Estimation of three-dimensional radar tracking using modified extended kalman filter

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Estimation of three-dimensional radar tracking using modified extended kalman filter

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Abstract. Kalman filter is an estimation method by combining data and mathematical models then developed be extended Kalman filter to handle nonlinear systems. Three-dimensional radar tracking is one of example of nonlinear system. In this paper developed a modification method of extended Kalman filter from the direct decline of the three-dimensional radar tracking case. The development of this filter algorithm can solve the three-dimensional radar measurements in the case proposed in this case the target measured by radar with distance r, azimuth angle θ, and the elevation angle ϕ. Artificial covariance and mean adjusted directly on the three-dimensional radar system. Simulations result show that the proposed formulation is effective in the calculation of nonlinear measurement compared with extended Kalman filter with the value error at 0.77% until 1.15%.

1. Introduction

Kalman filter known as linear quadratic estimation, is an algorithm that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone. Extensions to the method have also been developed, the extended Kalman filter is nonlinear version of Kalman filter which used for nonlinear systems. Algorithm is almost same with Kalman filter but in the extended Kalman filter, the state transition and observation models don't need to be linear functions of the state but may instead be differentiable functions[1]. Extended Kalman filter can be used for three-dimensional radar tracking system (one of example of nonlinear systems). Radar tracking control system that has been designed according to the needs with the level of accurate measurement there will be a noise. By reducing the noise in the measurement system and needed an approach that is more accurate than ever before. The approach taken is estimated to determine the level of noise [2]. Based on the results of research conducted by Jang Song and in 2001 on "Improved Kalman Filter Design for Three-Dimensional Radar Tracking" [3], is obtained new filter to overcome the problem of radar tracking. In that paper, the level of measurement accuracy compared to the method extended Kalman filter with new methods improved of design Kalman filter, and the results of the simulation indicated that the improved of Kalman filter is a better result. Extended Kalman Filter are also used in research Nousheen in 2015 [4] that discussed abaout it can not be used on the system parameters and / or measurement that is not linear. In the topic of this research, studied the relevant journal in which the estimate is not a linear measurement

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of three-dimensional radar tracking system. There are many research on Kalman filter such as control position of underwater vehicle [5], also mobile robot position [6], heat conduction and its development [7], about air pollution [8], also stirred tank reactor [9], extended preferred radar tracking [10] and the last is estimation of reduced model [11].

2. Dynamical model

Case of three-dimensional radar tracking is sensor measures the target in three dimension space including the range (distance) r, azimuth angle (turning angle) θ, and elevation angle (elevation angle) ϕ (cf. Fig.1)

Range r is the measured distance of target from the center of the tracker, then the azimuth angle θ shows direction of the tracker rotation motion in the direction of the x-axis and y-axis, and the elevation angle ϕ indicates the direction of tracker's motion elevation in the direction of x, y to z axis (the development of 2D radar) [10]. Then the equation can be described as follows:

$$r = \sqrt{x^2 + y^2 + z^2} \tag{1}$$

$$\theta = \tan^{-1} \frac{y}{x} \tag{2}$$

$$\phi = \tan^{-1} \frac{z}{(x^2 + y^2)^{\frac{1}{2}}} \tag{3}$$

Equation (1)-(3) describe the measurement model in this research. Notice that the three equations (1)-(3) are obtained from the spherical shape. Then we transform (1)-(3) into the shape of cartesian coordinates by using the following transformation:

$$x = r\cos\theta\cos\phi \tag{4}$$

$$y = r\sin\theta\cos\phi \tag{5}$$

$$z = r\sin\phi \tag{6}$$

where x, y and z is the scalar component of the cartesian coordinate measurement that is described as a position of a moving target in three-dimensional space [1]. Using the chain rule,

