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Modeling of Three-Dimensional Radar Tracking System and Its Estimation using Extended Kalman Filter

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***Abstract*—Nowadays, the three-dimensional radar tracking has a rapid development. Tracking filter designs commonly rely on a linear system, while the nonlinear systems mostly occur in everyday life. The development of this filter algorithm can solve the three-dimensional radar tracking problem by using some measurement data. In the case discussed in this paper, the target is measured by radar with distance r, azimuth angle θ, and the elevation angle ϕ. Notice that the data is not a linear measurement data. Thus, to address the nonlinearities inherent in the system model and the measurement model, we use the Extended Kalman Filter approach. Variables and parameters are adjusted directly on the three-dimensional radar system. The simulation results show that the proposed formulation is very effective in the calculation of nonlinear measurement with the error belongs to interval from 0.69% to 1.21%.**

I. Introduction

Radio detection and ranging (Radar) is a useful system of electromagnetic waves to detect and measure the distance to create the folder objects such as aircraft, motor vehicles and various weather (rain). Three-dimensional radar tracking is the extension of two-dimensional radar tracking. The two-dimensional radar tracking covers a distance or range and direction of the angle, whereas in three-dimensional radar tracking covers a distance or range, two angles, and derivatives as the state space [1]. Internal or external factors of the radar tracking can hamper the accuracy of the radar tracking to the target. Factors include internal or external shift direction angle, missing the target, and so on. In radar tracking control system that has been designed according to the needs with the level of accurate measurement, there is still a noise. Noise size is usually very small and the noise can occur in the radar tracking control system. By reducing the noise in the measurement system, we need an approach that is more accurate than ever before. The approach taken is estimated to determine the level of noise [2].

Based on the results of research conducted by Jang and Song in 2001 on "Improved Kalman Filter Design for Three-Dimensional Radar Tracking" [1], the authors obtained a new filter to overcome the problems depreciated. That paper becomes the main reference in this work. In that paper, the level of measurement accuracy is compared using the Kalman Filter method and improved design of Kalman Filter. Based on the results of another study conducted in 2015 on Extended Kalman Filter (EKF) for Radar Tracking Modeling with Missing Measurements [3], EKF is a good approach for 3D Radar. Extended Kalman Filters are also used in the research of Nousheen in 2015 [4]. Based on these studies, Kalman Filter cannot be used on the system parameters and/or measurement that is not linear [5]. There are many research on Kalman filter such as control position of underwater vehicle [6], mobile robot position [7], heat conduction [8], air pollution [9], stirred tank reactor [10] and estimation of reduced model [11], [12]. In this paper, we study the estimation of a nonlinear measurement of three-dimensional radar tracking system.

II. Dynamical Model of 3D Radar Tracking

We assumed the case of the three-dimensional radar tracking. The sensor measures the three-dimensional target including the range (distance) r, azimuth angle (turning angle) θ, and elevation angle (elevation angle) ϕ (cf. Fig.1)

Range r denotes the measured distance of the target from the center of the tracker, while the azimuth angle or swivel angle θ shows the direction of motion of the tracker rotation in the direction of the x-axis and y-axis, and the elevation angle ϕ indicates the direction of motion of trackers' elevation in the direction of x, y to z axis (the development of 2D radar) [13].

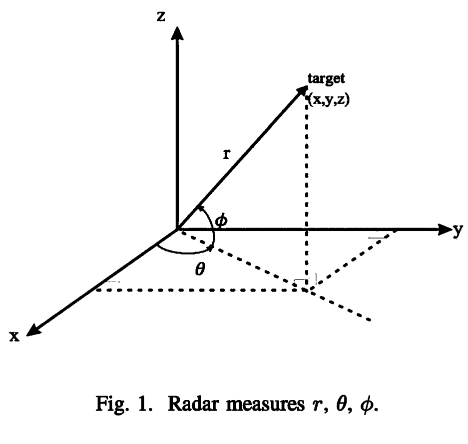

Fig. 1. Radar measures r, θ, ϕ.

