An Adaptive IMM Estimator for Aircraft Tracking

Emil Semerdjiev [1]   **Ludmila Mihaylova** [1]   **X. Rong Li** [*]

Bulgarian Academy of Sciences   *University of New Orleans*
Central Laboratory for Parallel Processing   *Department of Electrical Engi-
neering*
'Acad. G. Bonchev' Str., Bl. 25-A, 1113 Sofia, Bulgaria
Phone: (359 2) 979 66 20;   Fax: (359 2) 707 273   *New Orleans, LA 70148*
E-mail: lsm@bas.bg,   signal@bas.bg   *Phone: 504-280-7416, Fax: 504-280-3950*
E-mail: xli@uno.edu

Abstract–*An adaptive Interacting Multiple-Model (IMM) estimator using a small number of models is proposed for maneuvering aircraft tracking. It estimates the difference between the true target control parameter and the value currently used in the IMM models to improve the estimator's performance. The algorithm performance is compared with the performance of a standard IMM estimator for some maneuver scenarios via Monte Carlo simulations.*

Key Words: target tracking, IMM, adaptive estimation

1. Introduction

In recent years the design of reliable and effective multiple-model (MM) algorithms for maneuvering target tracking is a subject of extensive research (see e.g., [1-7]). These algorithms are used to overcome problems caused by structural and parametric uncertainties. The Interacting Multiple-Model (IMM) algorithm is the most commonly used MM estimation algorithm among them. The lack of knowledge about the target's control parameters is overcome in it by introducing a set of fixed control parameters. This set is expected to cover the range of possible parameter changes. A set of models represents the system behavior in each fixed control value. Kalman filters based on these models are running in parallel and their estimates are finally fused [1-3, 6]

to compute the overall estimate. When the range of the expected control parameter is wide, however, IMM needs a large number of models to provide consistent estimation.

One promising solution to this problem is to use variable-structure estimation algorithms [5-7]. An alternative, nontrivial solution is proposed in this paper. It requires a minimal number of models (one for rectilinear motion, one for right turn and one for left turn) to cover the range of all possible target maneuvers. The proposed adaptive IMM algorithm estimates the difference between the control parameter assumed in the current model and its real value. The method has been applied at first for marine targets tracking in [10], where the range of the control parameter is very narrow. To cover the very wide respective range for air targets an additional adaptation mechanism is applied. It is concerned with the IMM transition probabilities and the fudge factor and the noise covariance matrices of the maneuvering models. The algorithm's performance is evaluated by Monte Carlo simulations and the effectiveness is illustrated by a comparison with 3- and 5-model standard IMM algorithm versions.

2. Aircraft Models

The target motion is described in the horizontal plane xOy by the commonly used model [8]:

$$\dot{X} = V \sin \boldsymbol{j} \ ,$$
$$\dot{Y} = V \cos \boldsymbol{j} \ ,$$

$$\dot{v} = gn_T,$$

$$\boldsymbol{j} = -gn_N^* / v,$$

where $n_N^* = n_N \sin\boldsymbol{g}$, $\boldsymbol{g} = arc\,cos(1/n_N)$; (X, Y) are aircraft mass center coordinates, V and $\boldsymbol{j}$ are aircraft velocity and heading, n_N and n_T are normal and tangential g-load factors (NLF and TLF), $\boldsymbol{g}$ is roll angle; $g = 9.81$ is the load factor.

The respective discrete-time model has the form

$$X_k = X_{k-1} + Tv_{k-1}\sin\boldsymbol{j}_{k-1},$$

$$Y_k = Y_{k-1} + Tv_{k-1}\cos\boldsymbol{j}_{k-1},$$

$$\boldsymbol{j}_k = \boldsymbol{j}_{k-1} + Tgn_{N,k-1}^* / v_{k-1},$$

$$v_k = v_{k-1} + Tgn_{T,k},$$

where $n_{N,k}^* = n_{N,k}\sin\boldsymbol{g}_k$, $\boldsymbol{g}_k = arc\,cos(1/n_{N,k})$; k is the current discrete time; T is the radar sampling interval. The state vector $x_k = \begin{pmatrix} X_k & Y_k & \boldsymbol{j}_k & v_k \end{pmatrix}'$ should be estimated in the presence of unknown control parameters n_N and n_T based on radar measurements y_k, modeled as:

$$y_k = Hx_{i,k} + w_k,$$

where H is measurement matrix and w_k is white Gaussian noise with a covariance matrix R:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} \boldsymbol{s}_X^2 & 0 \\ 0 & \boldsymbol{s}_Y^2 \end{bmatrix}.$$