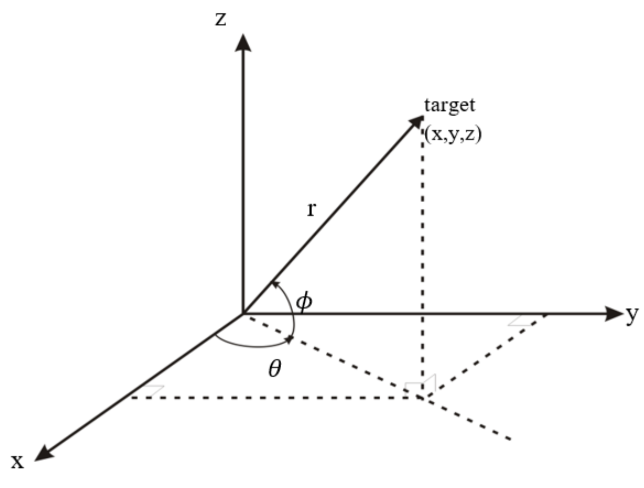

Figure 1. Radar measures r, θ, ϕ.

the derivation of (4)-(6) w.r.t. time is as follows:

$$\frac{dx}{dt} = \cos\theta\cos\phi\frac{dr}{dt} - r\sin\theta\cos\phi\frac{d\theta}{dt} - r\cos\theta\sin\phi\frac{d\phi}{dt} \tag{7}$$

$$\frac{dy}{dt} = \sin\theta\cos\theta\frac{dr}{dt} + r\cos\theta\cos\phi\frac{d\theta}{dt} - r\sin\theta\sin\phi\frac{d\phi}{dt} \tag{8}$$

$$\frac{dz}{dt} = \sin\phi\frac{dr}{dt} + r\cos\phi\frac{d\phi}{dt} \tag{9}$$

Equation (7)-(9) is a dynamical system that will be estimated by using extended Kalman filter and its modification, with derive equations and made a stochastic discrete model the final results are as follow:

$$x_{k+1} = \frac{x_k}{\sqrt{x_k^2+y_k^2+z_k^2}} - y_k - \frac{x_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{1k} \tag{10}$$

$$y_{k+1} = \frac{y_k}{\sqrt{x_k^2+y_k^2+z_k^2}} + x_k - \frac{y_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{2k} \tag{11}$$

$$z_{k+1} = \frac{z_k}{\sqrt{x_k^2+y_k^2+z_k^2}} + \sqrt{x_k^2+y_k^2} + w_{3k} \tag{12}$$

$$\begin{aligned} V_{k+1}^x = & \tfrac{-2y_k}{\sqrt{x_k^2+y_k^2+z_k^2}} - \tfrac{2x_k z_k}{\sqrt{x_k^2+y_k^2+z_k^2}\sqrt{x_k^2+y_k^2}} \\ & -2x_k + \frac{2y_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{4k} \end{aligned} \tag{13}$$

$$\begin{aligned} V_{k+1}^y = & \tfrac{2x_k}{\sqrt{x_k^2+y_k^2+z_k^2}} - \tfrac{2y_k z_k}{\sqrt{x_k^2+y_k^2+z_k^2}\sqrt{x_k^2+y_k^2}} \\ & -2y_k + \frac{2x_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{5k} \end{aligned} \tag{14}$$

$$V_{k+1}^z = \frac{2\sqrt{x_k^2+y_k^2}}{\sqrt{x_k^2+y_k^2+z_k^2}} - z_k + w_{6k} \tag{15}$$

We define the following measurement model:

$$Z_k = \begin{pmatrix} \sqrt{x_k^2+y_k^2+z_k^2} + v_{1k} \\ tan^{-1}\frac{y_k}{x_k} + v_{2k} \\ tan^{-1}\frac{z_k}{\sqrt{x_k^2+y_k^2}} + v_{3k} \end{pmatrix}$$

where:

- x_k : target position at x-axes
- y_k : target position at y-axes
- z_k : target position at z-axes
- V_k^x : target velocity at x-axes
- V_k^y : target velocity at y-axes
- V_k^z : target velocity at z-axes

Equations (10)-(15) will be used to generate the real system, whereas the model used for estimation is the linearized version. The linearized model produces a system matrix **A**, that will be used in modified extended Kalman filter algorithm (see the next section).

