Proceedings of the 2003 IEEE
International Symposium on Intelligent Control
Houston, Texas • October 5-8, 2003

Interacting Multiple Model Algorithm with Maximum Likelihood Estimation for FDI

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***Abstract*— In this paper we propose an approach to detect, identify and estimate failures, including partial failures, in a dynamic system. The approach (IM^{3}L) uses the interacting multiple model (IMM) estimators to detect and identify total and partial failures and the maximum likelihood estimator (MLE) to estimate the extent of failure. It provides an effective and integrated framework for fault detection, identification and state estimation. The hierarchical structure of the fault model set for FDI is further addressed and a two-level hierarchical IMM (HIMM) is implemented. By using an aircraft example, the proposed IM^{3}L approach is evaluated and its performance is compared with those of the HIMM and autonomous MM (AMM). The robustness of IM^{3}L is analyzed in the presence of the uncertain noise statistics. The results show that the proposed approach provides not only fast detection and proper identification but also good estimation of the failure extent as well as robust state estimation.**

I. Introduction

With increasing complexity of modern engineering systems, the requirement for reliability, availability and security is growing significantly. Fault detection and diagnosis (FDD) is becoming a major issue in safety critical systems such as modern flight control systems. In the last two decades many techniques have been developed for FDD. Different methods are surveyed in [2][6][8][9][22].

A conventional approach to FDD relies on hardware redundancy which use multiple sensors and actuators. This approach is expensive. In the analytical redundancy approach, most methods are model-based and detect faults based on a residual signal, which is the difference generated by observations and the system mathematical model. There are various approaches to the residual generation, such as diagnostic observers, multiple models, parity relation and parameter estimation [3][17]. However most of them do not deal with state estimation and fault estimation problem.

Recently multiple model (MM) techniques have been applied to fault detection and identification (FDI). It consists of a bank of elemental filters running in parallel, each based on a model matching a particular mode of the system. MM algorithms for FDI have been successfully applied to a number of particular problems under other names, such as multiple hypothesis testing [22], multiple adaptive estimate algorithm (MMAE) [14][13][15] and multiple dedicated observers [6]. All these approaches are based on autonomous (or "noninteracting") multiple filters, which does not fit well into the framework of FDI in a dynamic system because the structure or parameter of such a system does change as its component or subsystem changes. To make MM algorithm more suitable for the current problem, new IMM-FDI approaches have been proposed [20][4][18][19][24]. The difference between IMM and the "noninteracting" MM algorithms lies in that elemental filters in the IMM interact with each other which leads to improved performance. The IMM-FDI approach provides fast detection and proper identification. In addition, the IMM-FDI approach can be naturally extended to fault-tolerant control.

Prior studies of IMM-FDI show a good performance when the designed failure model matches the truth. However, in practice faults often occur with different magnitudes such as total failures and partial failures. To have reliable identification and good state estimation under the failure condition, it is very important to estimate the severity of failure. The focus of this paper is on the development of an approach to fast detection and accurate estimation of partial failures.

The rest of this paper is organized as follows. A general FDD approach based on IMM estimator is presented in section II. A hierarchical scheme for IMM-FDI (HIMM) is presented in section III. The proposed new approach IM^{3}L for FDI is presented in section IV. Performance comparison of different approaches for sensor and actuator failures of an aircraft is illustrated in section V. Conclusions are given in section VI.

II. Fault Detection Using IMM

A. The dynamic model for systems subject to failures

In flight control systems, there are several types of possible failures such as sensor failures, actuator failures and component failures. Therefore, in the IMM-FDI, a set of models (hypotheses) are used to represent the possible system structures due to different failures. Each failure hypothesis corresponds to a specific model with a corresponding set of F, G, H and covariance matrices Q and R. Let M denote the set of all designed system models and j a generic model in it. Then a linear dynamic system can be represented by

$$x_{k+1} = F_k^j x_k + G_k^j u_k + T_k^j w_k^j \tag{1}$$

$$z_k = H_k^j x_k + v_k^j \tag{2}$$

where x is the state vector; z is the measurement vector; u is the control input vector; w_k and v_k are independent discrete-time process and measurement noise with mean $\bar{w}_k$ and $\bar{v}_k$, and covariance Q_k and R_k, respectively. It is assumed that the initial state x_0 has a mean $\bar{x}_0$ and covariance P_0, and is independent of w_k and v_k. It is also assumed that the system mode sequence is a first order Markov chain with transition probabilities

$$\pi^{ij} = P\{m_{k+1}^j | m_k^i\} \tag{3}$$

where m_k^i denotes that the ith model is in effect at k.

