

## FUSION OF MULTIRADAR AND ADS DATA IN ATC SYSTEMS

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### INTRODUCTION

Data received from geographically dispersed radars, either primary (2D) or secondary (3D), are the only input information which qualify modern civil air traffic control (ATC) systems to automatically update central tracks of each surveilled aircraft.

In this sense, it is envisaged that the introduction and operation of Automatic Dependent Surveillance (ADS), ARINC (1), will have a considerable impact on future ATC performances and procedures. ADS is a function whereby the aircraft Flight Management Computer (FMC) automatically transmits to ground facilities, via a VHF, Mode S or satellite data link, navigational and meteorological data derived from on-board avionics equipment, including aircraft identification and three-dimensional position and speed.

In oceanic and large continental areas with limited surveillance availability, the ADS function will be the only reliable means of supporting an automated *Tactical Tracking* function. Nevertheless, the concept of ADS is not limited to this type of regions. The integration of the information it can provide with that received from already deployed radars can permit to fulfil tighter operational requirements in en-route, approach or airport areas: higher estimation accuracy for *Tactical Tracking*; longer extrapolation time for *Conflict Prediction*; or, thanks to the increased accuracy, improved airspace utilisation due to the reduced aircraft separation minima required to guarantee the same preestablished flight safety level.

In this contribution<sup>1</sup>, reformulated radar data fusion methods, extended to the multisensor+ADS case, are presented. The problem of optimally combining the information coming from the ADS-equipped aircraft and from en-route and/or approach radars is addressed, and a practical open-loop mode of operation of the ADS function, both in situations with and without radar coverage, is proposed.

### PRINCIPLES OF OPERATION OF ADS

Three different types of ADS contracts can be established between the aircraft and a control centre in its normal mode of operation(1). Both within and beyond multiradar coverage, the standard mode of operation of ADS will be the Periodic Contract, in

which ADS reports are sent periodically from the aircraft to the requesting control centre. Based on knowledge of flight progress, the report rate ( $T_{ADS}$ ), contract duration and message contents can be selected by the ground facility in an adaptive way as needed, through uplink Periodic Contract Request Messages. If the ADS function receives an Event Contract Request Message, it assembles and sends an ADS report when the specified event(s) occur(s). Finally, the ATC system will also be capable of requesting a single ADS report at any moment (Demand Contract).

ADS periodic reports composed of the Basic Group (aircraft latitude, longitude, altitude, time stamp and Figure of Merit (FOM)) and the Airframe Identification Group (aircraft ICAO ID) will be referred to as Basic Messages. The FOM, which is defined as the positioning accuracy (within 95% probability) of the selected measurement source (INS, VOR, DME, LORAN-C, or GPS, among others) is sent as a quality measure of the actual navigation performance. Those ADS reports containing additional groups of data will be named Extended Messages. In this contribution we will only consider those Extended Messages containing a Basic Message and the Earth Reference Group (aircraft true track, ground speed and vertical rate), which permits to obtain a 3D velocity plot.

### MULTIRADAR+ADS DATA FUSION SCHEME

A distributed fusion architecture, Farina and Studer (2), is proposed to integrate ADS as a new data source in existing civil multisensor surveillance systems, since deployed ATC sensors already form local tracks (updated with its own plots), which are then precombined and presented to the controller. These local tracks could be fused with an ADS track, updated with those ADS reports received from the same aircraft, to constitute the central track finally presented.

This situation can be found, for instance, at the control tower of (big) airports where local tracks received from close terminal area radar(s), local tracks that may be available from the surface movement radar, and ADS tracks could be fused to perform an accurate tracking during the final approach and surface movement phases of ADS equipped aircraft.

Due to the asynchronous local track updating instants of the various radars, central tracks will be built extrapolating monoradar and ADS tracks to a common time instant  $t_r$  (previously transformed to the central

<sup>1</sup> This work has been financed by Aena and Indra.

tracking coordinate system). Then, those tracks associated to the same aircraft (this association solved by means of a code correlation process), will be adequately combined. This track fusion procedure will be performed in two stages.

Firstly, correspondent filtered state variables of the 3D local tracks associated to the same aircraft, predicted to  $t_f$  according to the prediction equations of each local tracker, are linearly combined (per coordinate) to build a 3D multiradar track,  $\underline{x}_R(t_f)$ . For the case of position estimation<sup>2</sup>:

$$\hat{x}_R(t_f) = c_1 \hat{x}_1(t_f) + c_2 \hat{x}_2(t_f) + \dots + c_n \hat{x}_n(t_f)$$

where  $\hat{x}_i(t_f)$  ( $i=1,2,\dots,n$ ) represents a particular coordinate of the 3D position estimator of the local track received from the  $i$ -th radar, expressed in the central tracking coordinate system, predicted from  $t_i$  (time of last local updating) to the presentation/fusion instant  $t_f$ ; and  $c_i$  ( $i=1,2,\dots,n$ ) are the fusion coefficients for position estimation in that coordinate. To build the 3D central track  $\underline{x}_C(t_f)$ , periodically presented to the controller, the multiradar track can then be fused with the ADS track. Again, for position estimation:

$$\hat{x}_C(t_f) = c_R \hat{x}_R(t_f) + c_{ADS} \hat{x}_{ADS}(t_f)$$

in which  $\hat{x}_{ADS}(t_f)$  represents the considered coordinate of the 3D position estimation of the ADS track (already expressed in the central tracking coordinate system), predicted to  $t_f$  from the last update instant of the ADS track; and  $c_R$ ,  $c_{ADS}$  are the second stage fusion coefficients for position estimation in that coordinate.