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Then the equation can be described as follows:

$$r = \sqrt{x^2 + y^2 + z^2} \tag{1}$$

$$\theta = \tan^{-1} \frac{y}{x} \tag{2}$$

$$\phi = \tan^{-1} \frac{z}{(x^2 + y^2)^{\frac{1}{2}}} \tag{3}$$

Equation (1)-(3) describe the measurement model in this research. Notice that the three equations (1)-(3) are obtained from the spherical shape. Then we transform (1)-(3) into the shape of cartesian coordinates by using the following transformation:

$$x = r \cos\theta \cos\phi \tag{4}$$

$$y = r \sin\theta \cos\phi \tag{5}$$

$$z = r \sin\phi \tag{6}$$

where x, y and z is the scalar component of the cartesian coordinate measurement that is described as a position of a moving target in three-dimensional space [1]. By using the chain rule, the derivation of (4)-(6) w.r.t. time is as follows:

$$\frac{dx}{dt} = \cos\theta\cos\phi\frac{dr}{dt} - r\sin\theta\cos\phi\frac{d\theta}{dt} - r\cos\theta\sin\phi\frac{d\phi}{dt} \tag{7}$$

$$\frac{dy}{dt} = \sin\theta\cos\theta\frac{dr}{dt} + r\cos\theta\cos\phi\frac{d\theta}{dt} - r\sin\theta\sin\phi\frac{d\phi}{dt} \tag{8}$$

$$\frac{dz}{dt} = \sin\phi\frac{dr}{dt} + r\cos\phi\frac{d\phi}{dt} \tag{9}$$

Equation (7)-(9) is a dynamical system that will be estimated by using Extended Kalman Filter. By assuming $\frac{dr}{dt} = 1$, $\frac{d\theta}{dt} = 1$ and $\frac{d\phi}{dt} = 1$, equations (7)-(9) become:

$$\dot{x} = \cos\theta\cos\phi - r\sin\theta\cos\phi - r\cos\theta\sin\phi \tag{10}$$

$$\dot{y} = \sin\theta\cos\phi + r\cos\theta\cos\phi - r\sin\theta\sin\phi \tag{11}$$

$$\dot{z} = \sin\phi + r\cos\phi \tag{12}$$

because we do not estimate the change of position only, but we also estimate the change of velocity. The equation for the change of velocity is obtained from

$$\begin{aligned}\dot{V}^x &= -2\sin\theta\cos\phi - 2\cos\theta\sin\phi - 2\sin\theta\cos\phi \\ &\quad - 2r\cos\theta\cos\phi + 2r\sin\theta\sin\phi \end{aligned}\tag{13}$$

$$\begin{aligned}\dot{V}^y &= 2\cos\theta\cos\phi - 2\sin\theta\sin\phi + 2\cos\theta\cos\phi \\ &\quad - 2r\sin\theta\cos\phi - 2r\cos\theta\sin\phi \end{aligned}\tag{14}$$

$$\dot{V}^z = 2\cos\phi - r\sin\phi \tag{15}$$

We want to transform the equation above into cartesian coordinate. In order to do so, we use the following formula:

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}, \cos\theta = \frac{x}{\sqrt{x^2+y^2}},$$

$$\sin\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}, \cos\phi = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}},$$

$$r = \sqrt{x^2+y^2+z^2}$$

By using the above formula, equations (10)-(15) can be transformed into the following equations:

$$\dot{x} = \frac{x}{\sqrt{x^2+y^2+z^2}} - y - \frac{xz}{\sqrt{x^2+y^2}} \tag{16}$$

$$\dot{y} = \frac{y}{\sqrt{x^2+y^2+z^2}} + x - \frac{yz}{\sqrt{x^2+y^2}} \tag{17}$$

$$\dot{z} = \frac{z}{\sqrt{x^2+y^2+z^2}} + \sqrt{x^2+y^2} \tag{18}$$

$$\begin{aligned}\dot{V}^x &= \tfrac{-2y}{\sqrt{x^2+y^2+z^2}} - \tfrac{2xz}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}} \\ &\quad -2x + \frac{2yz}{\sqrt{x^2+y^2}}\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{V}^y &= \tfrac{2x}{\sqrt{x^2+y^2+z^2}} - \tfrac{2yz}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}} \\ &\quad -2y + \frac{2xz}{\sqrt{x^2+y^2}}\end{aligned} \tag{20}$$

$$V^z = \frac{2\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} - z \tag{21}$$