3. Extended Model and Adaptive IMM

The proposed IMM algorithm uses three models to cover the possible target motions in the horizontal plane: rectilinear motion ($i = 1$), right turn ($i = 2$) and left turn ($i = 3$):

$$X_{i,k} = X_{i,k-1} + Tv_{i,k-1}\sin\boldsymbol{j}_{i,k-1}, \quad (1)$$

$$Y_{i,k} = Y_{i,k-1} + Tv_{i,k-1}\cos\boldsymbol{j}_{i,k-1}, \quad (2)$$

$$\boldsymbol{j}_{i,k} = \boldsymbol{j}_{i,k-1} + Tg\left(n_{N_i}^* + \Delta n_{N_i,k-1}^*\right)/v_{i,k-1}, \quad (3)$$

$$v_{i,k} = v_{1,k-1} + Tgn_{T,k}, \quad (4)$$

$$\Delta n_{N_i,k}^* = \Delta n_{N_i,k-1}^*. \quad (5)$$

The i-th difference $\Delta n_{N_i,k-1}^*$ is a measure of the mismatch between the NLF currently used by i-th model and its true value. The extended state vector has the form $x_{i,k} = \begin{pmatrix} X_{i,k} & Y_{i,k} & \boldsymbol{j}_{i,k} & v_{i,k} & \Delta n_{N_i,k}^* \end{pmatrix}'$. It is also presumed for all IMM filters that $n_{T,k} \equiv 0$.

The EKF for the i-th model have the recursion:

$$\hat{x}_{i,k/k} = \hat{x}_{i,k/k-1} + K_{i,k}\boldsymbol{g}_{i,k}, \quad (6)$$

$$\hat{x}_{i,k/k-1} = f\left(\hat{x}_{i,k-1/k-1}, n_{N_i,k-1}^*\right), \quad (7)$$

$$\boldsymbol{g}_{i,k} = y_k - H_i\hat{x}_{i,k/k-1}, \quad (8)$$

$$P_{i,k/k-1} = \boldsymbol{f}_i f_i^x P_{i,k-1/k-1}\left(f_i^x\right)' + Q_{i,k}, \quad (9)$$

$$S_{i,k} = H_i P_{i,k/k-1} H_i' + R, \quad (10)$$

$$K_{i,k} = P_{i,k/k-1} H_i' S_i^{-1}, \quad (11)$$

$$P_{i,k/k} = P_{i,k/k-1} - K_{i,k} S_i K_{i,k}', \quad (12)$$

where $\hat{x}_{i,k/k}$ and $\hat{x}_{i,k/k-1}$ are the filtered and the predicted estimates of x_k; $f_i^{\hat{x}} = \P f_i / \P \hat{x}_i$ given by:

$$f_i^{\hat{x}} = \begin{bmatrix} 1 & 0 & T\hat{v}_{i,k/k-1}\cos\boldsymbol{j}_{i,k/k-1} & T\sin\boldsymbol{j}_{i,k/k-1} & 0 \\ 0 & 1 & -T\hat{v}_{i,k/k-1}\sin\boldsymbol{j}_{i,k/k-1} & T\cos\boldsymbol{j}_{i,k/k-1} & 0 \\ 0 & 0 & 1 & -\dfrac{Tg\left(n_{N_i}^* + \Delta \hat{n}_{N_i,k/k-1}^*\right)}{\hat{v}_{i,k/k-1}^2} & \dfrac{Tg}{\hat{v}_{i,k/k-1}} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boldsymbol{g}_i$ and $S_{i,k}$ are the filter residual process and its covariance matrix, $P_{i,k/k}$ and $Q_{i,k}$ is the error and the system noise covariance matrices, $K_{i,k}$ is the filter gain matrix, $\boldsymbol{f}_i > 1$ is a fudge factor (FF).

