Measles in Small Populations

Predictability in Highly Stochastic Systems

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[PRESS ASSOCIATION TELEGRAM.]

NEW PLYMOUTH, July 2.

There is a serious outbreak of measles at Parihaka. Several Natives have died. Dr. McCarrol, the hospital surgeon, has sent a supply of medicine. It is feared the disease will spread amongst the Natives all over the district, as a Parihaka meeting has just broken up, and the Natives have returned to their own settlements, probably carrying contagion with them.

The Press, 4 July 1887

Outline

Data from Iceland, the Faroe Islands, and Bornholm

TSIR, fitting, predictions for upcoming epidemics

Potential improvements

Disclaimer

The next slide is a **gross, insulting generalisation.**

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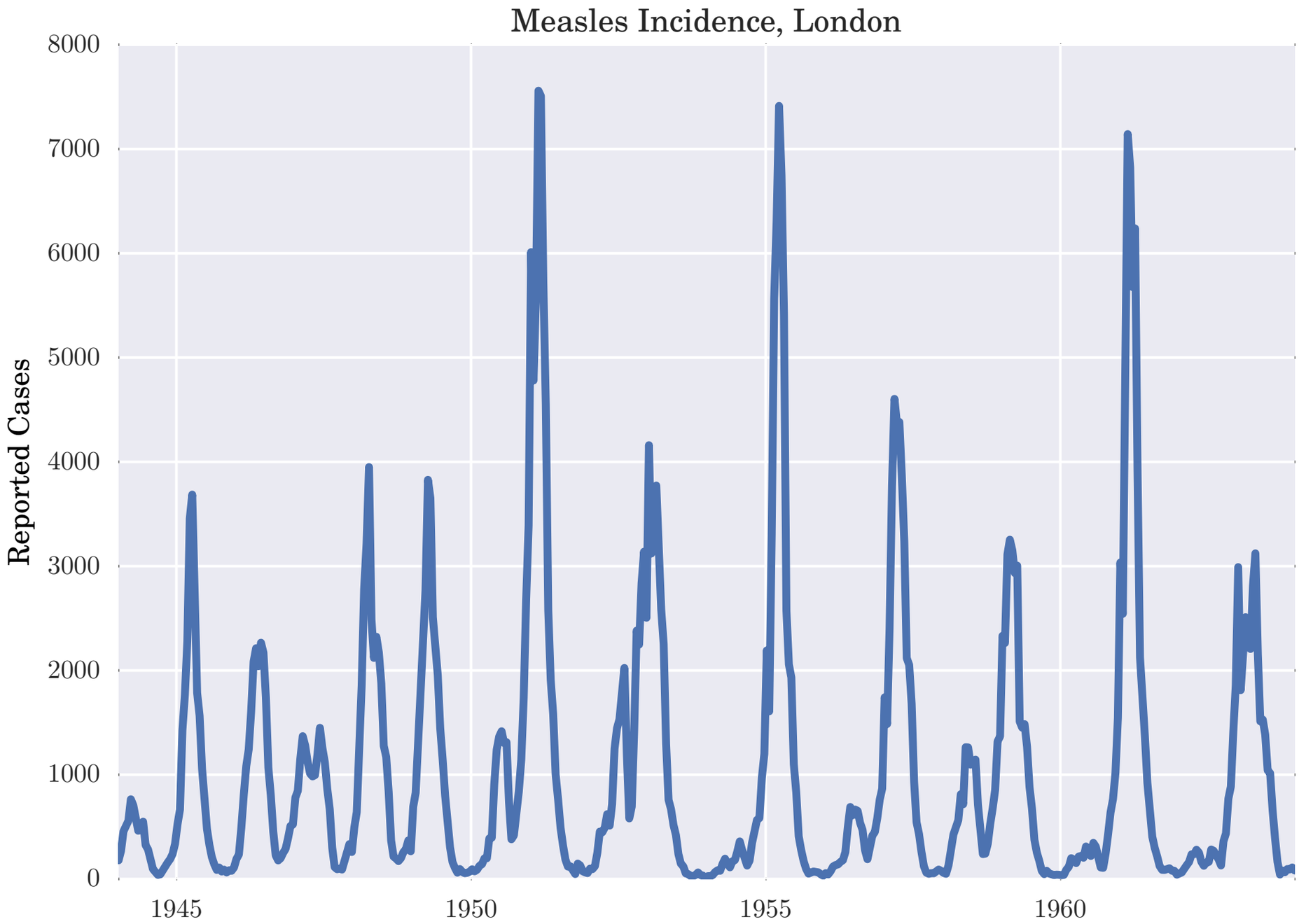
Typical Assumptions*



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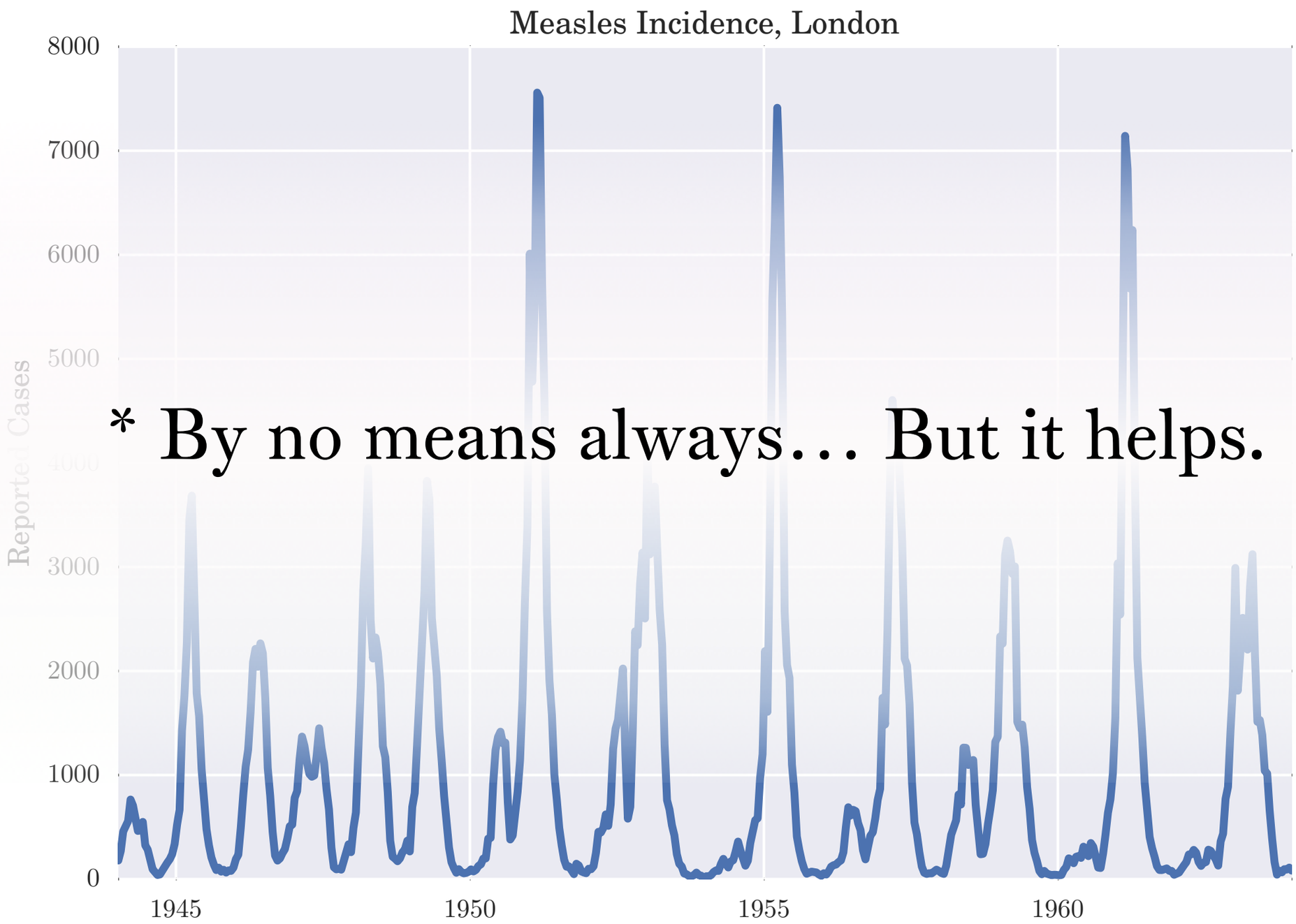
Typical Assumptions*



* By no means always… But it helps.