This two-stage fusion scheme (alternatively all tracks could have been combined altogether in one step) allows to monitor the quality of the multiradar track independently from the central one and, consequently, decide whether to activate/deactivate the ADS function (when the required accuracy is not/is achieved by the multiradar track).

On the other side, it is necessary to maintain an individual ADS track since the aircraft can fly through areas with no radar coverage. While  $\underline{x}_R(t_f)$  were unavailable,  $\underline{x}_{ADS}(t_f)$  would be the track presented.

The central track quality measure which will guide the adaptive management of the ADS report rate and message content of the Periodic Contract established between the aircraft and the ground facility will be defined before describing the proposed mode of operation of the ADS function. The basic aim of the design will be to guarantee a preestablished accuracy level of the central track (adjustable by the controller), defined as the maximum acceptable distance (per coordinate) between the real aircraft position and the presented one, with probability 0.95, in the worst case.

<sup>2</sup> The procedure would be analogous for central velocity/acceleration estimates.

This worst case implies maximum extrapolation time of the ADS track/local tracks to the fusion/presentation instant and a manoeuvre acceleration of  $0.5g \text{ m/s}^2$  in each coordinate (maximum realistic value in ATC applications). Considering gaussian measurement errors and linear filtering, the accuracy level (95% error) results  $\epsilon_{\tilde{x}(t_f)} = \left| b_{\tilde{x}(t_f)} \right| + 1.96 \sigma_{\tilde{x}(t_f)}$ , where  $\left| b_{\tilde{x}(t_f)} \right|$  represents the module of the bias of the predicted position estimation error and  $\sigma_{\tilde{x}(t_f)}$  is its standard deviation, both at the presentation/fusion instant  $t_f$ .

## DATA FUSION COEFFICIENTS DESIGN

To later illustrate the mode of operation of the ADS, let us assume that radar local tracks are updated and extrapolated (per coordinate) to the presentation instant by classical  $\alpha$ - $\beta$  filtering(2). Anyway, the fusion methodology that follows can be easily reformulated with any other filtering scheme.

ADS tracks during Periodic Contracts composed of Basic Messages will be also updated (per coordinate) using an  $\alpha$ - $\beta$  filter, while the proposed solution to build the ADS track during Periodic Contracts with Extended Messages is an  $\alpha$ - $\alpha$ - $\beta$  filter, whose filtering and prediction equations are:

$$\begin{cases} x_s(t) = x_p(t) + \alpha_1 [x_m(t) - x_p(t)] \\ v_s(t) = v_p(t) + \alpha_2 [v_m(t) - v_p(t)] \\ a_s(t) = a_p(t) + \frac{\beta}{T_{ADS}} [v_m(t) - v_p(t)] \end{cases} \quad \begin{cases} x_p(t + T_{ADS}) = x_s(t) + T_{ADS} v_s(t) \\ v_p(t + T_{ADS}) = v_s(t) + T_{ADS} a_s(t) \\ a_p(t + T_{ADS}) = a_s(t) \end{cases}$$

An essential difference with radar trackers arises from the fact that ADS messages arrive with a certain data-link dependent delay, (time difference between the measurement instant and the reception at the ground control centre). To avoid violating the recursive nature of the filtering scheme, it will be assumed that the maximum acceptable time delay through the data link should not be larger than the time between consecutive presentations, (something which may be translated into data link design restrictions). Otherwise, the received ADS report is rejected.

Under these assumptions, the accuracy level of the central track in steady state, worst case of prediction for time alignment and unity probability of detection can be shown to be:

$$\epsilon_{\tilde{x}_C} = \left| c_R b_{\tilde{x}_R} + c_{ADS} b_{\tilde{x}_{ADS/B/E}} \right| + 1.96 \sqrt{c_R^2 \sigma_{\tilde{x}_R}^2 + c_{ADS}^2 \sigma_{\tilde{x}_{ADS/B/E}}^2}$$

where  $b_{\tilde{x}_R} = \sum_{k=1}^n c_k b_{\tilde{x}_k}$ ,  $b_{\tilde{x}_k} = \frac{a T_k^2}{\beta_k}$  are the worst case

steady state biases of the predicted position estimation errors of the multiradar and local tracks at  $t_f$ , in which:  $T_k$  ( $k=1,2,\dots,n$ ) represent the scan rate of the  $k$ -th radar;  $a$  is the manoeuvre acceleration (if  $a=0 \text{ m/s}^2$   $\epsilon_{\tilde{x}_C}$  represents the steady state accuracy level of the central

track during constant velocity flight); and  $\sigma_{\tilde{x}_R}^2$  is the worst case steady state position error variance of the multiradar track:

$$\sigma_{\tilde{x}_R}^2 = \sum_{k=1}^{k=n} c_k^2 \sigma_{\tilde{x}_k}^2 \quad \sigma_{\tilde{x}_k}^2 = \frac{2\beta_k + \alpha_k \beta_k + 2\alpha_k^2}{\alpha_k (4 - 2\alpha_k - \beta_k)} \sigma_{\tilde{x}_{mk}}^2$$

which is a function of:  $\sigma_{\tilde{x}_{mk}}^2$ , projected measurement error variance of each radar in the considered central coordinate; and  $\alpha_k, \beta_k$  ( $k=1,2,\dots,n$ ), filter coefficients of the local tracker of the  $k$ -th radar.

