Supplementary File of "Activation Function-assisted Objective Space Mapping to Enhance Evolutionary Algorithms for Large-scale Many-objective Optimization"

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I. Algorithm Pseudocode

The procedure of I_{ϵ^+} indicator selection method in EAGOA is presented in Algorithm S1.

Algorithm S1. Q = **IndicatorSelect** (P, L_k , Q)

Input: P (current population), L_k (solutions set), Q (new population);
Output: Q (new population)

/* Utilize the I_{ϵ^+} indicator to select the last layer */
Calculate a Chebyshev distance matrix I between each two solutions in L_k;
$F(\boldsymbol{s}_1) = \sum_{\boldsymbol{s}_2 \in L_k \setminus \boldsymbol{s}_1} - e^{-\mathrm{I}(s_2,s_1)/\kappa}$ for all $\boldsymbol{s}_1 \in L_k$; /* Calculate fitness values of solutions in L_k, κ is a coefficient, usually 0.05 */
while |P| - |Q| < $|L_k|$ **do**
  Remove $\boldsymbol{s}_*$ with the smallest fitness value from L_k;
  $F(\boldsymbol{s}) = F(\boldsymbol{s}) + e^{-\mathrm{I}(s_*,s)/\kappa}$ for all $\boldsymbol{s} \in L_k$; /* Update fitness values */
end while
Q ← Q ∪ L_k.

Specifically, for any $\boldsymbol{s}_1, \boldsymbol{s}_2 \in L_k$, a distance matrix is defined as:

$$\mathrm{I}(\boldsymbol{s}_1, \boldsymbol{s}_2) = \max\left(f_j(\boldsymbol{s}_1) - f_j(\boldsymbol{s}_2), 1 \le j \le M\right. \quad (1)$$

where M is the dimensionality of objective space. Then, the fitness value of $\boldsymbol{s}_1$ is calculated as:

$$F(\boldsymbol{s}_1) = \sum_{\boldsymbol{s}_2 \in L_k \setminus \boldsymbol{s}_1} - e^{-\mathrm{I}(s_2,s_1)/\kappa} \quad (2)$$

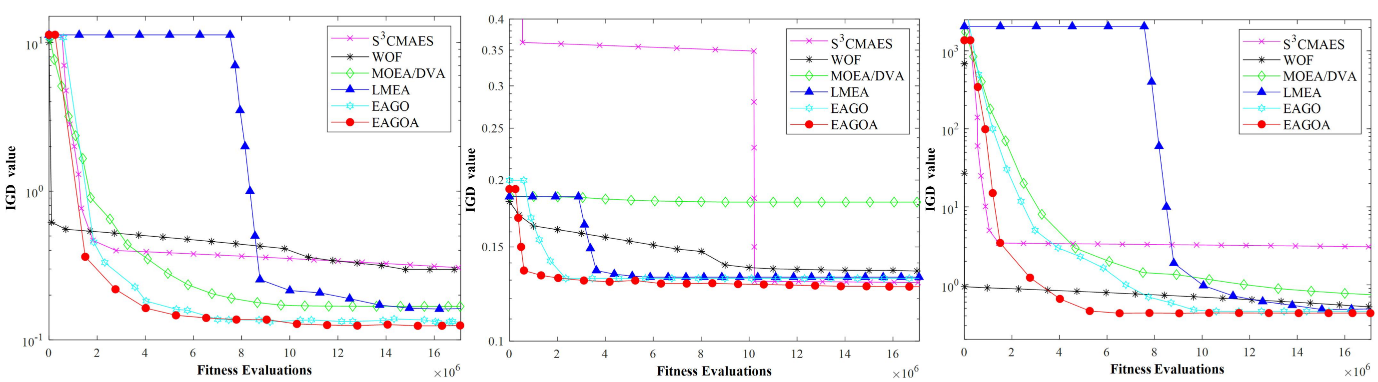
where κ is a fitness scaling factor, usually is 0.05. The solutions from L_k are selected by I_{ϵ^+} indicator until the size of new population Q is equal to the size of population P.

II. Additional Experimental Results

Fig. S1 shows the convergence process of IGD values of six algorithms under 5-objective and 1000-dimensional LSMOP1-9 problems. The abscissa is fitness evaluations (does not represent the algorithm time), and the ordinate is IGD. It can be clearly seen that EAGOA outperforms than its peers in most cases.

