Deep Generative Models: Part I

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[2]ENS Lyon

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April 11, 2025

A Few Words About Me

Academic

- 2018 - 2021 Ph. D. at Inria Saclay: Large-Scale Optimization for Neuroimaging
- 2021 - 2024 Postdoc at Mila and Université de Montréal
- 2024 - now Reasearcher at Inria Lyon, based in Laboratoire Hubert Curien

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Research interests:

- Numerical optimization
- Generative optimization
  - Retraining of generative models on their own data
- Multi-agent reinforcement learning
  - Emergence of cooperation in multi-agent settings

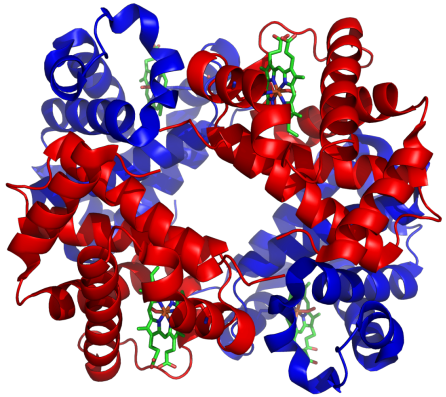
Outline

What are Generative Models?

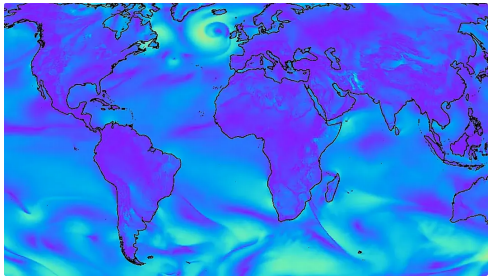
Generative Models are Powerful

Text-to-Image

Text-to-Text



Protein Design



Weather Forecast

Generative Models are Powerful

Text-to-Image

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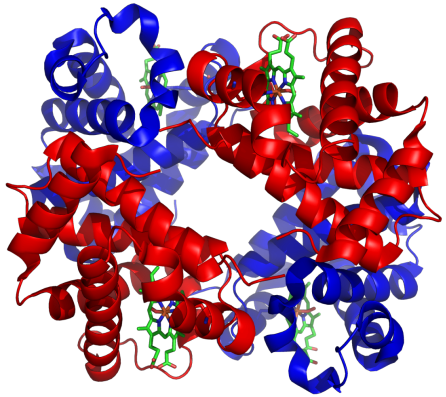

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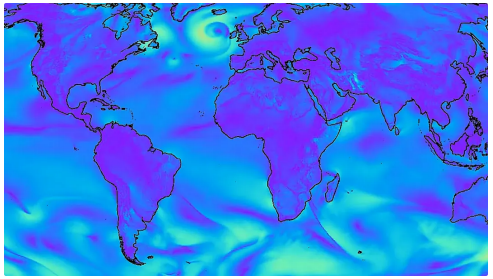

Weather Forecast

Academic References:

- Stability AI (2023)
- Watson et al. (2023)
- Lam et al. (2023)
- Xu et al. (2025)

What are Generative Models 1/3

Data: $x^{(1)}, \ldots, x^{(n)}$

Goal: new synthetic samples $\tilde{x}^{(1)}, \ldots, \tilde{x}^{(n)}$

Generative Models 101:

- **Setting**: Access to samples $\left(x^{(1)}, \ldots, x^{(n)}\right) \rightarrow$ unlabeled data

What are Generative Models 1/3

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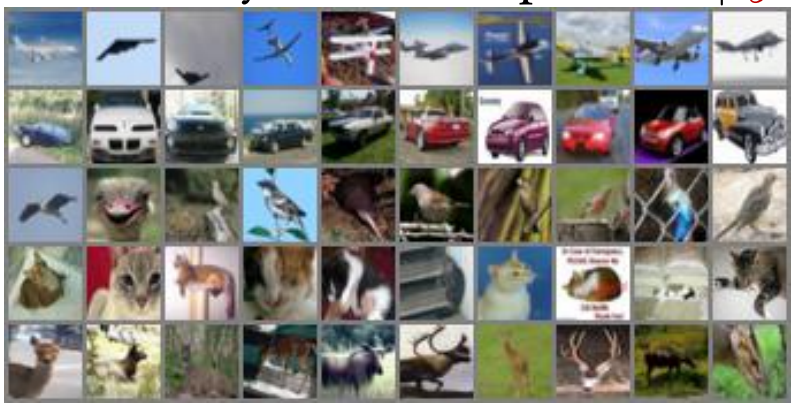
- **Goal**: Create new synthetic samples $\tilde{x}^{(i)} \sim p$
  - *e.g.*, generate new images