B. Failures

Here we consider complex failure situations, including total (hard) failures and partial (soft) failures of sensors/actuators. Partial sensor failures can be caused by a reduced sensor power. Partial actuator failures can arise from damage to a control surface resulting only a portion of control effectiveness delivered [13]. In order to better define such failures, an effectiveness factor, denoted as α ($0 \le \alpha \le 1$), is introduced to represent the extent of a failure. An effectiveness of 0% ($\alpha = 0$) indicates a "total failure" while an effectiveness of 100% ($\alpha = 1$) indicates "no failure"; $0 < \alpha < 1$ indicates "partial failures". Clearly α is an unknown parameter in the linear stochastic model of a dynamic system. In model-set design, the value of α is quantized to represent different magnitudes of failure.

Sensor failures can be modeled by multiplying the respective row of the H matrix by α. Soft sensor failures can also be modeled by changing the measurement noise mean or elements of the R matrix. This type of failures means the output of the sensor is corrupted by the noise. Similarly the actuator failures can be modeled by multiplying the respective column of the G matrix by α.

C. IMM estimator for FDI

As mentioned above, an IMM-FDI runs a bank of Kalman filters, each based on a different model of the system, where α represents the failure extent. In each cycle there are four major steps: 1) model-conditional reinitialization, 2) model-conditional filtering, performed in parallel by elemental filters, 3) model probability update, based on model-conditional likelihood functions, and 4) estimation fusion, which produces the overall state estimate as the probabilistically weighted sum of the outputs from all elemental filters. The first step is unique for IMM estimators compared to other MM algorithms. The reader is referred to [1][11][10] for details.

In IMM-FDI, the model probabilities are used as an indication of a failure because it provides a meaningful measure of how likely each fault mode is at a given time. The fault detection decision can be made by:

$$\mu_k^j = \begin{array}{l} \max_i \mu_k^i > \mu_T \Rightarrow H_j : \text{fault } j \text{ occurred} \\ \max_i \mu_k^i < \mu_T \Rightarrow H_1 : \text{no fault} \end{array} \tag{4}$$

where μ_T is a detection threshold.

A complete cycle of the IMM-FDD scheme with Kalman filter as its mode-matched filter is summarized in Table 1 of [24].

III. Hierarchical IMM-FDI

For many applications of the flight control systems, it is highly desirable to have reliable fault detection and identification with robust state estimation. In reality, failures quite often occur with different fault magnitudes. In order to get good state estimation, detection and estimation of partial failures are of major importance. This and next sections will focus on this issue.

In most prior research, the designed model set only considers normal and total failure models without explicit inclusion of the partial failure models. Although some partial failures can be covered in principle by combinations of the normal and total failure models, the quantization level of the failure degree is too crude to produce an accurate estimate of the extent of failure. A natural idea is to have a finer parameter quantization. This means that filters for partial failures should be added. There are different ways to implement it. One way is to use all filters including additional filters at each time. The computational load of this approach is obviously heavy. Instead, a hierarchical structure of IMM estimators can be considered to reduce the computational complexity [5][7][21]. Fig. 1 illustrates such a hierarchical structure.

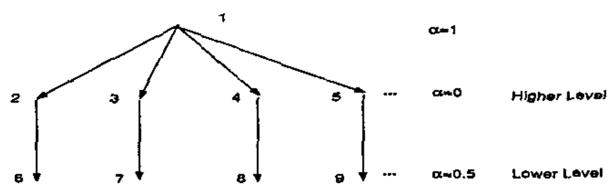


Fig. 1. A hierarchical structure

The hierarchical structure consists of layers of multiple models. It has two levels in Fig.1. At the higher level there are $N+1$ filters designed for one normal model ($\alpha = 1$) and N total sensors/actuators failure models ($\alpha = 0$). The higher level is used to detect which sensor or actuator failed. Once a specific fault is declared at the higher level, the lower level is brought on line to run instead of the higher level. The lower level is to test how severe that specific fault is by estimating the effectiveness factor α. The lower level consists of N model sets, each designed for a specific sensor or actuator failure at the higher level. In Fig.1, at the lower level each model set consists of one for total failure ($\alpha = 0$), one for half failure ($\alpha = 0.5$) and one for normal operation ($\alpha = 1$). Including the normal model at the lower level allows the algorithm to change back to the higher level when there is no failure in fact. The total expectation theorem is used to fuse the parameter estimates α_j at the lower level as well as the state estimates $\widehat{x}_{k|k}^j$, respectively:

