



QCRYPT  
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# Experimental Quantum Conference Key Agreement Using a Photonic Graph State

Joseph Ho | QCrypt, August 2021

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Engineering and  
Physical Sciences  
Research Council

## Outline

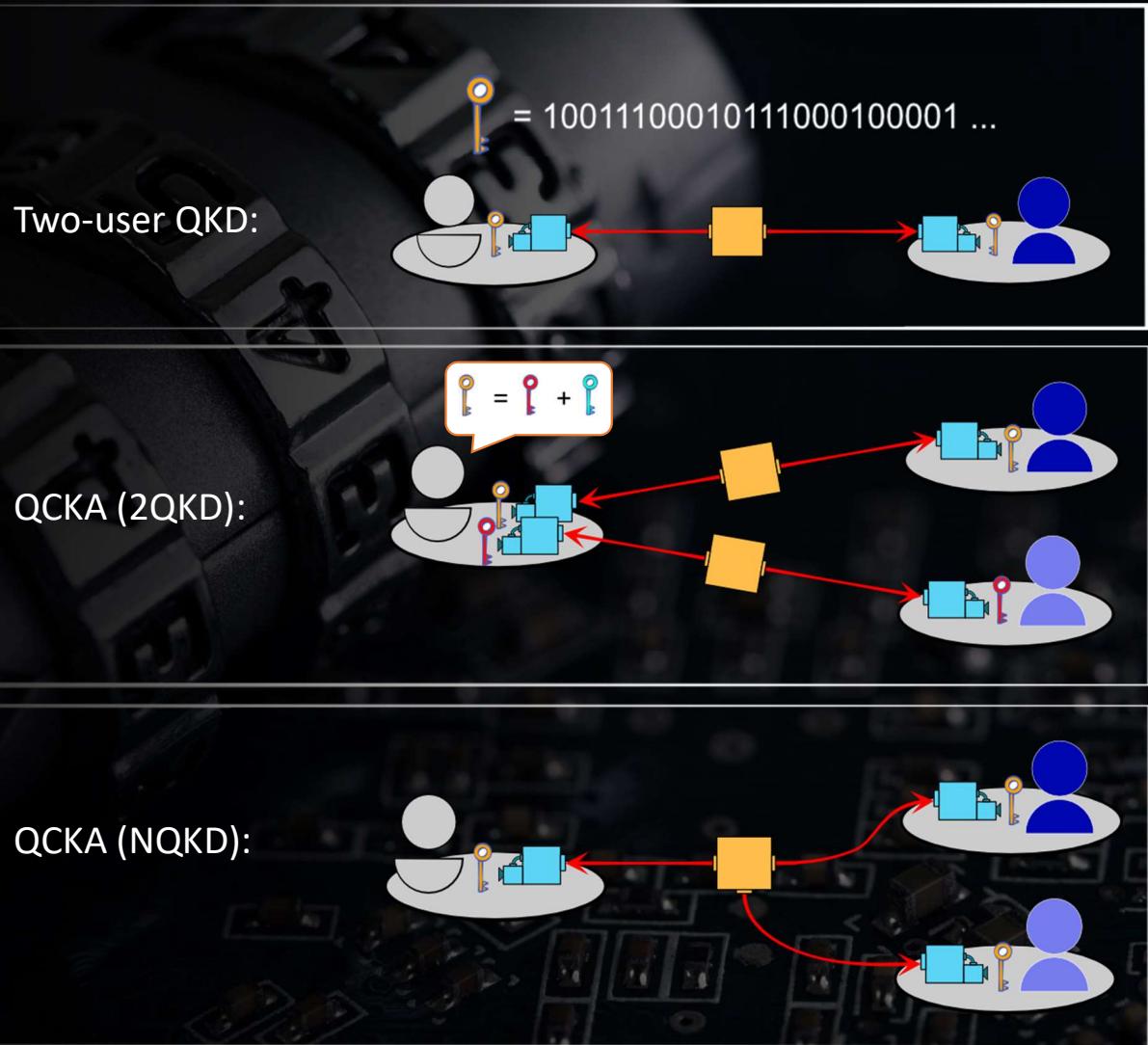
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- Conference key agreement at a glance
  - Previous experiment using GHZ states
- Quantum networks and graph states
  - Conference key agreement: NQKD vs 2QKD
- Experimental setup
  - 6-photon graph, GHZ states and Bell pairs
  - Results: measured key rate
- Summary

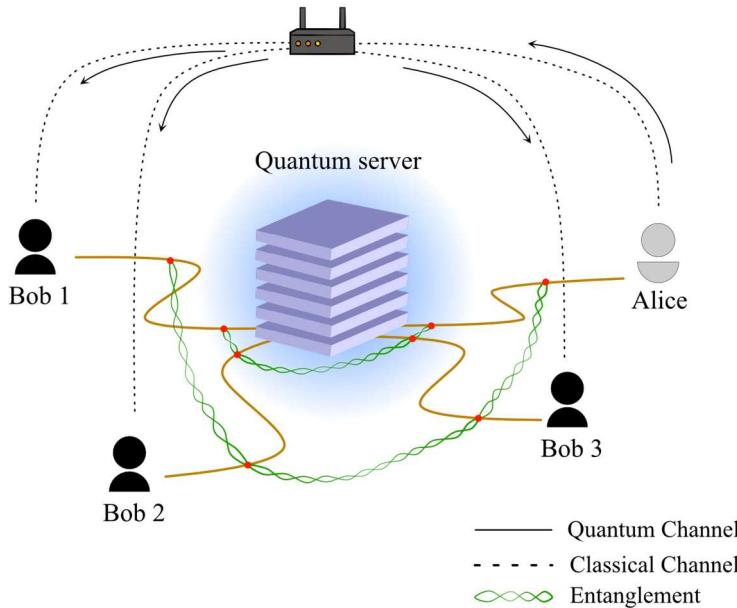


# Quantum Conference Key Agreement

Allows N users to share a common, secret key for group-wide encryption.