The estimate $\Delta \hat{n}_{N_i,k/k}^*$ has no significant physical meaning directly but it contains useful information about the maneuver's starting/final times and intensity. It allows us to develop an adaptive mechanism for estimation consistency improvement. They are arranged below according to their impact.

a) *The FF* is adaptively changed according to the ufinal estimate $\hat{x}(5)$ in the subsequent times k, k-1:

$$\boldsymbol{f}_{i,k} = \begin{cases} \boldsymbol{f}_0 = 1.06, & \text{for } i = 1; \\ 1 + \dfrac{|\hat{x}_k(5) + \hat{x}_{k-1}(5)|}{4n_{N,\max}^*} \le \boldsymbol{f}_0, & \text{otherwise} \end{cases} \quad (13)$$

This adaptive FF is introduced to improve the common filter consistency.

b) *The fifth diagonal element of the process noise covariance matrix* $Q_{i,k}$, $i = 1,2,3$ of the EKF is adaptively changed to provide faster response to the maneuvers:

$$Q_{i,k} = diag\left(\boldsymbol{s}_X^2 \quad \boldsymbol{s}_Y^2 \quad \boldsymbol{s}_j^2 \quad \boldsymbol{s}_v^2 \quad \left|\hat{x}_k(5) - \hat{x}_{i,k/k-1}(5)\right|\right) \quad (14)$$

c) *The transition probabilities* are computed as follows:

$$Pr_{11}(k) = Pr_{11}(0)e^{-|\hat{x}_k(5) - \hat{x}_{1,k}(5)|},$$

$$Pr_{jj}(k) = Pr_{jj}(0)e^{-|\hat{x}_k(5) - \hat{x}_{j,k}(5)|/n_{N,\max}^*} \quad j = 2,3; \quad (15)$$

$Pr_{ij}(k) = Pr_{il}(k) = (1 - Pr_{ii}(k))/2 ; i \neq j \neq l$,

$i, j, l \in [1,3]$.

where $n^*_{N,max}$ (it is set $= 7$) is the maximal expected value of the NLF $n^*_{N,k}$, and the standard IMM transition probability matrix has the form:

$$Pr_{ii}(k) = Pr_{ii}(0) = const, \qquad (16)$$

$Pr_{ij}(k) = Pr_{il}(k) = (1 - Pr_{ii}(k))/2 ; i \neq j \neq l$,

$i, j, l \in [1, i_{max}]$.

This adaptive transition probability matrix provides faster system mode transition.

4. Performance Evaluation

The performance of the adaptive IMM filter (denoted below as IMM3a) is evaluated by Monte Carlo simulation for 200 independent runs. Its performance is compared with that of a 3- and a 5-model standard IMM (denoted as IMM3 and IMM5). IMM3 and IMM5 use: the model (1)-(4) and the EKF equations (6)-(12), where it is set $\Delta \hat{n}^*_{N_i, k/k-1} \equiv 0$ and in the matrix $f_i^{\hat{x}}$ the last row and column are excluded.

It is preset:

$$\mathbf{s}_X = \mathbf{s}_Y = 100m, \mathbf{s}_y = 3^o, \mathbf{s}_v = 10m/s,$$

$$\mathbf{s}_{\Delta n_N} = 0.2, T = 1s. \qquad (17)$$

$$n^*_{N_i} = n_{N_i} \, sin(arccos(1/n_{N_i}))$$

IMM3 and IMM3a use $n_N = (1 \quad 4 \quad -4)'$, whereas IMM5 use $n_N = (1 \quad 3 \quad -3 \quad 6 \quad -6)'$.

Example 1.
To demonstrate the significant role of the proposed adaptation, the IMM3 and IMM5 are compared with a simplified version of IMM3a (denoted as IMM3as), where the adaptive mechanisms (13)-(15) and (16) are not included; that is, all algorithms use constant transition probabilities (16), a common FF $\mathbf{f} = 1.06$ and covariance matrices $Q_{i,k}$ with diagonal elements given in (17). All the IMM algorithms are running with the following initial conditions: $X_0 = Y_0 = 0 \, m$, $V_0 = 350 \, m/s$, $\mathbf{j} = 0^o$. The initial error covariance matrix $P(0)$, the initial mode probability vector $\mathbf{m}(0)$ and the transition probability matrices Pr are:

$P(0) = diag\{10000, 10000, 9, 4000, 0.5\}$ for IMM3as,

$P(0) = diag\{10000, 10000, 9, 4000\}$ for IMM3,

where $diag\{.\}$ denotes a diagonal matrix.

