Day 4

INTRODUCTION TO QUANTUM COMPUTATION

Topics

- Vector space Transformation
- Unitary Transformation
- Gates

Transformation

- Vector space Transformation

$$V \xrightarrow{T} W$$

$$T(v) \in W \;\; \forall \;\; v \in V$$

Transformation

- Defining Vectorspace
- Transformation: "**T**"

$$V \xrightarrow{T} W$$

$$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$v_1, v_2 \in \mathbb{C} \quad ; \quad w_1, w_2, w_3 \in \mathbb{C}$$

Transformation

- Transformation Defined

$$For \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ; \qquad T(x) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 + x_2 \end{bmatrix}$$

Transformation

- Checking for the transformation

$$For \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}; \qquad T(v) = \begin{bmatrix} v_1 \\ v_2 \\ v_1 + v_2 \end{bmatrix}$$

$$v_1, v_2 \,\epsilon\, \mathbb{C} \;\Rightarrow\; v_1 + v_2 \,\epsilon\, \mathbb{C} \;\Rightarrow\; T(v) \,\epsilon\, W$$

Transformation

- Matrix of the transformation

$$T_{3\times 2} \cdot v_{2\times 1} = w_{3\times 1}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_1 + v_2 \end{bmatrix}$$

Transformation

- **T** has been written as a matrix and it operates on v as

$$T(v) = T_{matrix}.v = w$$

- Not all transformation can be written as a matrix.

Unitary Transformation

- In QC we work with Unitary Transformation

$$U^{\dagger}(U(v)) = U(U^{\dagger}(v)) = v$$

$$U^{\dagger}_{mat}.U_{mat} = U_{mat}.U^{\dagger}_{mat} = \mathbb{I}$$

- Properties of Unitary Transformation
  - It conserves magnitude.
  - It conserves angle between vectors.

Gates

- In QM we call the transformations as Operator acting on a Quantum state.
- In QC we use the Gates for the transformation of one state to another.
- Each Gate is a separate and unique transformation

X Gate

- What a X Gate does:

$$\left.\begin{matrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} \\ a, d = 0; b, c = 1\end{matrix}\right\} X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Z Gate

- What a Z Gate does:

$$\left.\begin{array}{c}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix} \\ b, c = 0; a = 1; d = -1 \end{array}\right\} Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Checking for Unitary

- Z Gate

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, Z^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Z^\dagger . Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Checking for Unitary

- X Gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, X^\dagger = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{aligned} X^\dagger . X &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Let's see some new Gates

- Hadamard Gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Let's see some new Gates

- Hadamard Gate

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\cdot\begin{bmatrix}1\\ 0\end{bmatrix}=\frac{1}{\sqrt{2}}\cdot\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\cdot\begin{bmatrix}0\\ 1\end{bmatrix}=\frac{1}{\sqrt{2}}\cdot\begin{bmatrix}1\\ -1\end{bmatrix}$$

Basis Vector

- |0>,|1> can act as basis vector;

$$|0> = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1> = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- |+>,|-> can too

$$|+> = \frac{1}{\sqrt{2}} . \begin{bmatrix} 1 \\ 1 \end{bmatrix}, |-> = \frac{1}{\sqrt{2}} . \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Basis Vector Transformation

- H Gate acts as the transformation between the two basis.

$$H.|0>=|+>, H.|1>=|->$$

$$H.|+>=|0>, H.|->=|1>$$

END

- We would be lerarning about representation of qubit states on bloch sphere in the next session.