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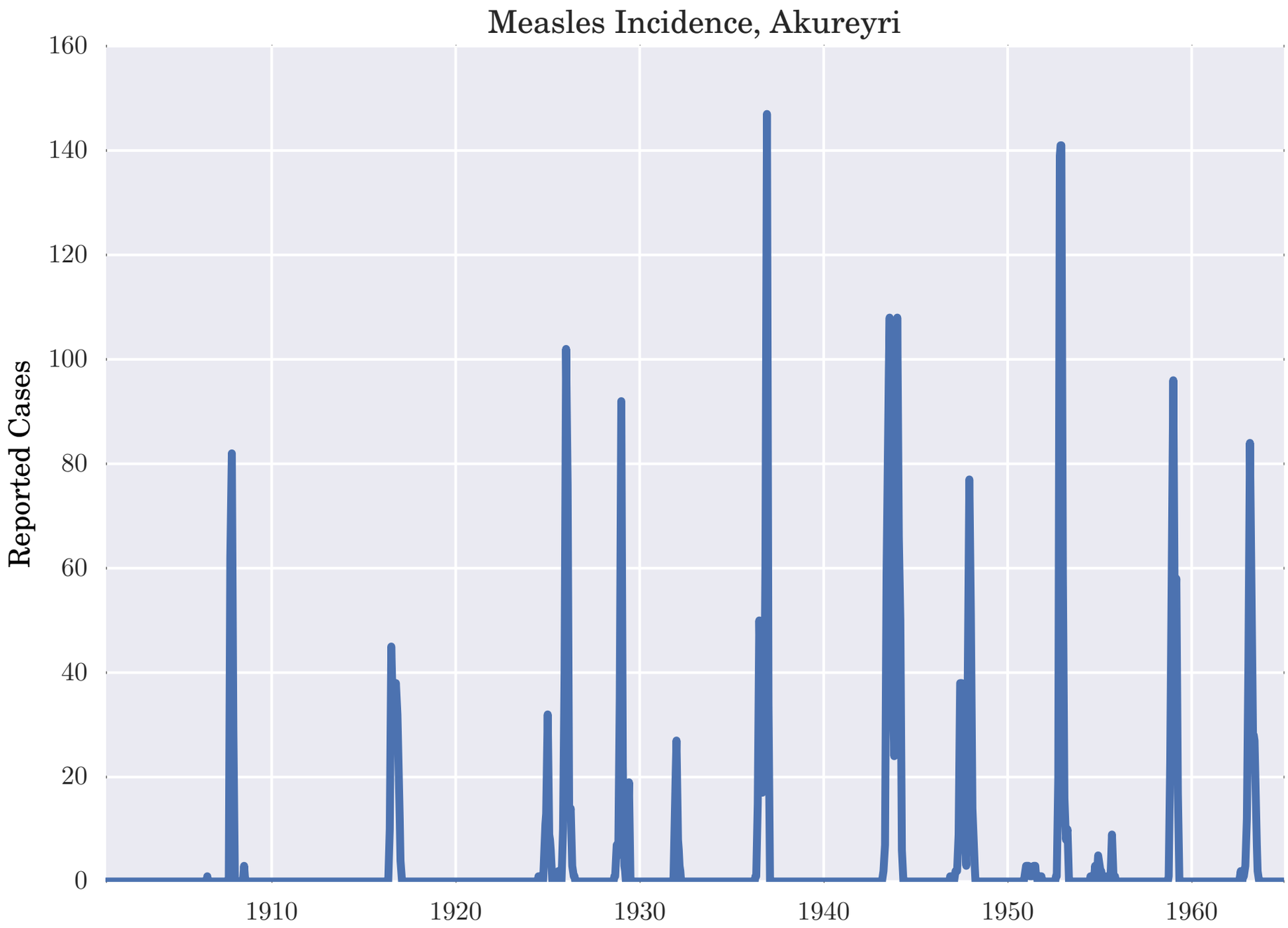
Small Populations



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Incidence Data

Parish and medical records, 1900 - 1990

- Monthly
- Iceland : from 47 medical districts
- Bornholm, Faroe : island / archipelago level
- Truncate at 1965 (prevaccination)

Cliff, A.D., Haggett, P., Ord, J.K., Versey, G.R. (1981).
Spatial Diffusion : an Historical Geography of Epidemics in an Island Community.

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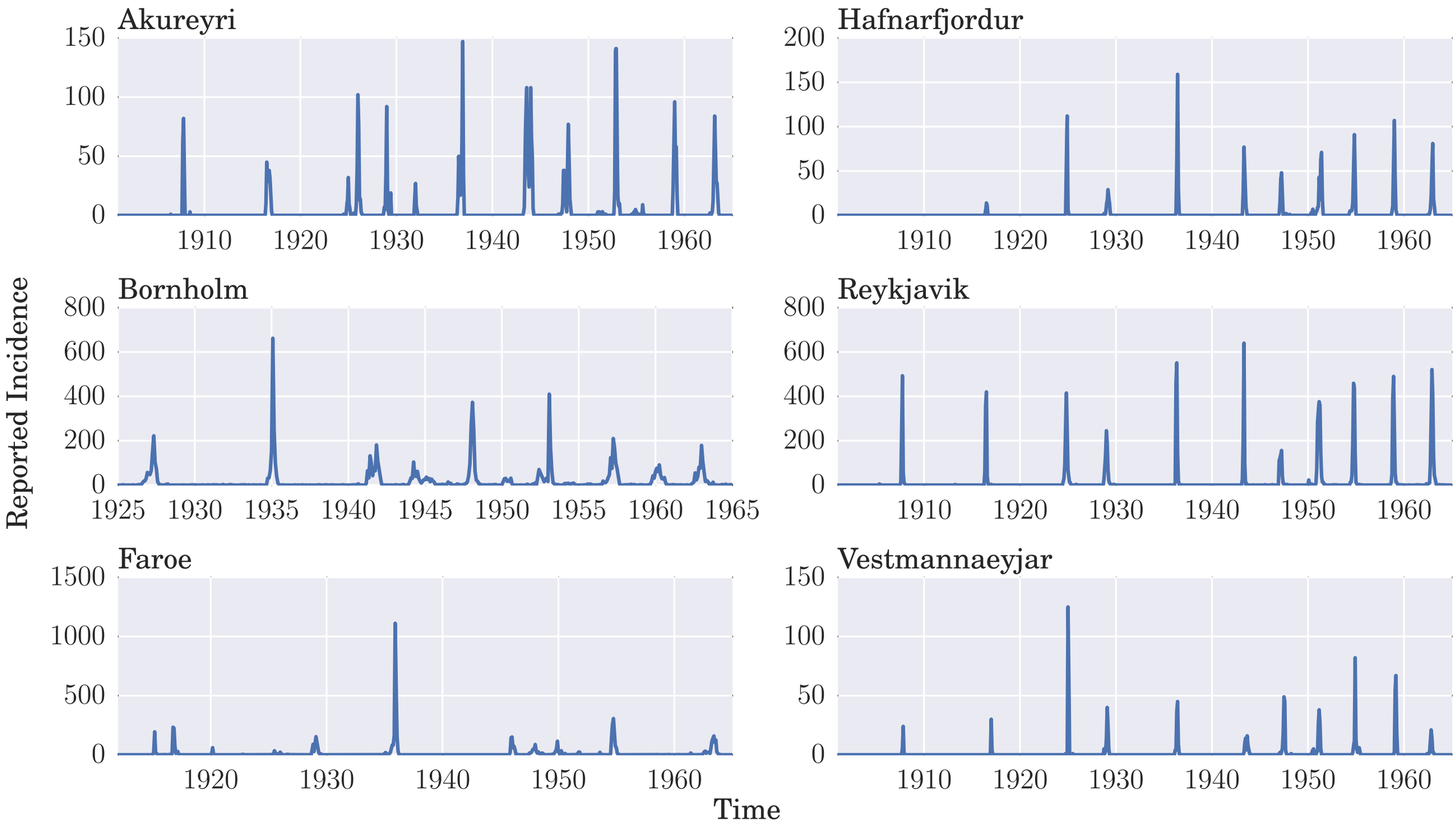
Incidence Data



