

Math 1553

Introduction to Linear Algebra

School of Mathematics
Georgia Institute of Technology

Introduction to Linear Algebra

Motivation and Overview

Linear. Algebra.

What is Linear Algebra?

Linear

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- ▶ from al-jabr (Arabic), meaning reunion of broken parts
- ▶ 9th century Abu Ja'far Muhammad ibn Muso al-Khwarizmi

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$$7x_1 + 2x_2 - 13x_3 - 7x_4 + 21x_5 + 8x_6 = 2567$$

$$-x_1 + 9x_2 + \frac{3}{2}x_3 + x_4 + 14x_5 + 27x_6 = 26$$

$$\frac{1}{2}x_1 + 4x_2 + 10x_3 + 11x_4 + 2x_5 + x_6 = -15$$

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In real life, the difficult part is often in recognizing that a problem can be solved using linear algebra in the first place: need *conceptual* understanding.

Large classes of engineering problems, no matter how huge, can be reduced to linear algebra:

$$Ax = b \quad \text{or}$$

$$Ax = \lambda x$$

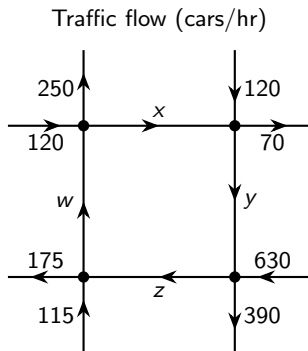
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“...and now it's just linear algebra”

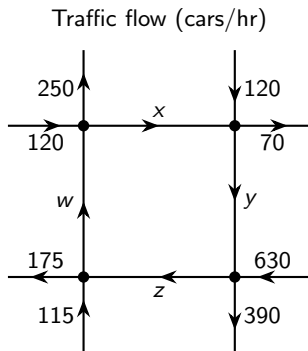
Civil Engineering: How much traffic flows through the four labeled segments?



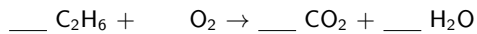
Applications of Linear Algebra

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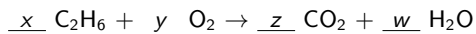


Chemistry: Balancing reaction equations



# Applications of Linear Algebra

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~~~~~> system of linear equations, one equation for each element.


Applications of Linear Algebra

Biology: In a population of rabbits. . .

- ▶ half of the new born rabbits survive their first year
- ▶ of those, half survive their second year
- ▶ the maximum life span is three years
- ▶ rabbits produce 0, 6, 8 rabbits in their first, second, and third years

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## Question

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**Geometry and Astronomy:** Find the equation of a circle passing through 3 given points, say  $(1, 0)$ ,  $(0, 1)$ , and  $(1, 1)$ .

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Very similar to: compute the orbit of a planet:

$$ax^2 + by^2 + cxy + dx + ey + f = 0$$

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Stay tuned!

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- ▶ Almost solve the equation  $Ax = b$ 
  - ▶ Find best-fit solutions to systems of linear equations that have no actual solution using least squares approximations.

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- ▶ About half the material focuses on how to do linear algebra computations—that is still important.
- ▶ The other half is on *conceptual* understanding of linear algebra. This is much more subtle: it's about figuring out *what question* to ask the computer, or whether you actually need to do any computations at all.

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**Piazza:** this is where to ask questions, and where I'll post announcements.