What are Generative Models 2/3

Data: $\left(x^{(1)}, y^{(1)}\right), ..., \left(x^{(n)}, y^{(n)}\right)$

Goal: new synthetic samples $\tilde{x}^{(i)} \mid y^{(i)}$



Generative Models 201: Class Conditional Generative Models

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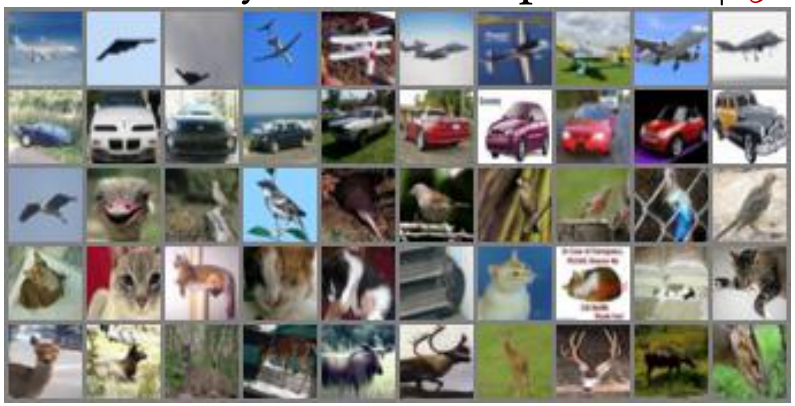

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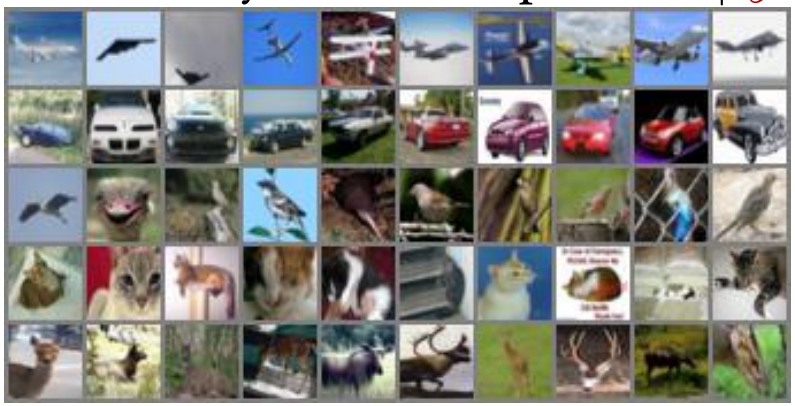

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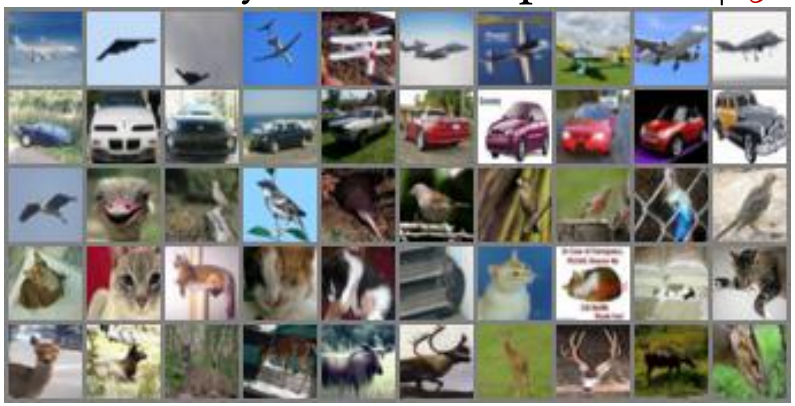