Equations (16)-(21) are defined over continuous time. However, the requirement for Extended Kalman Filter is that the system has to be defined in discrete time. Thus we have to discretize the continuous-time system above. The results are as follow:

$$x_{k+1} = \frac{x_k}{\sqrt{x_k^2+y_k^2+z_k^2}} - y_k - \frac{x_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{1k} \tag{22}$$

$$y_{k+1} = \frac{y_k}{\sqrt{x_k^2+y_k^2+z_k^2}} + x_k - \frac{y_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{2k} \tag{23}$$

$$z_{k+1} = \frac{z_k}{\sqrt{x_k^2+y_k^2+z_k^2}} + \sqrt{x_k^2+y_k^2} + w_{3k} \tag{24}$$

$$\begin{aligned}V^x_{k+1} &= \tfrac{-2y_k}{\sqrt{x_k^2+y_k^2+z_k^2}} - \tfrac{2x_k z_k}{\sqrt{x_k^2+y_k^2+z_k^2}\sqrt{x_k^2+y_k^2}} \\ &\quad -2x_k + \frac{2y_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{4k}\end{aligned} \tag{25}$$

$$\begin{aligned}V^y_{k+1} &= \tfrac{2x_k}{\sqrt{x_k^2+y_k^2+z_k^2}} - \tfrac{2y_k z_k}{\sqrt{x_k^2+y_k^2+z_k^2}\sqrt{x_k^2+y_k^2}} \\ &\quad -2y_k + \frac{2x_k z_k}{\sqrt{x_k^2+y_k^2}} + w_{5k}\end{aligned} \tag{26}$$

$$V^z_{k+1} = \frac{2\sqrt{x_k^2+y_k^2}}{\sqrt{x_k^2+y_k^2+z_k^2}} - z_k + w_{6k} \tag{27}$$

We define the following measurement model:

$$Z_k = \begin{pmatrix} \sqrt{x_k^2+y_k^2+z_k^2} + v_1k \\ tan^{-1}\frac{y_k}{x_k} + v_2k \\ tan^{-1}\frac{z_k}{\sqrt{x_k^2+y_k^2}} + v_3k \end{pmatrix}$$

where:

- x_k : target position at x-axes
- y_k : target position at y-axes
- z_k : target position at z-axes

- V_k^x : target velocity at x-axes
- V_k^y : target velocity at y-axes
- V_k^z : target velocity at z-axes

Equations (22)-(27) will be used to generate the real system, whereas the model used for estimation is the linearized version. The linearized model produces a system matrix $\mathbf{A}$, that will be used in Extended Kalman Filter algorithm (see the next section).

III. EXTENDED KALMAN FILTER ALGORITHM

Extended Kalman Filter method is the extension of the classical Kalman Filter [14] for nonlinear systems. In reality, many models are nonlinear. Since the model is nonlinear (22)-(27), we use Extended Kalman Filter. Given a nonlinear stochastic model:

$$X_{k+1} = f(X_k, u_k) + w_k$$
$$Z_k = h_k(X_k) + v_k$$

where the measurement model is nonlinear, $x_0 \sim N(\overline{X}_0, P_{x0})$, $v_k \sim N(0, R_k)$ has a normal distribution and assumed to be white. This means that there is no correlation between each other and also with initial condition X_0 [14].