When ADS reports Basic Messages:

$$b_{\tilde{x}_{ADS_B}} = \frac{aT_{ADS}^2}{\beta_{ADS}} \quad \sigma_{\tilde{x}_{ADS_B}}^2 = \frac{2\beta_{ADS} + \alpha_{ADS}\beta_{ADS} + 2\alpha_{ADS}^2}{\alpha_{ADS}(4 - 2\alpha_{ADS} - \beta_{ADS})} \sigma_{\tilde{x}_m}^2$$

If the ADS track is updated with Extended Messages:

$$b_{\tilde{x}_{ADS_E}} = \frac{aT_{ADS}^2}{2\alpha_1} \quad \sigma_{\tilde{x}_{ADS_E}}^2 = \frac{n_x}{d} \sigma_{\tilde{x}_m}^2 + \frac{n_{xv}}{d} \sigma_{\tilde{x}_m} \sigma_{\tilde{v}_m} + \frac{n_v}{d} \sigma_{\tilde{v}_m}^2$$

$$d = (\alpha_1 - 2)\alpha_1\alpha_2(2\alpha_2 + \beta - 4)(\alpha_1^2(\alpha_2 - 1) - \alpha_1\alpha_2 - \beta(1 - \alpha_1))$$

$$n_x = \alpha_1^2\alpha_2(\alpha_1^2(\beta(1 - \alpha_2) + 2\alpha_2(3 - \alpha_2) - 4) + 2\alpha_1(\alpha_2(\alpha_2 - 2) - \beta(4(1 - \alpha_1) - \alpha_2(\alpha_2 + 2)) + (1 - \alpha_1)\beta^2))$$

$$n_{xv} = 2\alpha_1\alpha_2(2\alpha_2 + \beta - 4)(\alpha_1\alpha_2 + \beta(1 - \alpha_1))T_{ADS}$$

$$n_v = (\beta(2\alpha_1(\alpha_1 - 2)) - \alpha_2(4 - \alpha_1(12 - 5\alpha_1) - \alpha_2(2 - \alpha_1(5 - 3\alpha_1))) + 2\alpha_1^2\alpha_2^2(\alpha_2 - 2 + \alpha_1(1 - \alpha_2)) + \alpha_2\beta^2(1 - \alpha_1))T_{ADS}^2$$

In both cases the position, crossed- and velocity measurement variances in the central coordinate are obtained projecting the current operative (FOMs/1.96)<sup>2</sup>.

The fusion coefficients  $c_i$  ( $i=1,2,\dots,n$ ) and  $c_R, c_{ADS}$  that respectively minimise the steady state mean square position estimation error (m.s.e.) of the multiradar and central track, assuming  $P_d=1.0$  and a constant acceleration value of a  $m/s^2$  result (the derivation procedure is included in the appendix):

$$c_i = \frac{\frac{1}{\sigma_{\tilde{x}_i}^2}}{\sum_{k=1}^{k=n} \frac{1}{\sigma_{\tilde{x}_k}^2}} \left( 1 + \frac{\sum_{k=1}^{k=n} \frac{b_{\tilde{x}_k}}{\sigma_{\tilde{x}_k}^2} \left( \sum_{k=1}^{k=n} \frac{b_{\tilde{x}_k}}{\sigma_{\tilde{x}_k}^2} - b_{\tilde{x}_i} \sum_{k=1}^{k=n} \frac{1}{\sigma_{\tilde{x}_k}^2} \right)}{\sum_{k=1}^{k=n} \frac{1}{\sigma_{\tilde{x}_k}^2} \left( 1 + \sum_{k=1}^{k=n} \frac{b_{\tilde{x}_k}^2}{\sigma_{\tilde{x}_k}^2} \right) - \left( \sum_{k=1}^{k=n} \frac{b_{\tilde{x}_k}}{\sigma_{\tilde{x}_k}^2} \right)^2} \right)$$

$$c_R = \frac{\sigma_{\tilde{x}_{ADS}}^2 + b_{\tilde{x}_{ADS}}^2 - b_{\tilde{x}_R} b_{\tilde{x}_{ADS}}}{\sigma_{\tilde{x}_R}^2 + \sigma_{\tilde{x}_{ADS}}^2 + [b_{\tilde{x}_R} - b_{\tilde{x}_{ADS}}]^2} \quad c_{ADS} = \frac{\sigma_{\tilde{x}_R}^2 + b_{\tilde{x}_R}^2 - b_{\tilde{x}_R} b_{\tilde{x}_{ADS}}}{\sigma_{\tilde{x}_R}^2 + \sigma_{\tilde{x}_{ADS}}^2 + [b_{\tilde{x}_R} - b_{\tilde{x}_{ADS}}]^2}$$