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3. Modified Extended Kalman Filter

Extended Kalman filter method is the extension of the classical Kalman Filter [12] for nonlinear systems. In reality, many models are nonlinear. Since the model is nonlinear, we use extended Kalman filter. Given a nonlinear stochastic model:

$$X_{k+1} = f(X_k, u_k) + w_k$$
$$Z_k = h_k(X_k) + v_k$$

where the nonlinear measurement model , $x_0 \sim N(\overline{X}_0, P_{x0})$, $v_k \sim N(0, R_k)$ has a normal distribution and assumed to be white that means there is no correlation between each other and also with initial condition X_0 [13].

Before the estimation process, linearization process is carried out since the system is not linear. Linearization process is done by defining it as follows:

$$X^*_{k+1} = f(\widehat{X}_k, u_k)$$
$$Z^*_{k+1} = h(X^*_{k+1})$$
$$\mathbf{A} = [A_{i,j}] = [\frac{\partial f_i}{\partial X_j}(\widehat{X}_k, u_k)]$$
$$\mathbf{H} = [H_{i,j}] = [\frac{\partial h_i}{\partial X_j}(X^*_{k+1})]$$

where $\mathbf{A}$ and $\mathbf{H}$ are the Jacobi matrix derived from a differential in f and h to the direction of X. A modification of Kalman filter algorithm is called extended Kalman filter algorithm [14].

From research conducted by Song Taek Park [1] modification extended Kalman filter is insert error covariance and mean-made into the algorithm as follows:

$$R^p_k \approx \begin{pmatrix} \sigma_r^2 + \frac{r_k^2(\sigma_\theta^4 + \sigma_\phi^4)}{2} & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_\phi^2 \end{pmatrix} \tag{16}$$

$$\mu^p_k \approx \begin{pmatrix} r_k[(\theta^m_k)^2 + (\phi^m_k)^2 - \sigma_\theta^2 - \sigma_\phi^2] \\ 0 \\ 0 \end{pmatrix} \tag{17}$$

where:
σ_θ: parameters of angle noise θ
σ_ϕ : parameters of angle noise ϕ
θ^m_k : $\theta_k + v^\theta_k$
ϕ^m_k : $\phi_k + v^\phi_k$
Error covariance R^p_k in Equation (16) and mean-made μ^p_k in Equation (17) which insert in modified extended Kalman filter algorithm, modified extended Kalman filter algorithm given by Table 1.

Table 1. Modified extended Kalman filter

	Description
`Model system`	$x_{k+1} = f(x_k, u_k) + w_k$
`Measurement model`	$Z_{k+1} = h(x_{k+1}) + v_k$
`Assumption`	$x_0\ N(\overline{x}_0, P_{x_0}), w_k\ N(0, Q_k)$, $v_k\ N(0, R_k)$
`Initialization`	$\widehat{x}_0 = \overline{x}_0, P_0 = P_{x_0}$
`Predict`	$\mathbf{A} = [\frac{\partial f_i}{\partial x_j}]$ Estimation: $\widehat{x}_{k_{\overline{k}}} = f(\widehat{x}_k, u_k)$ Error Covariance: $P_{k+1} = \mathbf{A}P_k + P\mathbf{A}^T + G_k Q_k G_k^T$
`Update & Gain Kalman:`	$K_{k+1} = P_{k+1}\mathbf{H}^T[\mathbf{H}_k P_{k+1}\mathbf{H}^T + \overline{R}_{k+1}^p]^{-1}$ Estimation: $\widehat{x}_{k+1} = \widehat{x}_{k+1} + K_{k+1}(z_{k+1} - h(\widehat{x}_{k+1}) - \overline{\mu}_{k+1}^p)$ Error Covariance: $P_{k+1} = [\mathbf{I} - K_k\mathbf{H}]P_{k+1}$

4. Results
Simulation is given initial conditions and parameters (see Table 2) with measurements model r, θ,and ϕ.

Table 2. Initialization

No	Variables	Values
1	x	15 kilometers
2	y	12 kilometers
3	z	5 kilometers
4	V^x	-60 meters/sec
5	V^y	-70 meters/sec
6	V^z	-40 meters/sec
	Parameters	
7	σ_r	0.003 kilometers
8	σ_θ	0.0261799 rad/sec
9	σ_ϕ	0.0261799 rad/sec
10	P_0	0.05
11	Q_k	0.0001
12	R_k	0.00002
13	dt	0.0001

