Section 1.7

Linear Independence

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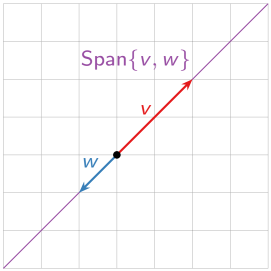

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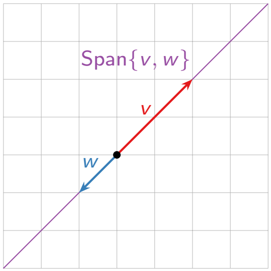

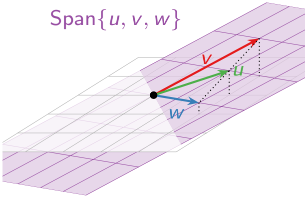

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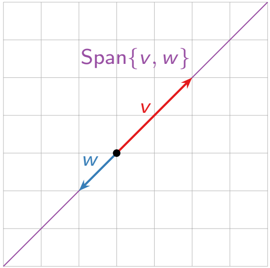

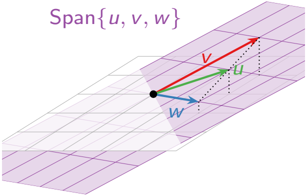

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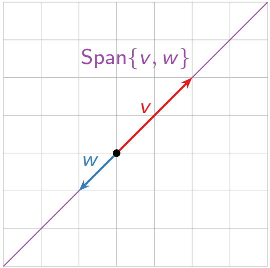

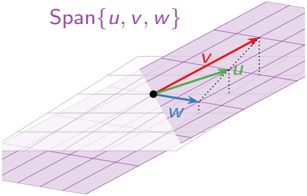

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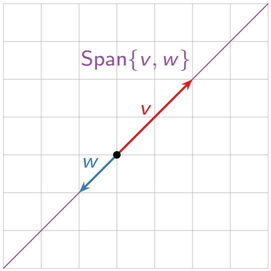

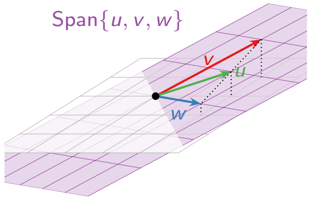

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Notice in each case that one vector in the set is already in the span of the others—so it doesn’t make the span bigger.

Today we will formalize this idea in the concept of *linear (in)dependence*.

Linear Independence

Definition

A set of vectors $\{v_1, v_2, \ldots, v_p\}$ in $\mathbf{R}^n$ is **linearly independent** if the vector equation

$$x_1 v_1 + x_2 v_2 + \cdots + x_p v_p = 0$$

has only the trivial solution $x_1 = x_2 = \cdots = x_p = 0$.

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Like span, linear (in)dependence is another one of those big vocabulary words that you absolutely need to learn. Much of the rest of the course will be built on these concepts, and you need to know exactly what they mean in order to be able to answer questions on quizzes and exams (and solve real-world problems later on).

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has only the trivial solution $x_1 = x_2 = \cdots = x_p = 0$. The set $\{v_1, v_2, \ldots, v_p\}$ is **linearly dependent** otherwise.

> Note that linear (in)dependence is a notion that applies to a *collection of vectors*, not to a single vector, or to one vector in the presence of some others.

Checking Linear Independence

Question: Is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Checking Linear Independence

Question: Is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Linear Independence and Matrix Columns

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- The vectors $v_1, v_2, \ldots, v_p$ are linearly independent if and only if the matrix with columns $v_1, v_2, \ldots, v_p$ has a pivot in each column.
- Solving the matrix equation $Ax = 0$ will either verify that the columns $v_1, v_2, \ldots, v_p$ of A are linearly independent, or will produce a linear dependence relation.