# Quantum Conference Key Agreement - NQKD



## N-BB84 Protocol



Distribute GHZ state  
each round



Perform sequence of  
measurements



Estimate security  
parameters



Error correction and  
privacy amplification  
on raw key

Distribute  $|GHZ\rangle = \frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$  for a total of L rounds

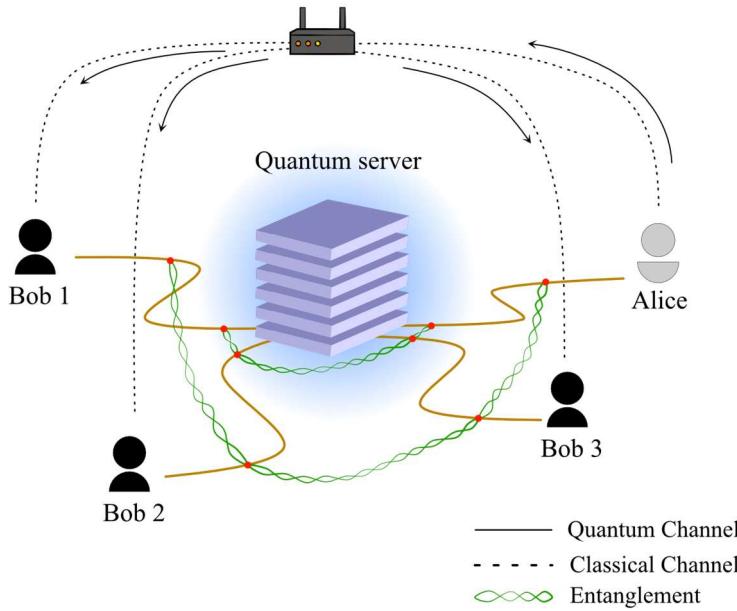
In each round, measure qubit according to preshared sequence:

Type-I:  $\hat{Z}$  for key generation

Type-II:  $\hat{X}$  for parameter estimation,  $m = L \cdot p$

Epping et al., NJP, 19, 093012 (2017)  
Grasselli et al., NJP, 20, 113014 (2018)

# Quantum Conference Key Agreement - NQKD



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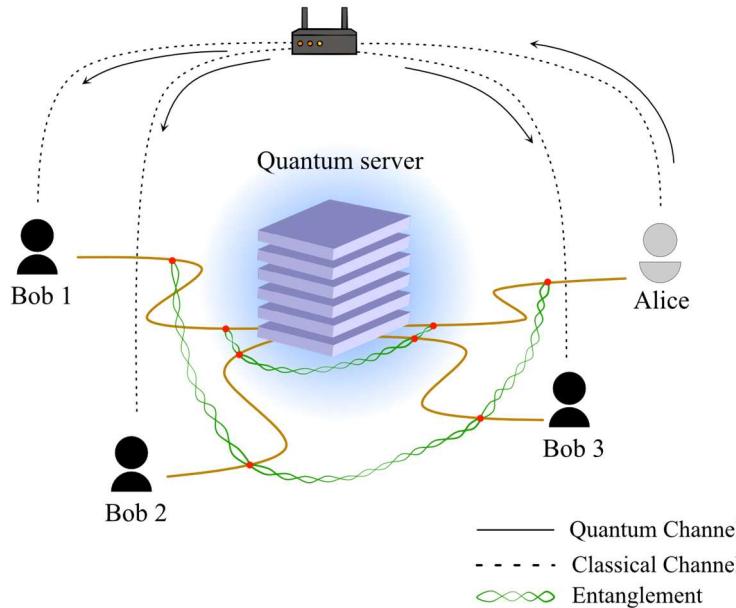
Estimate security parameters:

- Disclose subset of  $m$  type-I rounds
- Disclose all  $m$  type-II rounds
- Evaluate:

$$QBER = \max \{Q_{AB_i}^m\} \quad \text{where, } Q_{AB_i}^m = \left(1 - \langle \sigma_Z^A \sigma_Z^{B_i} \rangle\right)/2$$

$$Q_X^m = (1 - \langle \sigma_X^{\otimes N} \rangle)/2$$

# Quantum Conference Key Agreement - NQKD



## N-BB84 Protocol



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Raw key:  $n = L - 2m$

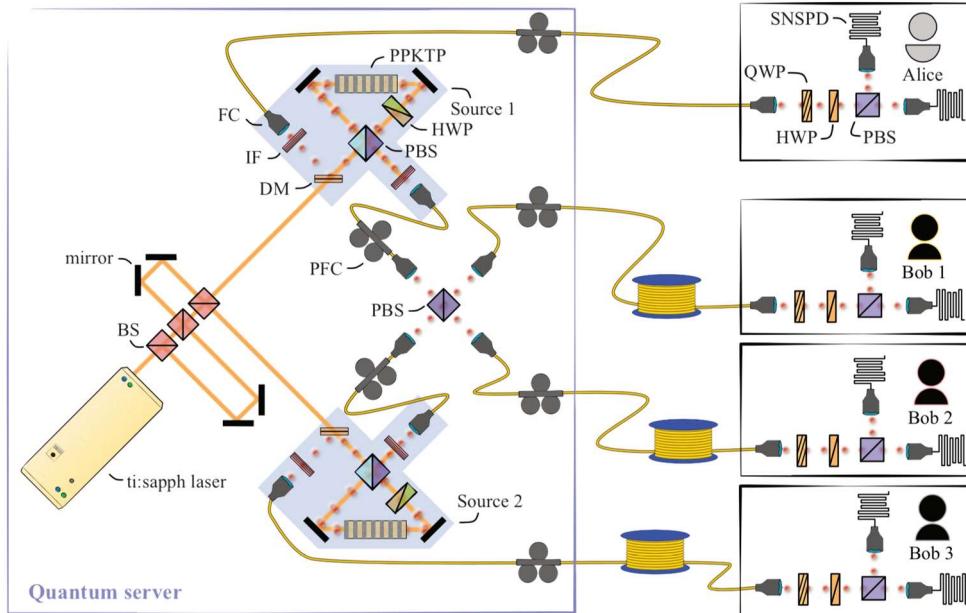
Apply multi-user error correction and privacy amplification

Fractional secure key rate (asymptotic limit):