Before the estimation process, linearization process is carried out since the system is not linear. Linearization process is done by defining it as follows:

$$X^*_{k+1} = f(\widehat{X}_k, u_k)$$
$$Z^*_{k+1} = h(X^*_{k+1})$$
$$\mathbf{A} = [A_{i,j}] = [\frac{\partial f_i}{\partial X_j}(\widehat{X}_k, u_k)]$$
$$\mathbf{H} = [H_{i,j}] = [\frac{\partial h_i}{\partial X_j}(X^*_{k+1})]$$

where $\mathbf{A}$ and $\mathbf{H}$ are the Jacobi matrix derived from a differential in f and h to the direction of X. A modification of Kalman Filter algorithm is called Extended Kalman Filter algorithm [9]. The EKF algorithm works as follows. First of all, the initial estimation for state variables and the corresponding covariance is given. Then, at each time step there are two main steps:

- Time update: we determine the estimation of state variables by using the original nonlinear model and the corresponding covariance by using the following formula

$$\widehat{x}_{k_{\overline{k}}} = f(\widehat{x}_k, u_k)$$
$$P_{k+1} = \mathbf{A}P_k + P\mathbf{A}^T + G_k Q_k G_k^T$$

- Measurement update: we update the estimation of state variables by using the measurement and the corresponding covariance. The equations used in this step are as follows:

$$K_{k+1} = P_{k+1}\mathbf{H}^T[\mathbf{H}_k P_{k+1}\mathbf{H}^T + \overline{R}_{k+1}]^{-1}$$
$$\widehat{x}_{k+1} = \widehat{x}_{k+1} + K_{k+1}(z_{k+1} - h(\widehat{x}_{k+1}))$$
$$P_{k+1} = [\mathbf{I} - K_k\mathbf{H}]P_{k+1}$$

IV. SIMULATION RESULTS

We apply the EKF algorithm to several case studies. In all case studies, the initial states are $x = 50$ kilometers, $y = 50$ kilometers, $z = 715$ kilometers, $V^x = 711$ km/h, $V^y = 711$ km/h, $V^z = 711$ km/h.

A. Simulation 1

In this simulation, the measured state variables are r, θ, and ϕ. In Fig. 2, the red color in the graph shows the real value and the estimated value obtained using EKF is shown in blue. As we can see in Fig. 2, the estimated position and the real value are very close to each other. The root mean squared error (RMSE) between the real value and the EKF estimation is 0.0083.

In Fig. 3, the red color of the graph shows the real value and the estimated value of EKF is shown in blue. We can see that the estimated value closely follows the real value for position y. In this case, the maximal difference between the real value and the EKF estimation is 0.0069.

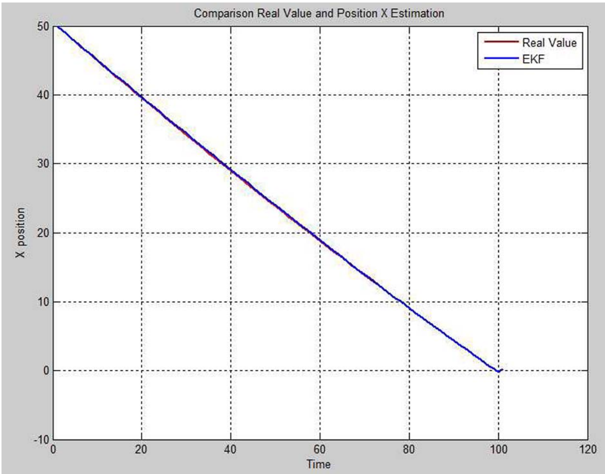

Fig. 2. Comparison of real value and estimated value of position x.

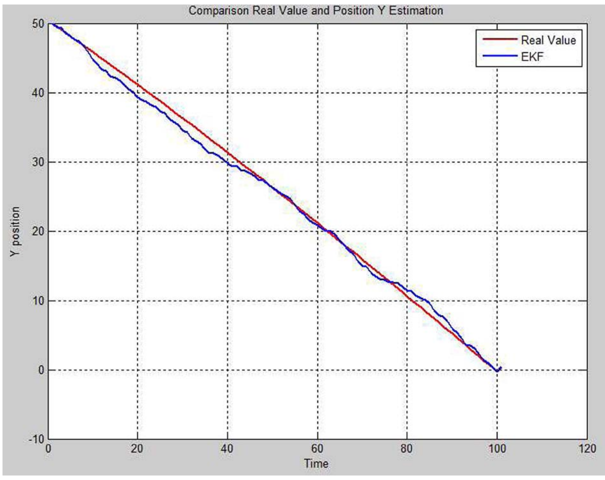

Fig. 3. Comparison of real value and estimated value of position y.