Suppose that one of the vectors $\{v_1, v_2, \ldots, v_p\}$ is a linear combination of the other ones (that is, it is in the span of the other ones):

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Then the vectors are linearly *de*pendent:

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Theorem

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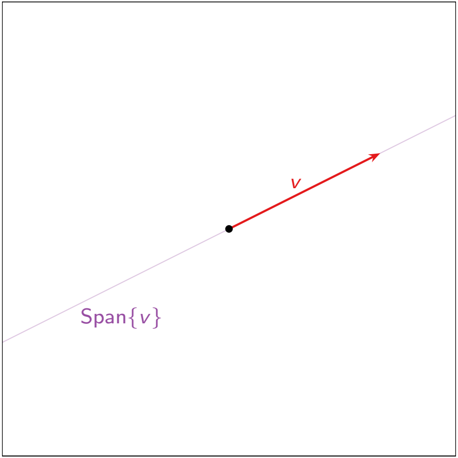
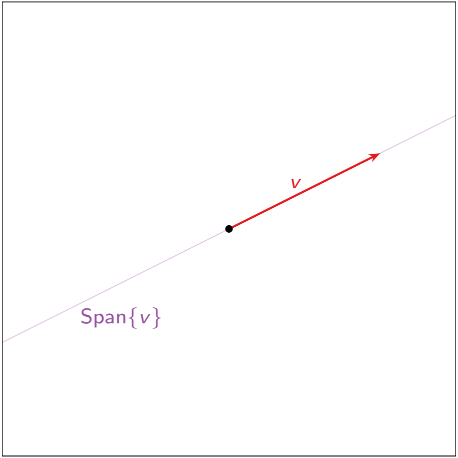
Linear Independence

Pictures in $\mathbf{R}^2$



In this picture

One vector $\{v\}$:



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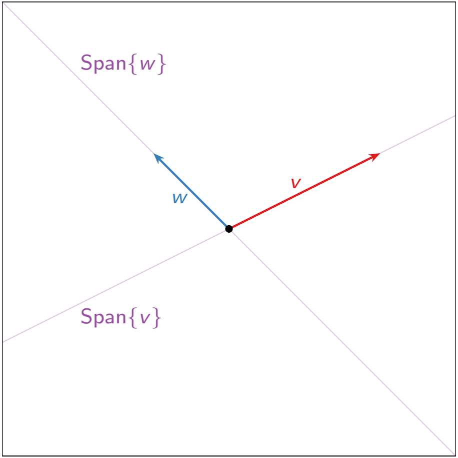

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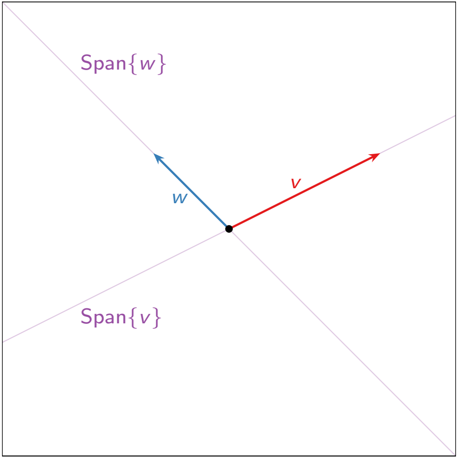

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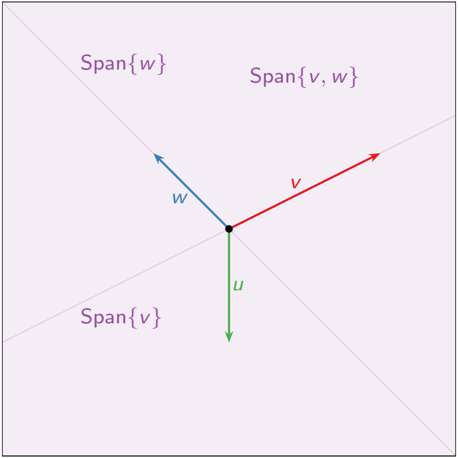

