Fault Detection of Aircraft Plant Using KALMAN FILTER

Ravi Kumar M G
Reva University

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Ravi Kumar M. G. and Dr. Rama Murthy T.V*

Abstract: Estimation of the parameters of the safety critical system as in aircraft is very important for fault detection. Before implementing this system, simulation analysis, performance prediction and the control application has to be carried out. Joint state and parameter estimation using Kalman filter to detect the fault in the safety critical system has been proposed in this paper. When the norm of the state transition matrices reaches a threshold value the fault will be detected. Thus, by this method, the complicated issue regarding fault detection is resolved in the very short span of time. Since Kalman filter is used, the fault detection becomes robust even in the presence of noise. The proposed scheme is evaluated using PID control of longitudinal dynamics of an aircraft model configured for pitch control and simulated using matlab.

Key words: Kalman filter, Fault Detection, Aircraft dynamics, pitch controller, Cramer's method, Joint state and parameter Estimation.

1. INTRODUCTION

Fault detection, isolation and reconfiguration is very important in safety-critical applications like aircraft control, nuclear reactors and electric traction. Faults occurring in the system due to sensor or actuator failures or plant variations due to damages will have serious consequences leading to aborting the flight in case of fighter aircraft. Plant variations may also result in instability. Conventional approaches are reported in literature for detection and identification of failures in dynamic systems [1], [2]. Luenberger observers have been used for generating signals for analytical redundancy purposes [3],[4]. Kalman filters and its various advanced versions have been reported [5], [6]. Neural networks are also used by some researchers for fault detection [7],[8].

Kalman filters are generally used for state estimation in the presence of noise and the plant is assumed invariant. However, in safety-critical system plant is not assumed invariant because the operating condition like flight condition (mach number or altitude) keep changing. The changes in the plant parameters at any given flight condition, are considered faulty and have to be detected using parameter estimation in addition to state estimation. For example, a sensor failure (if not corrected) will lead to wrong interpretation of variation in plant parameter. (A matrix). Similarly, an actuator failure leads to wrong estimation of parameters of B matrix. Similarly, plant variation due to damages of the aircraft structure will lead to wrong values of A, B, C and D matrices. Thus, parameter estimation in addition to state estimation leads to fault detection of the dynamic system. In this paper a Kalman filter is used for recursively estimating the states and model parameters. The input given to the system is considered as deflection angle, the implementations will yield pitch angle as the output. These angles along with the covariance matrices of noise are used for the above fault detection purposes.

Reva Institute of Technology, Yelahanka-560064, Bangalore, Karnataka, India
* e-mail:ravimg9591@gmail.com

A model of the feedback system consisting of the various parameters such as angle of attack, also the pitch rate along with pitch angle as states are used in the state-space form. Kalman filter algorithm is implemented in Matlab to compute the states and parameters with additive white Gaussian noise. The noise is typical of sensor noise and wind disturbance noise. Once, the recursive estimation reaches error free state condition, the parameters of the system is obtained by Cramer's method. Cramer's method is used to obtain the model parameters giving the best estimate of the states and model parameters even in the presence of noise. This becomes useful when faults occur. The model parameters deviates from the healthy values and these deviations, once exceed a threshold can trigger a fault annunciator.

2. SYSTEM MODELING

An ideal technique for fixed-wing aircraft flight control system contains flight control surfaces, each of the cockpit controls, connecting linkages, and the essential functioning techniques to control the aircraft's direction in flight. With the variation of speed, aircraft engine controls become a part of the flight controls.

The general coordinate axes and forces on a plane are shown in the Fig. 1. In smaller or older aircraft, the lines represent actual cables that link controls to control surfaces. In modern aircraft, a computer monitors control movements and sends it electronically to control surface actuators (fly-by-wire). The equations regarded for the movement of an plane are a collection of six nonlinear coupled differential equations. Ideally, with several conditions, they may be able to be decoupled and linearized into longitudinal and lateral equations. Aircraft pitch is ruled through the longitudinal dynamics. In this problem we will consider a system that controls the pitch of an aircraft.