$$\widehat{x}_{k|k} \triangleq E[x_k | z^k] = \sum_{j=1}^{n} \widehat{x}_{k|k}^j \mu_k^j \tag{5}$$

$$\widehat{\alpha}_k \triangleq E[\alpha_k | z^k] = \sum_{j=1}^{n} \alpha_j \mu_k^j \tag{6}$$

where $\widehat{x}^j_{k|k}$ is the state estimate from the jth elemental filter, μ^j_k is the corresponding model probability, α^j is the designed failure effectiveness factor for the model and n is the number of models in the model set in effect at time k at the lower level.

For the hierarchical MM estimators only one level is running at a time. Since the lower level is in effect only when the corresponding failure is detected at the higher level, it reduces the number of elemental filters running in parallel at each time. When switching from the higher level to the lower level, each filter at the lower level needs to be initialized. When a specific failure is declared at the higher level, the estimate and covariance of the lower level filters are set equal to those of the corresponding filter at the higher level at the previous time. As such, the lower level filters have the same initial conditions. Their model probabilities are set equal.

IV. The IM^{3}L Scheme

In the above, a smaller model set for an identified failure is opened up to estimate the extent of failure when a specific failure is detected at the higher level. Here we propose a different way to estimate the partial failure based on the maximum likelihood estimation (MLE). It is well known that MLE is most popular for parameter estimation. For the detection problem formulated in this context, the only unknown parameter is the effectiveness factor α. So if we can estimate α by MLE, a new failure model can be designed based on the estimate $\widehat{\alpha}$ for that specific failure. This model is expected to generate better state estimation if $\widehat{\alpha}$ is accurate so that the new model is close to the truth.

The likelihood function is the basis of MLE. MLE chooses as the estimate the "most likely" value of the true parameter given the observations by maximizing the likelihood function. Under the assumption of the Kalman filtering, it is easy to get an analytical formula for $\widehat{\alpha}$. Assume at time k, a failure is detected by the IMM estimator, which is done in the same way as in the HIMM higher level. The likelihood function of $\alpha = [\alpha_1, ..., \alpha_k]'$ is

$$f(z^k|\alpha) = f(z_k|z^{k-1}, \alpha_k) \prod_{j=1}^{k-1} f(z_j|z^{j-1}, \alpha_j) \tag{7}$$

Since the fault occurs at time k and estimates of α_j at all previous times are assumed available, α_k is the only unknown parameter. Then we only need to deal with the conditional likelihood function $f(z_k|z^{k-1}, \alpha_k)$. Assuming

$$z_k \sim \mathcal{N}(\widehat{z}_{k|k-1}, S_k)$$

where $\widehat{z}_{k|k-1} = E[z_k|z^{k-1}]$ and S_k is the associated covariance. We have

$$f(z_k|z^{k-1}, \alpha_k) = \frac{1}{|2\pi S|^{1/2}} e^{-\frac{1}{2}(z_k - H_k\widehat{x}_{k|k-1})' S_k^{-1}(z_k - H_k\widehat{x}_{k|k-1})} \tag{8}$$

Then the MLE of α can be obtained (see appendix A).

HIMM and IM^{3}L have the same mechanism for fault detection. However, they differ in estimating the effectiveness factor α after the failure is detected. The HIMM uses an additional model set to estimate α, and so the estimation accuracy depends on the quantization level of that model set. The estimation of partial failures and state could be improved if more models are included at the lower level, which would cost more computation. The IM^{3}L obtains $\widehat{\alpha}$ from MLE directly, which mainly depends on observations, and updates the state estimate by the new model based on $\widehat{\alpha}$

$$\widehat{\alpha}_k = \arg\max_{\alpha_k} f(z_k|z^{k-1}, \alpha_k)$$
$$\widehat{x}_{k|k} \triangleq E[x_k|z^k] = \sum_{j=1}^{n} \widehat{x}^j_{k|k}\mu^j_k$$

Note that the updated state estimate from the new model based on $\widehat{\alpha}_k$ replaces the old one assuming $\alpha_k = 0$ (total failure).