Table S-I and S-II shows the statistical results of the IGD values obtained by six algorithms on DTLZ1–DTLZ7 problems. The Wilcoxon rank sum test is adopted at a significance level of 0.05. The symbols "+", "-" and "≈" respectively indicate that the results are significantly better, significantly worse and similar to the results obtained by EAGOA. Friedman's test is adopted and "AvgRank" represents the average order of algorithms. When D = 100 and 1000, fitness evaluation count is 1 000 000 and 17 000 000, respectively. It can be seen that the proposed EAGOA outperforms or at least reaches its peers' best level.

To verify the computational efficiency of EAGOA, Fig. S2 summarizes the runtime of the six compared algorithms on test instances with 1000 variables. As illustrated in Fig. S2, our proposed EAGOA is computationally more efficient than its peers on all the test instances.



(a) (b) (c)

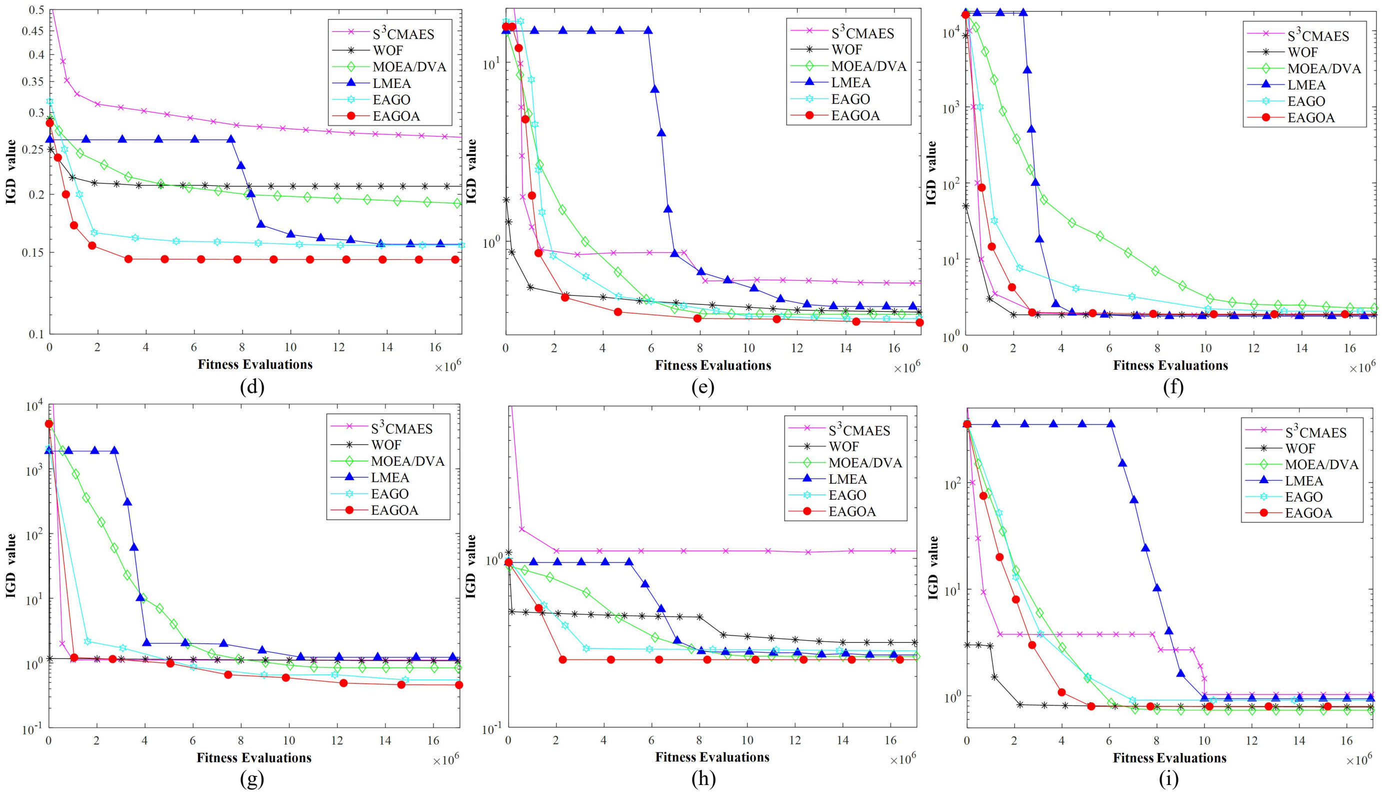


Fig. S1. The IGD convergence iterative process obtained by six algorithms on five-objective 1000-dimensional LSMOP1-9 problems. (a) LSMOP1. (b) LSMOP2. (c) LSMOP3. (d) LSMOP4. (e) LSMOP5. (f) LSMOP6. (g) LSMOP7. (h) LSMOP8. (g) LSMOP9.

TABLE S-I

IGD metric values of six algorithms on DTLZ1–DTLZ7 problems, where the best results on each test instance are bolded

Problem	M	D	MOEA/DVA	LMEA	WOF	S^3-CMA-ES	EAGO	**EAGOA(ours)**
DTLZ1	5	100	1.0056E+01(5.72E-05) −	**6.2360E-02(4.22E-04)** ≈	2.0477E-01(2.36E-02) −	5.5986E+02(3.24E+01) −	2.8348E-01(4.04E-03) −	6.9006E-02(2.56E-03)
	10	100	3.0555E+01(1.79E-02) −	**1.0524E-01(4.87E-03)** +	1.2431E+02(3.41E+01) −	4.3005E+02(4.68E+01)−	1.3804E-01(7.03E-03) +	1.6688E-01(2.37E-02)
DTLZ2	5	100	2.9042E-01(9.26E-08) −	1.9126E-01(2.14E-03) −	1.9499E-01(3.24E-02) −	1.9844E-01(3.31E-02) −	1.9291E-01(7.50E-05) −	**1.8747E-01(1.71E-02)**