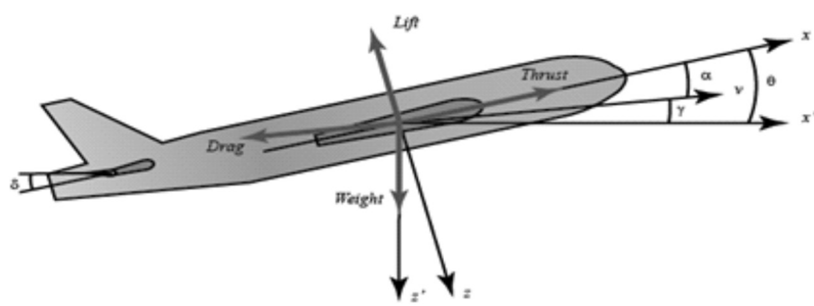

Figure 1: The general coordinate axes and forces acting on an aircraft [9]

δ = Elevator deflection angle

α = Angle of attack

θ = Pitch angle of the aircraft

γ = Flight path angle

q = pitch rate

The aircraft is assumed (i) to be in steady-cruise at fixed altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the *x*- and *y*-directions. (ii) such that an alteration in pitch angle will not alter the speed of a plane under any circumstance (the difficult situation is evaluated and simplified just a little). With these conditions, the longitudinal equations of motion for the aircraft are obtained. Now, several Numerical values are plugged in prior to the evaluation of the transfer function and state-space models. This is done for simplifying the modeling equations shown above:

$$\dot{\alpha} = -0.313\alpha + 56.7q + 0.232\delta \tag{1}$$

$$\dot{q} = -0.0139\alpha - 0.426q + 0.0203\delta \tag{2}$$

$$\dot{\theta} = 56.7q \tag{3}$$

Applying the Laplace transform, the above modeling equations can be expressed in terms of the Laplace variable s.

$$sA(s) = -0.313A(s) + 56.7Q(s) + 0.232\Delta(s) \tag{1}$$

$$sQ(s) = -0.0139A(s) - 0.426Q(s) + 0.0203\Delta(s) \tag{2}$$

$$s\theta(s) = 56.7Q(s) \tag{3}$$

We arrive at the following open-loop transfer function from the above equations (4)-(6),

$$P(s) = \frac{\theta(s)}{\delta(s)} = \frac{1.151s+0.1774}{s^3+0.739s^2+0.921s} \tag{7}$$

The structure of the control system has the form shown in the Figure.2.

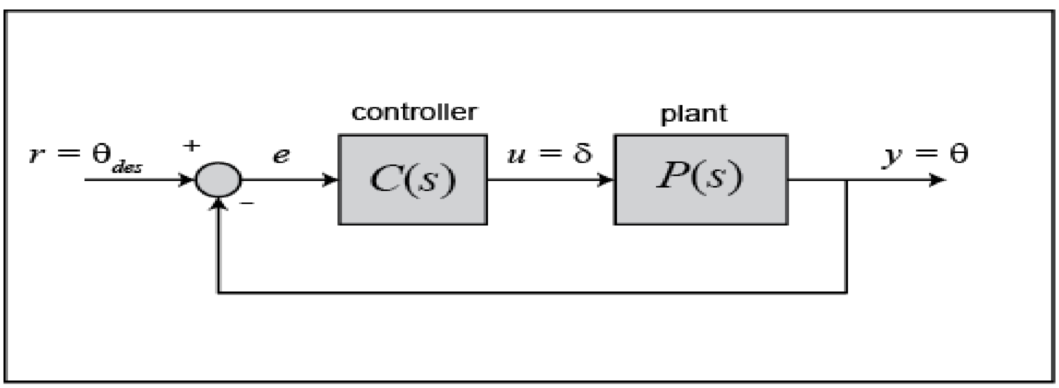

Figure 2: Structure of the control system [9]

We arrive at the following closed-loop transfer function for the control system as shown above.

$$P(s) = \frac{\theta(s)}{\delta(s)} = \frac{1.151s+0.1774}{s^3+0.739s^2+2.072s+0.1774} \tag{8}$$