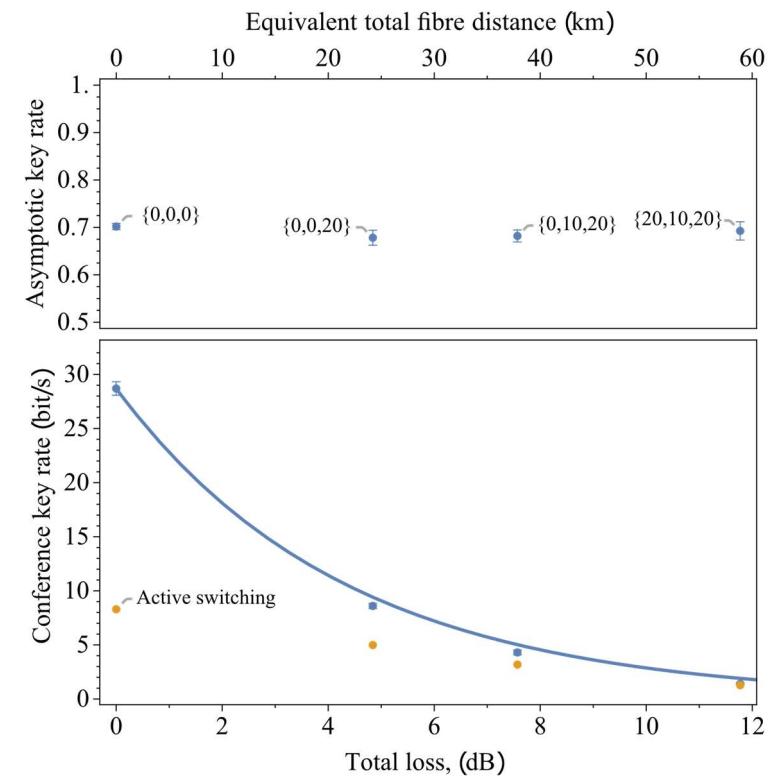
$$AKR = \frac{\ell}{L} = 1 - h(QBER) - h(Q_X)$$

Epping et al., NJP, 19, 093012 (2017)  
Grasselli et al., NJP, 20, 113014 (2018)

# Quantum Conference Key Agreement - Experiment



Proietti, et al., Sci Adv, eabe0395 (2021)



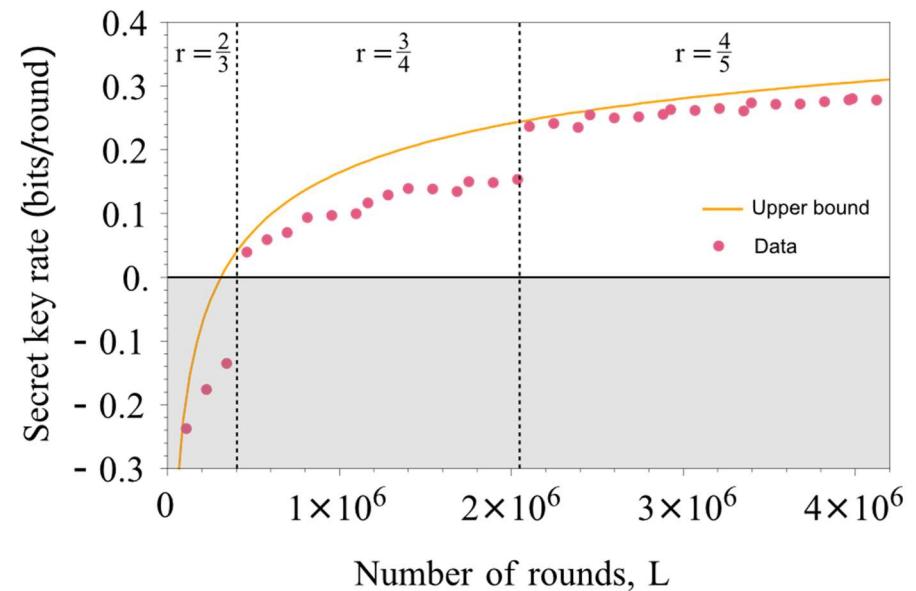
# Quantum Conference Key Agreement - Experiment

Finite key rate:

$$\begin{aligned}\frac{\ell}{L} = & \frac{n}{L}[1 - h(Q_X^m + 2\xi_X) \\ & - h(\text{QBER}^m + 2\xi_Z)] - \log_2 \left[ \frac{2(N-1)}{\epsilon_{EC}} \right]^{\frac{1}{L}} \\ & - 2 \log_2 \left[ \frac{1 - 2(N-1)\epsilon_{PE}}{2\epsilon_{PA}} \right]^{\frac{1}{L}} - h(p),\end{aligned}$$

see Grasselli et al., NJP, 20, 113014 (2018).

- Multi-party error correction, LDPC codes
- Standard privacy amplification, Toeplitz matrix