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Demography

7000

49000

6000

3500

28000

47000

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Demographic Data

Iceland, 1901 - 1990

Live births :

- Monthly
- Aggregated at country level

Population :

- Decennial
- Across 262 municipalities
- Many missing values

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Demographic Data

Bornholm, 1925 - 1965

Live births :

- Annual

Population :

- Pentennial

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Demographic Data

The Faroe Islands, 1912 - 1970

Live births and Population :

- Annual
- Aggregated at country level
- Note : “average” population…

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Data Matching

Births :

- Interpolated cubically, rounded, then scaled by integral

Population :

- Interpolated cubically to 24 time points per year, rounded

Incidence :

- Interpolated linearly from 12 to 24 time points per year

Iceland : name and location matched

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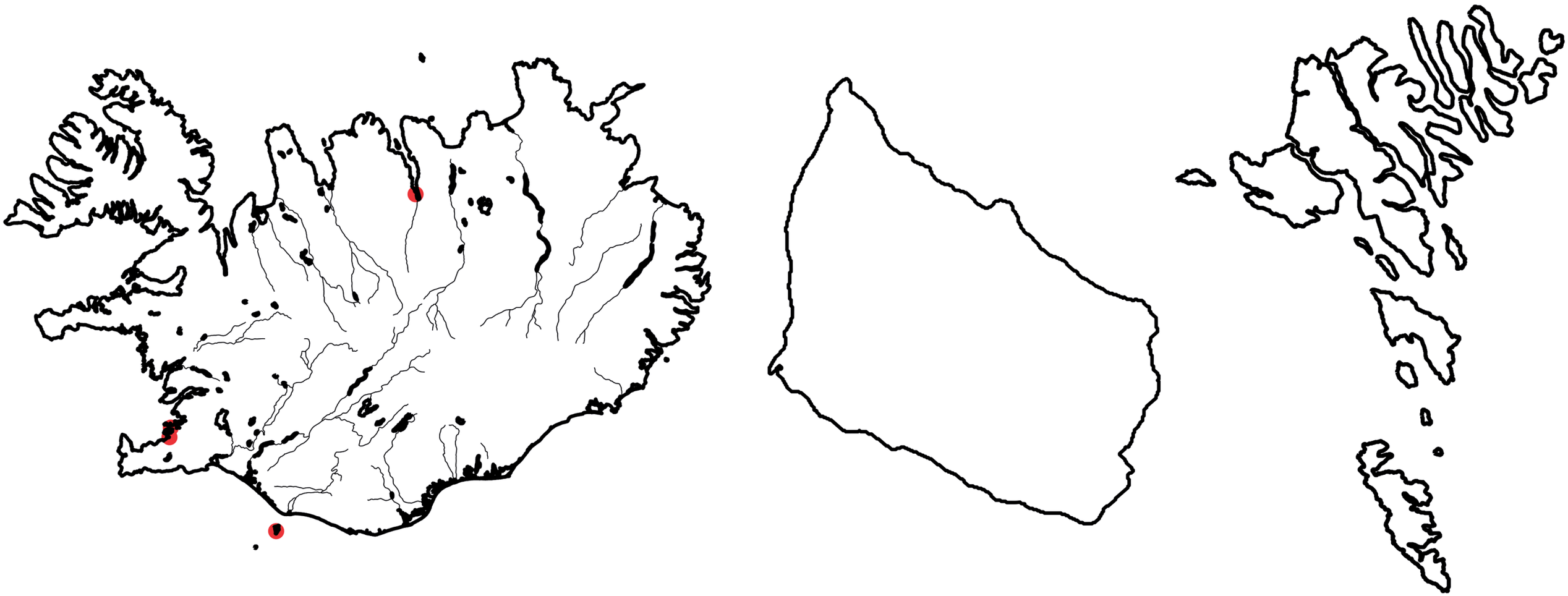
Geography



Model-Friendly

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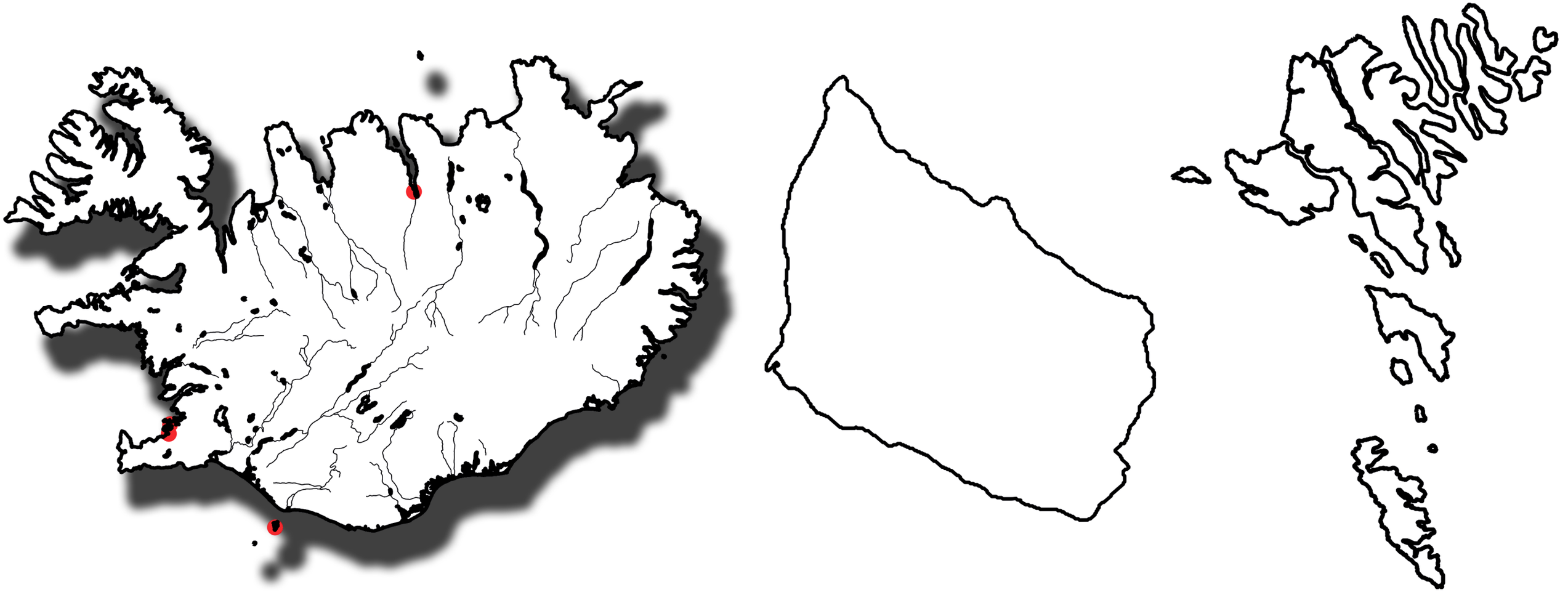
Geography