In Figure 2-7 the green color of the graph shows the real value, the estimated value of eKf is blue and red color is value estimation results meKf. In Figure 2, the results showed the difference in value between the real value and eKf estimation is 0.0094249, and the difference in value between the real value and meKf estimation is 0.0088085. For Figure 3, the results show the difference in value between the real value and eKf estimation is 0.0150247, and the difference in value between the real value and meKf estimation is 0.0073667. In Figure 4, the results show the difference in value between the real value and eKf estimation is 0.0111491, and the difference in value between the real value and meKf estimation is 0.0093434. In Figure 5, the results show the difference in value between the real value and ekf estimation is 0.01824580, and the difference in value between the real value and meKf estimation is 0.0114886. In Figure 6, the results obtained show the difference in value between the real value and eKf estimation is 0.0107461, and the difference in value between the real value and meKf estimation is 0.01008190. In Figure 7, the results show the difference in value between the real value and eKf estimation is 0.0115598, and the difference in value between the real value and meKf estimation is 0.0105467.

Figure 8 shows a graph of the error between the real and estimated values of all variables. The smallest error in the estimated value of the modification extended Kalman filter (red line) in each variable with the indicated value of each variable RMSE.

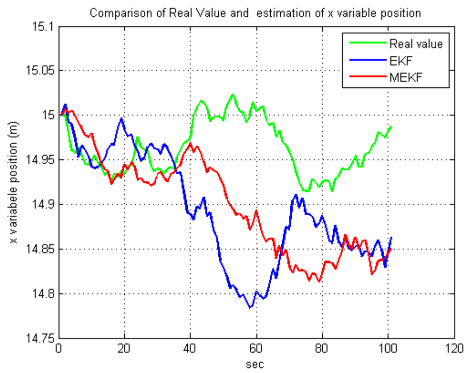

Figure 2. Comparison graphs real value and estimated x variable position

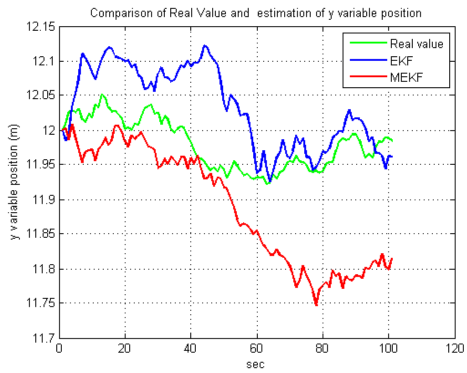

Figure 3. Comparison graphs real value and estimated y variable position

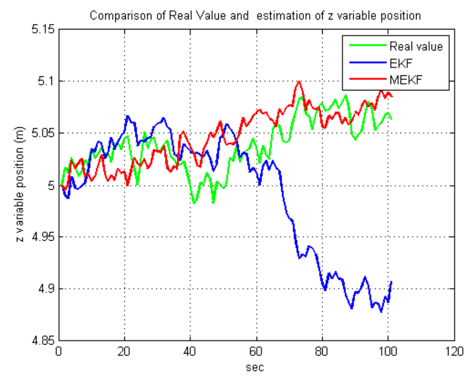

Figure 4. Comparison graphs real value and estimated z variable position

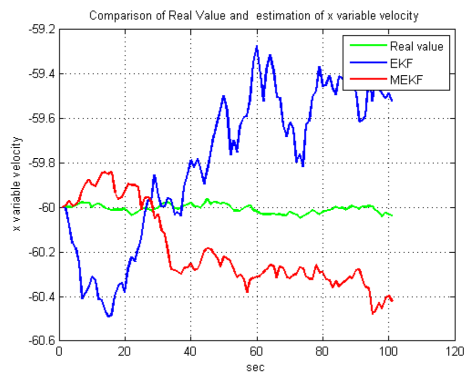

