Predictability in Highly Stochastic Systems

Measles in Small, Heterogeneous Populations

Quentin Caudron, Ayesha Mahmud, Magnús Gottfreðsson, Bryan Grenfell

Ecology and Evolutionary Biology

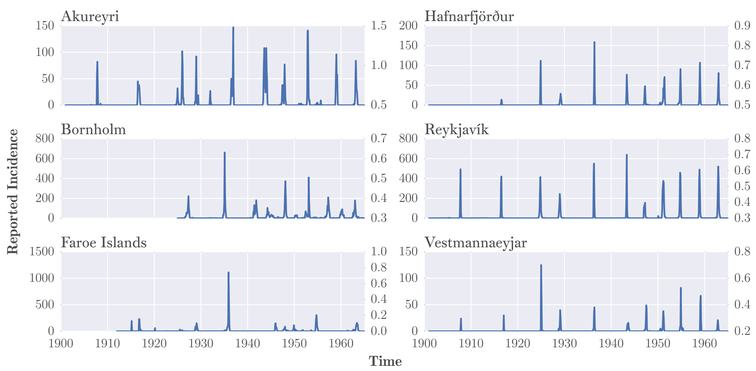
qcaudron@princeton.edu

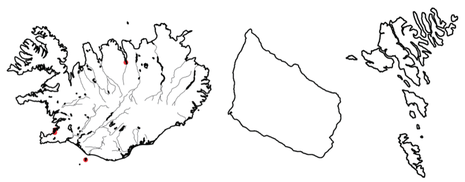
PRINCETON UNIVERSITY

Abstract

Measles is a strongly immunising, acute, directly transmitted infection, making it an exemplar of SIR-like diseases. However, in small, heterogeneous populations, dynamics become difficult to model. Epidemics are highly stochastic, driven by introduction from outside the population, and may only involve distinct metapopulations. In Iceland, the Faroe Islands, and Bornholm, we show that some predictability can be found, despite geographical isolation, spatially inconsistent demographic data, and small population sizes.

Data





Iceland : inconsistent demographic and medical district borders

Bornholm : single population, no spatial barriers, representative data

Faroe Islands : aggregate incidence data, but distinct spatial processes; potential significant metapopulation effect

Model Fitting

$$I_t = r_t \, I_{t-1}^{\alpha} \, S_{t-1} \, \varepsilon_t \qquad \mathbb{E}\left[\varepsilon_t\right] = 1$$

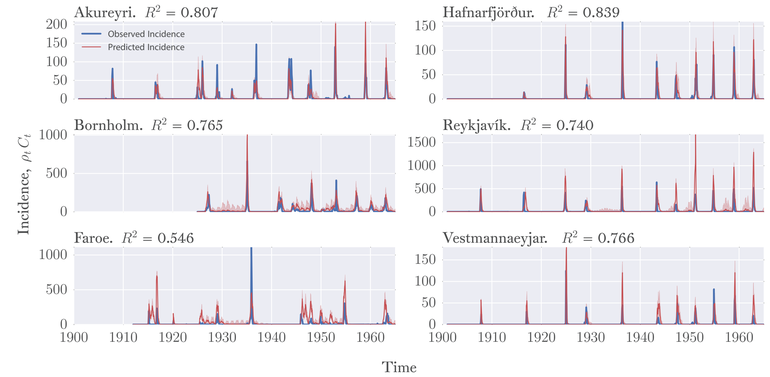
$$S_t = B_{t-d} + S_{t-1} - I_{t-1} + u_t \qquad \mathbb{E}\left[u_t\right] = 0$$

r_t Periodic seasonality, $r_t = r_{t+P}$

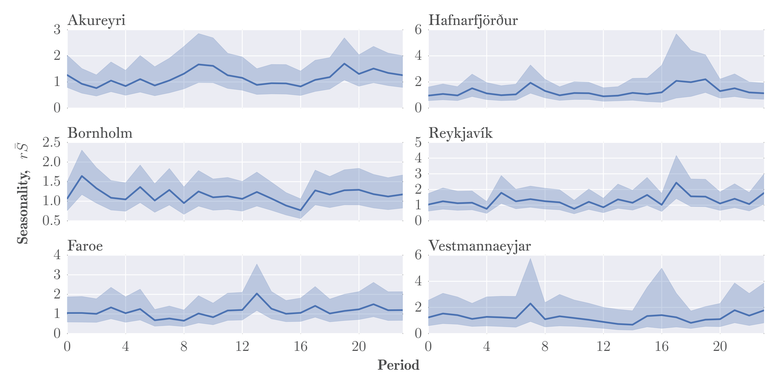
α Inhomogeneity parameter

d Delay due to maternal immunity

Simulation Predictions

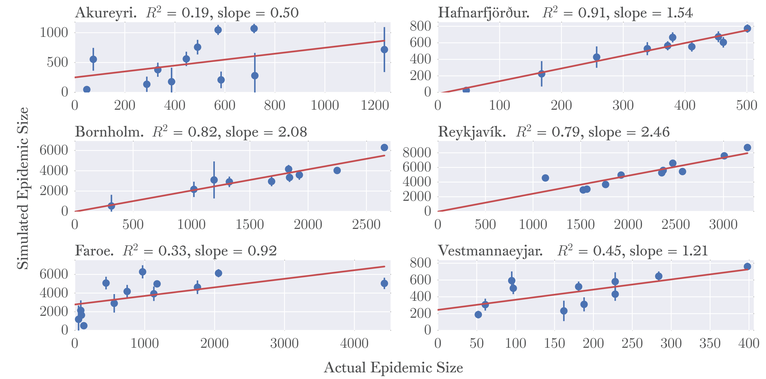


Seasonality



Results

Predicting Epidemic Sizes



Bornholm : single population and simple geography yield a good fit between observed and predicted epidemic sizes.

Iceland : depending on overlap between medical and municipal borders, fits vary significantly in quality. Hafnarfjorður, a small region, shows good agreement (except for one point), possibly indicating matching medical and municipal borders. Reykjavík, probably having a large number of municipalities in the medical district, fits poorly.

Faroe Islands : distinct metapopulations cause a poor fit. Without improved observation and demographic data, little could be said about the size of upcoming epidemics.

Some signal is found in predicting epidemic sizes. Improved data, especially related to spatial processes, would allow for more predictive power.

References

Time series modelling of childhood diseases : a dynamical systems approach. Finkenstädt and Grenfell, J R Stat Soc C : Appl Stat **49** (2), 2000.

Spatial diffusion : an historical geography of epidemics in an island community. Cliff, Haggett, Ord, and Versey, 1981, Cambridge University Press.