Iceland



Matching incidence data to changing demographic boundaries

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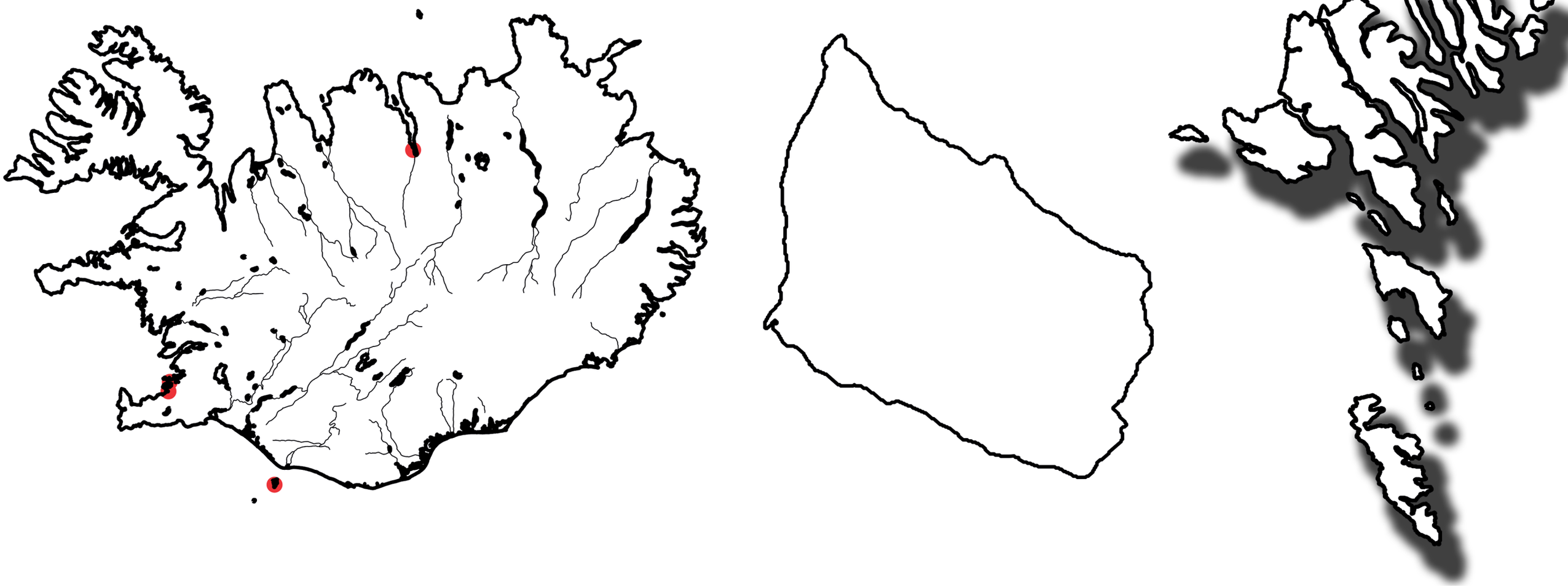
Faroe Islands



Aggregate data, but strong population inhomogeneity

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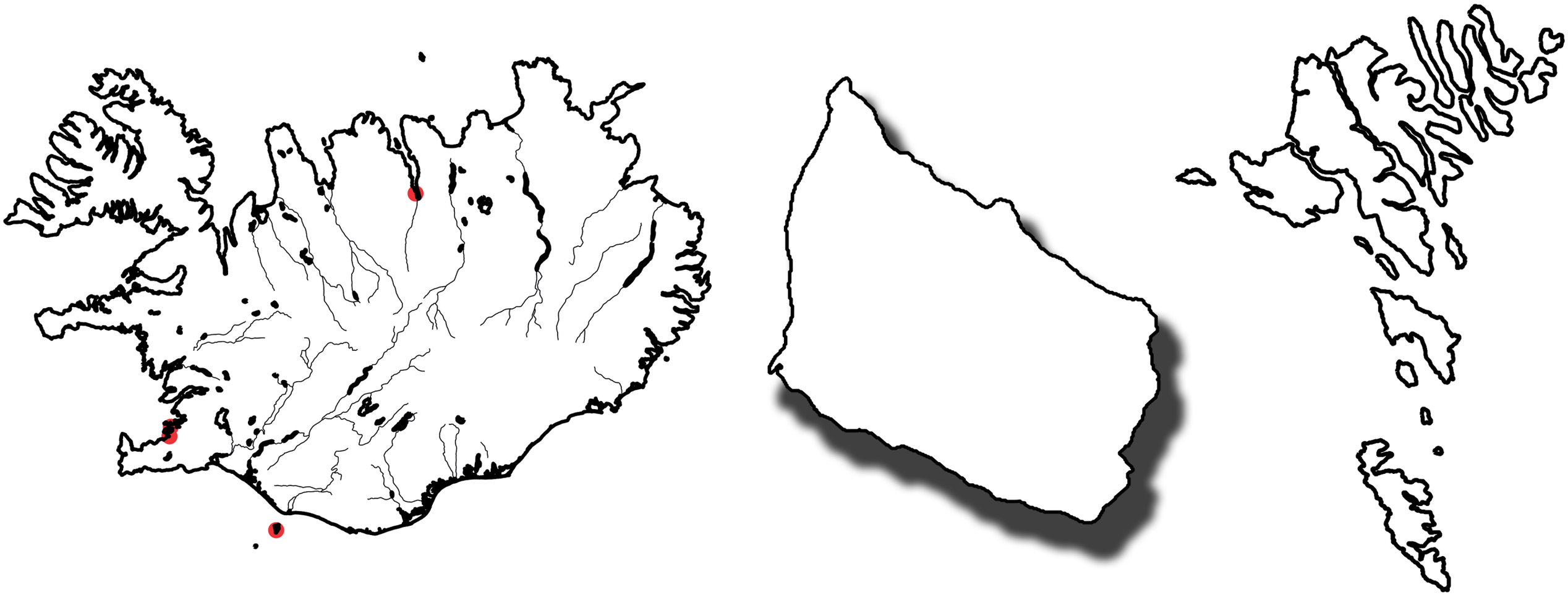
Geography

Bornholm



Single, mixed population; no spatial barriers; isolated

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Time-Series SIR

Dynamics

$$I_t = r_t \, I_{t-1}^{\alpha} \, S_{t-1} \, \varepsilon_t \qquad \mathbb{E}\left[\varepsilon_t\right] = 1$$

$$S_t = B_{t-d} + S_{t-1} - I_{t-1} + u_t \qquad \mathbb{E}\left[u_t\right] = 0$$

r_t Periodic seasonality, $r_t = r_{t+P}$

α Inhomogeneity parameter

d Delay due to maternal immunity

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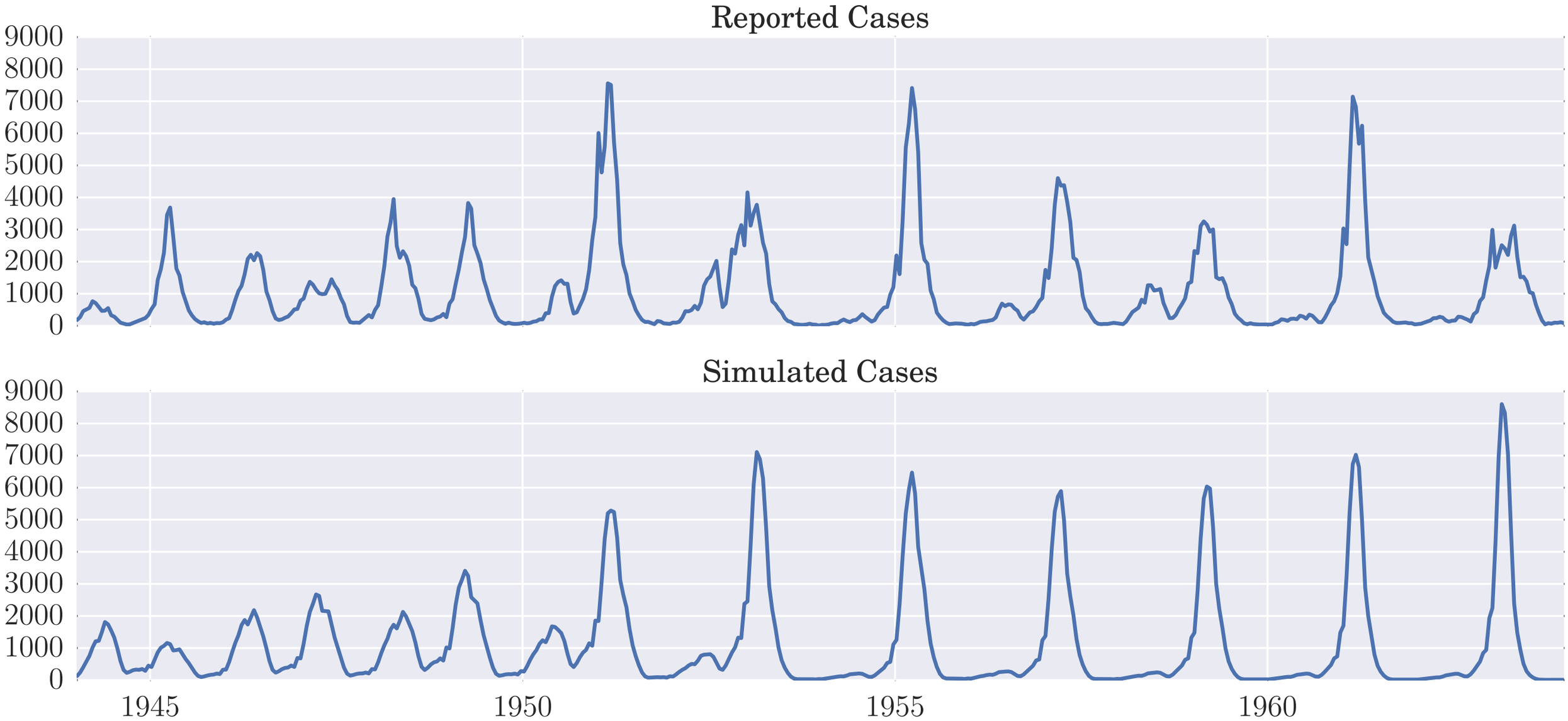
Time-Series SIR