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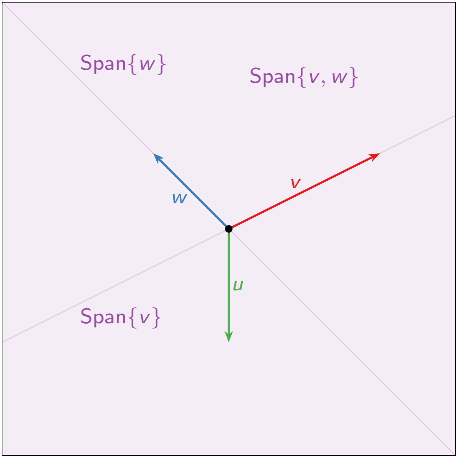

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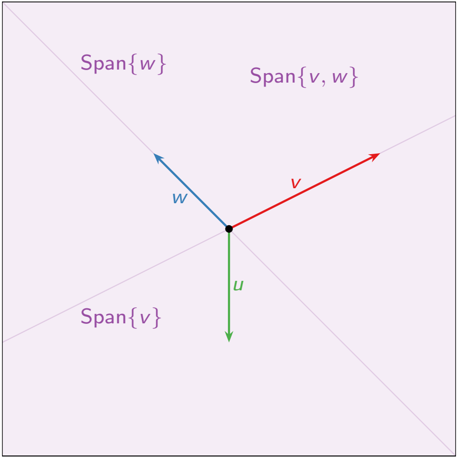

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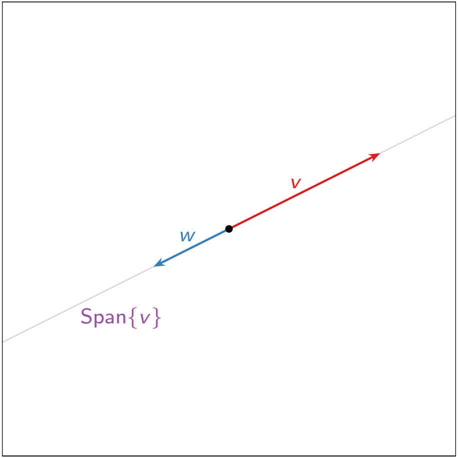
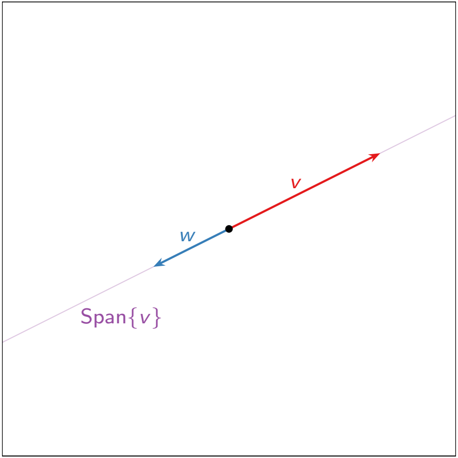
Also v is in Span$\{u, w\}$ and w is in Span$\{u, v\}$.



Two collinear vectors $\{v, w\}$:



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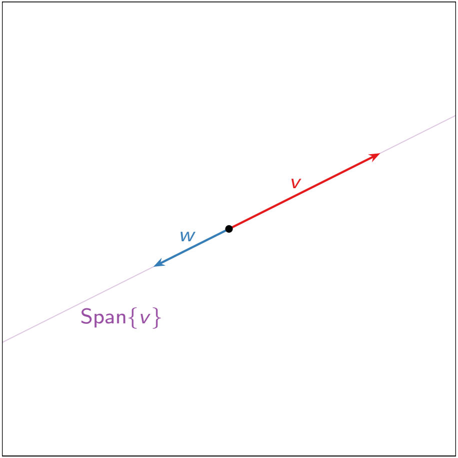
Linearly dependent: w is in Span$\{v\}$ (and vice-versa).