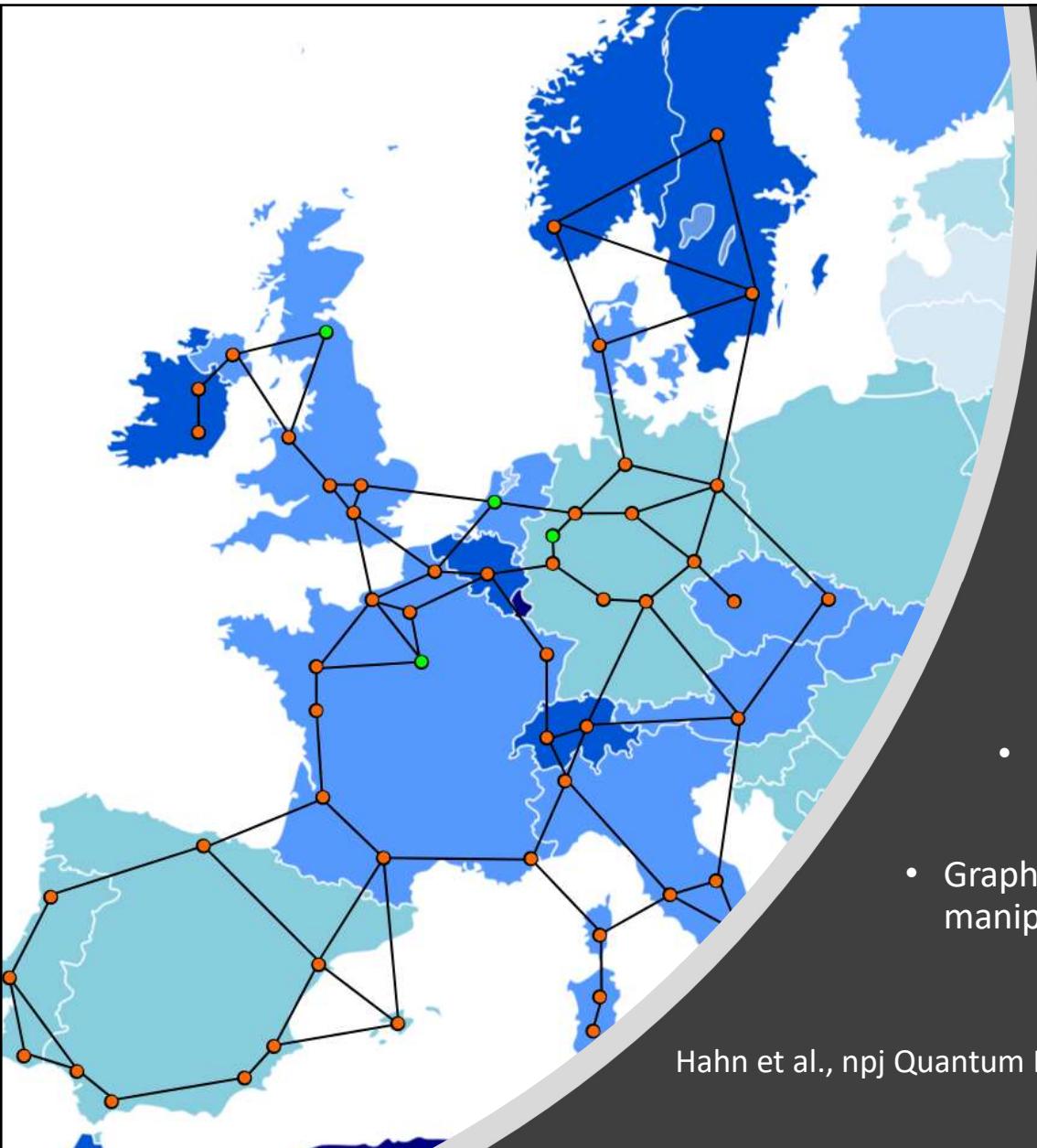


# QCKA in Networks

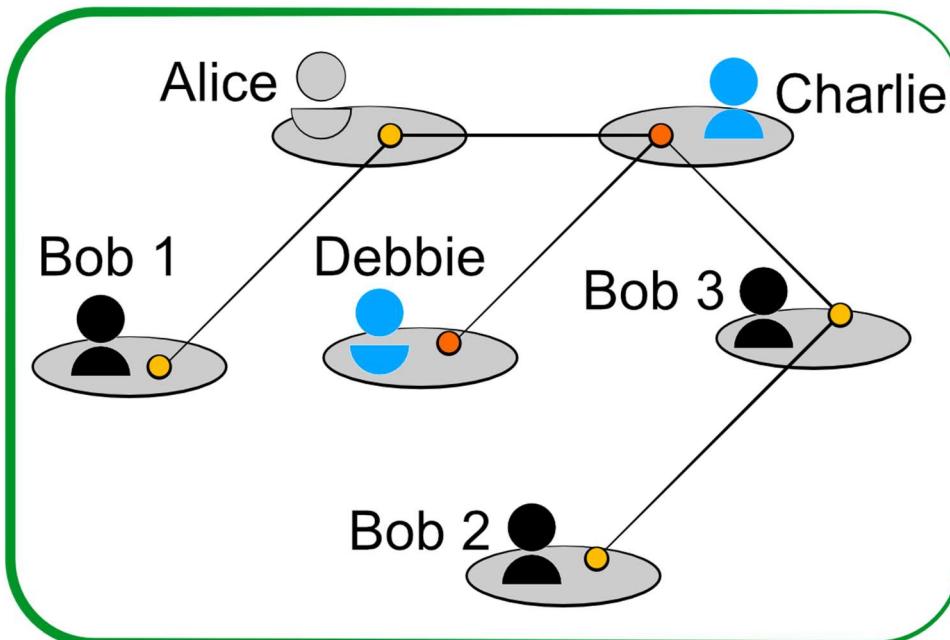
- Future multi-node quantum networks will have finite channels
- In constrained networks, NQKD can use GHZ states to reduce congestion versus 2QKD with Bell pairs
- Delivering different entanglement resources to connected users poses challenges
- Graph states provide a useful framework for describing and manipulating complex entanglement in networks

Hahn et al., npj Quantum Inf. 5, 76 (2019)

Epping et al., NJP, 19, 093012 (2017)