	10	100	6.1598E-01(5.13E-02) −	3.9818E-01(1.48E-02) −	4.2554E-01(2.87E-02) −	4.1839E-01(2.43E-02) −	3.9480E-01(1.30E-04) −	**3.7656E-01(4.51E-02)**
DTLZ3	5	100	2.9242E+01(5.92E-05) −	1.9540E-01(2.14E-03) −	2.1673E+00(6.21E-01) −	2.3358E+03(3.75E+02) −	3.6499E-01(6.54E-03)−	**1.8151E-01(3.78E-02)**
	10	100	1.2187E+02(3.77E-02) −	3.9735E-01(3.77E-02) −	4.6357E+02(4.83E+01) −	2.2424E+03(2.75E+02) −	4.0103E-01(1.91E-03) −	**3.8359E-01(3.40E-02)**
DTLZ4	5	100	6.3366E-01(1.29E-01) −	**2.5338E-01(1.55E-02)** +	1.9509E-01(8.16E-02) +	5.8834E-01(2.74E-02) −	3.3175E-01(2.42E-02) ≈	3.6427E-01(2.52E-02)
	10	100	7.1535E-01(3.33E-02) −	6.7294E-01(2.47e-02) −	**4.2389E-01(4.68E-02)** +	6.5566E-01(3.45E-02) −	6.2721E-01(4.65E-03) −	6.1183E-01(2.34E-02)
DTLZ5	5	100	4.8810E-01(5.06E-04) −	4.3544E-03(1.44E-04) −	1.4902E-01(7.22E-02) −	4.1256E-03(3.54E-04) −	4.3206E-03(7.06E-05) −	**3.6412E-03(1.68E-03)**
	10	100	3.1085E+00(1.87E-04) −	2.5612E-03(6.95E-05) +	6.2533E-01(1.69E-02) −	3.6875E-02(2.58E-01) −	**2.4812E-03(8.00E-05)** +	1.8172E-02(1.45E-03)
DTLZ6	5	100	5.9687E-01(2.43E-06) −	3.9526E-03(2.14E-04) +	5.0083E-01(1.82E-02) −	6.5184E+01(5.71E+00) −	**3.9479E-03(3.68E-04)** +	1.1485E-02(2.34E-03)
	10	100	2.1714E+00(4.09E-02) −	2.2089E-03(5.11E-04) +	1.0390E+01(2.86E+00) −	6.0823E+01(3.43E+00) −	**1.7114E-03(1.40E-05)** +	9.3941E-03(1.46E-03)
DTLZ7	5	100	3.8346E-01(2.51E-06) −	3.2417E-01(1.10E-02) −	3.2324E-01(4.15E-02) −	3.1684E-01(3.34E-02) −	3.2803E-01(2.45E-03) −	**3.1242E-01(1.37E-02)**
	10	100	1.0258E+00(7.84E-02) −	8.9741E-01(6.40E-03) −	1.0207E+00(2.37E-01) −	1.0308E+00(4.78E-01) −	**8.3919E-01(3.02E-02)** +	8.5510E-01(1.46E-02)
+/−/≈			**0 / 14 / 0**	**5 / 8 / 1**	**2 / 12 / 0**	**0 / 14 / 0**	**5 / 8 / 1**	/
AvgRank			**5.2857**	**2.5000**	**3.8571**	**4.8571**	**2.5000**	**2**