For IMM3 and IMM3as it is preset:

$$Pr = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}; \qquad \mathbf{m}(0) = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix},$$

+For the IMM5 it is preset:

$$Pr = \begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.05 & 0.8 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.8 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.8 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.8 \end{bmatrix}, \qquad \mathbf{m}(0) = \begin{pmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{pmatrix}$$

A target maneuver with $n_N = 7$ is the worst case for IMM3a and IMM3as. The true target trajectory and the NLF change are given in Figs. 1, 2.

The Normalized Estimation Error Squared (NEES) [2] is the most informative and integral measure of performance. So, the respective NEES plots for all algorithms are computed and presented in Fig. 3 ('1' - IMM3, '2' - IMM3as, '3' - IMM5). Here and below the NEES is computed for the first four components of the state vector.

Obviously the standard 3-model IMM does not provide consistent estimates during the maneuver at all, while the consistency of IMM3as is better than the IMM5.

Example 2.
The designed IMM3a (denoted by '1') is compared with IMM5 (denoted by '2') for three types of maneuvers: a fast maneuver with $n_N = 7$, a moderate maneuver with $n_N = 3$ and a weak maneuver with $n_N = 1.2$. A noise covariance matrix is introduced in the EKF equation (9), with elements $j = \overline{1,4}$, given in (17). The fifth element of Q in IMMa is adaptively changed according to (14). Obviously, the presence of separate models with $n_N = 3$ and $n_N = 6$ gives advantages to the IMM5 over the IMM3 and IMM3a in the first two test scenarios.
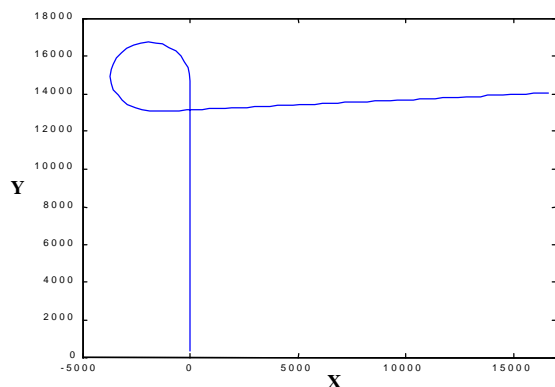
Fig. 1 True aircraft trajectory

The respective results for the fast maneuver are shown in Figs. 4-14. The NEES is given in Fig. 4. The mean errors (ME) and the root-mean square errors (RMSE) of the state vector are shown in Figs. 5-8 and Figs.9-11, respectively. The average model probabilities are presented in Figs. 12-13 and the average FF behavior is given in Fig. 14.   These results show that IMM3a have better consistency, accuracy and faster response to abrupt maneuvers.
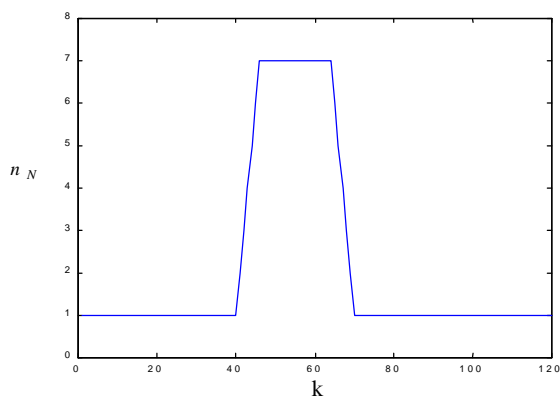