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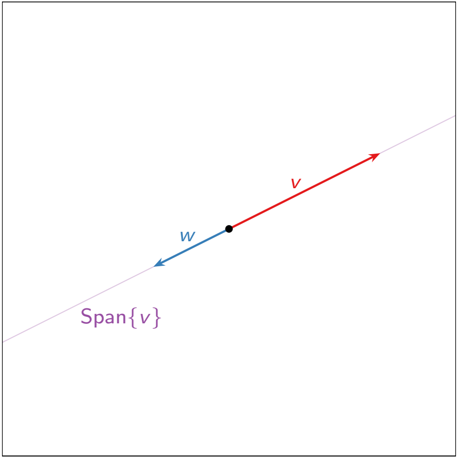
Observe: *Two* vectors are linearly *de*pendent if and only if



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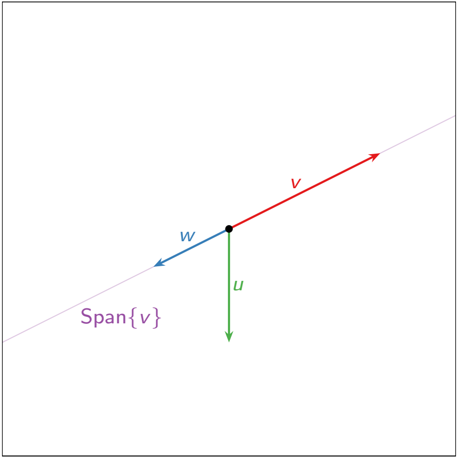

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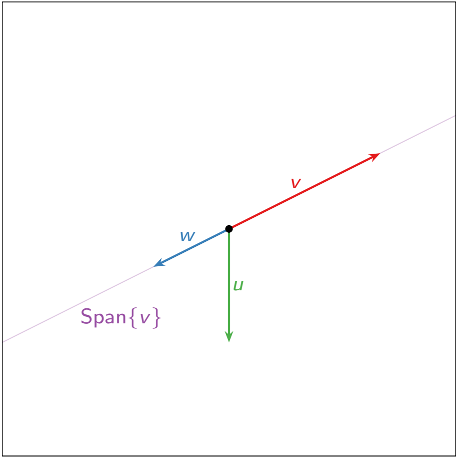

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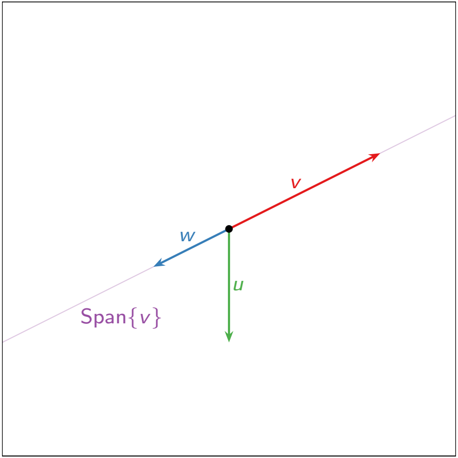
Linear Independence

Pictures in $\mathbf{R}^2$



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Three vectors $\{v, w, u\}$:
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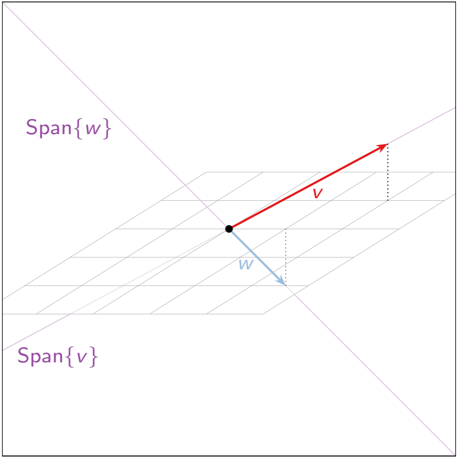
Observe: If a set of vectors is linearly dependent, then so is any larger set of vectors!