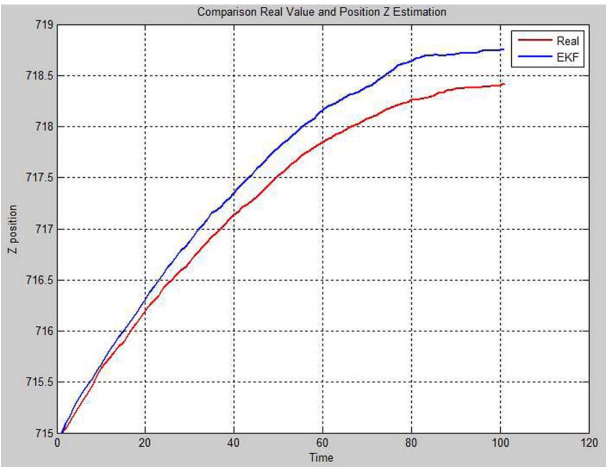

Fig. 4. Comparison of real value and estimated value for z position.

In Fig. 4, the red color of the graph shows the real value and the estimated value of EKF is shown in blue. The results are quite similar for the position z because the estimated value is close to the real value. The maximal difference between the real value and EKF estimation is 0.0121.

Fig. 5 shows a graph of the error between the real and estimated values of all variables. As we can see in Fig. 5, the error is upper bounded by 1 kilometer, which is a satisfactory result.

Table I shows that the RMSE value of each variable is relatively small at intervals of $0.0069 < error < 0.0121$, or it can be said that the percentage of mistakes is between 0.69% to 1.21% for the Extended Kalman Filter. In summary, the Extended Kalman Filter is a suitable estimation method for the three-dimensional radar tracking.

TABLE I
ROOT MEAN SQUARE ERROR OF EACH VARIABLE.

Steps	x position	y position	z position
100	0.0083	0.0069	0.0121

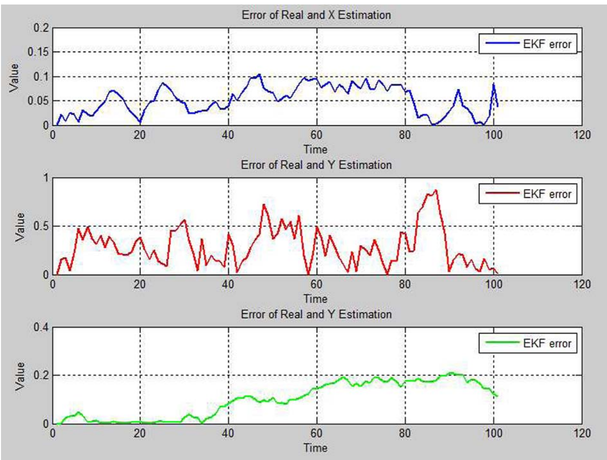

Fig. 5. Comparison of error for all variables.

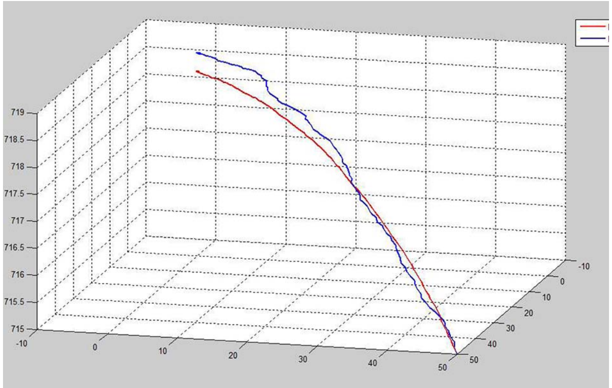

Fig. 6. Comparison of error for all variables.