Dynamics



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Time-Series SIR

Reporting Rate

$$I_t = \rho_t \, C_t$$

$$\text{Let } X_t = \sum_{i=1}^{t} C_i \text{ and } Y_t = \sum_{i=1}^{t} B_{i-d}$$

$$\text{Then } Y_t = -Z_0 + \rho_t \, X_t + Z_t$$

ρ_t Reporting rate

Z_t Susceptible dynamics, $\mathbb{E}[Z_t] = 0$

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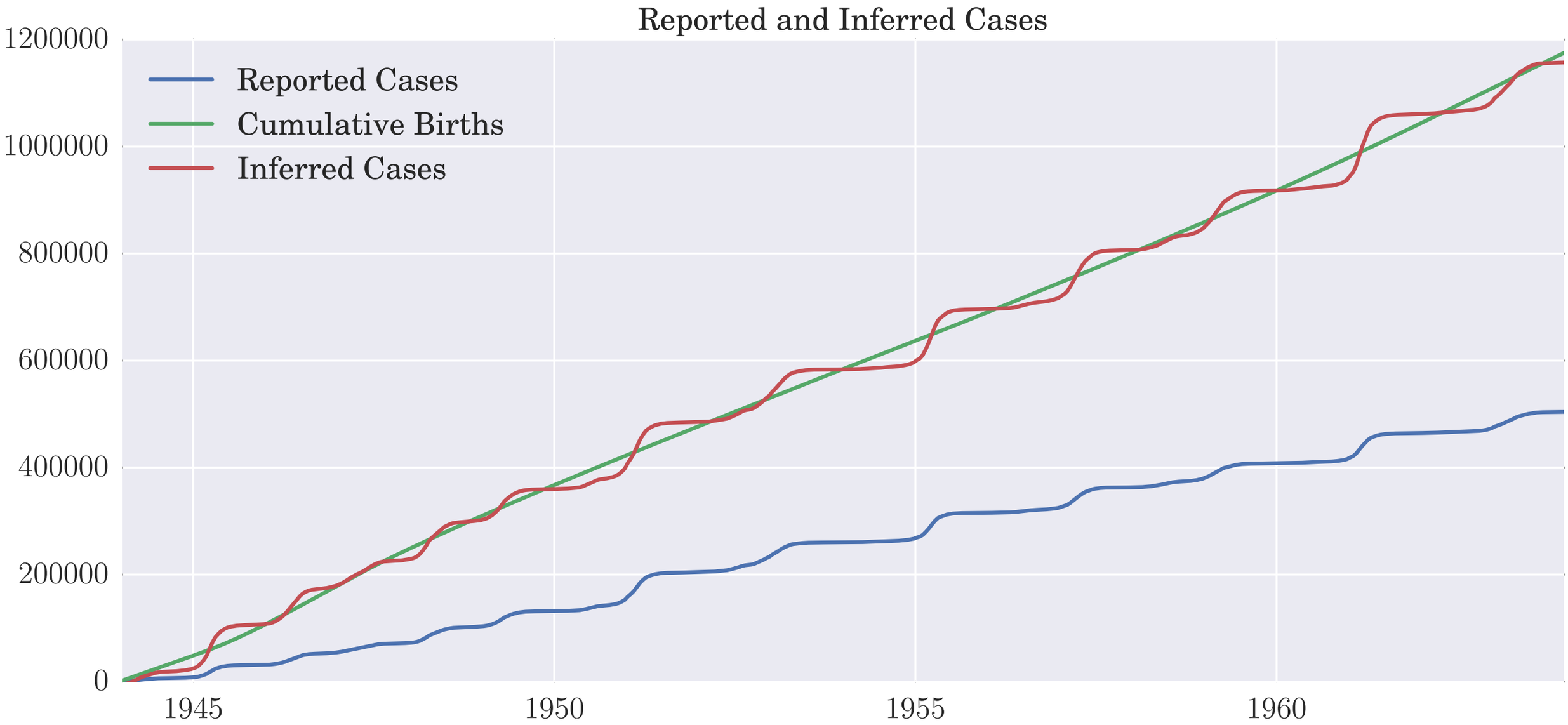
Reporting Rate



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Time-Series SIR

Susceptible Reconstruction

$$\text{Let } \mathbb{E}\left[S_t\right] = \bar{S} \ : \ S_t = \bar{S} + Z_t$$

$$\text{Then } Z_t \text{ are the residuals of the regression :}$$

$$Y_t = -Z_0 + \rho_t X_t + Z_t$$

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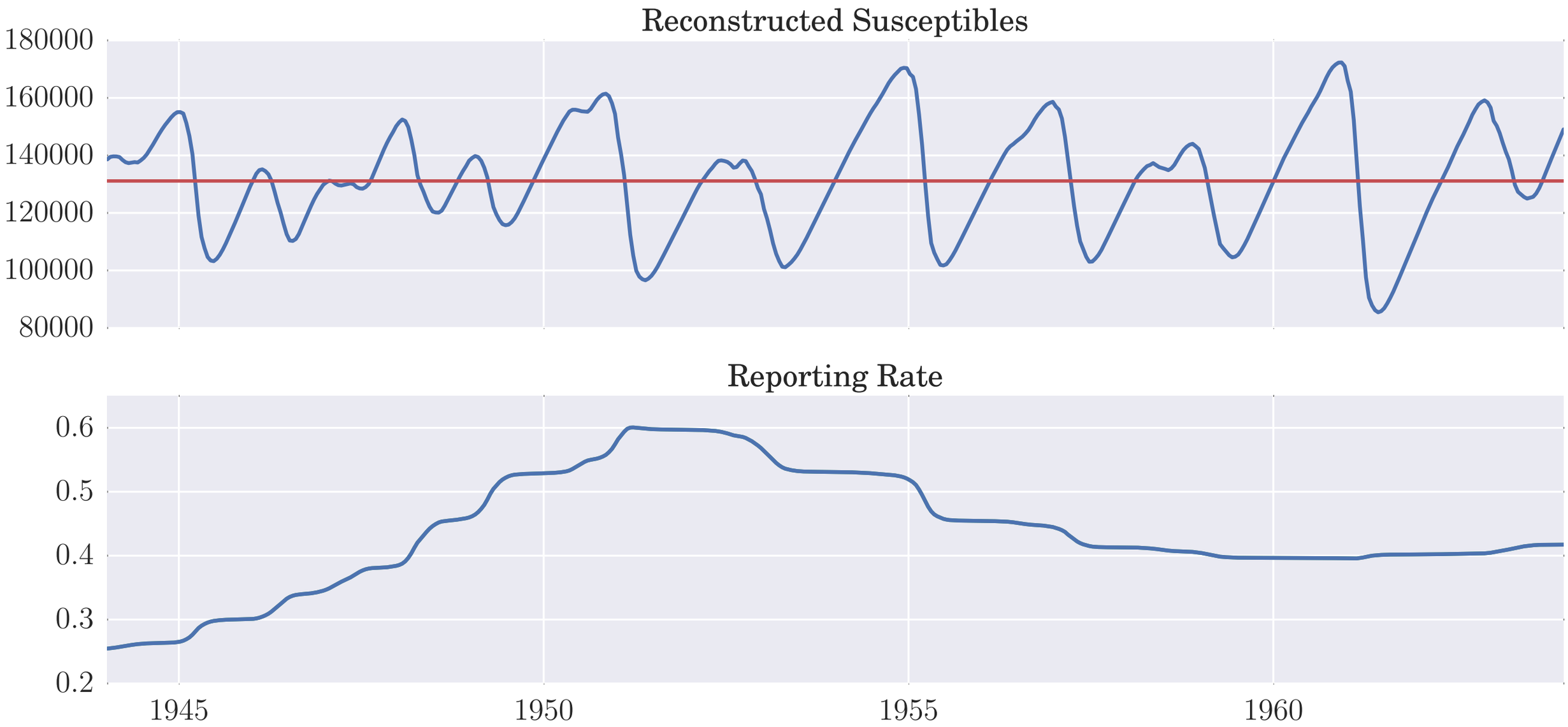
Susceptible Reconstruction