# Six-photon Graph for QCKA

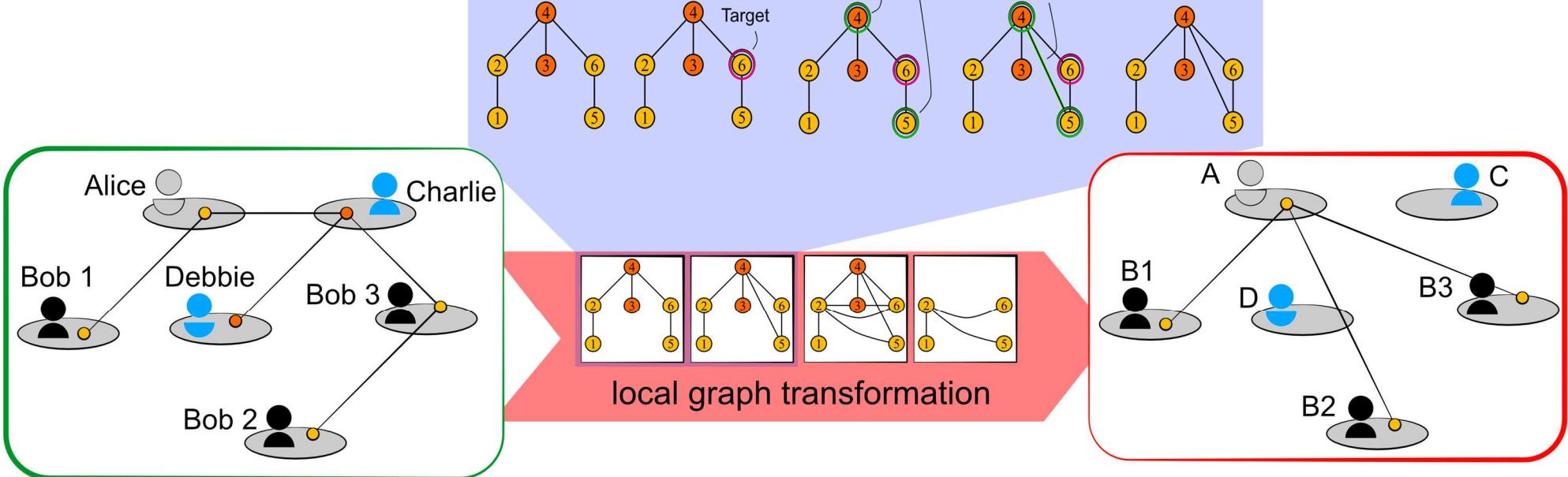


- Graph state created in a network
  - Nodes → qubit-encoded photon
  - Edges → pairwise interaction
- Using local complementation (LC) techniques to transform graph and distribute entanglement resources

Hahn et al., npj Quantum Inf. 5, 76 (2019)

Adcock et al., Quantum Sci. Tech. 4, 015010 (2019)

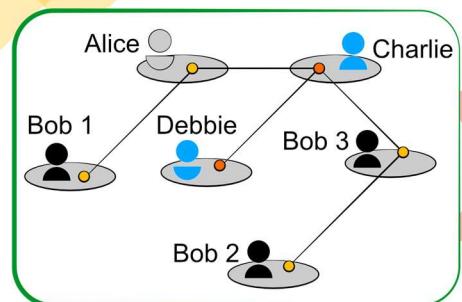
# Six-photon Graph for QCKA



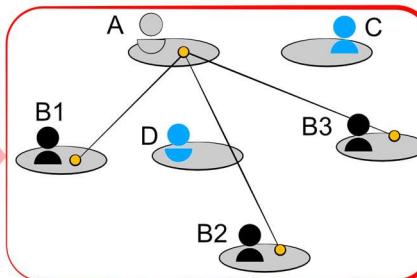
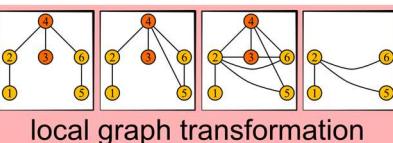
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# Six-photon Graph for QCKA



**NQKD**

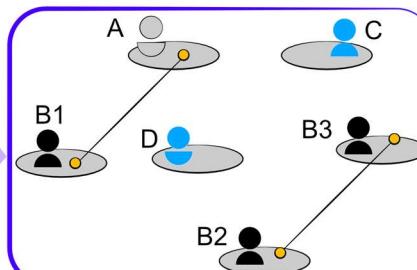
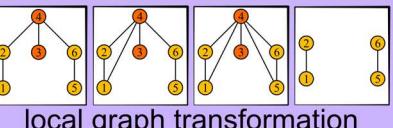


**NQKD protocol**

- Obtain GHZ state
- Measure security parameters
- Evaluate AKR from N-BB84 scheme



**2QKD**

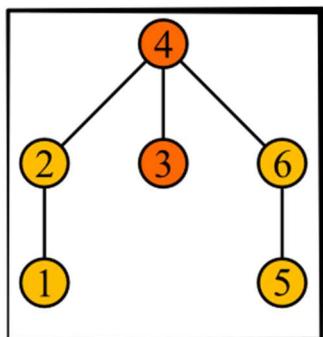


**2QKD protocol**

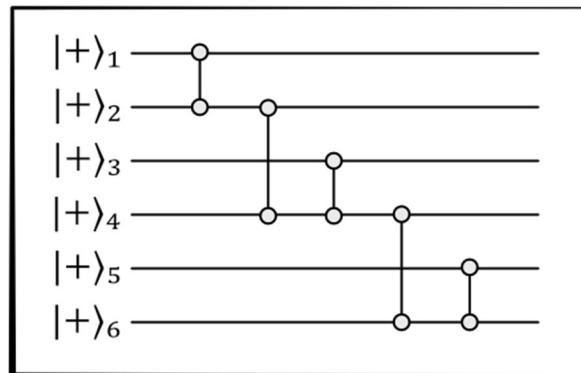
- Obtain three Bell pairs
- Measure security parameters
- AKR of each Bell pair from BB84,  $r_{AB_1}, r_{B_2B_3}, r_{AB_2}$ , then evaluate,

$$AKR_{2QKD} = \frac{1}{\frac{1}{r_{AB_3}} + \max \left\{ \frac{1}{r_{AB_1}}, \frac{1}{r_{B_2B_3}} \right\}}$$

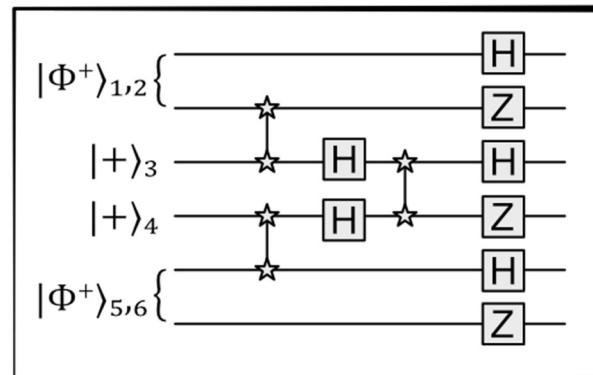
# From Graph to Optics Circuit



graph



equivalent circuit model



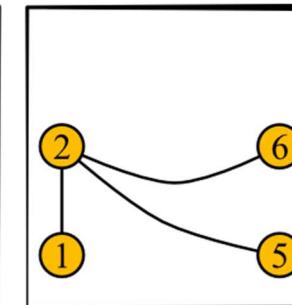
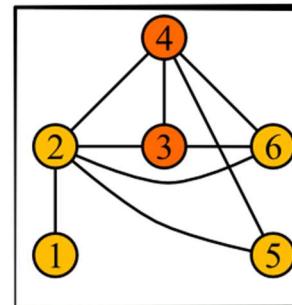
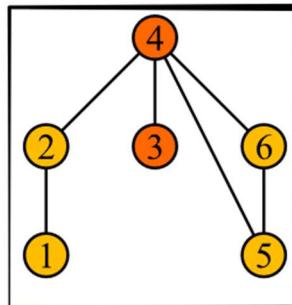
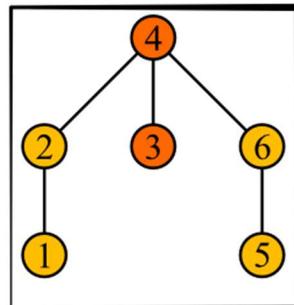
circuit optimisation