Fig. 2 True aircraft normal load factor
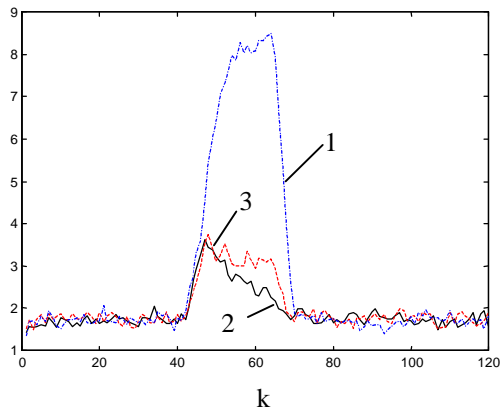
Fig. 3 Normalized Estimation Error Squared
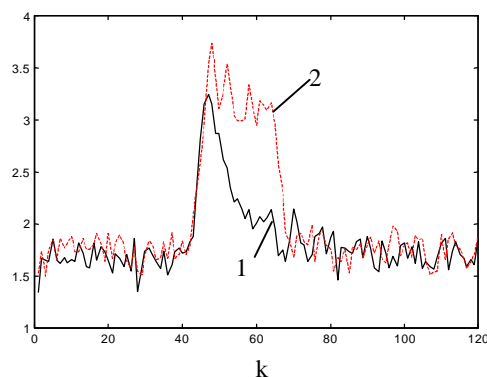
Fig. 4 Normalized Estimation Error Squared
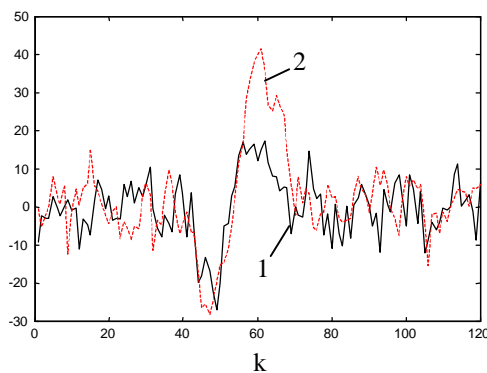
Fig. 5 X Position ME
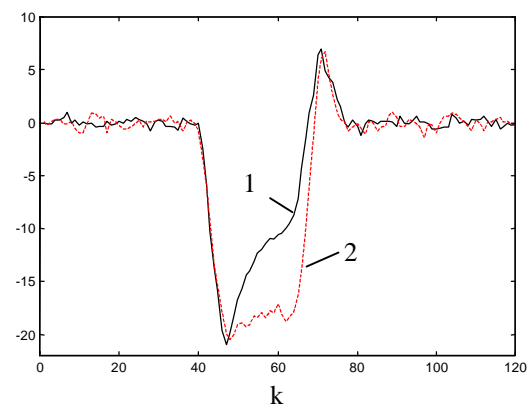
Fig. 6 Heading ME
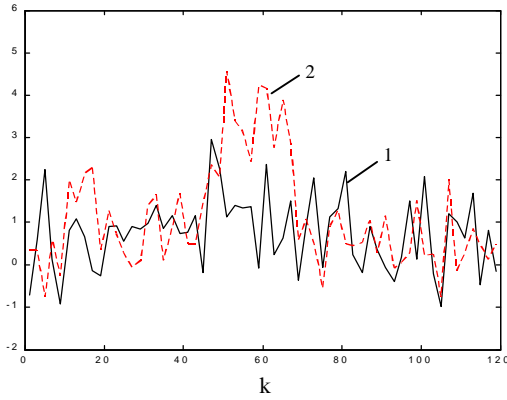
Fig. 7 Velocity ME

The true trajectory and the true NLF change for the moderate maneuver are represented in Figs. 15-16. The NEES and average FF behavior are given in Figs. 17 and 18, respectively.