Analogous expressions can be applied to fuse local velocity or acceleration estimates. The values of  $\alpha_{ADS}, \beta_{ADS}$  have been chosen for each value of  $T_{ADS}$  to minimise the steady state 95% error of the ADS track assuming  $P_d=1.0$  and worst case acceleration of  $a=0.5g$   $m/s^2$ , fixed  $\beta_{ADS}$  with the Benedict-Bordner condition(2), which maintains the  $\alpha$ - $\beta$  filter near critical damping for  $0 < \alpha < 1$ . In this way,  $\epsilon_{\tilde{x}_c}$  represents

the worst case error of the central track including the transient error. The  $\alpha$ - $\beta$  filter coefficients were designed for each  $T_{ADS}$  following a similar methodology. Fixed  $\beta = \alpha_2^2/(2 - \alpha_2)$  (Benedict-Bordner),  $\alpha_2$  is chosen to minimise the steady state 95% error of the filtered velocity:

$$\epsilon_{\tilde{v}_{ADS_E}} = \frac{aT_{ADS}^2}{\beta} + 1.96 \sqrt{\frac{2\beta + \alpha_2\beta + 2\alpha_2^2}{\alpha_2(4 - 2\alpha_2 - \beta)}} \sigma_{\tilde{v}_m}$$

and then  $\alpha_1$  minimising  $\epsilon_{\tilde{v}_{ADS_E}} = b_{\tilde{x}_{ADS_E}} + 1.96 \sigma_{\tilde{x}_{ADS_E}}$ .

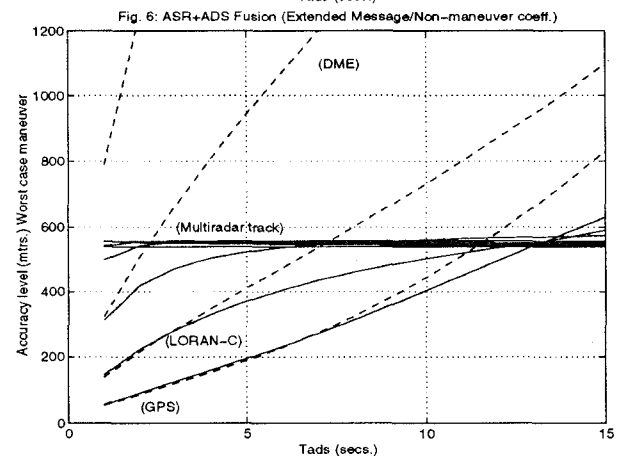
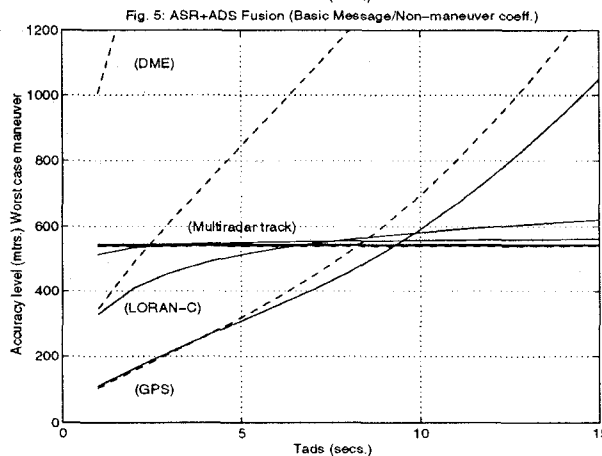
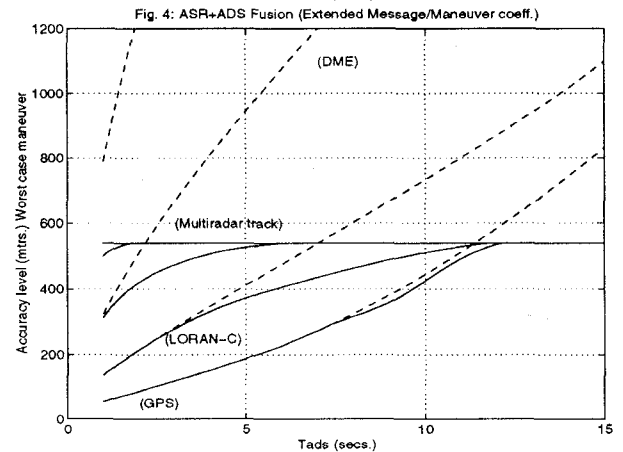
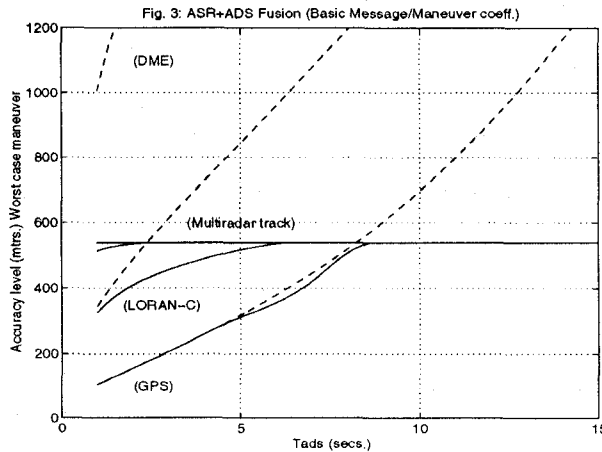
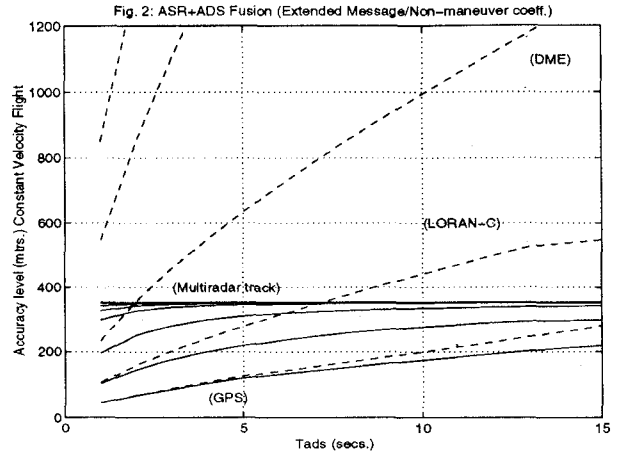
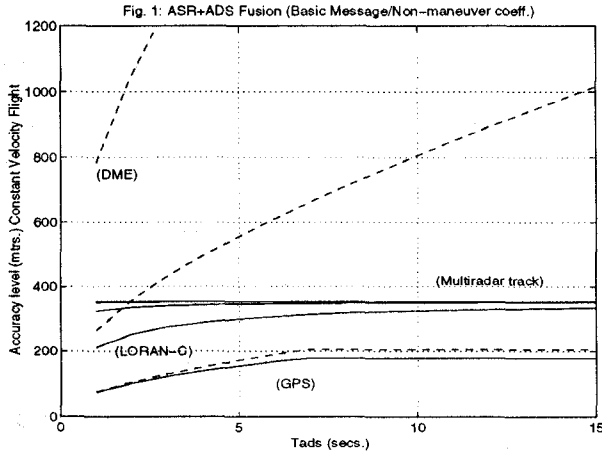
## MANAGEMENT OF THE ADS PERIODIC CONTRACT