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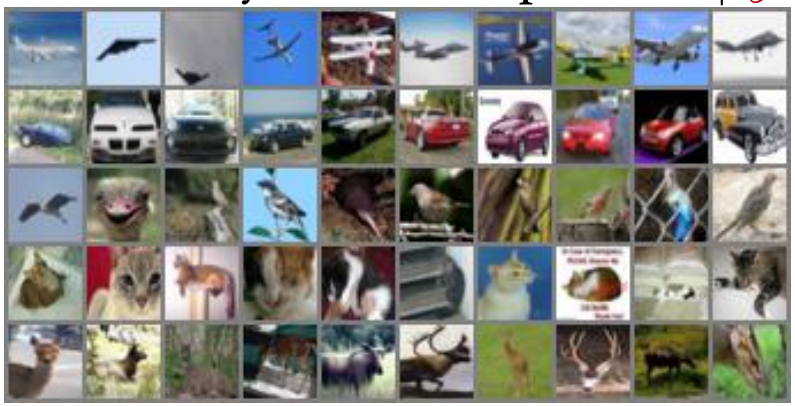

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  - *e.g.*, generate specifically new images of planes

What are Generative Models 3/3

An avocado chair

A photo of Einstein

A house made of sushi

Generative Models 301: Text-to-Image Models

- **Setting**: Access to samples $\left(x^{(1)}, y^{(1)}\right), \ldots, \left(x^{(n)}, y^{(n)}\right) \rightarrow$ pairs (image, caption)

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- **Goal**: Create new synthetic samples $\tilde{x}^{(i)} \sim p\left(\cdot \mid \text{caption } y^{(i)}\right)$
- **Trick**: Use a "meaningful" text/caption representation

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- **Text-to-Text Models**
  - Conditional text generation w.r.t. the previous text sequence

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  - No cross-validation
  - **Very challenging** for text-to-image and text-to-text models

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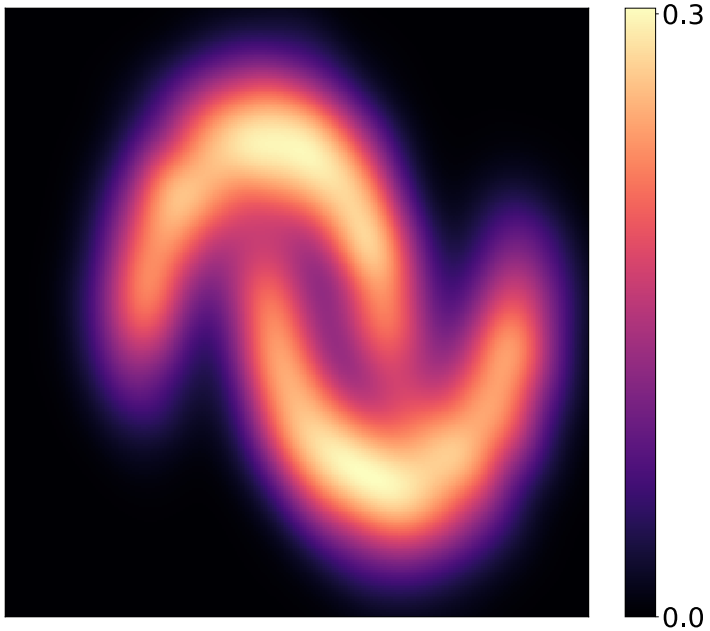
  - Conditional text generation w.r.t. the previous text sequence
- **Evaluation?**
  - Quality of the generated images?
  - No cross-validation
  - **Very challenging** for text-to-image and text-to-text models
  - Often use other models to assess the quality e.g., GPT

“Implicit” Generative Models: Problem Setting 1/4

Goal: estimate the unknown complex density p_{data}



Unknown density p_{data}

“Implicit” Generative Models: Problem Setting 2/4

- **Unknown complex** data distribution p_{data}
- **Data**: what do you have access to?