The dynamic equations of closed-loop control system in state-space form are given as

$$\dot{x} = \begin{bmatrix} -0.739 & -2.0720 & -0.1774 \\ 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r \tag{9}$$

Since the output is pitch angle, the output equation can be written as

$$y = \theta = \begin{bmatrix} 0 & 1.5110 & 0.1774 \end{bmatrix} x \tag{10}$$

These state-space equations have the standard form shown below where the state vector is xand the input u = r.

$$\dot{x} = A_C x + B_C u \tag{11}$$

$$y = C_C x \tag{12}$$

For the essential requirement of the discrete Kalman filter, it is required to process the continuous state-space model of equation (11) to discrete state-space model conversion.

3. KALMAN FILTER

The discrete Kalman filter is applied to the discrete state space model of the pitch control system of the aircraft. Because the number of the parameters to be identified is three, a third order system is defined as

$$x = A_C x + B_C u + w \tag{13}$$

Where, w is the system noise and u is the input signal. The output equation can be defined as

$$y = C_C x + v \tag{14}$$

The highest Eigen value of matrix, Ω = 0.3255 rad/sec.

Hence, for conversion of continuous to discrete state space form, a sampling interval of 0.2 >>2Π/(5Ω) sec is used.

The discrete state space form is as follows:

$$x_k = Ax_{k-1} + Bu_k + W_{k-1} \tag{15}$$

$$y_k = Cx_k + v \tag{16}$$

$$\text{Where } A = \begin{bmatrix} 0.8251 & -0.3833 & -0.0325 \\ 0.1834 & 0.9606 & -0.0034 \\ 0.0189 & 0.1973 & 0.9998 \end{bmatrix} \text{ and } \qquad B = \begin{bmatrix} 0.1834 \\ 0.0189 \\ 0.0013 \end{bmatrix}$$

Where, x_k is the state vector at k=1,2,3,…n and the measurement noise is a zero v_k mean white Gaussian noise.

The algorithm of the Kalman filter is explained in this section: At the k^{th} iteration, the estimations of the state vector and the parameter matrices are calculated from the input u= δ. There is a consideration of two ideal equations. Time Update *(prediction)* and Measurement Update *(correction)*. Both the equation sets are applied at each k^{th} state. The Kalman filter principle is detailed as the following set of estimations below [10].

Time update:

1. $\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k$
2. $\overline{P}_k = AP_{k-1}A^T + Q$

Measurement update:

1. $K_k = \overline{P}_k C^T (C\overline{P}_k C^T + R)^{-1}$
2. $\hat{x}_k = \hat{x}_k^- + K_k(z_k - C\hat{x}_k^-)$
3. $P_k = (I - K_k C)\overline{P}_k$

$\hat{x}$:Estimation of the state vector

$\hat{x}^{-}$:Prediction of the state vector

z :Measurement values vector

K :Kalman gain

$\overline{P}$: Prediction of the error covariance

P : Error covariance update

I : Unit matrix

Q : Covariance matrix of system noise

R : Covariance matrix of measurement noise

4. RESULTS

Conditions for the Experiment

In the experiment, the feedback controller design is based on [9]. The feedback controller processed for the various conditions obtained to each iteration of pitch angle, the actual pitch angle overshoots less than 10%, has a rise time of less than 2 seconds, a settling time of less than 10 seconds, and a steady-state error of less than 2%. By considering a sample set of values such as the reference is 0.2 radians (11 degrees), then the pitch angle will not exceed approximately 0.2 rad, will increase gradually over 0.05 rad to 0.18 rad within 8 seconds, will remain in the range of 2% of its steady-state value within 10 seconds, and will settle between 0.18 and 0.20 radians in steady-state. The sampling period was 0.2 sec.Then this data is applied to Kalman filter algorithm. The faulty conditions are introduced by changing plant matrices A and B, explained in section 5.