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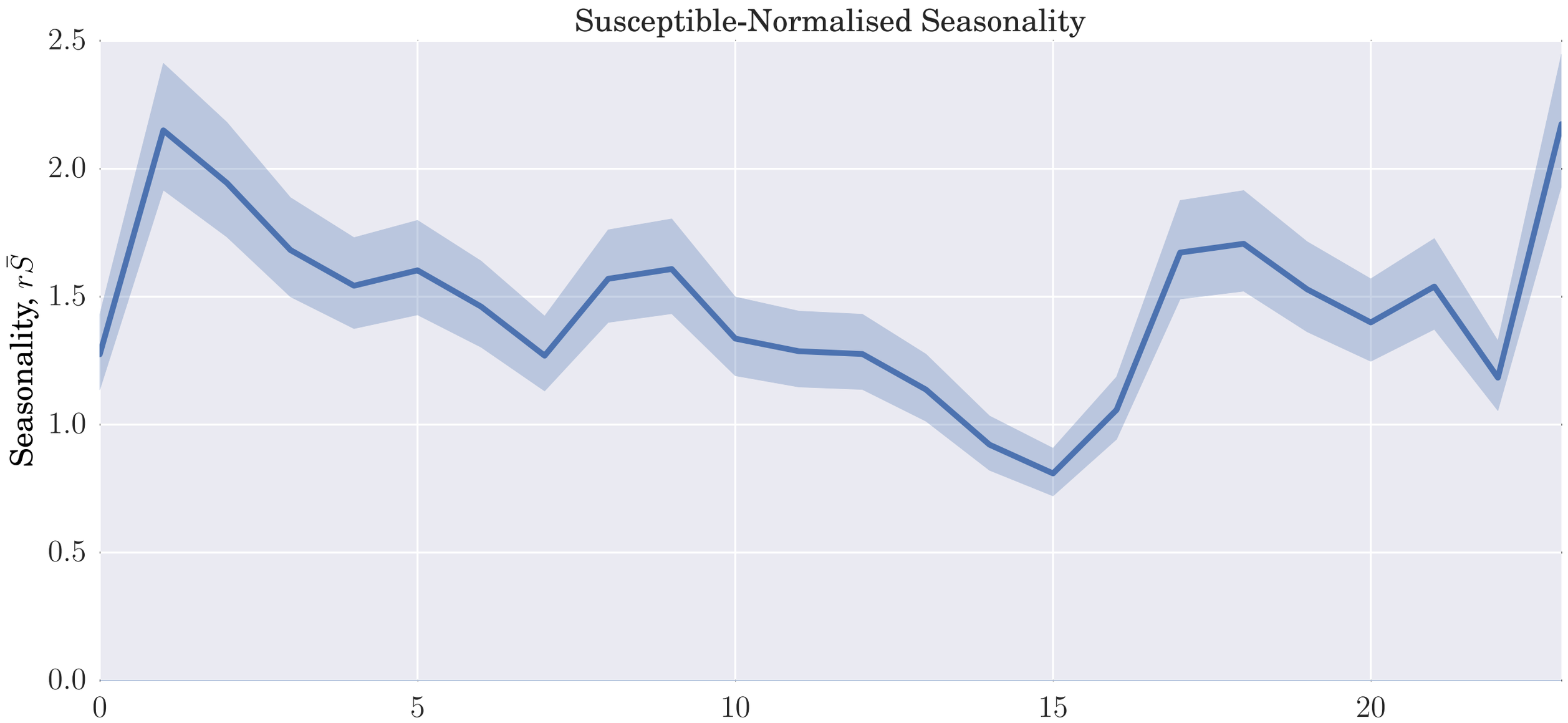
Time-Series SIR

Seasonality



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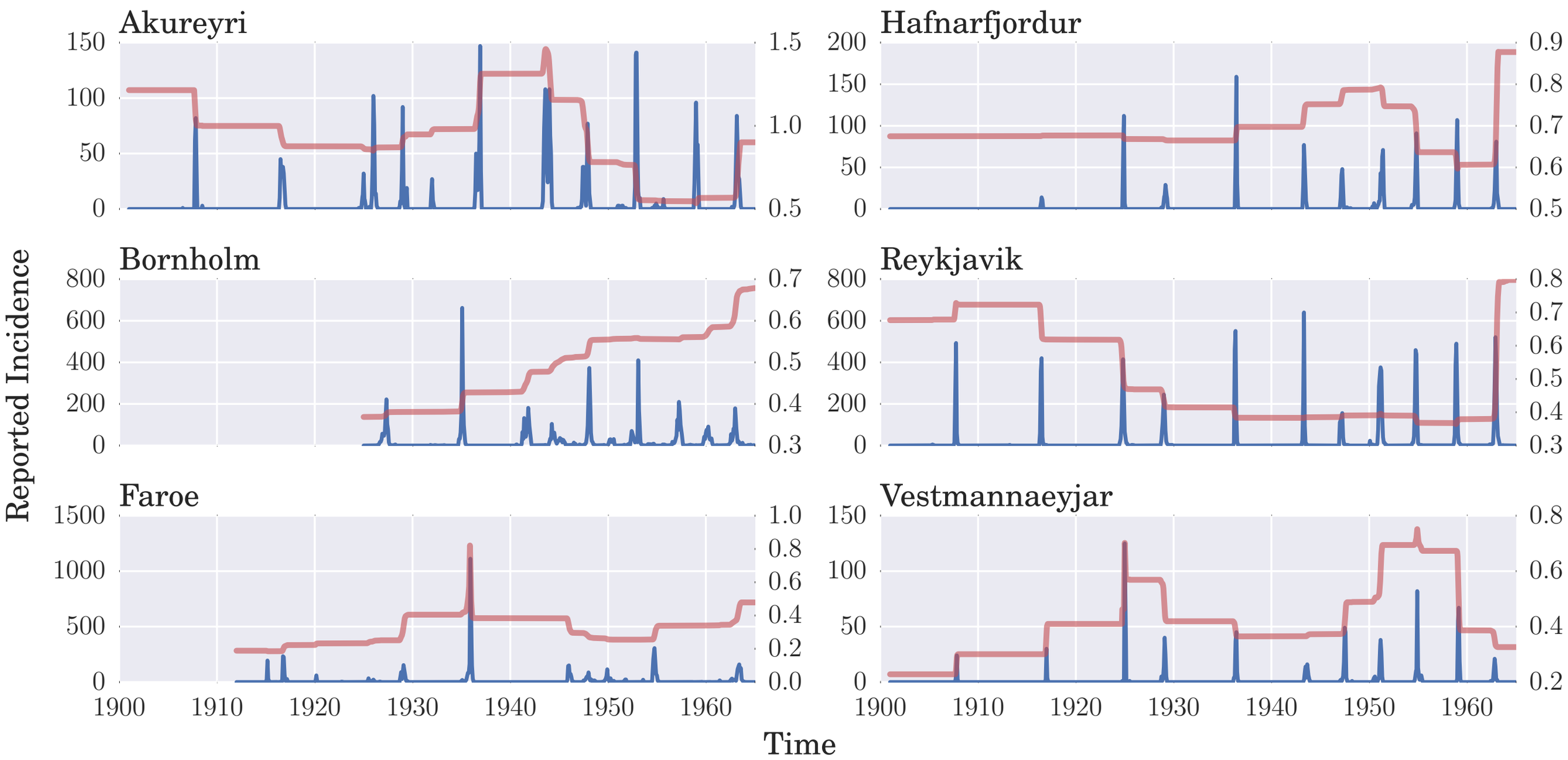
Model Fitting