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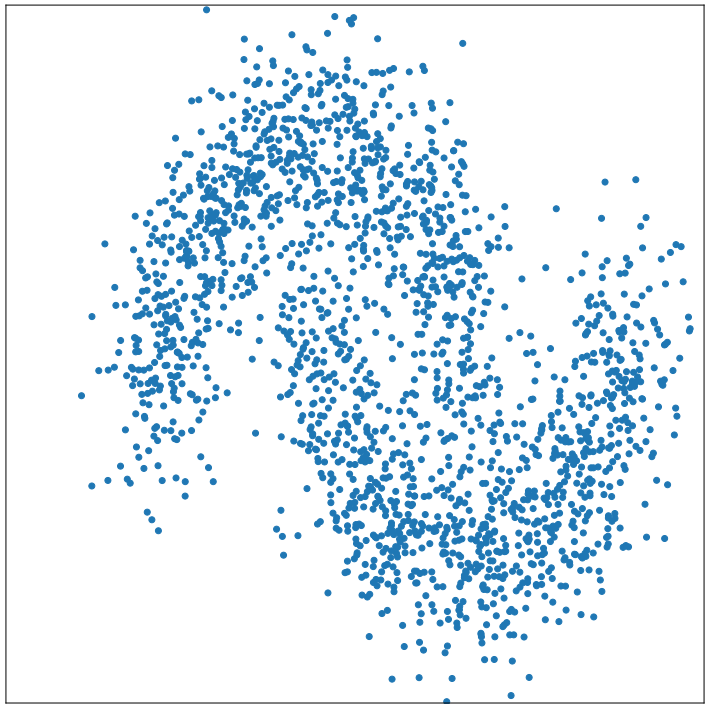
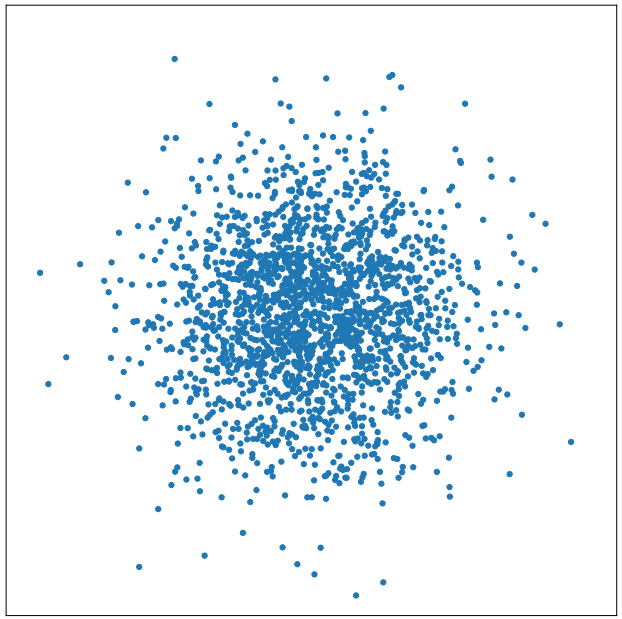


Data points $x^{(i)} \in \mathbb{R}^2$, $x^{(i)} \sim p_{\text{data}}$

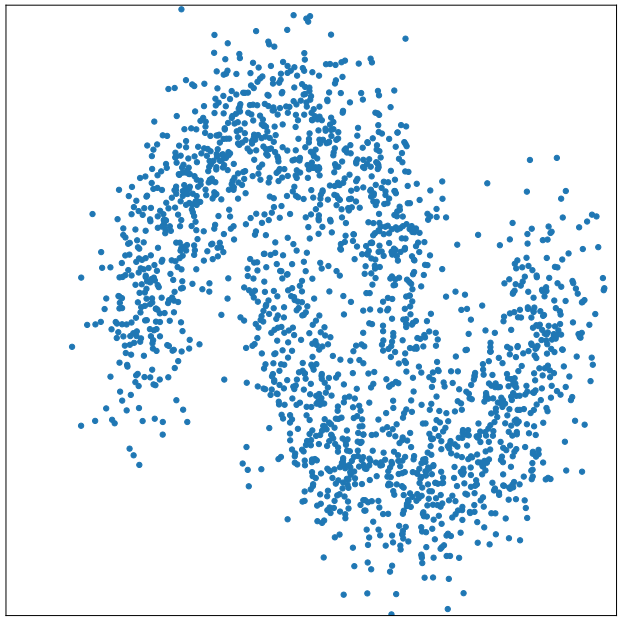
"Implicit" Generative Models: Problem Setting 3/4