V. Simulation Results

A longitudinal vertical takeoff and landing (VTOL) aircraft model [16][24][23] is used in this section to demonstrate the performance of our proposed schemes.

A. Aircraft model

The linearized model of VTOL can be described by

$$\dot{x}(t) = Ax(t) + Bu(t) + \xi(t) \tag{9}$$
$$z(t) = Cx(t) + v(t) \tag{10}$$

where $x = [V_h, V_v, q, \theta]'$, $u = [\delta_c, \delta_l]'$, and

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.420 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

The state consists of horizontal velocity V_h, vertical velocity V_v, pitch rate q, and pitch angle θ. The control inputs are collective pitch control δ_c and longitudinal cyclic pitch control δ_l.

Discretizing (9) and (10) yields

$$x_{k+1} = Fx_k + Gu_k + w_k \tag{11}$$
$$z_k = Hx_k + v_k \tag{12}$$

where $F = e^{AT}$, $G = (\int_0^T e^{AT} d\tau)B$, $H = C$, and sampling period $T = 0.1$s.

The transition probability matrix was designed for different scenarios. The following parameters were used for all scenarios. $Q = (0.01)^2 I$, $R = (0.2)^2 I$, $x_0 = [250, 50, 10, 8]'$, $u = [100, 100]'$.

B. Performance Indices

In order to evaluate the performance of different approaches, the following performance measures are used in this work: false alarm (FA), missed detection (MD), percentages of correct detection and identification (CDI), incorrect fault identification (IFI), no mode detection (NM) (for details see [24]). In addition, the flops in one cycle is also used. The RMS error of the effectiveness factor at time k is defined as

$$\text{RMSE}(\widehat{\alpha}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\alpha_k^i - \widehat{\alpha}_k^i)^2} \tag{13}$$

where N is the number of Monte Carlo runs, and the superscript i stands for quantities on run i.

C. Simulation Results

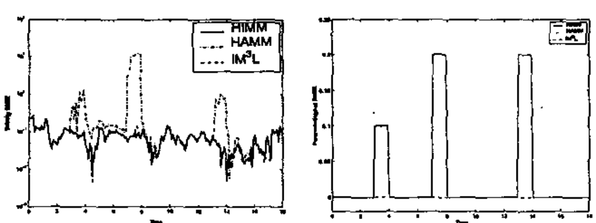
All results presented here are averages over 100 Monte Carlo runs. HIMM uses a robust IMM implementation [12]. HAMM represents a hierarchical autonomous MM algorithm with a 10^{-3} lower bound for each model probability. IM^{3}L represents an implementation of the proposed approach.

1) Scenario 1. Sequential partial sensor failures

For HIMM and HAMM, a total of nine models are used (one for normal, four for total failures plus four for half failures). For IM^{3}L, a total of five models are used (four for total failures plus one for normal). In Case 1, three different failures with different magnitudes were simulated. There is a 10% horizontal velocity V_h failure during $[30, 39]$ (i.e., between $k = 30$ and $k = 39$), a 20% vertical velocity V_v failure during $[70, 79]$, and a 20% pitch rate q failure during $[130, 139]$. Fig. 2 shows the RMS velocity errors (i.e., the RMS errors of $[V_h, V_v]'$) and RMS errors of $\widehat{\alpha}$.

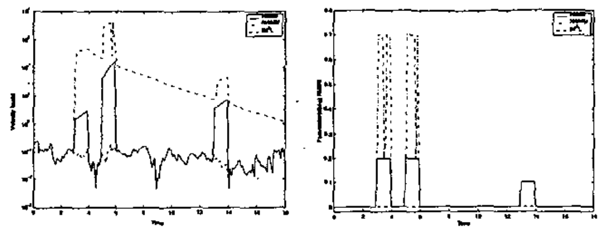
Case 1 is for a small true effectiveness ($\alpha = 20\%$), that is, the failure condition is quite severe. In this case all algorithms make the correct detection, and the difference for the RMS velocity error is not large. However, as α increases, the performance of HAMM and HIMM deteriorates, especially HAMM. This is shown in Case 2 for a 30% horizontal velocity failure during $[30, 39]$, a 30% vertical velocity failure during $[50, 59]$, and a 40% pitch rate failure during $[130, 139]$. Fig. 3 shows the RMS velocity errors and $\widehat{\alpha}$ errors. Note the tremendous improvement of IM^{3}L $\widehat{\alpha}$ errors over the HIMM and HAMM.