Using these filters,  $\epsilon_{\tilde{x}_c}$  with ADS periodically reporting Basic or Extended Messages is represented versus  $T_{ADS}$  in Fig. 1 and Fig. 2 for a constant velocity flight in a certain approach scenario (ASR radar with  $T_{ASR}=4.5$  secs.,  $\sigma_\theta=0.11^\circ$  and aircraft at 40 nmi). The fusion coefficients  $c_R, c_{ADS}$  have been applied, in which all biases have been considered 0 (constant velocity fusion coefficients). In Fig. 3 and Fig. 4 the accuracy level of the central track is shown during a manoeuvre of  $a=0.5g$   $m/s^2$ , obtained applying the expressions of the fusion coefficients including the bias terms. In all cases, dotted lines represent the ADS track and the continuous lines represent the central track. The horizontal line (independent of  $T_{ADS}$ ) represents the multiradar track. The FOM values adopted are(1): GPS=47 m. LORAN-C=236 m. and DME=945 m. ( $\sigma$  errors). A FOM value for the velocity measurement of  $\sigma=10$  m/s has been assumed, Siouris (3), (upper limit of measurement accuracy achievable by present avionics).

A manoeuvre detector is required in order to apply the proper fusion coefficients. In this sense the ADS Event Contract can be considered the ideal mechanism to warn the beginning of manoeuvres. Alternatively, if the constant velocity fusion coefficients are also applied during manoeuvres, the central track accuracy level obtained is represented in Fig. 5 and Fig. 6 for each type of ADS message. As it can be seen, to the lowest FOMs and the practical values of the ADS report rate (those of the same order of magnitude of the scan rate of ground radars), the advantage (reduced error) of using the fusion coefficients that minimise the m.s.e. during manoeuvres disappears.

The ADS report rate ( $T_{ADS}$ ) and the type of message (Basic or Extended), known the operative FOM, can be selected to achieve a demanded accuracy level with the shortest in length report (minimum occupation of the data-link) straightforwardly from the curves.

Consider a demanded accuracy level of 300 m. and ADS reporting with FOM LORAN-C. During constant velocity flight we obtain from Fig. 1 and Fig. 2 that the ADS report rate should be 5 secs. with Basic Message or 15 secs. with Extended Message. Detected a

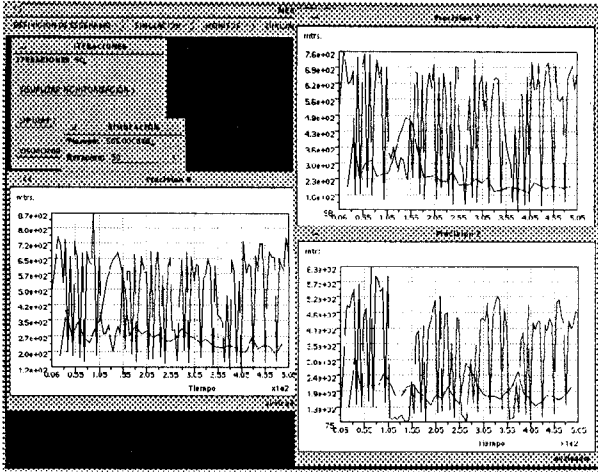


manoeuvre, a new Periodic Contract should be established with a report rate of 1 sec. (Basic Message) or 4 secs. (Extended Message), to maintain the required accuracy level.