The same inferences can be drawn for the next two scenarios. The true trajectory and the true NLF change for the weak maneuver are presented in Figs. 19-20. The NEES and average FF behavior are given in Figs. 21 and 22, respectively.
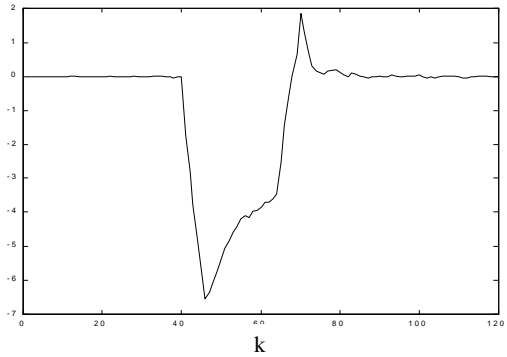
Fig. 8 $\Delta \hat{n}^*_{N_i,k/k}$ ME
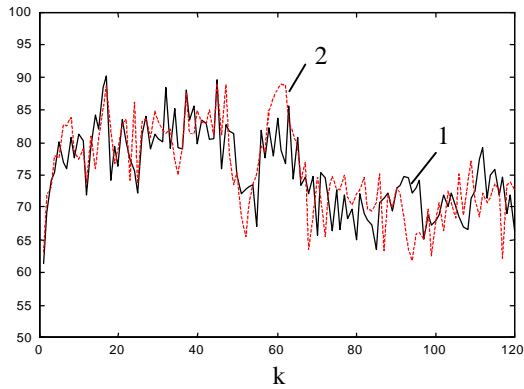
Fig. 9 X Position RMSE
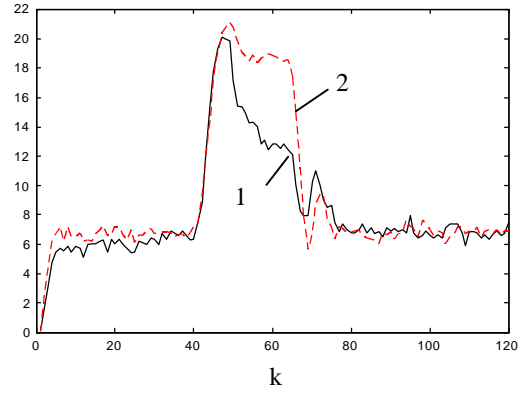
Fig. 10 Heading RMSE
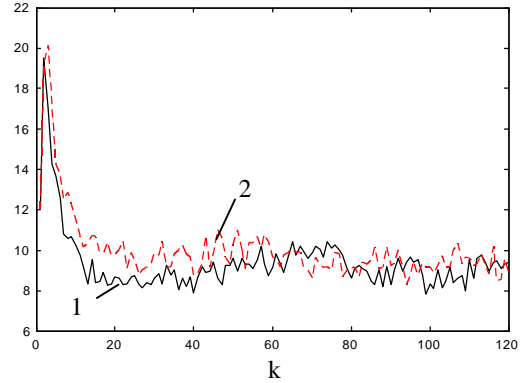
Fig. 11 Velocity RMSE
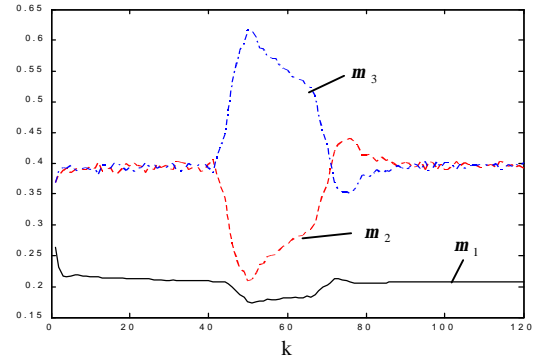
Fig. 12 Mean of Model Probabilities for IMM3
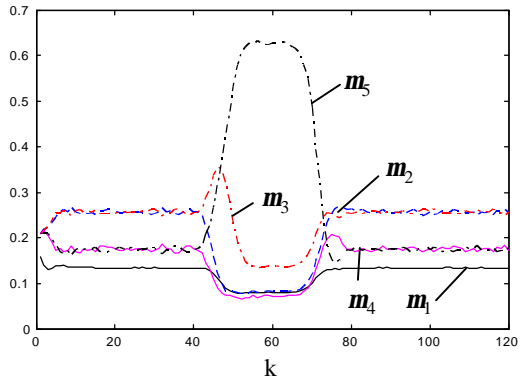
Fig. 13 Mean of Model Probabilities for IMM5
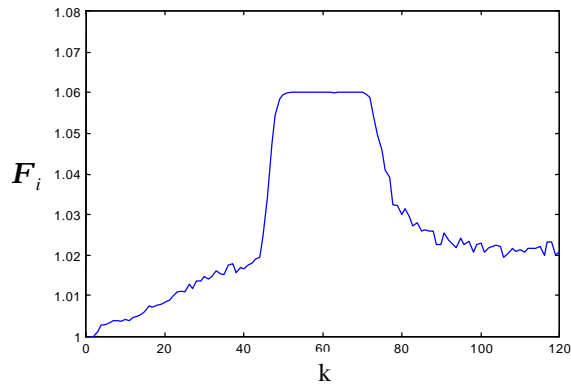
Fig. 14 Average fudge factor
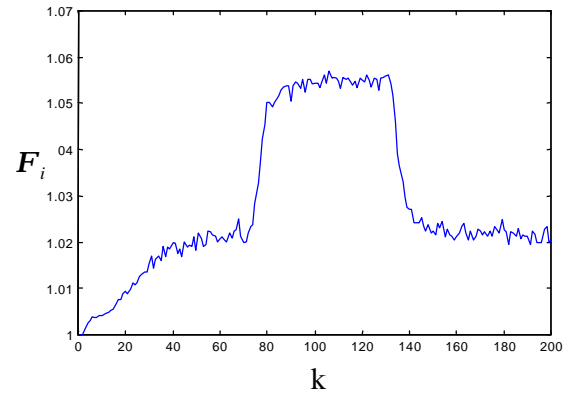
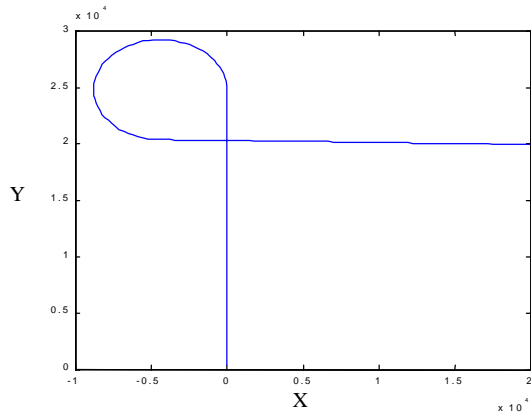**Fig. 18** Average fudge factor