The total failure and normal models were used to detect sensor partial failures in the above simulations. However,



(a) RMS velocity errors (b) RMS $\widehat{\alpha}$ errors

Fig. 2. Case 1 – sensor failures (severe, total failure models)

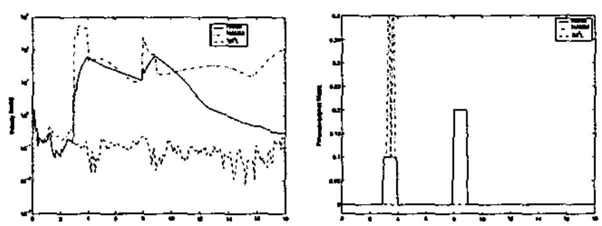


(a) RMS velocity errors (b) RMS $\widehat{\alpha}$ errors

Fig. 3. Case 2 – sensor failures (mild, total failure models)

the total failure model has a limited detection range (i.e., $\alpha < 0.5$), outside of which the fault cannot be detected. If the total failure models at the higher level are replaced with half failure models, a greater detection range is obtained. In this case, the IM^{3}L still has excellent performance while the HIMM becomes worse and the HAMM worst, as seen by a comparison of Fig. 3 and Fig. 4.

Case 3 represents the situation when there is a 10% horizontal velocity failure during $[30, 39]$ and a 70% pitch rate failure during $[80, 89]$. Fig. 4 shows the performance comparison results. Table 1 presents the comparison of the detection range: HIMM1 (with total failure models at the higher level) and HIMM2 (with half failure models at the higher level). Similar results were observed for AMM estimators.



(a) RMS velocity errors (b) RMS $\widehat{\alpha}$ errors

Fig. 4. Case 3 – sensor failures (half failure models)

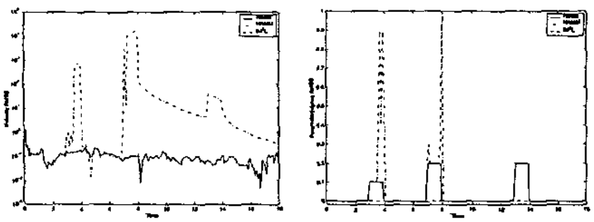
Table 1. Detection ranges

α	Total Failure	Half Failure	No Failure
HIMM1	$[0, 0.3)$	$[0.3, 0.5)$	$[0.5, 1]$
HIMM2	$[0, 0.25]$	$(0.25, 0.75)$	$[0.75, 1]$

In order to evaluate the performance impact due to the uncertainties in noise statistics, Case 4 was tested. It differs from Case 1 in that the noise matrices Q and R used for the filter are 20 times the true ones. Fig. 5 shows the RMS velocity error and $\widehat{\alpha}$ errors. Cases with other uncertainties in noise statistics were also simulated. It shows that HIMM and IM^{3}L keep their performance but the performance of HAMM deteriorates greatly. Table 2 shows the FDD results for all tested cases based on the partial sensor failures.

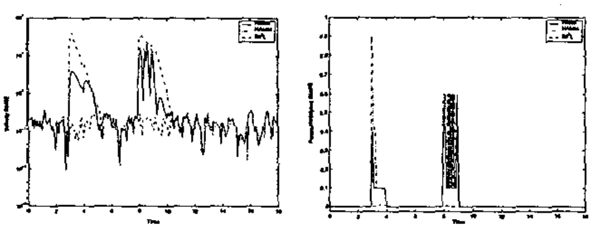
2) Scenario 2. Sequential partial actuator failures

There are two hypothesized actuator failures for VTOL. The HIMM or HAMM use a total of five models (one for normal, two for total failures and two for half failures); IM^{3}L uses three models (two total failure models plus one for normal). In Case 1, two different failures with different magnitudes were simulated. There is a 10% collective pitch control δ_c failure during $[30, 39]$ and a 40% longitudinal cyclic pitch control δ_l failure during $[80, 89]$. Fig. 6 shows the RMS velocity error



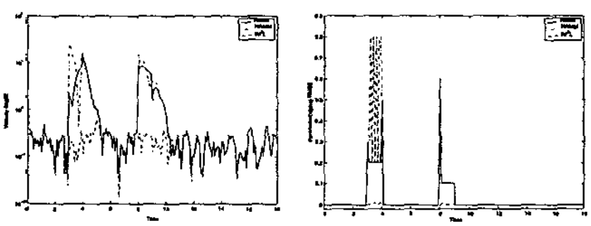
(a) RMS velocity errors (b) RMS $\widehat{\alpha}$ errors