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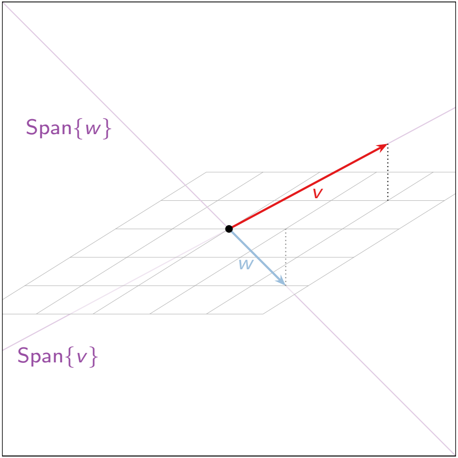

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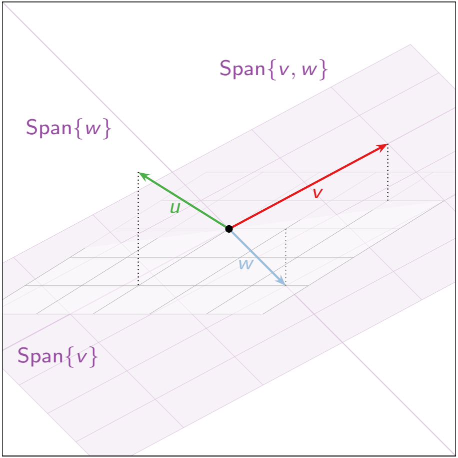

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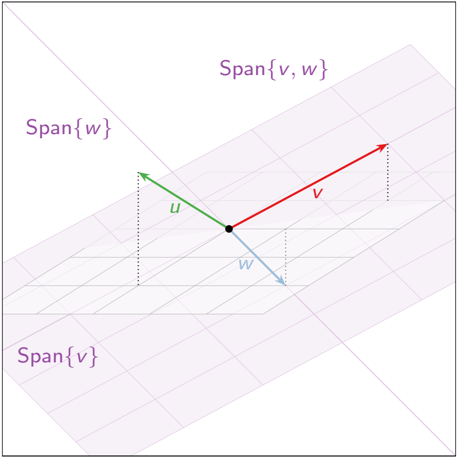
Three vectors $\{v, w, u\}$:

Linear Independence

Pictures in $\mathbf{R}^3$



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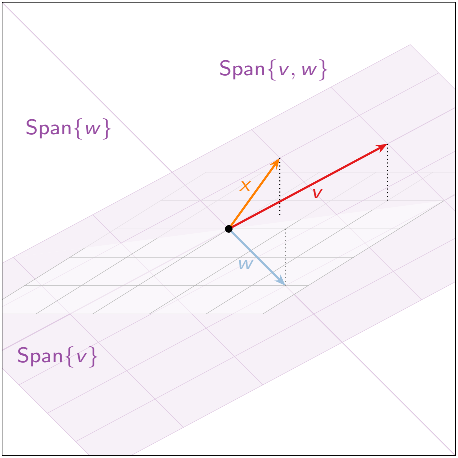
Three vectors $\{v, w, u\}$:
Linearly independent: no one is in the span of the other two.



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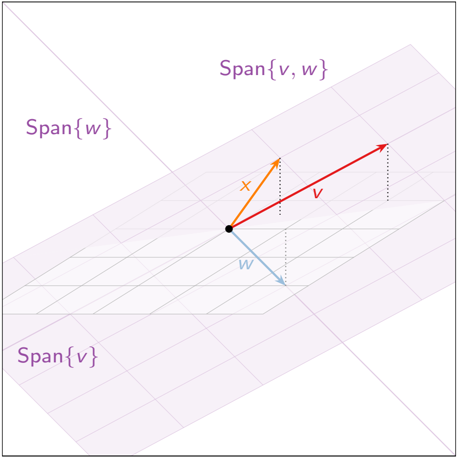
Three vectors $\{v, w, x\}$:



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Two vectors $\{v, w\}$:
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Three vectors $\{v, w, x\}$:
Linearly dependent: x is in $\text{Span}\{v, w\}$.