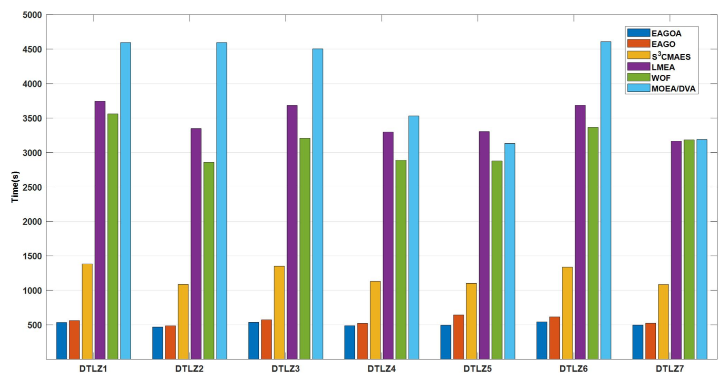
"+", "−" and "≈"respectively indicate that the result is better, worse and similar to that of EAGOA.

"AvgRank" represents the average ranking results through Friedman test.

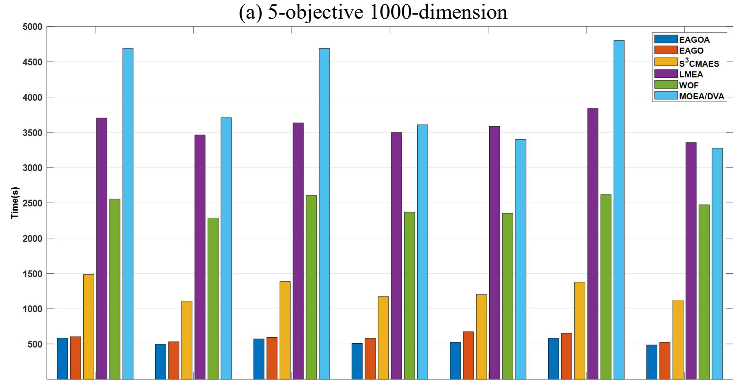
TABLE S-II
IGD METRIC VALUES OF THE SIX ALGORITHMS ON DTLZ1–DTLZ7 PROBLEMS, WHERE THE BEST RESULTS ON EACH TEST INSTANCE ARE BOLDED