If no manoeuvre detector is included a more conservative mode of operation can be employed, applying constant velocity flight fusion coefficients in every situation. To guarantee the accuracy level of 300 meters during all phases of flight a value of  $T_{ADS}=1$  sec. for Basic Message or of  $T_{ADS}=4$  secs. for Extended Message is selected from Fig. 5 and Fig. 6 (assuming that a worst case manoeuvre could appear). During constant velocity flight, a better performance can be expected if these periodic contracts are maintained (200 meters in both cases, deduced from Fig. 1 and Fig. 2).

In those occasions that the required accuracy level can be obtained with the Basic Message, at a certain report rate, or with the Extended Message, at a slower rate, the decision of the type of contract to request can be made dependent on the data-link occupation cost.

If a radar with lower scan rate is considered ( $T_{ASR}=12$  secs.), it is depicted in the following figure vs time, the accuracy level in each coordinate of the central track and the ASR local track, at each presentation instant (screen presentation period of 5 secs.), for an aircraft reporting Basic Messages initially located at 30 nmi and flying at 35000 feet which performs a transversal manoeuvre of  $a=0.3g$   $m/s^2$  from  $t_0=100$  secs. to  $t_1=150$  secs. and two small height changes of 20 secs. duration at constant vertical speed at  $t_2=250$  secs. and  $t_3=350$  secs..



If a 95% error of 300 meters. is to be maintained for the central track in every situation and a manoeuvre detector is available,  $T_{ADS}=6$  secs. during constant speed phases and  $T_{ADS}=1$  sec. during detected manoeuvres (values respectively obtained from equivalent curves to those in Fig. 1 and Fig. 3 considering the new performances of the radar). Notice how the 95% error of the presented central track takes the value of the accuracy level of the ADS track when no ASR local track is available to be fused: 650 m. at  $T_{ADS}=6$  s. (see Fig. 1) and 350 m. at  $T_{ADS}=1$  s. (see Fig. 3).

In real operation, in addition to aircraft manoeuvres, several other reasons may require a convenient rise or reduction of the ADS report rate: changes in the radar measurement accuracy (due to the aircraft movement); changes in the accuracy level demanded by the controller; changes of FOM (navigation means); or changes in the type of coverage (the aircraft moves into areas covered by a different number of radars).

## CONCLUSIONS

ADS has lots of potential possibilities to work in a cooperative manner with radar sensors in the context of future CNS/ATM applications. Innovative data fusion + sensor management methods like those proposed in this contribution are required to achieve a synergistic integration of ADS into the *Tactical Tracking* function of existing ATC surveillance systems.

Future lines of challenging research include: the integration of the proposed mode of operation into other multiradar fusion schemes (plot fusion or hybrid architectures); or the proposal of adequate operation methodologies for the Demand and Event contracts. The solution of some problems inherent to ADS, such as the possible delay establishing contracts (which may impose a limitation on its adaptive capabilities), should also be addressed.

## APPENDIX

The fusion coefficients  $c_i$  ( $i=1,2,...,n$ ) are obtained minimising the m.s.e. of the fused position estimator of

the multiradar track  $E\{(c_1\tilde{x}_1+...+c_n\tilde{x}_n)^2\}$  subject to the constraint  $c_1+...+c_n=1$  (the estimation error  $\tilde{x}_i$  ( $i=1,2,...,n$ ) is to be independent of the estimated variable  $x$ ). Applying Lagrange multipliers, the following linear system of  $n+1$  equations, obtained from the partial derivatives of the m.s.e. with respect to  $c_i$  ( $i=1,2,...,n$ ), has to be solved:

$$2b_{\tilde{x}_i} \sum_{j=1}^{i=n} c_j b_{\tilde{x}_j} + 2c_i \sigma_{\tilde{x}_i}^2 + \lambda = 0 \quad (i=1,2,...,n) \quad [1]$$

$$c_1 + c_2 + ... + c_n = 1$$

in which  $b_{\tilde{x}_i} = E\{\tilde{x}_i\}$  and  $\sigma_{\tilde{x}_i}^2 = E\{\tilde{x}_i^2\} - b_{\tilde{x}_i}^2$ . From each one of the  $n$  first of equations of [1] it can be obtained:

$$c_i = \frac{-\lambda}{2\sigma_{\tilde{x}_i}^2} - \frac{b_{\tilde{x}_i}}{\sigma_{\tilde{x}_i}^2} \sum_{j=1}^{i=n} c_j b_{\tilde{x}_j} \quad (i=1,2,...,n) \quad [2]$$

Substituting the  $n$  equations in [2] into the last equation of [1],  $\lambda$  can be deduced, which substituted back into [2] yields the following set of  $n$  equations:

$$c_i = \frac{1 + \sum_{k=1}^{k=n} \frac{b_{\tilde{x}_k}}{\sigma_{\tilde{x}_k}^2} \sum_{j=1}^{i=n} c_j b_{\tilde{x}_j}}{\sigma_{\tilde{x}_i}^2 \sum_{j=1}^{i=n} \frac{1}{\sigma_{\tilde{x}_j}^2}} - \frac{b_{\tilde{x}_i}}{\sigma_{\tilde{x}_i}^2} \sum_{j=1}^{i=n} c_j b_{\tilde{x}_j} \quad (i=1,2,...,n) \quad [3]$$

in which  $c_i$  is expressed as a function of  $\sum_{j=1}^{i=n} c_j b_{\tilde{x}_j}$ .

Multiplying each  $c_i$  in [3] by  $b_{\tilde{x}_i}$  and adding the result yields:

$$\sum_{i=1}^{i=n} c_i b_{\tilde{x}_i} = \frac{\sum_{i=1}^{i=n} \frac{b_{\tilde{x}_i}}{\sigma_{\tilde{x}_i}^2} \left( 1 + \sum_{k=1}^{k=n} \frac{b_{\tilde{x}_k}}{\sigma_{\tilde{x}_k}^2} \sum_{j=1}^{i=n} c_j b_{\tilde{x}_j} \right)}{\sum_{j=1}^{i=n} \frac{1}{\sigma_{\tilde{x}_j}^2}} - \sum_{i=1}^{i=n} \frac{b_{\tilde{x}_i}^2}{\sigma_{\tilde{x}_i}^2} \sum_{j=1}^{i=n} c_j b_{\tilde{x}_j} \quad [4]$$

from which  $\sum_{j=1}^{i=n} c_j b_{\tilde{x}_j}$ , can be obtained, which substituted

into [3] yields the general equation for  $c_i$  ( $i=1,2,...,n$ ) as a function of  $b_{\tilde{x}_i}$  and  $\sigma_{\tilde{x}_i}^2$  ( $i=1,2,...,n$ ) that appears in the text.  $c_R$  and  $c_{ADS}$  are obtained as a particular case of this general expression with  $n=2$ .

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# IMPROVED ASSOCIATION OF ESM MEASUREMENTS WITH RADAR TRACKS

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**ABSTRACT:** Radar-to-ESM association is the basis of radar-to-ESM data fusion which is one of the key subjects in military multisensor data fusion. Based on fuzzy synthetic function and statistical theory, a multi-threshold radar-to-ESM association algorithm is presented, which is suitable for the situations where both radar and ESM tracks are specified by different numbers of measurements. Simulation results show the feasibility of the algorithm.

**Key Words :** radar, ESM, association, fuzzy synthetic function.

## 1. INTRODUCTION

In recent years, the discipline of multisensor data fusion has received considerable attention<sup>[1]</sup>. One important aspect of it is the fusion of data from both a radar and an ESM sensor with the same target<sup>[2-5]</sup>. The key issue of radar-to-ESM data fusion is association of a set of measurements from ESM with the correct target in a radar system. The problem of radar-to-ESM association is essentially a pattern recognition problem.

Trunk and Wilson<sup>[4]</sup> proposed an algorithm of correlating radar tracks with ESM tracks. The method is suitable for the situations where both radar and ESM tracks are specified by different numbers of measurements. In this method, the decision was based on the cumulative probability of the random variable with a chi-squared density. Using the fuzzy synthetic function and statistical theory, we develop a new multi-threshold radar-to-ESM association algorithm which is also suited for the situations where both radar and ESM tracks are specified by different numbers of measurements. The paper is organized as follows. Section 2 is the problem formulation. Section 3 describes the new radar-to-ESM correlation algorithm. The simulation results and the conclusions are given in section 4 and 5, respectively.

## 2. PROBLEM FORMULATION

Radar is the most important sensor, and can provides target location information represented in a polar coordinate system. Unlike radar, an ESM sensor is basically a passive sensor, and can only provide angle measurements and additional non-kinematic target identification information, but does not directly measure range. Since a target can carry multiple emitters, i.e., multiple ESM tracks can be associated with a radar track, each ESM track association can be considered by itself, thus resulting in disjoint correlation problems. Consequently, an equivalent problem is: given an ESM track specified by  $n$  ESM bearing measurements, associate the ESM track with no radar track or one of  $m$  radar tracks. Thus, the problem reduces to the multiple hypothesis testing problem<sup>[4]</sup>

$H_0$ : ESM track associates with no radar track;

$H_j$ : ESM track associates with  $j$ th radar track.

In order to simplify the analysis, some notations and assumptions are made. Assume that  $n_j$  is the maximum number of ESM measurements possible for  $j$ th radar track,  $\theta_{ei}$  is the ESM measurement at time  $t_i$ ,  $\theta_{ji}$  is the predicted azimuth of radar track  $j$  at time  $t_i$ , and the ESM measurement errors are independent and Gaussian distributed with zero mean and constant variance, designated  $\sigma^2$ . With the notations and assumptions on hand, the algorithm is derived in the sequel.

## 3. RADAR-ESM ASSOCIATION ALGORITHM

### 3.1 The discriminant function between an ESM track and a radar track

Since an ESM sensor is a passive sensor and can only provide angle measurements in position information, but does not directly measure range, the only information used in radar-to-ESM correlation is angle information. Let

$$\varepsilon_{ji} = \frac{\theta_{ei} - \theta_{ji}}{\sigma} \quad (1)$$

We make a mapping  $d_{ji} : \varepsilon_{ji} \rightarrow [0,1]$ , which indicates the degree of similarity measure between  $\theta_{ei}$  and  $\theta_{ji}$ . Let

$$d_{ji} = f(\varepsilon_{ji}) \quad j = 1, \dots, m; i = 1, \dots, n_j \quad (2)$$

where  $f(\cdot)$  has the features: (1)  $f(0)=1$ ; (2)  $f(x) = f(-x)$ ; (3)  $\forall x_1 > x_2 \geq 0$ , we have  $f(x_1) \leq f(x_2)$ . Thus,  $\forall i = 1, \dots, n_j$ , after  $d_{ji}$  is got, the similarity vector  $D_j$ , which is the similarity measure between the ESM track and the  $j$ th radar track, is obtained as

$$D_j = (d_{j1}, d_{j2}, \dots, d_{jn_j}) \quad 1 \leq j \leq m \quad (3)$$

In order to make maximum utilization of the data when one is not willing to predict the target position over (possible) long periods into the past, the correlation decision should be made on the similarity measure vector  $D_j$ .  $\forall j = 1, 2, \dots, m$ , since the dimensions of vector  $D_j$  may be different, the normal of the vector  $D_j$  can not be used to compare the closeness degree among the ESM track and the radar tracks. In this case, the synthetic closeness degree among the ESM track and the radar tracks can be obtained by using the fuzzy synthetic function and can be used in the radar-to-ESM correlation situation. Let

$$d_j = M_{n_j}(D_j) = M_{n_j}(d_{j1}, d_{j2}, \dots, d_{jn_j}) \quad (4)$$

where  $M_{n_j} : [0,1]^{n_j} \rightarrow [0,1]$  is a fuzzy synthetic function and has the characteristics of [6]:

(1) order-preserving. That is, assuming  $D_j^* = (d_{j1}^*, d_{j2}^*, \dots, d_{jn_j}^*)$ , if  $d_{ji}^* \leq d_{ji}$ ,  $\forall i = 1, \dots, n_j$ ,

then  $M_{n_j}(D_j^*) \leq M_{n_j}(D_j)$ ; and

(2) synthesis. That is,  $M_{n_j}(\cdot)$  satisfies  $\min_{1 \leq i \leq n_j} \{d_{ji}\} \leq M_{n_j}(D_j) \leq \max_{1 \leq i \leq n_j} \{d_{ji}\}$ .

Thus,  $d_j$  can be treated as a discriminant function of radar-to-ESM correlation when each track is specified by different number of measurements.

It is obvious that if  $f(\cdot)$  and  $M_{n_j}(\cdot)$  take different forms, then  $d_j$  will consequently have different forms too. If  $f(\cdot)$  and  $M_{n_j}(\cdot)$  are selected such

$$d_{ji} = f(\varepsilon_{ji}) = \exp(-\varepsilon_{ji}^2) \quad (5)$$

and

$$d_j = M_{n_j}(D_j) = \left( \prod_{i=1}^{n_j} d_{ji} \right)^{\frac{1}{n_j}} \quad (6)$$

respectively, then  $d_j$  is obtained as

$$d_j = \exp(-\varepsilon_j) \quad (7)$$

where

$$\varepsilon_j = \frac{1}{n_j} \sum_{i=1}^{n_j} \varepsilon_{ji}^2 \quad (8)$$

As the fuzzy synthetic similarity degree  $d_j$  is a monotonically decreasing function of  $\varepsilon_j$ , selecting maximum synthetic similarity  $d_j$  is equivalent to selecting minimum  $\varepsilon_j$ . Thus,  $\varepsilon_j$  given by Eq.(8) can be used as a discriminant function when each radar track being specified by different number of measurements.

### 3.2 Decision-making rules

**3.2.1 Double-threshold decision making.** Firstly,  $\exists s \in \{1, \dots, m\}$  such that

$$d_s = \max \{d_j | j = 1, \dots, m\} \quad (9)$$

then the radar track  $s$  is considered to be most likely associated with the ESM track. After this, we choose thresholds  $t_H$  and  $t_L$ , and the double-threshold decision rule is generated as follows

$$\begin{aligned} \text{Decide } H_s : & \quad \text{if } \varepsilon_s \leq t_L; \\ \text{Uncertainly:} & \quad \text{if } t_L < \varepsilon_s \leq t_H; \\ \text{Decide } H_0 : & \quad \text{if } \varepsilon_s > t_H \end{aligned}$$

**3.2.2 Triple-threshold decision-making.** After  $d_s$  is found and the thresholds  $t_H$ ,  $t_M$  and  $t_L$  are selected, the triple-threshold decision rule is generated as:

$$\begin{aligned} \text{Decide } H_s : & \quad \text{if } \varepsilon_s \leq t_L; \\ \text{Tentatively Decide } H_s : & \quad \text{if } t_L < \varepsilon_s \leq t_M; \\ \text{Tentatively Decide } H_0 : & \quad \text{if } t_M < \varepsilon_s \leq t_H; \\ \text{Decide } H_0 : & \quad \text{if } \varepsilon_s > t_H \end{aligned}$$

**3.2.3 Quadri-threshold decision -making.** Assume  $\exists s, t \in \{1, \dots, m\}$ , such that

$$s = \arg \min_{1 \leq j \leq m} \{\varepsilon_j\} \quad (10)$$

and

$$t = \arg \min_{1 \leq j \leq m, j \neq s} \{\varepsilon_j\} \quad (11)$$

Let

$$P_s = \Pr\{Z_s \geq n_s \varepsilon_s\} \quad (12)$$