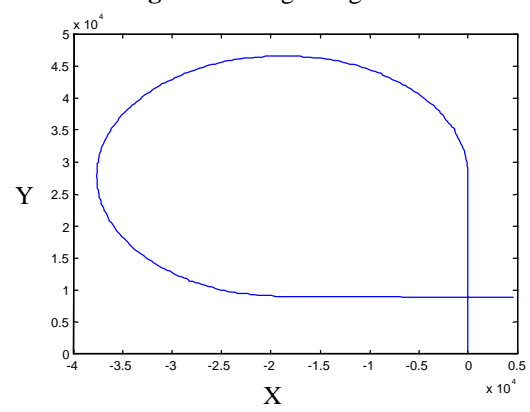**Fig. 15** True aircraft trajectory for $n_N = 3$

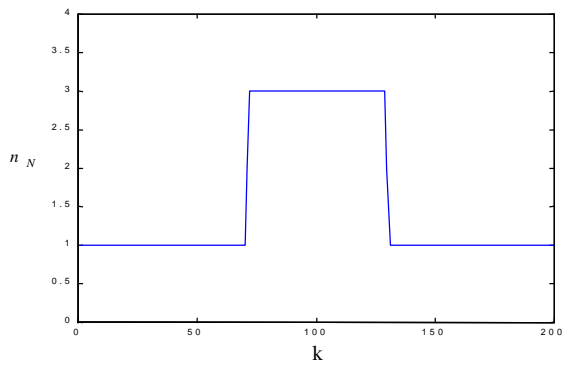**Fig. 19** True aircraft trajectory for $n_N = 1.2$

Fig. 16 True aircraft normal load factor
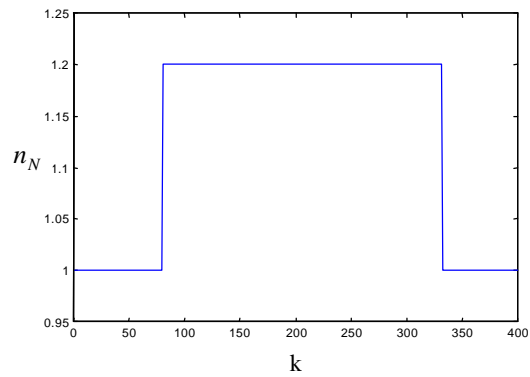
Fig. 20 True aircraft normal load factor
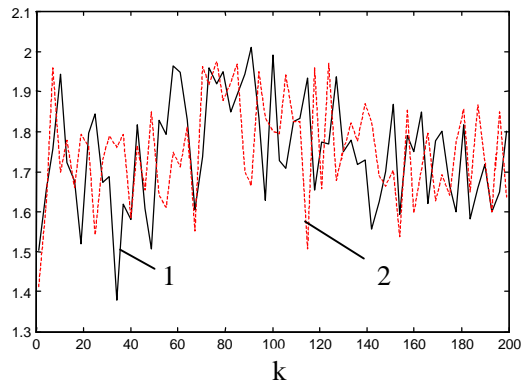
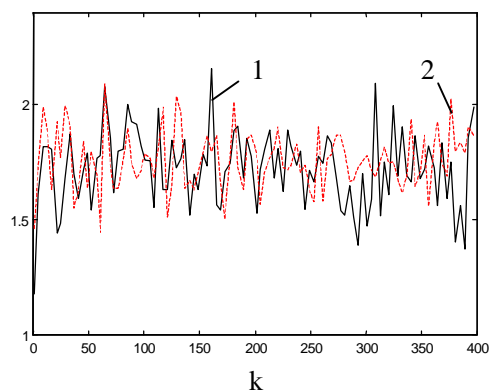**Fig. 17** Normalized Estimation Error Squared