Poll

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Are there four vectors u, v, w, x in $\mathbf{R}^3$ which are linearly dependent, but such that u is *not* a linear combination of v, w, x? If so, draw a picture; if not, give an argument.

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Linear dependence of $\{v_1, \ldots, v_p\}$ means *some* v_i is a linear combination of the others, not *any*.

Theorem

A set of vectors $\{v_1, v_2, \ldots, v_p\}$ is linearly *de*pendent if and only if one of the vectors is in the span of the other ones.

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Take the largest j such that v_j is in the span of the others.

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Theorem

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Better Theorem

A set of vectors $\{v_1, v_2, \ldots, v_p\}$ is linearly dependent if and only if there is some j such that v_j is in $\mathrm{Span}\{v_1, v_2, \ldots, v_{j-1}\}$.

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Translation

A set of vectors is linearly independent if and only if, every time you add another vector to the set, the span gets bigger.

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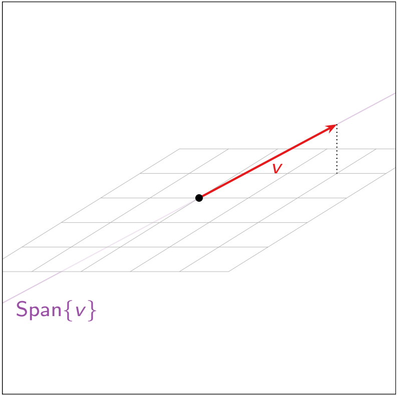

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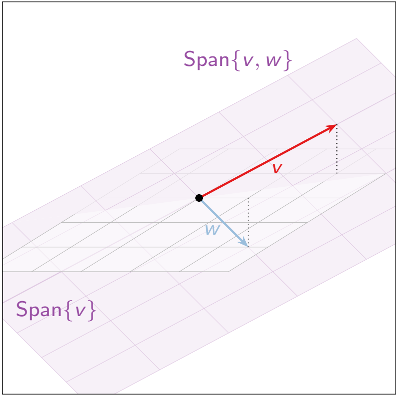

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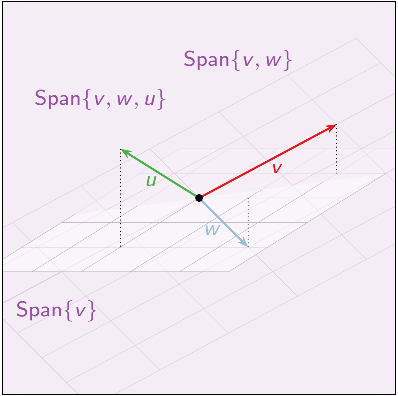

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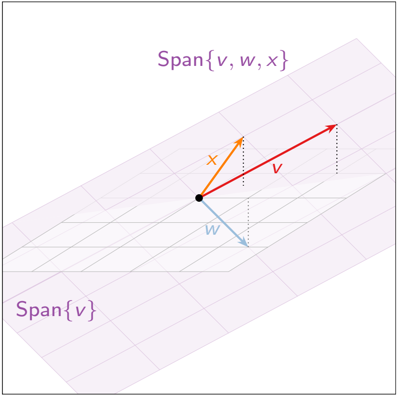

One vector $\{v\}$:
Linearly independent: span got bigger (than $\{0\}$).

Two vectors $\{v, w\}$:
Linearly independent: span got bigger.

Three vectors $\{v, w, x\}$:
Linearly dependent: span didn't get bigger.

Linear Independence

Two more facts

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