Lab 03. Optimization

Introduction to Computer Vision, Lab 03.

Today

- **Gauss-Newton method**

Gauss-Newton method

- Approximate second derivative

$$\hat{x} = \mathrm{arg}\min_{x} \parallel R(x) \parallel_2^2$$

  – Approximate R

$$\|R(x_k + \Delta x)\|_2^2 \approx \|R(x_k) + J_R \Delta x\|_2^2$$
$$= \|R(x_k)\|_2^2 + 2R(x_k)^T J_R \, \Delta x + \Delta x^T J_R^T J_R \Delta x$$

  – Optimal Δx satisfies

$$J_R^T J_R \Delta x + J_R^T R(x_k) = 0$$

  – Optimal direction

$$\Delta x = -(J_R^T J_R)^{-1} J_R^T R(x_k)$$

Gauss-Newton method

- Optimal direction

$$\Delta x = -(J_R^T J_R)^{-1} J_R^T R(x_k)$$
$$= -(J_R^T J_R)^{-1} J_F^T$$

- Compared with Newton
  - Newton step: $\Delta x = -H_F^{-1} J_F^T$
  - Here use to $J_R^T J_R$ approximate Hessian H_F

Gauss-Newton method

- Advantage
  - No need to compute Hessian
  - Fast to converge
- Disadvantage
  - If $J_R^T J_R$ is singular, the algorithm becomes unstable

Gauss-Newton method

$$J(x)^T J(x)\Delta x = -J(x)^T R(x)$$

$$H\Delta x = g$$

1. 给定初始值 $\boldsymbol{x}_0$。
2. 对于第 k 次迭代，求出当前的雅可比矩阵 $\boldsymbol{J}(\boldsymbol{x}_k)$ 和误差 $f(\boldsymbol{x}_k)$。
3. 求解增量方程：$\boldsymbol{H}\Delta\boldsymbol{x}_k = \boldsymbol{g}$.
4. 若 $\Delta\boldsymbol{x}_k$ 足够小，则停止。否则，令 $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \Delta\boldsymbol{x}_k$，返回 2.

Fitting an ellipse

- We provide 753 3d points with noise.
- We want to fit these points using ellipse function.
- Ellipse function is:

$$\frac{x^2}{A^2}+\frac{y^2}{B^2}+\frac{z^2}{C^2}=1$$

Fitting an ellipse

- You should solve this with Gauss-Newton method
- Let’s see jupyter notebook now.
- If the provided functions restrict you, feel free to implement your own pipeline. But you should add clear comments to your own pipeline.

Questions?