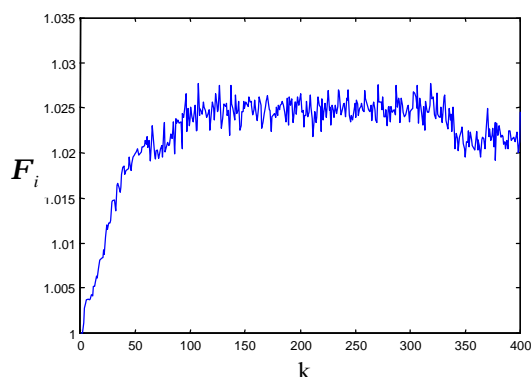**Fig. 21** Normalized Estimation Error Squared

Fig. 22 Average fudge factor

6. Conclusions

An adaptive IMM algorithm using a small number of models and covering a wide range of possible aircraft maneuvers is proposed. It estimates the difference between the real control parameter and its value used in the current model in real time. This makes it possible to introduce an additional adaptation mechanism to cover a wide range of possible air target maneuvers. This mechanism tunes the IMM transition probabilities, the EKF's fudge factor and the EKF's covariance. The algorithm's efficiency is demonstrated for the worst cases maneuvers.

References

[1]   Bar-Shalom, Y., Ed., *Multitarget-Multisensor Tracking: Applications and Advances*, vol. 2, Artech House, 1992.

[2]   Bar-Shalom, Y., and X. R. Li. *Estimation and Tracking Principles, Techniques and Software*, Artech House, 1993.

[3]   Bar-Shalom, Y., and X. R. Li. *Multitarget-Multisensor Tracking: Principles and Techniques*, YBS Publishing, 1995.

[4]   Jilkov, V., P. Angelova and Tz. Semerdjiev. Design and Comparison of Mode-Set Adaptive IMM for Maneuvering Target Tracking, *IEEE Trans. Aerosp. and Electr. Systems*, 35(1): 343-350, Jan. 1999.

[5]   Jilkov, V. P., L.Mihaylova, X. R. Li, An Alternative IMM Solution to Benchmark Radar Tracking Problem, 1998 Intern. Conf. on Information Fusion (Fusion'98), vol.2, 924-929, Las Vegas, Nevada, USA, July 6-9, 1998.

[6]   Li, X. R. *Hybrid Estimation Techniques. In Control and Dynamic Systems*. (Ed., C. T. Leondes), vol. 76, 213-287, Academic Press, 1996.

[7]   Li, X. R., and Y. Bar-Shalom. Multiple-Model Estimation with Variable Structure, *IEEE Trans. Automatic Control,* 41(4): 478-493, 1996.

[8]   Ostoslavsky, I., Strajeva, I., *Flight Dynamics.* Flying Aircraft Trajectories. Mashinostroenie, Moscow, 2nd edition, 1969 (in Russian).

[9]   Semerdjiev, E., L. Mihaylova, Tz. Semerdjiev. Manoeuvring Ship Model Identification and IMM Tracking Algorithm Design, in Proc. 1998 Intern. Conf. on Information Fusion (Fusion'98), Vol. 2, 968-973, Las Vegas, Nevada, USA, July 6-9, 1998.

[10]   Semerdjiev, E., L. Mihaylova, Adaptive IMM Algorithm for Manoeuvring Ship Tracking, in Proc. 1998 Intern. Conf. on Information Fusion (Fusion'98), Vol. 2, 974-979, Las Vegas, Nevada, USA, July 6-9, 1998.