The main sources of noise in the aircraft are sensors, and wind disturbance when the aircraft is in motion [11]. By varying the deflection angle the pitch angle of an aircraft can be controlled. The various values obtained are set such as the covariance matrices of system noise Q and measurement noise R, from the noise like sensor noise and wind disturbance. The acceptance of the system noise and the measurement noise are white Gaussian. The variances of these noises are computed. Next by the consideration of the condition of no correlation, the covariance matrices of the system noise Q and of the measurement noise R are set to

$$Q = \begin{bmatrix} 0.0050 & 0 & 0.0000 \\ 0 & 0.0400 & 0.0000 \\ 0.0000 & 0 & 0.1000 \end{bmatrix} \quad (17)$$

$$R = [0.0200] \quad (18)$$

The Initial values of the estimation of the state vector and parameters are fixed to some desired values. For the covariance matrix of the estimation error,

$$P_0 = \begin{bmatrix} 0.0278 & 0 & 0 \\ 0 & 0.0078 & 0 \\ 0 & 0 & 0.0460 \end{bmatrix} \quad (19)$$

can be used.

Figure 4 will give us the behavior of estimated values of, and Figure 5 shows the behavior of the parameter values of, $A = \begin{bmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{bmatrix}$, calculated with the Kalman filter algorithm in presence of additive white Gaussian noise. The error between the actual values and the estimated values is very small (0.005). The Cramer's method is implemented to get the system parameters by having the information about the input and output values [12].

For a step input (u_k =1), equation (15) is written as

$$x_k = Ax_{k-1} + Bu_k + w_{k-1} \tag{20}$$

$$x_{k+1} = Ax_k + Bu_{k+1} + w_k \tag{21}$$

By taking the difference between the above equations (20) and (21), and noting that $u_k = u_{k+1}$,

$$x_{k+1} - x_k = A(x_k - x_{k-1}) + (w_k - w_{k-1}) \tag{22}$$

It is seen from Figure.4 that after t>0.2 sec, steady state is reached and the effect of additive noise is minimal. Hence, in equation (21), the matrix elements of $A = \begin{bmatrix} 0.8251 & -0.3833 & -0.0325 \\ 0.1834 & 0.9606 & -0.0034 \\ 0.0189 & 0.1973 & 0.9998 \end{bmatrix}$ are estimated by Cramer's rule. Computationally efficient methods are available to avoid applying Cramer's rule.