- Nodes denote qubit,  $|+\rangle = \frac{(|0\rangle + |1\rangle)}{\sqrt{2}}$
- Edges indicate CZ gate between nodes

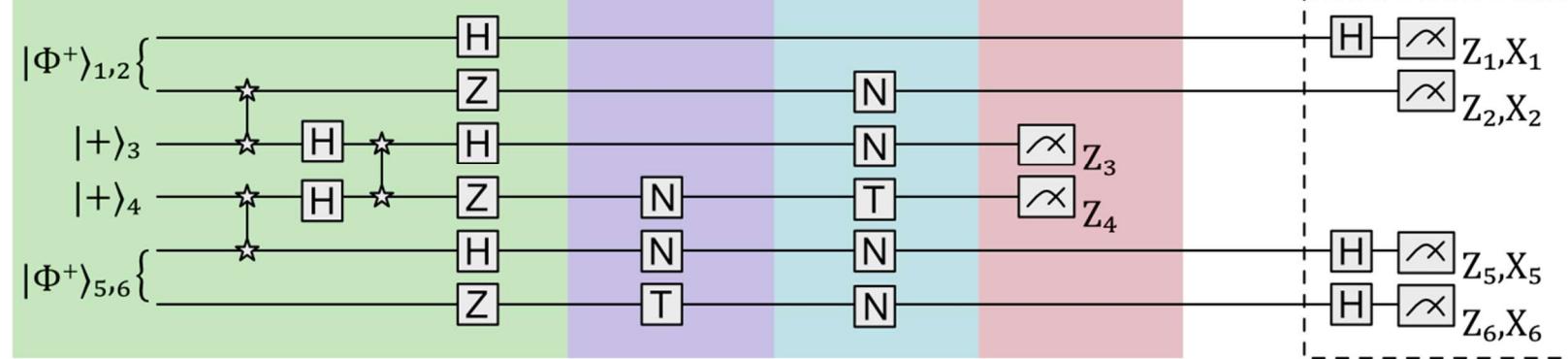
Optimum linear optics circuit exploiting:

- Offline entanglement
- Fusion gates
- Local unitaries

# Implementing Local Complementation



measurements  
for N-BB84



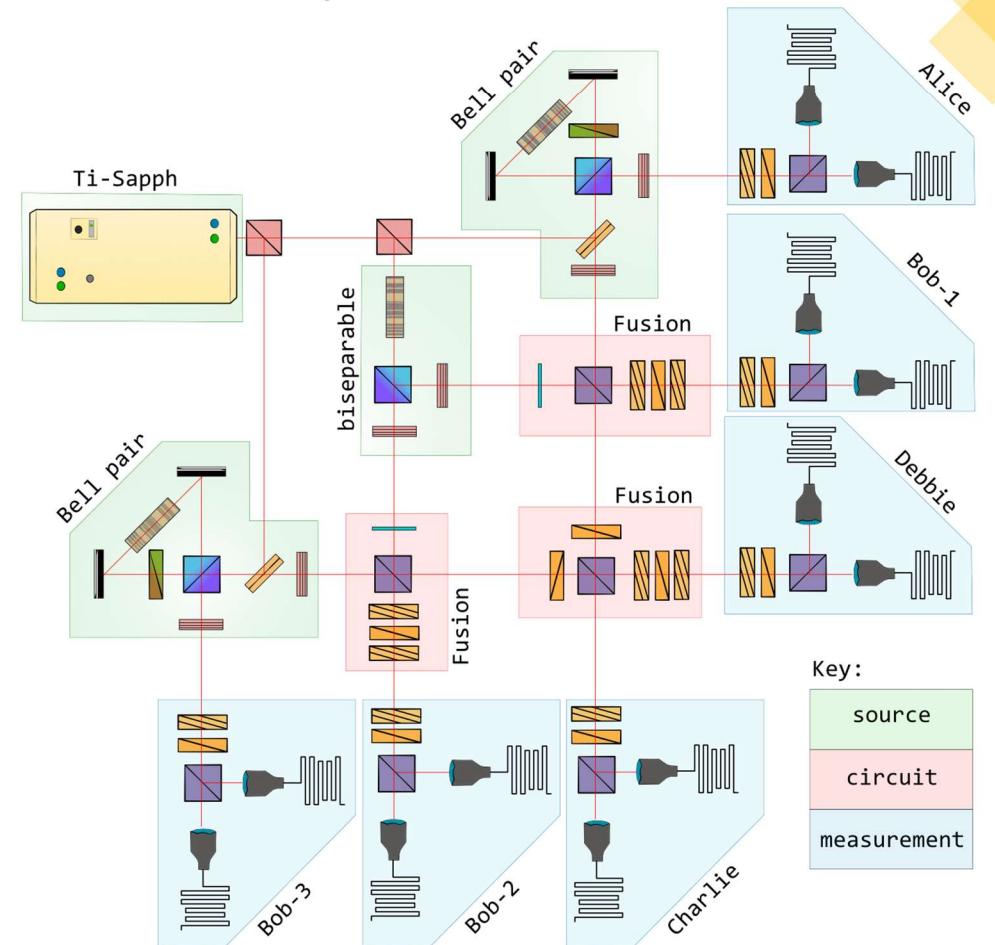
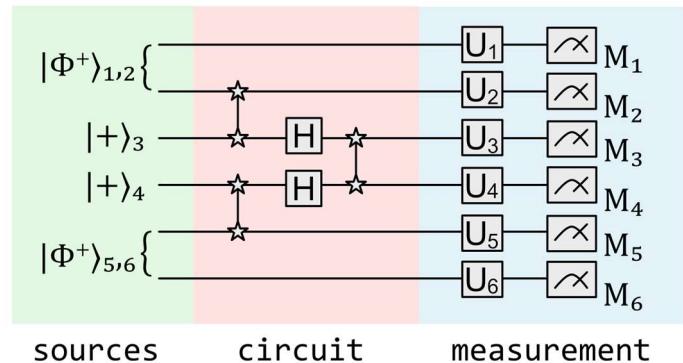
$$\boxed{N} = \sqrt{-iX}$$

$$\boxed{T} = \sqrt{iZ}$$

Adcock et al., Quantum Sci. Tech. 4, 015010 (2019)