Problem	*M*	*D*	MOEA/DVA	LMEA	WOF	S^3-CMA-ES	EAGO	**EAGOA(ours)**
DTLZ1	5	1000	8.7046E+01(4.66E-01) −	**6.4562E-02(4.19E-04)** +	1.0191E+01 (2.58E+00) −	6.6053E+03(3.34E+02) −	6.4086E-01(5.93E-03) +	7.5846E-01(3.56E-02)
	10	1000	2.3983E+02(1.20E+01) −	**1.2219E-01(8.75E-04)** +	3.7028E+02(4.61E+01) −	5.0124E+03(4.69E+02)−	1.2774E-01(2.20E-04) ≈	1.4721E-01(2.36E-02)
DTLZ2	5	1000	2.9047E-01(5.00E-06) −	1.9088E-01(1.10E-03) −	1.9492E-01(2.54E-02) −	1.9694E-01(3.57E-02) −	1.8915E-01(5.00E-05) −	**1.8558E-01(1.51E-02)**
	10	1000	6.0197E-01(1.60E-02) −	3.9338E-01(8.69E-04) −	4.2274E-01(2.91E-02) −	4.2879E-01(6.45E-02) −	3.9100E-01(5.30E-04) −	**3.8021E-01(6.81E-02)**
DTLZ3	5	1000	2.7859E+02(1.06E+01) −	**2.0518E-01(3.35E-04)** +	3.9683E+00(6.39E-01) −	2.7071E+04(7.35E+03) −	3.5745E+00(3.12E-01) −	2.1406E+00 (2.35E-01)
	10	1000	1.0482E+03(7.50E+00) −	4.1485E-01(2.90E-04) −	5.2477E+03(2.97E+02) −	2.8511E+04(2.15E+03) −	3.9108E-01(1.13E-03) −	**3.8033E-01(3.35E-02)**
DTLZ4	5	1000	6.3474E-01(1.00E-03) −	5.7542E-01(6.99E-02) −	**1.9521E-01(9.56E-02)** +	5.8474E-01(4.35E-02) −	2.6215E-01(1.41E-03) −	2.5976E-01(4.82E-02)
	10	1000	7.0918E-01(2.27E-02) −	4.9019E-01(2.23E-02)−	**4.2259E-01(5.48E-02)** +	6.1386E-01(5.63E-02) −	5.5746E-01(1.77E-03) −	4.8745E-01(2.36E-02)
DTLZ5	5	1000	5.7320E-02(3.47E-02) −	4.2209E-03(1.73E-04) +	2.5384E-01(8.42E-02) −	3.2554E-02(5.34E-03) −	**4.1726E-03(1.02E-04)** +	**9.9011E-03(1.45E-03)**
	10	1000	1.1377E+00(1.53E-01) −	**2.4686E-03(7.35E-06)** +	5.6923E-01(3.37E-02) −	4.3524E-02(2.24E-03) −	2.8748E-03(4.20E-05)) +	1.1708E-02(1.26E-03)
DTLZ6	5	1000	3.4444E+00(9.43E-06) −	4.4128E-03(3.16E-04) +	1.2457E+00(4.26E-01) −	7.4285E+02(2.85E+01) −	**3.9438E-03(1.34E-04)**+	1.1202E-02(2.24E-03)
	10	1000	4.4972E+01(2.72E+00) −	2.8410E-03(1.27E-03) +	3.1356E-01(5.47E-02) −	7.4437E+02(3.75E+01) −	**1.5603E-03(4.23E-05)**+	1.0608E-02(1.32E-03)
DTLZ7	5	1000	3.8148E-01(7.57E-07) −	3.2892E-01(6.80E-03) −	3.2923E-01(3.15E-02) −	**2.9414E-01(3.14E-02)** +	3.1768E-01(8.55E-04) −	3.0298E-01(1.96E-02)
	10	1000	9.6057E-01(3.70E-03) −	8.8562E-01(1.96E-02) −	9.3212E-01(7.37E-02) −	1.0044E+00(4.53E-01) −	8.8936E-01(1.24E-02) −	7.5298E-01(1.34E-02)
+/−/≈			**0 / 14 / 0**	**7 / 7 / 0**	**2 / 12 / 0**	**1 / 13 / 0**	**5 / 8 / 1**	/
AvgRank			**5.2857**	**2.2857**	**4**	**5.0714**	**2.2143**	**2.1429**

"+", "−" and "≈"respectively indicate that the result is better, worse and similar to that of EAGO.
"AvgRank" represents the average ranking results through Friedman test



(a) 5-objective 1000-dimension



(b) 10-objective 1000-dimension

Fig. S2. The total execution time of EAGOA, EAGO, S^3CMAES, LMEA, WOF and MOEA/DVA on DTLZ1-7 problems. (a) Execution time on 5-objective 1000-dimensional DTLZ1-7 problems. (b) Execution time on 10-objective 1000-dimensional DTLZ1-7 problems.