- **Unknown complex** data distribution p_{data}
- **Goal**: transform a **simple** data distribution p_0 in the **complex** p_{data}

Points $x_0 \sim p_0$ drawn from a **simple** data distribution $\mathcal{N}(0, 1)$



Data points $x^{(i)} \in \mathbb{R}^2$ from a **complex data distribution** $x^{(i)} \sim p_{\text{data}}$



"Implicit" Generative Models: Problem Setting 4/4

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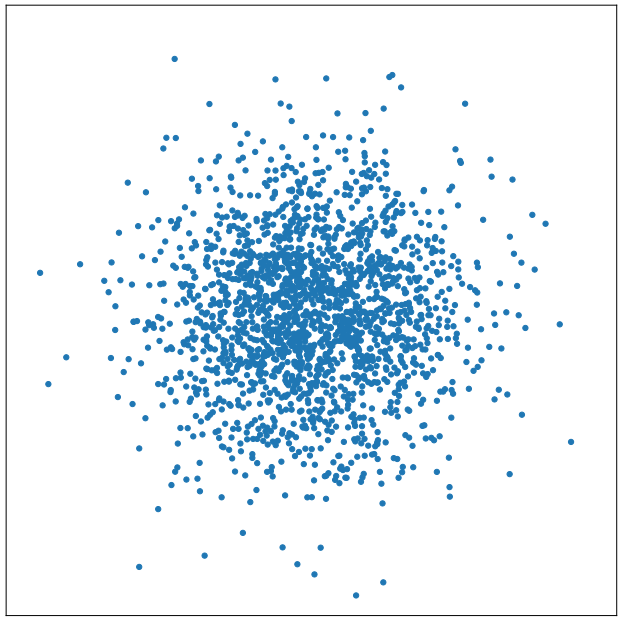

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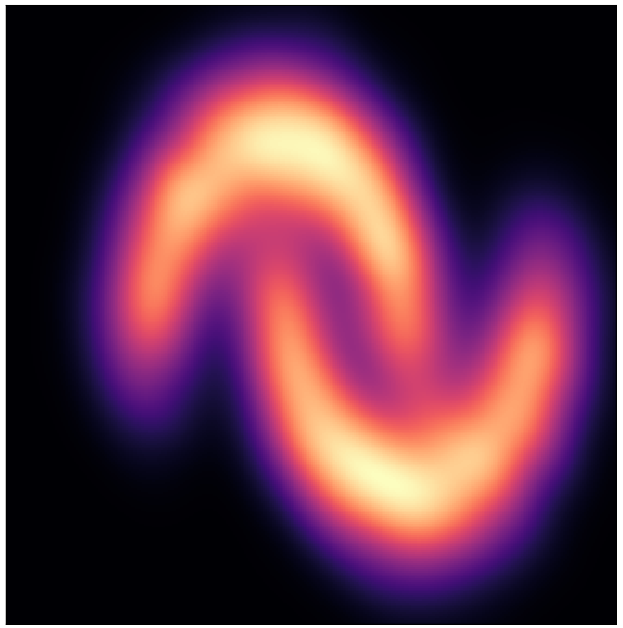
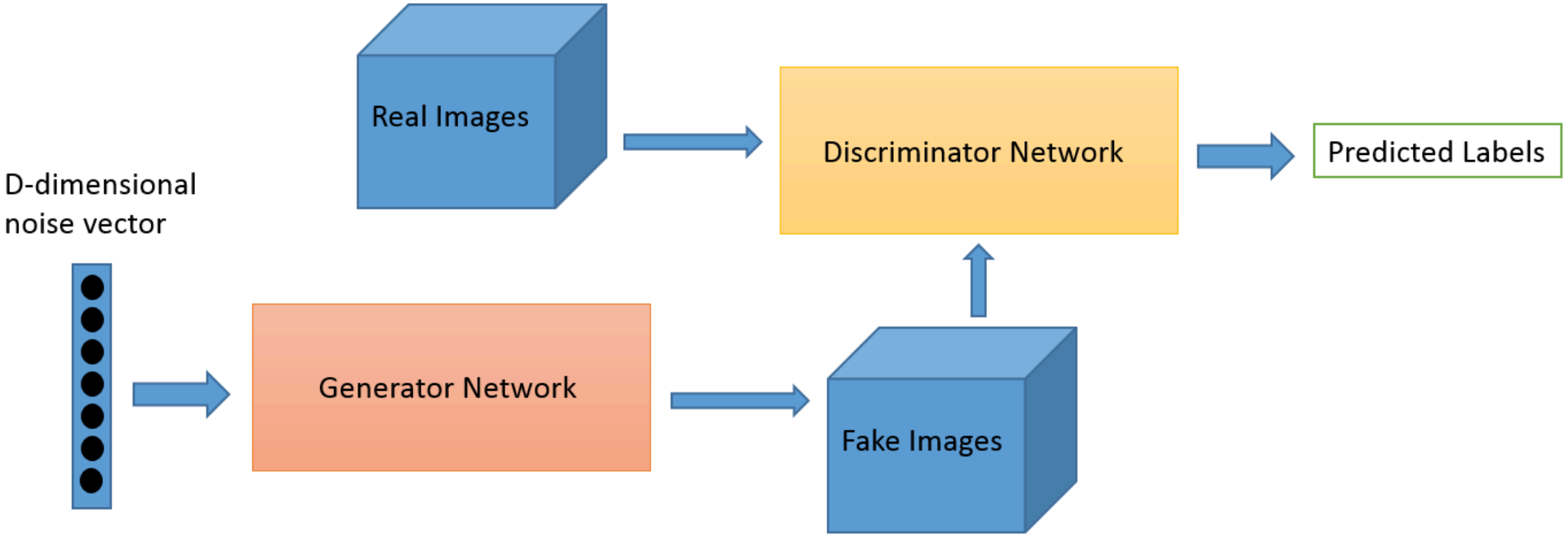


