Let  $\mathbb{E}[S_t] = \bar{S} : S_t = \bar{S} + Z_t$ 

 $Y_t = -Z_0 + \rho_t X_t + Z_t$ 

Then 
$$Z_t$$
 are the residuals of the regression: