IMM Algorithm Based on the Analytic Solution of Steady State Kalman Filter for Radar Target Tracking

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SUMMARY & CONCLUSIONS

Recently, the Interacting Multiple Model(IMM) algorithm based on the steady state Kalman filters has been proposed as a very attractive method for real-time implementation. But when the tracking filter is designed in the Cartesian coordinates, the covariance matrix of radar measurement error varies according to the range and bearing of the target. Therefore, the steady state Kalman gain and the covariance matrix calculated off-line may become inappropriate. In this paper, the IMM tracker is formulated in the Cartesian coordinate frame based on the analytic solution of the steady state Kalman filter in which gain and covariance matrix are calculated on-line.

The performance of the proposed approach is compared with the conventional IMM tracker in terms of the Root Mean Square Error (RMSE) and the Normalized Position Error (NPE) via simulation. The simulation results indicate that this approach not only improves the accuracy but also reduces computational load.

1. *INTRODUCTION*

During the past two decades, a great number of algorithms for tracking a maneuvering target have been suggested. Among them, the IMM algorithm, proposed by Blom in 1988 and 1989[1][2], has been accepted as the suitable approach for tracking maneuvering targets.

The IMM algorithm uses a bank of parallel filters. The filters, instead of working independently, react with each other in a probabilistic manner. Due to this interaction, the individual filters can adjust their parameters and provide optimum output corresponding to the input. A weighted average of the individual filter outputs may be taken as the system output[3].

As for the choice of the tracker frame, the spherical and the Cartesian coordinate systems have been widely used. It is generally accepted that tracking in the Cartesian coordinate is more accurate but is more computationally intensive. The vantage of the Cartesian coordinate is that the state equation is linear, while its drawback is that the corresponding measurement equation is non-linear[4]. The non-linearity of the measurements can be handled by converting the measurements to Cartesian coordinates prior to filtering[5]. Although this procedure is only a bit more complex than that of the commonly used classical conversion, it provides better estimation accuracy than the classical conversion filter or the mixed coordinate extended Kalman filter in general[6].

In modern military surveillance system, the tracker needs an ability to track a great number of targets at the same time. The conventional IMM algorithm based on Kalman or adaptive filters still can not stand so big a data load. Using steady-state filters instead of the complicate optimal or adaptive filters can greatly reduce the computational load[7].

In recent literature[7][8], the IMM tracker based on the fixed gain has been proposed as a very attractive method for the real-time implementation. Its advantage is to remarkably reduce the computational load by allowing all the computation associated with covariance matrix to be done in off-line. However, since the covariance matrix of radar measurement errors varies according to range and bearing of the target the steady state Kalman gain and state error covariance matrix should vary accordingly. This makes the off-line computation of the filter difficult.

In this paper, the IMM tracker is designed in the Cartesian coordinate system utilizing the analytic solution of the steady state Kalman filter in order to consider the variation of the steady state gain and covariance. The analytic steady state solution of the filter is based on the Castella-Dunnebacke's model[9] and the Ramachandra II model[3].

The performance of the proposed method is compared with conventional IMM algorithm and the result is analyzed in terms of RMSE and NPE[10].

2. *ANALYTIC SOLUTION OF STEADY STATE KALMAN FILTER*

2.1. Castella-Dunnebacke's Model

Castella and Dunnebacke developed the two-dimensional tracking filter which estimates the position and the velocity of an aircraft or similar vehicle[9].

The target is assumed to be moving with a constant velocity motion perturbed by a zero mean random acceleration.

$$\begin{aligned} x_{n+1} &= x_n + \dot{x}_n T + v_n T^2 / 2 \\ \dot{x}_{n+1} &= \dot{x}_n + v_n T \end{aligned} \tag{1}$$

where

x_n : the target position at scan n

$\dot{x}_n$: the target velocity at scan n

v_n : the acceleration acting on the target at scan n

T : the interval between observations

It is assumed that v_n is a zero mean white Gaussian noise and the target range and bearing are measured by two-dimensional TWS radar at uniform sampling intervals of time and all measurements are noisy.

The measurement equation can be written as:

$$\mathbf{z}_n = H\mathbf{x}_n + \mathbf{v}_n \tag{2}$$

where

$$\mathbf{z}_n = \begin{bmatrix} x_n^o \\ y_n^o \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{x}_n = \begin{bmatrix} x_n & y_n & \dot{x}_n & \dot{y}_n \end{bmatrix}^T$$

and

$$\mathbf{v}_n = \begin{bmatrix} v_n^x & v_n^y \end{bmatrix}^T$$

x_n^o : the measurement on x-axis at scan n

y_n^o : the measurement on y-axis at scan n

v_n^x : the random noise on x measurement at scan n

v_n^y : the random noise on x measurement at scan n

Since the radar measurements are expressed in polar coordinate system and tracking is made in Cartesian coordinate system, the measurements in Eq. (2) are coupled and the measurement errors are correlated. The measurement error covariance matrix in the Cartesian coordinate system can be described as follows:

$$R = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{bmatrix} \tag{3}$$

This matrix is calculated from the measurement error covariance at bearing zero and the transformation matrix by:

$$R = D \cdot R_o \cdot D^T \tag{4}$$

where

$$R_o = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & r^2\sigma_\theta^2 \end{bmatrix}, \quad D = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

The steady state characteristics of the filter are analytically determined under the assumption of a white noise maneuver model in the two dimensions (see Reference 3 in more details).

The predicted steady state error covariance matrix is described as follows:

$$\tilde{P}_{ss} = D_2 \cdot \tilde{P}_o \cdot D_2^T \tag{5}$$

where

$$D_2 = \begin{bmatrix} D & 0_{2\times2} \\ 0_{2\times2} & D \end{bmatrix}$$

$\tilde{P}_o$: the predicted state error covariance matrix at bearing zero

The updated steady state covariance matrix is described as:

$$\hat{P}_{ss} = D_2 \cdot \hat{P}_o \cdot D_2^T \tag{6}$$

where

$\hat{P}_o$: the updated state error covariance matrix at bearing zero

The steady state Kalman gain is obtained by:

$$K_{ss} = D_2 K_o D^T \tag{7}$$

where

K_o : the steady state Kalman gain at bearing zero

2.2. Ramachandra's Model II

This two-dimensional model estimates the position, velocity, and acceleration of an aircraft moving with a constant acceleration perturbed by a zero mean plant noise which makes account for maneuver and/or other random factors.

$$\begin{aligned} x_{n+1} &= x_n + \dot{x}_n T + \ddot{x}_n T^2/2 + w_n T^3/6 \\ \dot{x}_{n+1} &= \dot{x}_n + \ddot{x}_n T + w_n T^2/2 \\ \ddot{x}_{n+1} &= \ddot{x}_n + w_n T \end{aligned} \tag{8}$$

where

x_n : the target position at scan n

$\dot{x}_n$: the target velocity at scan n

$\ddot{x}_n$: the target acceleration at scan n

w_n : the rate change of acceleration at scan n

T : the interval between observations

Assuming w_n is zero mean white Gaussian noise, the measurement equation may be given by:

$$\mathbf{z}_n = H\mathbf{x}_n + \mathbf{w}_n \tag{9}$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The measurement error covariance matrix is the same as equations (3) and (4), and the predicted steady state covariance matrix can be expressed as:

$$\widetilde{P}_{ss} = D_3 \cdot \widetilde{P}_o \cdot D_3^T \tag{10}$$

where

$$D_3 = \begin{bmatrix} D & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & D & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & D \end{bmatrix}$$

The updated steady state covariance matrix is written as:

$$\hat{P}_{ss} = D_3 \cdot \hat{P}_o \cdot D_3^T \tag{11}$$

Finally, the steady state Kalman gain can be obtained as:

$$K_{ss} = D_3 \cdot K_o \cdot D^T \tag{12}$$

3. *INTERACTING MULTIPLE MODEL ALGORITHM*

The IMM estimator is a suboptimal hybrid filter. This estimator has the ability to estimate the state of a dynamic system with several behavior modes which can switch from one to another. This can be considered as a self-adjusting variable bandwidth filter and hence very well suited for tracking maneuvering targets.

This approach consists of running filters with respect to each model, a model probability evaluator, and an estimate combiner at the output of the filters. Each of the filters uses a mixed estimate at the beginning of each cycle and a measurement in order to compute a new estimate and the likelihood of the model within the filter. Sequentially, the likelihoods, the prior model probabilities, and the model switching probabilities are used to calculate new model probabilities. After then, the overall state estimate is computed with the new state estimates (see model probabilities shown in Figure 1).

3.1 Calculation of Mixing Probability

Assuming there are *m* interacting models in place, the probability that model *i* was in effect at the previous time step *k*, given that model *j* is in effect at the current time step *k+1*(conditioned on all measurements up to *k*) is calculated.

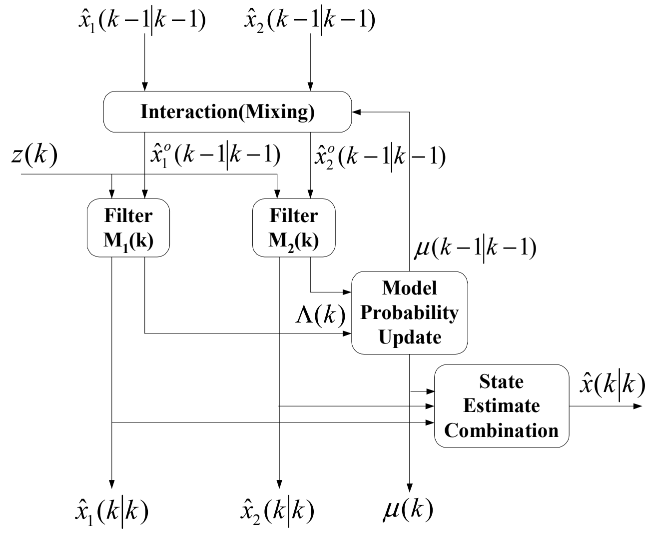

Figure 1. IMM Algorithm

$$\mu_{i|j}(k \mid k) = \frac{1}{\bar{c}_j} p_{ij}\mu_i(k), \; i,j = 1 \sim m \tag{13}$$

where

$\bar{c}_j$: the normalizing constant

μ_i : the model probability

p_{ij} : the model transition probability

m : the model number

3.2 Interaction

The mixed states which are matched to each mode (i.e. one of the two filter models in this case) are calculated by using the state estimates [$\hat{x}_i(k \mid k)$] from the previous time step:

$$\hat{x}_{0j}(k \mid k) = \sum_{i=1}^{m} \hat{x}_i(k \mid k)\mu_{i|j}(k \mid k), \;\; j = 1 \sim m \tag{14}$$

where m=2 in this case.

3.3 Mode Matched Filtering

The above estimates are then applied as inputs for the m models to yield $\hat{x}_j(k+1 \mid k+1)$, $(j = 1 \sim m)$ as the outputs. Then, the likelihood functions corresponding to the m filters are calculated as

$$\Lambda_j(k) = \frac{1}{\sqrt{2\pi|S_j(k)|}} e^{-\frac{1}{2}\tilde{z}_j^T(k)S_j(k)^{-1}\tilde{z}_j(k)} \tag{15}$$

where

$\hat{z}_j$: the measurement residuals

S_j : the residual covariance matrix

3.4 Mode Probability Update

The probability of each mode is updated

$$\mu_j(k+1) = \frac{1}{c}\Lambda_j(k+1)\bar{c}_j \quad j = 1 \sim m \tag{16}$$

where

c : the normalization constant

$$c = \sum_{i=1}^{m} \Lambda_i(k+1)\bar{c}_i$$

3.5 State Estimate and Covariance Matrix Combination

The m estimates are finally combined as following Equation (17). These values are used as the actual outputs of the system, but are not actually part of the IMM algorithm recursions.

$$\hat{x}(k+1|k+1) = \sum_{j=1}^{m} \hat{x}_j(k+1|k+1)\mu_j(k+1) \tag{17}$$

4. *COMPUTER SIMULATION*

The simulated target has a very complicated motion with varying velocity, sinusoidal acceleration and coordinated turn making account for real operational environments. For the radar measurement generation, a target trajectory is first generated in the two-dimensional coordinate system with a 3.75 second sampling interval and then converted to the polar coordinate system.

It is assumed that the radar measurement errors are known to be Gaussian with standard deviation of 130m and 1 degree for range and bearing, respectively.

The true trajectory and the velocity of the target in the x-y plane are depicted in Figures 2 and 3.

The initial speed of the target is about 124 knots. However, the target speed varies according to the target motion to the maximum speed of about 290 knots.

In order to implement the IMM algorithm, the Markov chain transition parameter and the initial model probability should be appropriately set. In this contribution, the Markov chain transition parameter and the initial model probability are set as:

$$p = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}, \quad \mu = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$$

RMS errors of position and velocity estimation are illustrated in Figures 4 and 5. In these figures, the top thick lines represent the RMS errors of the radar measurements in the Cartesian coordinate system. The RMS errors of the conventional IMM filter estimation errors and the RMS errors of the proposed IMM filter estimation errors are plotted by the solid and dotted lines, respectively.

It is seen from these results that the conventional IMM filter seems to converge a little faster than the proposed method. Nevertheless, the latter is more robust rather than the former.

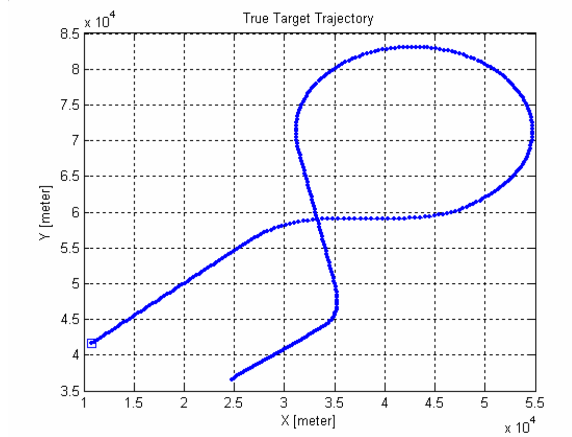

Figure 2. True target trajectory

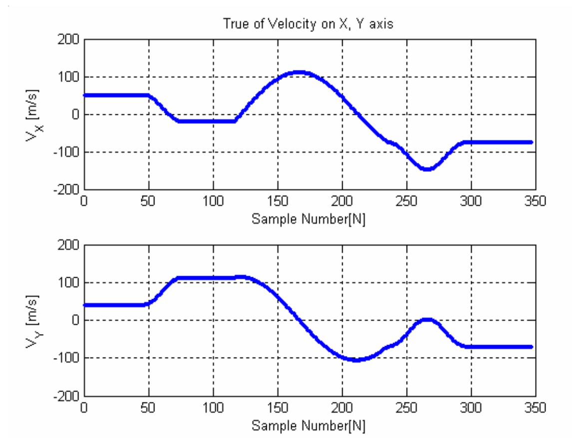

Figure 3. True target velocity on x and y axis

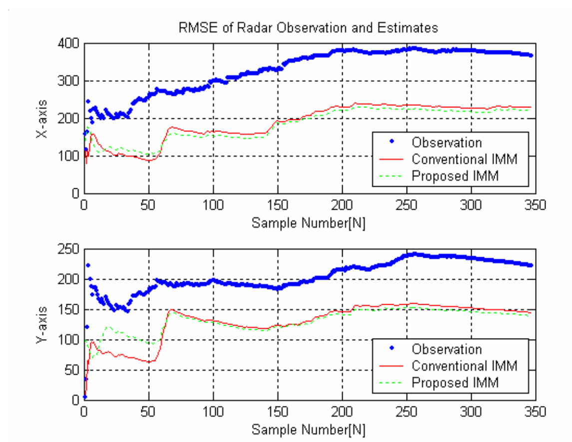

Figure 4. Position estimate error

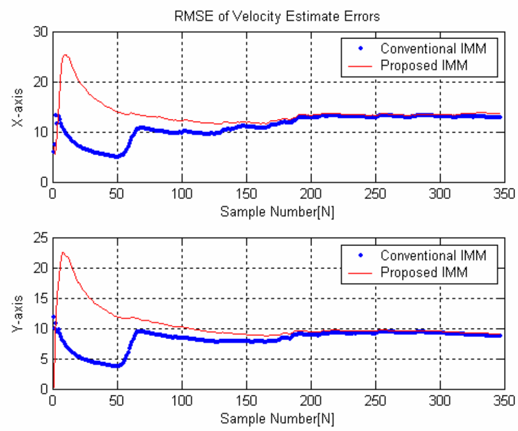

Figure 5. Velocity estimate error

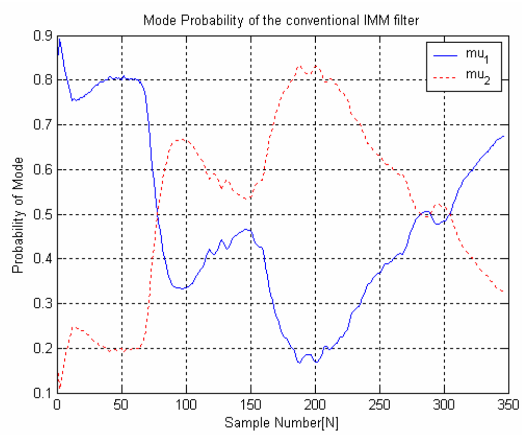

Figure 6. The transition of the mode probability of the conventional IMM filter

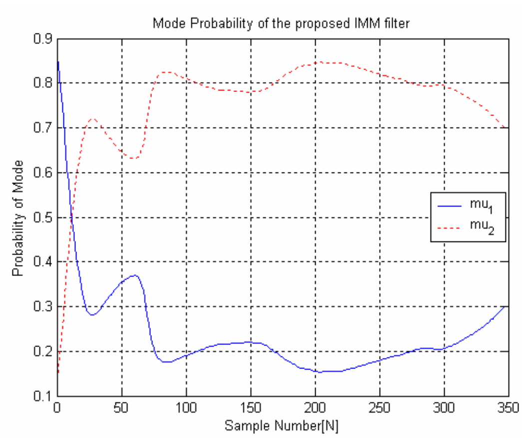

Figure 7. The transition of the mode probability of the proposed IMM filter

Figures 6 and 7 illustrate the change of mode probability of conventional IMM filter and the proposed method.

