1_features, im2_features):
2
3         matches = []
4         confidences = []
5         threshold = 0.9
6         from sklearn.neighbors import KDTree
7         kd_tree = KDTree(im1_features, leaf_size=3)
8
9         dist, ind = kd.query(im2_features, 2)

```

```

10
11     for i in range(len(ind)):
12         index = ind[i]
13         distances = dist[i]
14
15         if distances[0] / distances[1] < threshold:
16             matches.append([index[0], i])
17             confidences.append(1 - distances[0])
18 matches = np.array(matches)
19 confidences = np.array(confidences)
20 return matches, confidences

```