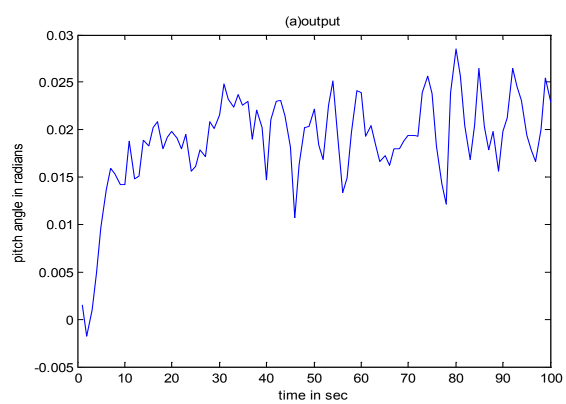

Figure 3: Result of Output

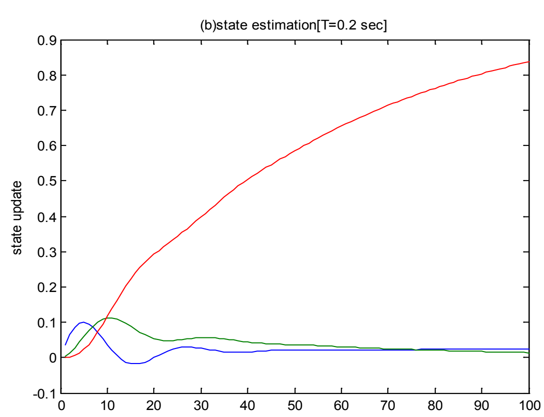

Figure 4: Result of State estimation

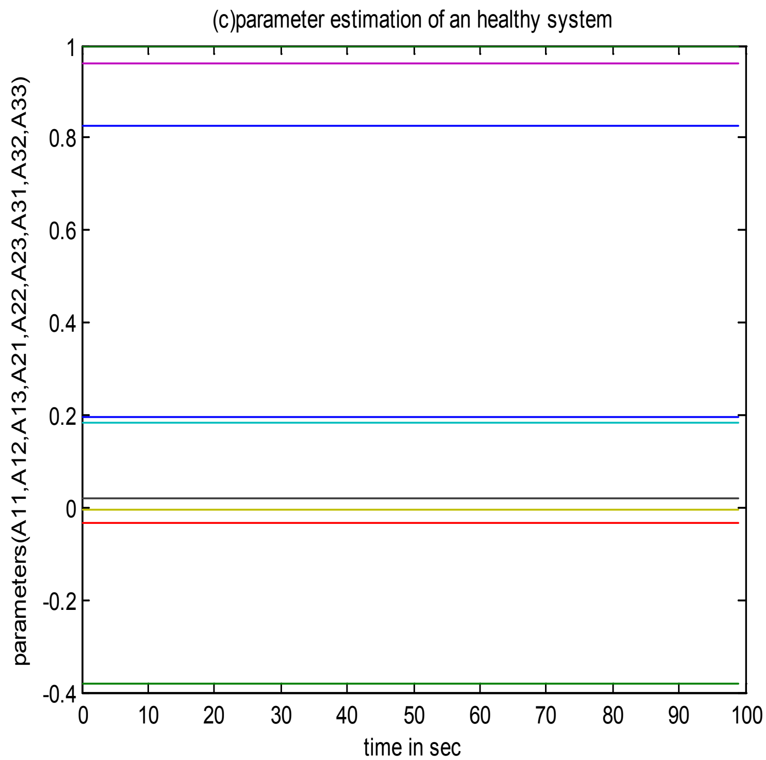

Figure 5: Result of parameter estimation of a healthy control system

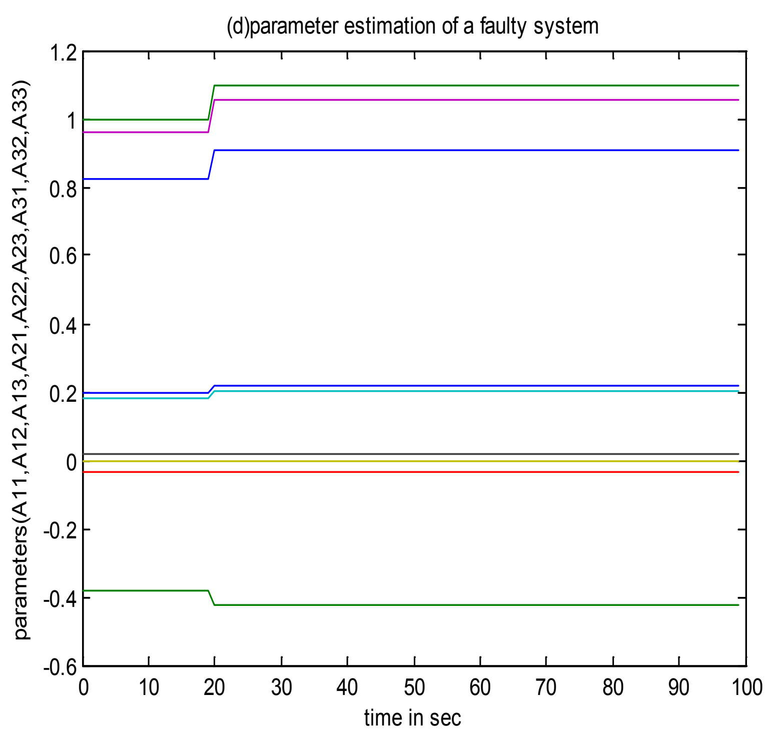

Figure 6: Result of parameter estimation of a faulty control system

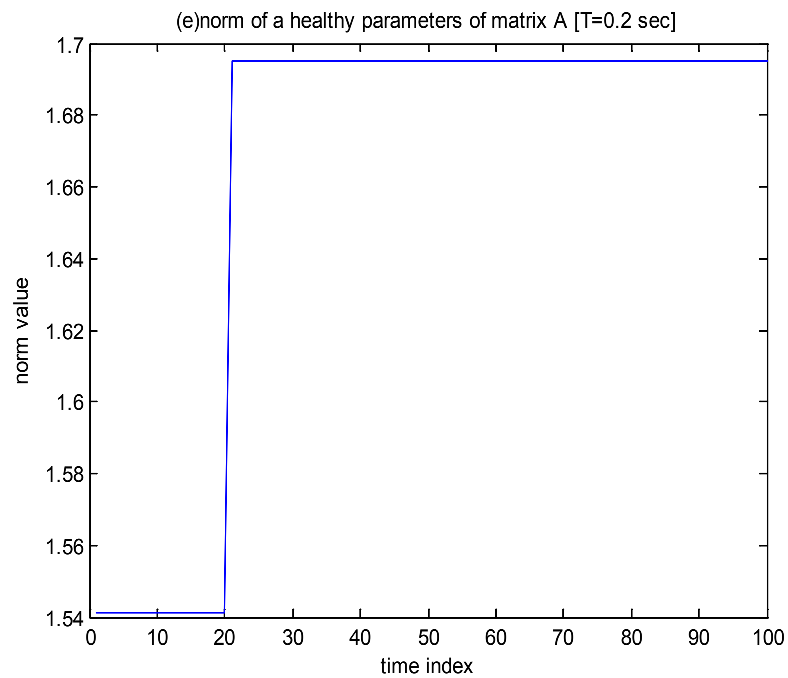

Figure 7: Norm of a faulty parameters of matrix A

5. DISCUSSION OF RESULTS ABOUT FAULT DETECTION

After converting to the discrete time the plant matrices A and B are given by by A_f =

$$\begin{bmatrix} 0.8251 & -0.3833 & -0.0325 \\ 0.1834 & 0.9606 & -0.0034 \\ 0.0189 & 0.1973 & 0.9998 \end{bmatrix} \text{ and } \qquad B_f = \begin{bmatrix} 0.1834 \\ 0.0189 \\ 0.0013 \end{bmatrix}$$

Now assume that the plant matrices A, B are perturbed due to faulty condition. The plant matrices in time domain are assumed to be given by A_f =A.GI and B_f = GIB, where G is a scalar (=1.1) and I is 3 × 3 identity matrix. The parameter estimation of A_f after simulation is (healthy at 20 sec) given by

$$\begin{bmatrix} 0.8251 & -0.3833 & -0.0325 \\ 0.1834 & 0.9606 & -0.0034 \\ 0.0189 & 0.1973 & 0.9998 \end{bmatrix}$$

The parameter values of faulty system at 20 sec

$$= \begin{bmatrix} 0.9076 & -0.4216 & -0.0325 \\ 0.2017 & 1.0566 & -0.0037 \\ 0.0208 & 0.2171 & 1.0998 \end{bmatrix}$$

The "norm" of a matrix A, is defined as

$$\|A\|_1 = \max \textstyle\sum_{i=1}^{m} |a_{ij}| \qquad \text{where } 1 \le j \le n \tag{23}$$

The norm [norm (A_e, 1) in Matlab] of faulty matrix A_f at 20 sec jumps from 1.54 to 1.6953. Setting a suitable threshold (>1) will trigger a fault announciator. The norm of a healthy model will have a constant norm, while any fault will trigger a sudden step in the value of the norm. The figure 7 displays this jump in value of the norm of the matrix.

6. CONCLUSION

This simulation experiment shows that a Kalman filter can be used to provide a joint estimate of the states and the parameters of the longitudinal dynamic model of the aircraft. The computation is robust because the results are presented in the presence of the noise. The states and the parameters are estimated using the measurements of the linearized longitudinal plant. This property is exploited to detect the changes in the plant due to faulty conditions and algorithms successfully detect the changes in the plant parameters. This faulty condition can alert the pilot to minimize the damage and can initiate 'safe home' mode. The estimation of the parameters of the model is indicated in equations (20) to (22). It is observed that as steady state is approached the differences in the estimated states at k and k-1 becomes smaller leading to errors in regression analysis. This can be avoided by recursive estimation of the parameters instead of using regression. Recursive estimation will also remove the restriction of using the step response data.

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