B. Simulation 2

In this experiment, the measurement data is variables x, y and z. The purpose of this simulation is to determine the trajectory of the target from the initial position to the end for 100 time steps. After 100 time steps, the position of the real system becomes $(0, 0, 718.5)$, whereas the estimated position is $(0, 0, 718)$. We conclude that the estimated position is closely follow the real position.

V. CONCLUSION

Based on the analysis and discussion presented in this paper, we can conclude:

1) The dynamical model of three-dimensional radar tracking is constructed in Section II.
2) The estimation results indicate that each variable (position and velocity x, y, and z) of the three-dimensional tracking radar by using Extended Kalman Filter is suitable, which is indicated by the error rate of only 0.69% to 1.21%.

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REFERENCES

[1] S.-T. Park and J. Lee, "Improved Kalman filter design for three-dimensional radar tracking," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 37, no. 2, pp. 727–739, 2001.

[2] R. Sani, "Estimasi variabel keadaan gerak longitudinal pesawat terbang menggunakan metode fuzzy Kalman filter," *Jurnal Sains dan Seni ITS*, vol. 5, no. 2, 2016.

[3] S. Aishaa and P. Keerthana, "Extended Kalman filter modelling for tracking radar with missing measurements," *International Journal of Innovative Research in Science, Engineering and Technology*, vol. 4, no. 9, 2015.

[4] N. Fahmedha, P. Prakash, A. Pooja, and R. Rachana, "Estimation of system parameters using Kalman filter and extended Kalman filter," *International Journal of Advanced Technology and Engineering Exploration*, vol. 2, no. 6, p. 84, 2015.

[5] D. Arif, Widodo, Salmah, and E. Apriliani, "Construction of the Kalman filter algorithm on the model reduction," *International Journal of Control and Automation*, vol. 7, no. 9, pp. 257–270, 2014.

[6] Z. Ermayanti, E. Apriliani, H. Nurhadi, and T. Herlambang, "Estimate and control position autonomous underwater vehicle based on determined trajectory using fuzzy Kalman filter method," in *International Conference on Advanced Mechatronics, Intelligent Manufacture, and Industrial Automation (ICAMIMIA)*, pp. 156–161, 2015.

[7] E. Apriliani, F. Yunaini, and S. Hartini, "Estimation and control design of mobile robot position," *Far East Journal of Mathematical Sciences*, vol. 77, no. 1, p. 115, 2013.

[8] D. Arif, M. Widodo, E. Apriliani, and Salmah, "Distribution estimation of heat conduction using Kalman filtering which implemented on reduction model," in *3rd International Conferences and Workshops on Basic and Applied Sciences*, p. M007, 2011.

[9] E. Apriliani, D. Arif, and B. Sanjoyo, "The square root ensemble Kalman filter to estimate the concentration of air pollution," in *International Conference on Mathematical Applications in Engineering (ICMAE'10)*, 2010.

[10] R. Fitria and D. Arif, "State variable estimation of nonisothermal continuous stirred tank reactor using fuzzy Kalman filter," *International Journal of Computing Science and Applied Mathematics*, vol. 3, no. 1, pp. 16–20, 2017.

[11] K. Mustaqim, D. Arif, E. Apriliani, and D. Adzkiya, "Model reduction of unstable systems using balanced truncation method and its application to shallow water equations," *Journal of Physics: Conference Series*, vol. 855, no. 1, p. 012029, 2017.

[12] T. Lesnussa, D. Arif, D. Adzkiya, and E. Apriliani, "Identification and estimation of state variables on reduced model using balanced truncation method," *Journal of Physics: Conference Series*, vol. 855, no. 1, p. 012023, 2017.

[13] D. Leskiw, *The Extended Preferred Ordering Theorem for Radar Tracking Using the Extended Kalman Filter*. PhD thesis, Syracuse University, 2011.

[14] L. Kleeman, "Understanding and applying Kalman filtering," in *Proceedings of the Second Workshop on Perceptive Systems, Curtin University of Technology, Perth Western Australia (25-26 January 1996)*, 1996.