Reporting Rates



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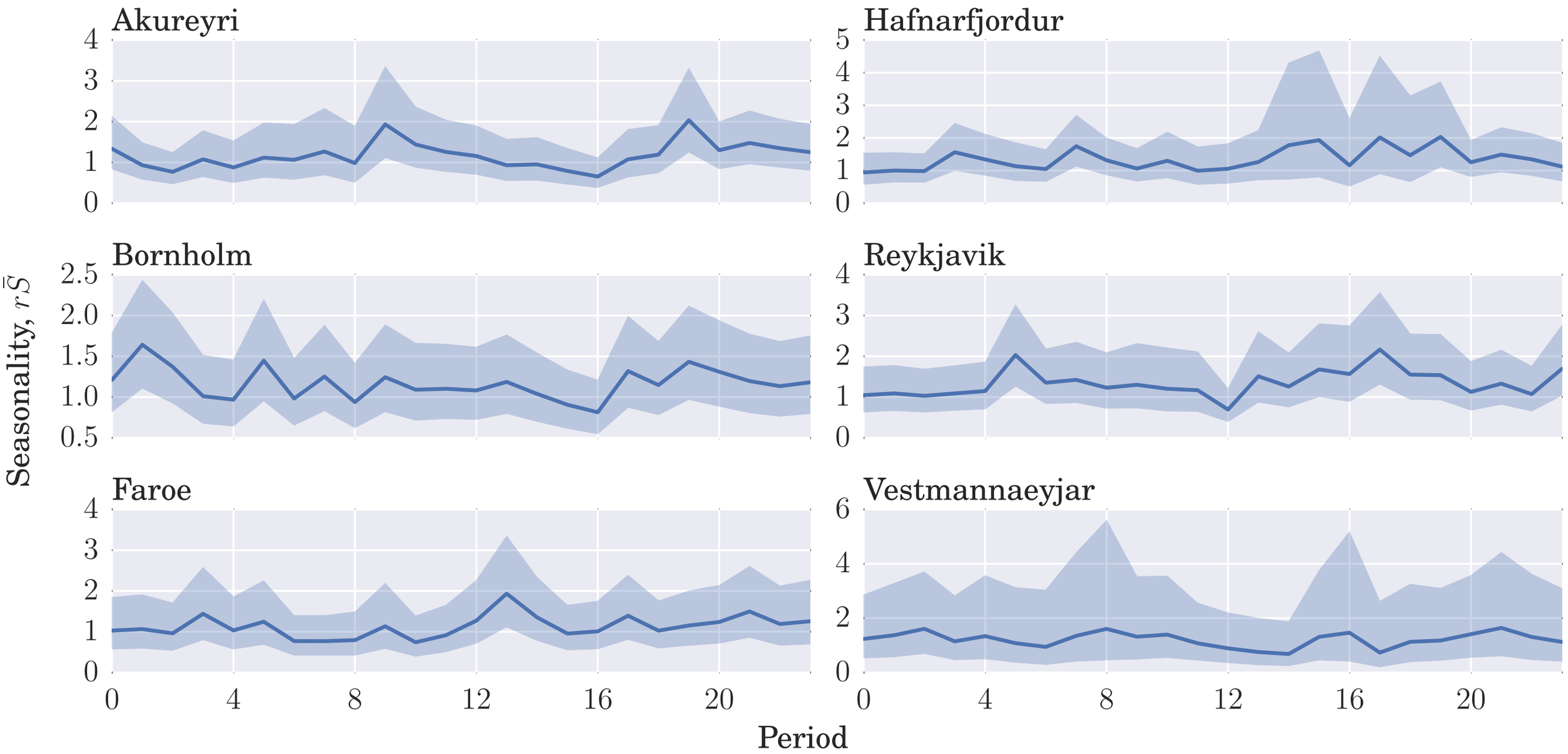
Model Fitting

Seasonality



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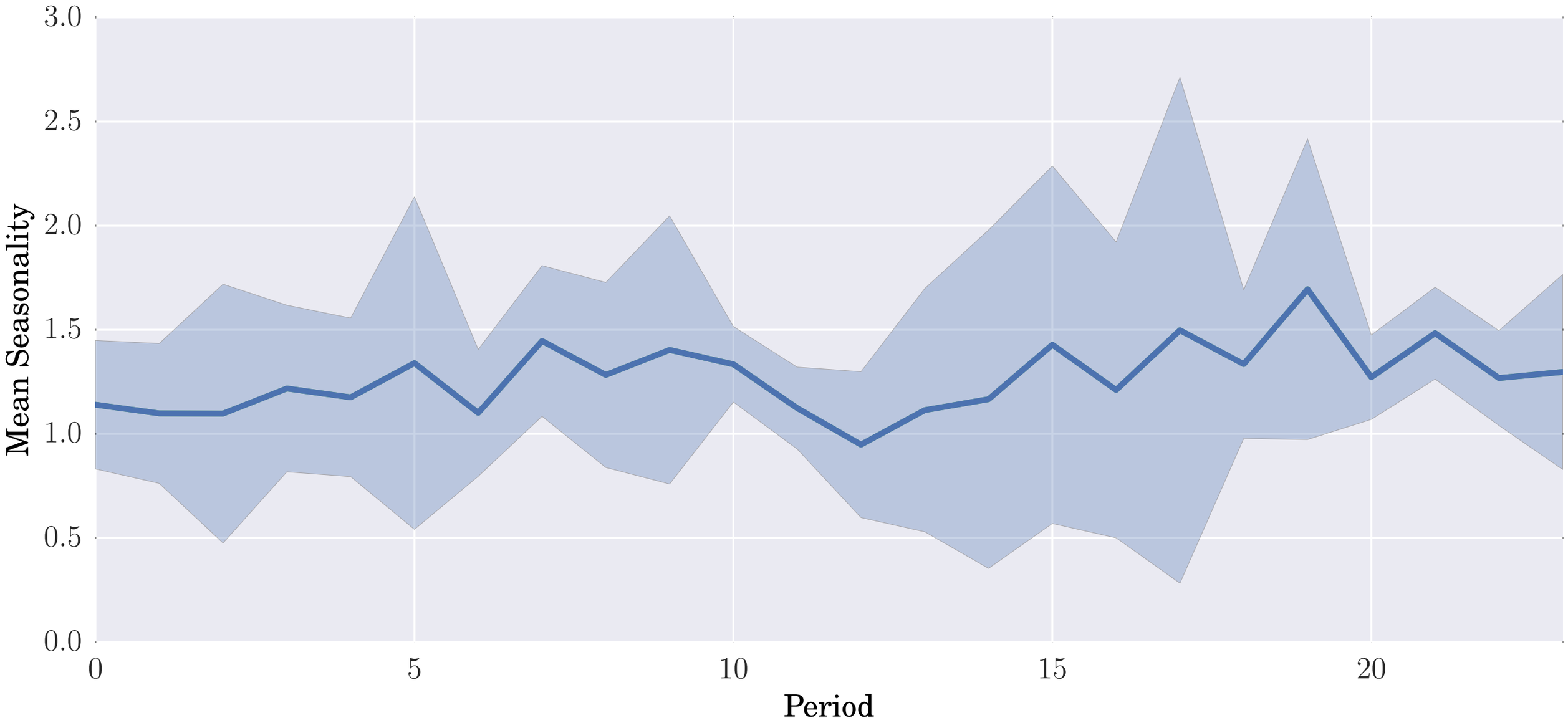
Seasonality