TABLE S-III
HV METRIC VALUES OF TWO ALGORITHMS ON LSMOP1–9 PROBLEMS, WHERE THE BEST RESULTS ON EACH TEST INSTANCE ARE BOLDED

Problem	*M*	*D*	EAGO	**EAGOA(ours)**
LSMOP1	5	1000	**9.1514E-01(7.12E-02)** +	8.6372E-01(3.85E-02)
LSMOP2	5	1000	9.2673E-01(3.48E-02) −	**9.5726E-01(1.45E-02)**
LSMOP3	5	1000	3.2359E-01(5.67E-02) −	**3.8226E-01(2.78E-02)**
LSMOP4	5	1000	9.3651E-01(2.78E-02) ≈	**9.6378E-01(4.77E-02)**
LSMOP5	5	1000	6.7088E-01(2.26E-02) −	**6.7727E-01(4.25E-02)**
LSMOP6	5	1000	**2.0392E-05(9.12E-05)** +	3.1247E-06(1.39E-05)
LSMOP7	5	1000	**5.8637E-01(4.39E-02)** +	4.2541E-01(1.78E-02)
LSMOP8	5	1000	6.2281E-01(3.78E-02) ≈	**6.8961E-01(4.25E-02)**
LSMOP9	5	1000	3.1228E-03(6.48E-02) −	**2.3657E-01(5.18E-02)**
+/−/≈			**3 / 4 / 2**	/

"+", "−" and "≈"respectively indicate that the result is better, worse and similar to that of EAGOA.

Besides, we present HV values on 5-objective 1000-dimensional LSMOP1-9 problems for a comprehensive comparison. The fitness evaluation count is 17,000,000. Table S-III shows that EAGOA outperforms EAGO in most test problems.

Next, we use some common activation functions, i.e., *Tanh*, *Sigmoid*, *ReLU*, to replace activation functions we have defined, thus resulting EAGOA-T, EAGOA-S and EAGOA-R, respectively. A numerical comparison has been performed to study the influence of different activation functions. The IGD values for four algorithms are given in Table S-IV. It is clear that activation functions we have defined for objective space mapping can help to generate a better offspring population than common activation functions, e.g., *Tanh*, *Sigmoid* and *Relu*.

TABLE S-IV
IGD METRIC VALUES OF FOUR ALGORITHMS ON LSMOP1–LSMOP9 PROBLEMS, WHERE THE BEST RESULTS ON EACH TEST INSTANCE ARE BOLDED

Problem	*M*	*D*	EAGOA-S	EAGOA-T	EAGOA-R	EAGOA
LSMOP1	5	1000	1.3929E-01(3.47E-02) ≈	1.8437E-01(6.12E-02) −	1.4912E-01(2.34E-02) −	**1.3591E-01(1.31E-01)**
LSMOP2	5	1000	1.3851E-01(4.85E-02) ≈	1.4173E-01(1.69E-02) −	1.3712E-01(3.57E-02) ≈	**1.3445E-01(2.36E-03)**
LSMOP3	5	1000	**5.1712E-01(5.02E-02)** +	9.5883E-01(2.69E-02) +	5.5584E-01(2.41E-02) +	1.3886E+00(9.44E-01)
LSMOP4	5	1000	**1.2906E-01(1.67E-02)** +	1.3485E-01(4.35E-02) ≈	1.3072E-01(1.49E-02) +	1.5213E-01(3.11E-02)
LSMOP5	5	1000	8.1522E-01(49.04E-02) −	4.5565E-01(3.79E-02) −	9.5013E-01(4.72E-02) −	**4.1443E-01(7.50E-02)**
LSMOP6	5	1000	2.5593E+00(3.76E-01) −	2.9522E+00(5.17E-01) −	2.0304E+00(4.87E-01) ≈	**1.9572E+00(9.37E-01)**
LSMOP7	5	1000	5.9473E-01(6.48E-02) ≈	**5.7464E-01(6.98E-02)** +	1.4121E+00(8.47E-01) −	6.1441E-01(5.17E-02)
LSMOP8	5	1000	4.0417E-01(3.45E-02) −	2.9505E-01(2.17E-02) −	3.4344E-01(3.65E-02) −	**2.8727E-01(1.02E-02)**
LSMOP9	5	1000	1.1915E+00(3.45E-01) −	1.2085E+00(6.46E-01) −	1.3787E+00(2.45E-01) −	**9.4756E-01(1.23E-01)**
+/−/≈			**2 / 4 / 3**	**2 / 6 / 1**	**2 / 5 / 2**	/
AvgRank			**2.3333**	**2.8889**	**2.8889**	**1.8889**