Fig. 5. Case 4 – sensor failures (robustness)



(a) RMS velocity errors (b) RMS $\widehat{\alpha}$ errors

Fig. 6. Case 1 – actuator failures (total failure models)

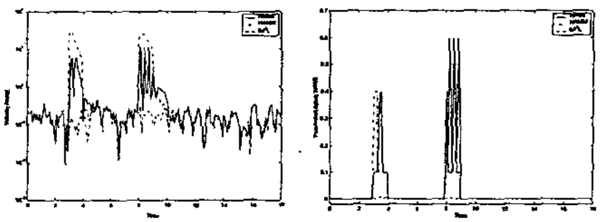


(a) RMS velocity errors (b) RMS $\widehat{\alpha}$ errors

Fig. 7. Case 3 – actuator failures (half failure models)

and $\widehat{\alpha}$ errors. Note that here the Q matrix had to be tuned (increased) to get the satisfactory performance for the HAMM. Otherwise it would fail to detect all simulated actuator failures, as shown in Case 2. In this case, the performance of the HIMM also decreases while the IM^{3}L still maintains the superior performance.

Similar to Case 3 of sensor failures in which the total failure models were replaced by the half failures at the higher level, Fig. 7 shows the performance comparison results for a 20% collective pitch control failure during [30, 39] and a 60% longitudinal cyclic pitch control failure during [80, 89]. It can be seen that the IM^{3}L outperforms the HIMM and HAMM significantly. The detection ranges of the HIMM and HAMM were small ($\alpha \leq 0.2$), significantly smaller than for sensor failures, when total failure models were used at the higher level. There is little improvement if these models are replaced by half failure models and different actuator failures have different detection ranges. Instead, the IM^{3}L had a greater detection range from $[0, 0.5)$ to $[0, 0.7]$ with excellent performance. Case 4 was simulated to evaluate the robustness of the proposed algorithms, which differs from Case 1 in that the noise matrices Q and R used for the filter are 20 times the true ones. Fig. 8 presents the RMS velocity errors and $\widehat{\alpha}$ errors. The results show that the IM^{3}L approach is robust to the uncertainties in noise statistics. Table 3 shows the FDD results for all four cases of partial actuator failures.



(a) RMS velocity errors (b) RMS $\widehat{\alpha}$ errors

Fig. 8. Case 4 – actuator failures (robustness)

Table 2. FDD results for sequential sensor failures

	%	MD	FA	CDI	NMD	IFI	Flops
Case 1	HIMM	0	0	100	0	0	1.135
	HAMM	0	0	100	0	0	0.933
	IM^{3}L	0	0	100	0	0	1.071
Case 2	HIMM	0	0	100	0	0	1.135
	HAMM	8.33	0	90.56	0	1.11	0.892
	IM^{3}L	0	0	100	0	0	1.071
Case 3	HIMM	0	0	99.44	0.56	0	1.139
	HAMM	0	0.12	98.17	0.04	1.67	0.937
	IM^{3}L	0	0	99.44	0.56	0	1.097
Case 4	HIMM	0	0	99.92	0.08	0	1.127
	HAMM	1.11	1.11	93.34	0	4.44	0.925
	IM^{3}L	0	0	99.92	0.08	0	1.063

Table 3. FDD results for sequential actuator failures

	%	MD	FA	CDI	NMD	IFI	Flops
Case 1	HIMM	2.78	0	97.22	0	0	1.151
	HAMM	3.89	0	96.11	0	0	0.989
	IM^{3}L	0	0	100	0	0	1.071
Case 2	HIMM	6.11	0	93.89	0	0	1.142
	HAMM	11.11	0	88.89	0	0	0.922
	IM^{3}L	0	0	100	0	0	1.096
Case 3	HIMM	0.55	0.57	98.79	0.09	0	1.180
	HAMM	3.33	0.57	96.01	0.09	0	0.999
	IM^{3}L	0	0.01	99.9	0.09	0	1.099
Case 4	HIMM	4.44	0	95	0.56	0	1.146
	HAMM	0	0	100	0	0	1.014
	IM^{3}L	0	0	100	0	0	1.089