Generative Adversarial Networks

Generative Adversarial Networks: General Idea



https://datascience.eu/machine-learning/a-beginners-guide-to-generative-adversarial-networks-gans/

Generative Adversarial Networks: Illustration

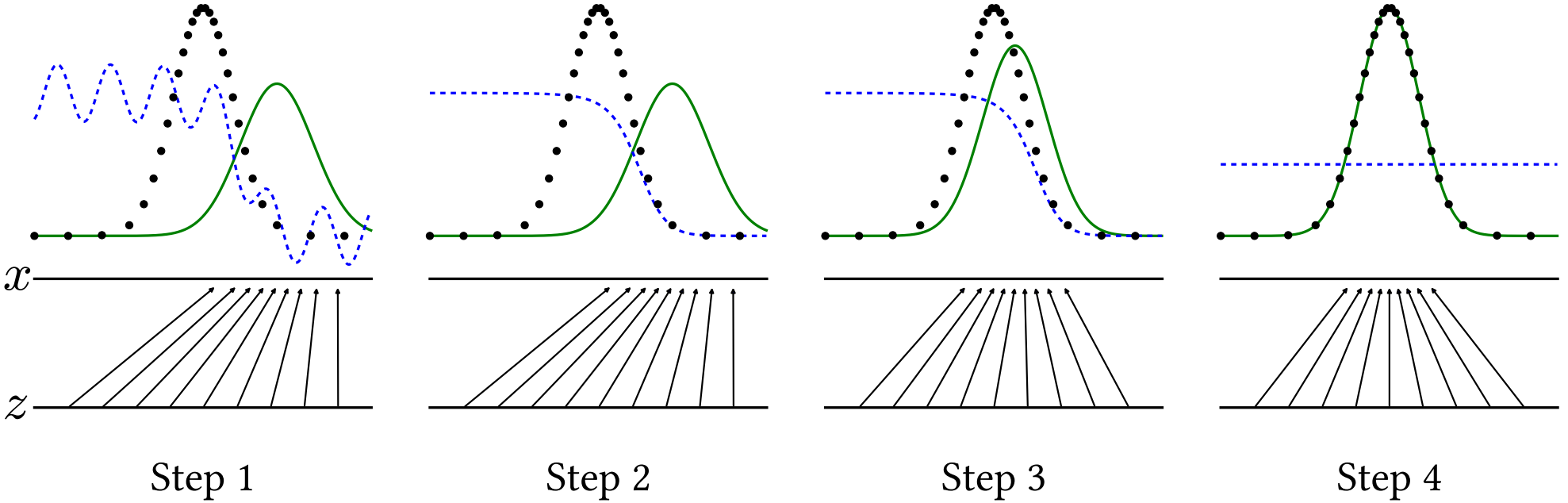


Image from Goodfellow et al. (2014)

How to Derive the GAN Loss?

Generative Adversarial Networks: Derive the Loss 1/2

We want to train:

Generative Adversarial Networks: Derive the Loss 1/2

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- 2 networks

Generative Adversarial Networks: Derive the Loss 1/2

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- 2 networks
  - A generator $G : \mathbb{R}^d \mapsto \mathbb{R}^d$

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    - Tell how much an image is fake or not

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Key Idea:

- Maximum log-likelihood

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Key Idea:

- Maximum log-likelihood
  - For a biased coin:
    - $\mathbb{P}(\text{head}|D) = D$

Generative Adversarial Networks: Derive the Loss 1/2

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Key Idea:

- Maximum log-likelihood
  - For a biased coin:
    - $\mathbb{P}(\text{head}|D) = D$, $\mathbb{P}(\text{tail}|D) = 1 - D$

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    - Likelihood $\mathbb{P}(x|D) = D^x(1 - D)^{1-x}$

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  Maximum log-likelihood: for a given sequence of observations x_i

$$\max_D \sum_i x_i \cdot \log D + \sum_i (1 - x_i) \cdot \log(1 - D)$$

Generative Adversarial Networks: Derive the Loss 2/2

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$$\mathbb{E}_{x \sim p_{\text{data}}}(\log(D(x))) + \mathbb{E}_{x_{\text{fake}} \sim p_G}(\log(1 - D(x_{\text{fake}})))$$

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We want to train:

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What problem can you foresee?

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Hence