It can be seen from these results that the mode probability of the conventional IMM filter changes more rapidly in response to the maneuver of the target.

Tracking accuracy may be expressed in terms of the NPE (which is the ratio of the root mean square of position estimate error to the root mean square of measurement error) for each scan. The NPE is computed by:

$$NPE = \sqrt{\frac{\sum_{i=1}^{N}[(x_i(k)-\hat{x}_i(k))^2+(y_i(k)-\hat{y}_i(k))^2]}{\sum_{i=1}^{N}[(x_i(k)-z_i^x(k))^2+(y_i(k)-z_i^y(k))^2]}} \tag{18}$$

The NPEs of the conventional IMM filter and the proposed method are given in Figure 8. The result shows that the proposed method is more accurate than the conventional IMM filter.

The computational time can be measured by Visual C++ functions such as QueryPerformanceFrequency( ) and Query-PerformanceCounter( ) which make measure of the elapsed time by using the CPU clock. The measured computational times of the conventional IMM filter and the proposed method are 3.93 usec and 3.06 usec, respectively, indicating that the proposed method takes less computation load.

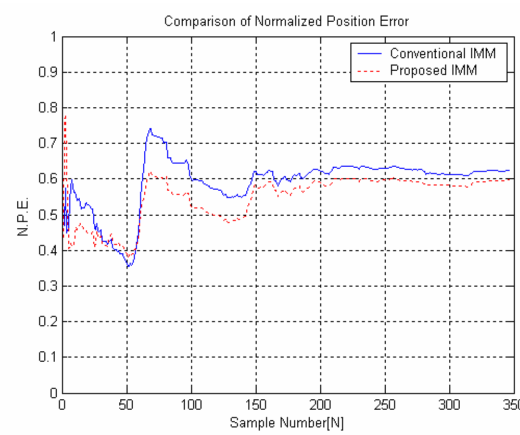

Figure 8. Normalized position error

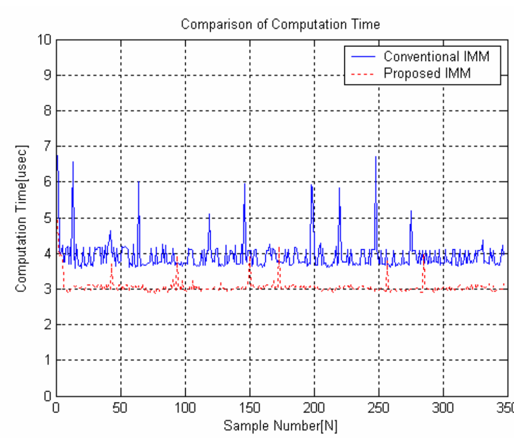

Figure 9. The comparison of computation time

5. CONCLUSION

In this paper, the IMM tracker was designed in the Cartesian coordinate system by utilizing the mode dependant analytic solution of the steady state Kalman filter in order to consider the variation of the steady state gain and covariance due to target maneuver. It was shown via simulation that the proposed method provides more accurate and robust performance compared to the conventional IMM algorithm. The convergence time of the proposed IMM filter may be improved by adding adequate gain scheduling logic.

The proposed method is also more efficient in terms of the computational load. Thus, the proposed approach is regarded as a very attractive method for tracking multiple targets with a Track-While-Scan (TWS) radar system.

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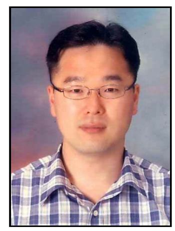
BIOGRAPHIES

Byung-Doo Kim was born in GyeongBuk, South Korea in 1971. He received the M.S. degree in control and instrumentation engineering from Ajou University, Suwon, Korea in 1998.



From September, 2002 to April, 2004 he was with MteQ Systems(Currently STX Radarsys), working on signal and data processing of sea surveillance radar systems. From May, 2004, He joined Telematics Research Division of ETRI.

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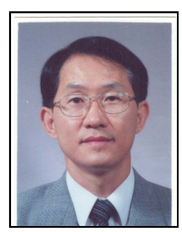

From 1981 to 1991 Dr. Lee was with the Space Systems Division of Rockwell International Corporation at Seal Beach, California, where he worked in the areas of spacecraft navigation, attitude determination and target tracking. In 1992, he joined the faculty of Ajou University, Suwon, Korea, where he teaches control and estimation theories until today.

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