D. Discussions

The IM^{3}L outperforms significantly the HIMM and HAMM in all tested scenarios. The IM^{3}L provides accurate estimates of the extent of failure and robust state estimates even during the failure. The reason is that the new model based on $\widehat{\alpha}$ is very close to the truth, which is verified by the histogram of measurement residuals (not shown). For the HIMM or HAMM, when the designed failure models do not match the truth well, it cannot provide good estimates of the effectiveness factor and thus good state estimates. Moreover, to obtain satisfactory performance, the HAMM has to be tuned in many cases, such as Kalman filtering tuning and state reinitialization for the higher level models to take advantages of the information from the lower level. The IMM-based approach (HIMM and IM^{3}L) does not need such heuristic adjustments. The IM^{3}L has a lighter computational load than the HIMM, and their computation is slightly more than that of the HAMM.

As mentioned before, soft sensor failures can also be modeled by changing the mean of the measurement noise or R matrix. For the tested scenarios of VTOL, the IM^{3}L can detect and estimate increased noise-mean sensor failures when the noise-mean represented increases within some range. In addition, the proposed IM^{3}L approach is also applicable to the detection of faults caused by a change in F matrix (i.e. change in physical components). However, the application of IM^{3}L to such failures by changes in R or F matrix is more complicated and we are not addressing it in this paper.

In the HIMM approach, the lower level is opened (i.e., in effect) when a fault is detected at the higher level and closed back to the higher level after the updated estimates of α and state are obtained. An alternative was also implemented: Once the lower level is opened, it remains to be open until the normal model gets the highest model probability over the threshold; then the lower level is closed and estimation goes back to the higher level. The simulations showed that the performance of this alternative is worse compared to the one proposed first.

The IM^{3}L presented here successfully estimates the failure severity as well as state when the partial failures occur sequentially. In practice, multiple failures exist in which a second failure occurs during the first failure. This problem is currently under investigation. Furthermore, the research here based on IM^{3}L focuses on the sensor failures or actuator failures separately. In fact it is applicable to sequential sensor and actuator failures. We only need to design a whole model set containing all failure hypotheses of interest.

VI. Conclusions

We have presented a new approach for detection and estimation of partial as well as total failures based on IMM and ML estimation. The hierarchical IMM-FDI scheme has also been discussed. The simulation results based on different scenarios tested demonstrate that the proposed IM^{3}L approach is a powerful technique for FDD. It outperforms the other two approaches (HIMM and HAMM) in terms of correct detection, accurate estimation of the extent of failure and robust state estimation in the presence of partial failures (unmodeled failures) and uncertainties of noise statistics.

Appendix A: MLE of α for sensor partial failures

The log-marginal likelihood function is

$$\ln f(z_k|z^{k-1}, a_k)$$
$$= -\frac{1}{2}\ln(|2\pi S_k|) - \frac{1}{2}(z_k - H_k\widehat{x}_{k|k-1})' S_k^{-1}(z_k - H_k\widehat{x}_{k|k-1})$$

Taking derivative and setting to zero with $\tilde{z}_{k|k-1} = z_k - H_k\widehat{x}_{k|k-1}$, $A_k = S_k^{-1}$, we have

$$\frac{d\ln f(z_k|z^k, \alpha_k)}{d\alpha_k} = 0 \Longrightarrow \frac{d\tilde{z}'_{k|k-1}A_k\tilde{z}_{k|k-1}}{d\alpha_k} = 0 \quad (14)$$

Since

$$\frac{d\tilde{z}'_{k|k-1}A_k\tilde{z}_{k|k-1}}{d\alpha_k} = \tilde{z}'_{k|k-1}(A'_k + A_k)\frac{d\tilde{z}_{k|k-1}}{d\alpha_k}$$

$$\frac{d\tilde{z}_{k|k-1}}{d\alpha_k} = \frac{d(z_k - H_k\widehat{x}_{k|k-1})}{d\alpha_k} = -(\frac{dH_k}{d\alpha_k})\widehat{x}_{k|k-1}$$

(14) becomes

$$\tilde{z}'_{k|k-1}(A'_k + A_k)(\frac{dH_k}{d\alpha_k})\widehat{x}_{k|k-1} = 0$$

Since $\frac{dH_k}{d\alpha_k}$ is easy to obtain by the formula of matrix derivation with respect to a scalar and other terms are given by Kalman filter directly, the MLE of α can be obtained. The actuator failure effectiveness factor can be estimated similarly.

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