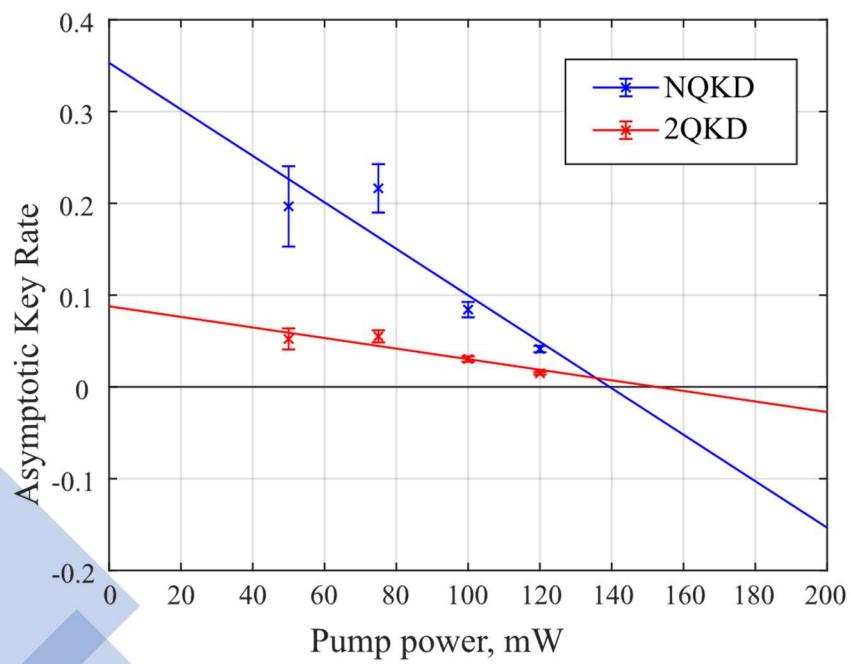
# Experimental QCKA Using Photonic Graph

- Ti-sapph laser: 80 MHz @ 774.9 nm, 1.3 ps
- 30 mm aperiodically-poled KTP crystal<sup>1</sup> for Type-II SPDC, 1550 nm photon pairs
- Non-deterministic Fusion gates,  $P_{suc} = \frac{1}{8}$

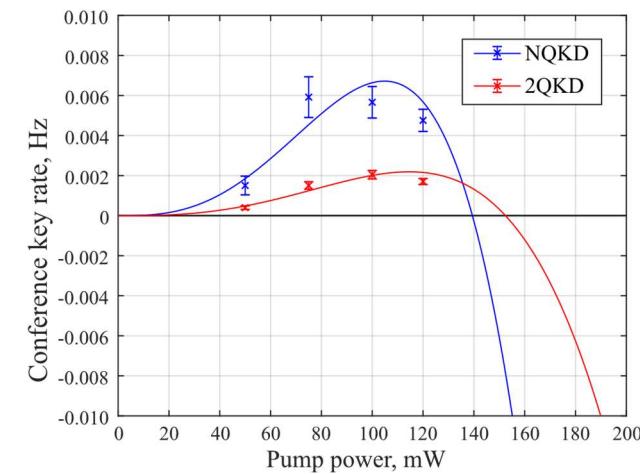
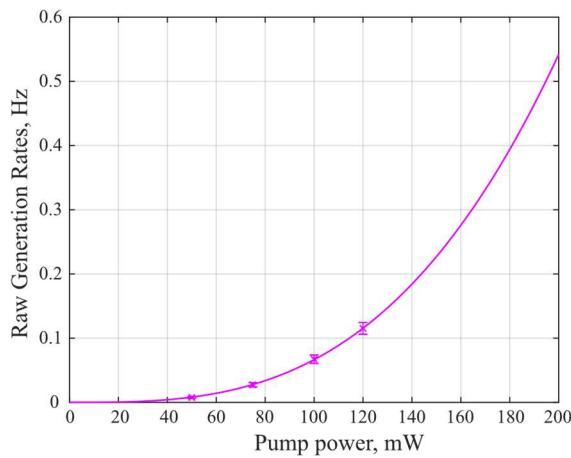


<sup>1</sup>Pickston et al., Opt. Express 29, 6991-7002 (2021)

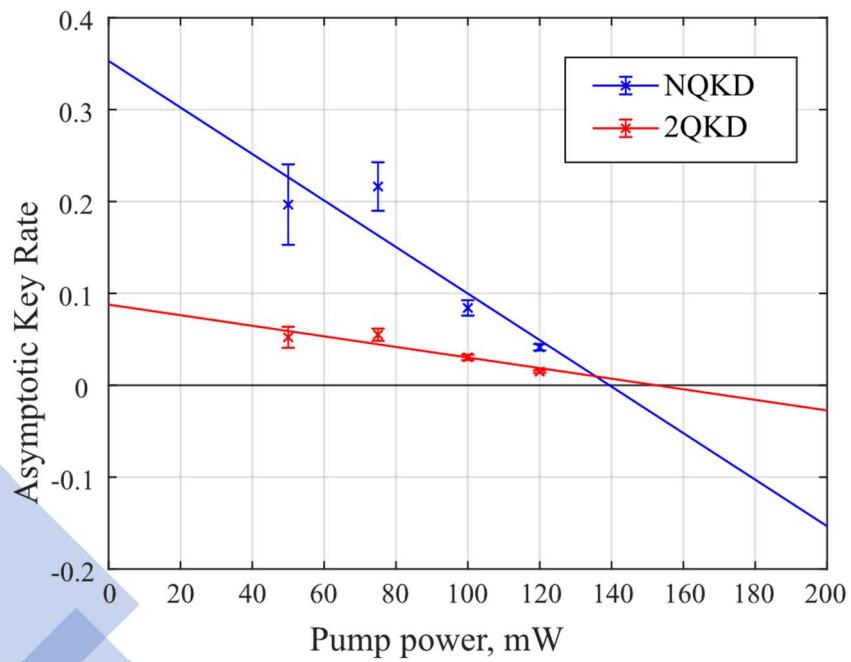
# Experimental QCKA Using Photonic Graph - Results