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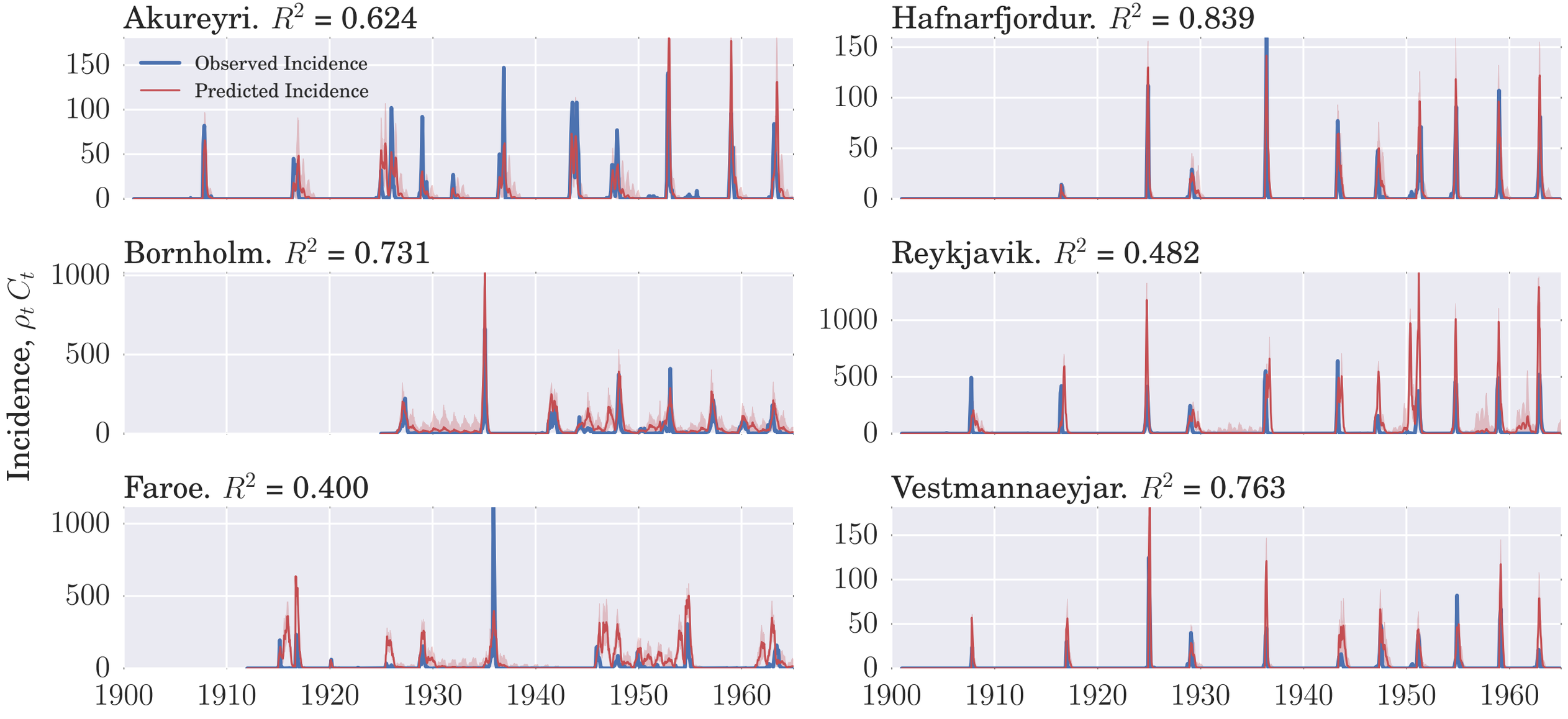
Model Fitting

Predictions



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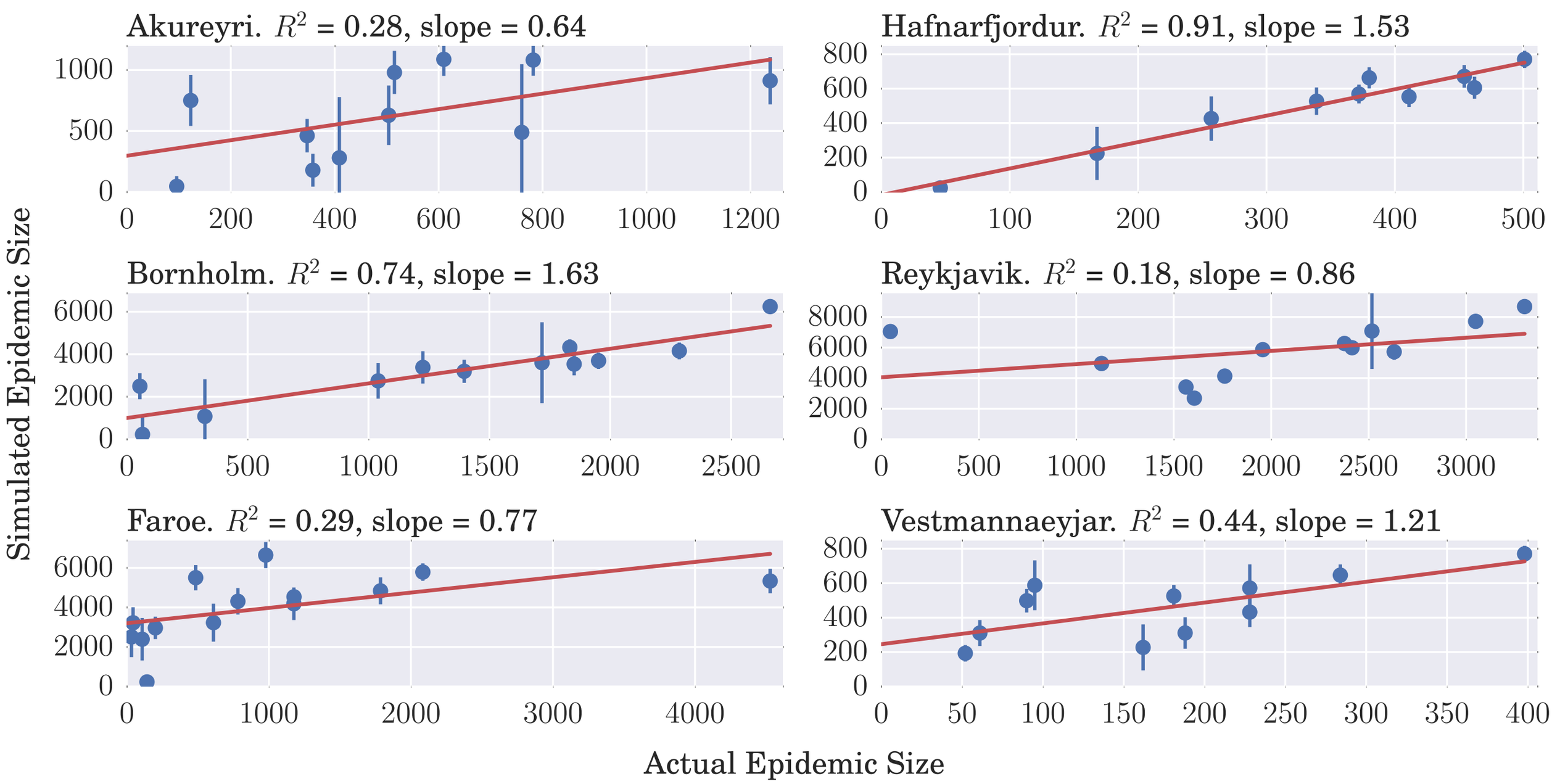
Model Fitting

Epidemic Sizes



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Summary

Fit TSIR to small population data

- Integer-factor interpolation on incidence data
- Gaussian process regression on reporting rate
- Convolution-based peak detection

Some signal found in the time-series

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Future Work

Find better data !

(Obtain and abuse government connections)

Trajectory matching / SSM over incidence

(Improve quality of current incidence data)

Fitting of hyperparameters

(Zero-sensitivity, GP, inhomogeneity, delay, TSEIR …)

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