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Wasserstein Generative Adversarial Networks (WGANs): Idea

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Rewrite the Wasserstein distance, and after computations

$$\min_G \max_F \mathcal{L}_{\text{WGAN}}(G, F) = \min_G \max_{F,\ \text{Lip}(F) \leq 1} \mathbb{E}_{x \sim p_{\text{data}}} F(x) - \mathbb{E}_{x_{\text{fake}} \sim p_G} F(x_{\text{fake}})$$

GANs vs WGANs

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WGANs: How to Enforce the Constraint? $\mathrm{Lip}(F) \leq 1$ 1/2

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while θ has not converged **do**
  for $t = 0, \ldots, n_{\text{critic}}$ **do**
    Sample $\{x^{(i)}\}_{i=1}^{m} \sim \mathbb{P}_r$ a batch from the real data.
    Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
    $g_w \leftarrow \nabla_w \left[\frac{1}{m}\sum_{i=1}^{m} f_w(x^{(i)}) - \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))\right]$
    $w \leftarrow w + \alpha \cdot \mathrm{RMSProp}(w, g_w)$
    $w \leftarrow \mathrm{clip}(w, -c, c)$
  end for
  Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
  $g_\theta \leftarrow -\nabla_\theta \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))$
  $\theta \leftarrow \theta - \alpha \cdot \mathrm{RMSProp}(\theta, g_\theta)$

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LAB 1: WGANs

Pytorch recalls?

Pytorch 101 Before the LAB

```
def train_one_epoch(training_loader):
    for i, data in enumerate(training_loader):
        # Every data instance is an input + label pair
        inputs, labels = data

        # Zero your gradients for every batch!
        optimizer.zero_grad()

        # Make predictions for this batch
        outputs = model(inputs)

        # Compute the loss and its gradients
        loss = loss_fn(outputs, labels)
        loss.backward()

        # Adjust learning weights
        optimizer.step()
```

WGAN and WGAN-gp Losses

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