- Measured noise parameters, QBER and QX, to evaluate AKR when using N-BB84 for NQKD and 2QKD
- Increasing source brightness led to reduction in fractional key rates owing to added noise



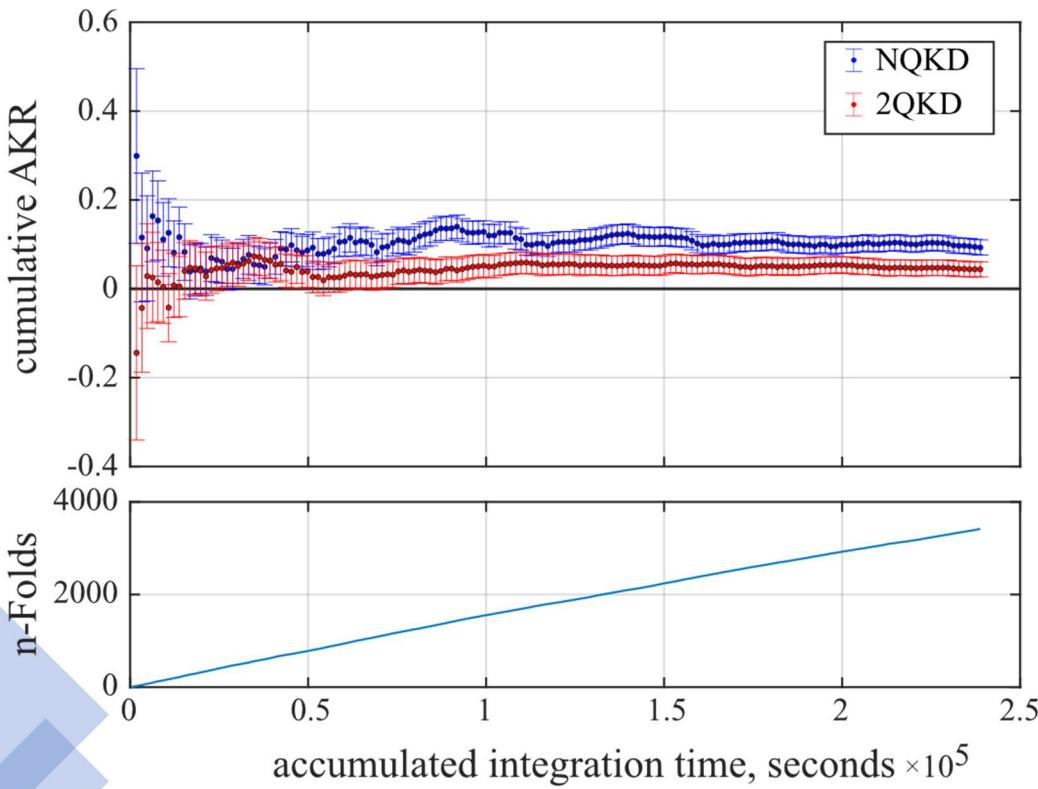
# Experimental QCKA Using Photonic Graph - Results



- Measured noise parameters, QBER and QX, to evaluate AKR when using N-BB84 for NQKD and 2QKD
- Increasing source brightness led to reduction in fractional key rates owing to added noise
- NQKD outperforms 2QKD by a factor greater than 2 in our measurement regime

Pump power	AKR – NQKD	AKR – 2QKD	Ratio
50 mW	$0.19 \pm 0.02$	$0.052 \pm 0.004$	$3.8 \pm 0.4$
75 mW	$0.216 \pm 0.009$	$0.055 \pm 0.003$	$3.9 \pm 0.3$
100 mW	$0.084 \pm 0.003$	$0.030 \pm 0.001$	$2.8 \pm 0.2$
120 mW	$0.041 \pm 0.002$	$0.0148 \pm 0.0005$	$2.8 \pm 0.2$

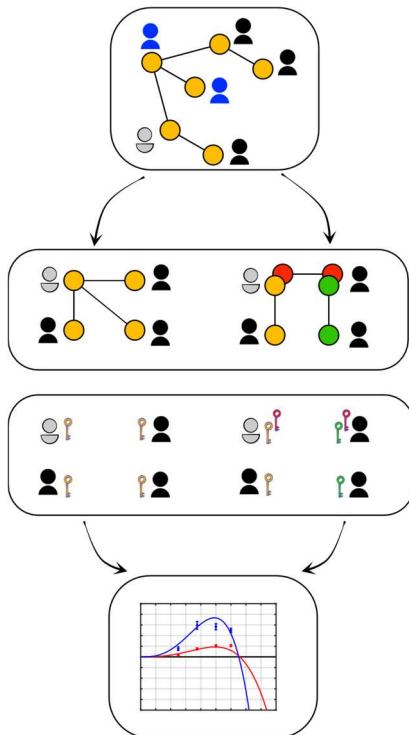
# Experimental QCKA Using Photonic Graph - Results



- Repeated measurement of noise terms for each protocol to assess long-term performance
- Measurement over 17 days without stabilisation or re-optimisation of state preparation
- Mean ratio of NQKD vs 2QKD rates of complete dataset,

$$AKR_{NQKD} : AKR_{2QKD} = 2.13 \pm 0.06$$

# Summary



- Experimentally implemented a 6-photon graph suitable for quantum conference key agreement
- We used local operations to distil resource states for NQKD and 2QKD (GHZ states and Bell pairs respectively)
- For a range of source brightness we observed the key rate advantage for NQKD over 2QKD
- In extended measurement run we measured a key rate advantage of NQKD:2QKD =  $2.13 \pm 0.06$

## Outlook:

- Improve 6-photon rates for other tasks
- Robustness of graph states to noise, e.g., photon loss, gate errors, channel dephasing