Figure 5. Comparison graphs real value and estimated x variable velocity

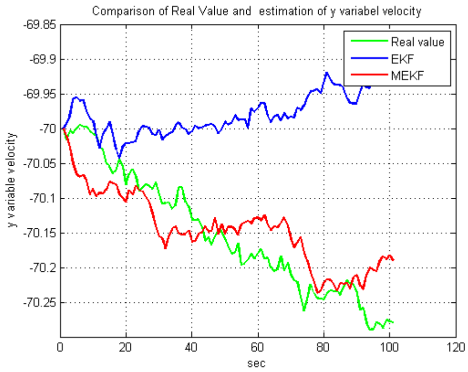

Figure 6. Comparison graphs real value and estimated y variable velocity

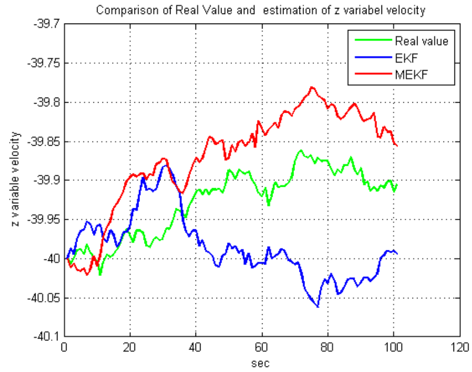

Figure 7. Comparison graphs real value and estimated z variable velocity

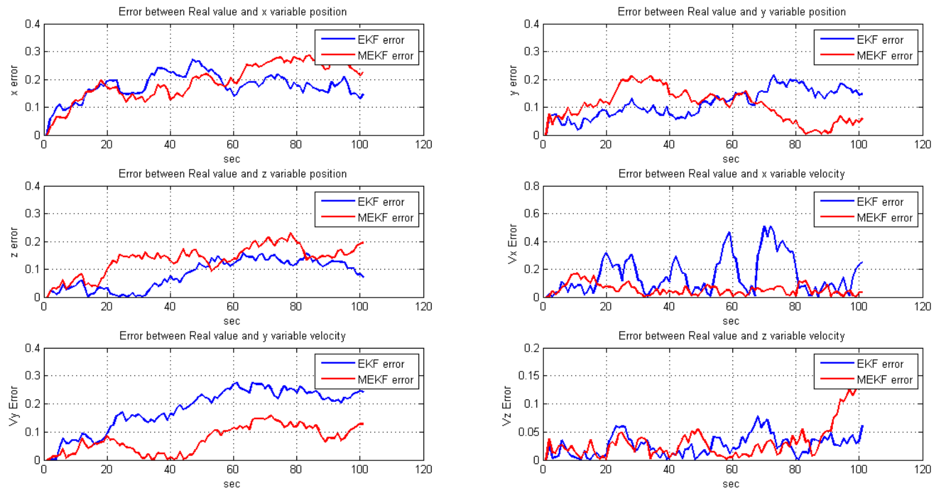

Figure 8. Comparison graphs error each variable

Table 3. Average value of rmse each variable

	RMSE of EKF	RMSE of MEKF
x position	0.00942490	0.00880850
y position	0.01502470	0.00736670
z position	0.01114910	0.00934340
V^x velocity	0.0182458	0.0114886
V^y velocity	0.0107461	0.0100819
V^z velocity	0.0115598	0.0105467

In Table 3 shows that the RMSE value of each variable is relatively small in the error value at intervals of $0.00736670 < RMSE < 0.0115598$, or it can be said mistakes by 0.77% up to 1.15% to modification of Extended Kalman Filter and $0.00942490 < RMSE < 0.01824580$, or it can be said mistakes of 0.94% to 1.82% for the extended Kalman filter. So overall it can be said that the modified extended Kalman filter method is suitable for estimating the three-dimensional radar tracking system in this case specifically for the measurement is not linear.

5. CONCLUSIONS

Based on the analysis and discussion presented in the previous chapter, we can conclude:

1. The dynamic model of three-dimensional radar tracking is formed as showed in chapter 2.
2. The estimation results indicate that each variables (position and velocity x, y, and z) of the tracking radar three dimensions by modification of extended Kalman filter better than methods extended Kalman filter with indicated by the error rate of only 0.77% up to 1.15% for modification of extended Kalman filter, while the error rate of extended Kalman filter is of 0.94% to 1.82%.
3. Based on the computation time show that the combined method of eKf and its modification takes 4.5770319 seconds.

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