\$ STORY IN THE STO

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp



Target tracking with fast adaptive revisit time based on steady state IMM filter



Farhad Masoumi-Ganjgah*, Reza Fatemi-Mofrad, Nader Ghadimi

Faculty of Electrical and Computer Engineering, Malek Ashtar University of Technology, Tehran, Iran

ARTICLE INFO

Article history: Available online 16 June 2017

Keywords:
Phased array radar
Estimation filters
Adaptive tracking
IMM filters and steady state filters

ABSTRACT

Time is a valuable resource in phased-array radar and proper use of it enables the radar to track more targets simultaneously. Appropriate usage of time in the tracking means the radar can minimize the revisit rate of the tracked targets while the accuracy of targets position estimation is still acceptable. In this paper, a new method for determination of target revisiting time is proposed which has relatively low computational cost and uses steady state filters with Interactive Multiple Models structure. The proposed algorithm allows the radar designer to determine the filter parameters by some criteria such as the desired accuracy and targets maneuver. The performance of the proposed method is compared with the conventional filters for standard maneuverable flying targets. Simulations show the better accuracy and reliability of the proposed method beside its less target revisit time and computational load.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

Phased-array radar can steer the beam position of its antenna electronically for fast beam direction changing. This enables the radar to perform simultaneous operations like surveillance, target tracking and fire control. The radar must perform these operations such fast that it can, handle all its tasks in overload condition. In tracking operation, overload occurs when the radar has to track several targets at the same time. If the target's revisit time is increased, the radar will be able to track more targets simultaneously. Therefore, time management is substantial in phased-array radars. Beside time, processing power is another important resource of radar. For example, in the case of multi-targets tracking, phased-array radar should estimate the position of all targets in a limited time and hence algorithms with high and complicated calculations are restrictive.

One suitable way to determine the proper revisit time of targets is the adaptive method in which targets revisit time is varying adaptively according to effective tracking parameters like target maneuver type. Tracking filters estimate target's kinematic model from its noisy position data. Optimum revisit time is determined so that the estimation error is acceptable. Therefore all the effective factors on the filter's estimation error such as target's maneuver, range and position, measurement accuracy and even environmental conditions can affect the adaptive revisit time.

E-mail addresses: Farhad@mut.ac.ir (F. Masoumi-Ganjgah), Fatemi@mut.ac.ir (R. Fatemi-Mofrad), Ghadimi@mut.ac.ir (N. Ghadimi).

A conventional method among adaptive techniques is choosing the revisiting time from a certain set of possible values [1–4]. In this way, different times are assessed and the best is chosen. Usually revisit time for highly dynamic targets is larger and for slower targets is lower [5]. It is clear that in this method, for every assessment, the estimation filter must be executed, so high computational burden is a drawback of this approach. There are also some other methods developed for adaptive determination of the revisit time which have lower computational load but lesser accuracy [6–10].

Maneuvering targets exhibit different kinematic dynamics during the flight such as constant velocity, accelerating or turning motions. Using a single kinematic model in the estimator's structure increases tracking error and may even lead to filter divergence in maneuvering phase. Another approach for increasing the revisit time is using estimators that adapt their state equations with target's dynamic. This enables the radar to estimate the target's path with desired accuracy through fewer observations resulting in reduced observation rate. For realizing filters with adaptive state equation, multiple kinematic models can be used in the structure of the filter that a popular one is Interactive Multiple Model (IMM) filter [11–15].

In this paper, a new filter is proposed with the ability to determine the revisit time adaptively which maintains good performance and high reliability beside rather low computational load than other adaptive IMM filters. In the proposed filter for increasing accuracy and performance, IMM structure with 3 dynamic models is used and for reducing the computational load of the final filter, steady-state filters are used in the structure of IMM instead

^{*} Corresponding author.

of conventional Kalman filter. State model used in steady-state filters is of constant velocity type with different noise levels leading to steady-state filters with several coefficients. This difference in the noise level of the models enables the filter to cover various maneuvers of the target without the concern about filter divergence.

In the proposed filter, coefficients of the steady-state filter are calculated based on maximum expected error. This simplifies the estimator design which depends on radar's mission type or selected targets priorities. Also this filter's state equations are defined in the spherical coordination so without the need to nonlinear coordinate conversion; radar measurement data are directly processed. To further reducing the processing burden of the proposed filter, the method of revisit time determination is designed in such a way that by a single execution of the filter, adaptive revisit time is calculated. In the rest of the paper, proposed filter is described and its performance is simulated and compared against two conventional filters with different capabilities in a scenario of flying targets with different maneuvers.

2. Steady-state filter

Steady-state filter is actually a Kalman filter that is in steady-state condition. Equations of this filter for constant velocity model are as follows [16]:

$$\hat{X}(k \mid k) = \hat{X}(k \mid k-1) + W[y(k) - H\hat{X}(k \mid k-1)]$$
 (1)

$$\hat{X}(k+1\mid k) = F(T)\hat{X}(k\mid k) \tag{2}$$

Where $\hat{X}(k \mid k)$ and $\hat{X}(k+1 \mid k)$ are the estimation and the one step prediction of state vector, respectively. W is the gain of the steady-state filter. F and H are, respectively, system and measurement matrix and T is sampling time.

$$F(T) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{3}$$

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{4}$$

Unlike Kalman filter, steady-state filter gain is fixed, so computational burden would be less than Kalman filter, however this filter's optimality can't be proved when it's not in steady conditions. For constant velocity dynamic, steady-state filter's gain is a 2-dimensional vector.

$$W = \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix} \tag{5}$$

Where α and β are coefficients of position and target velocity gain. In [17], the optimal value of α and β is calculated as follows:

$$\alpha = \frac{1}{8} \left(\lambda^2 + 8\lambda - (\lambda + 4)\sqrt{\lambda^2 + 8\lambda} \right) \tag{6}$$

$$\beta = \frac{1}{4} \left(\lambda^2 + 4\lambda - \lambda \sqrt{\lambda^2 + 8\lambda} \right) \tag{7}$$

 $\boldsymbol{\lambda}$ is the target's maneuver index and is defined as:

$$\lambda = \frac{\sigma_{\nu}}{\sigma_{w}} T^{2} \tag{8}$$

Where σ_v and σ_w are standard deviation of process and measurement noise respectively and T is sampling Time. Based on (6) and (7), for the gain of the constant-coefficient filters, we have:

$$\alpha = \sqrt{2\beta} - \frac{1}{2}\beta \tag{9}$$

$$\beta = -2\alpha + 4 - 4\sqrt{1 - \alpha} \tag{10}$$

$$\lambda = \sqrt{\frac{\beta}{1 - \alpha}} \tag{11}$$

3. Review of adaptive methods for revisit time

In choosing target revisit time, two main points should be considered:

- Revisit time should be chosen so that the prediction error is small enough such that the radar antenna points toward the next position of the target with acceptable accuracy. Consequently, revisit time should be as short as possible.
- The longer the revisit time, the more time phased array radar will have to do other tasks (such as tracking other targets) and target tracking time as an important resource of the radar is saved more. Hence from the radar resource allocation view, revisit time should be as long as possible.

Considering these two points, revisit time must be calculated as the largest possible time to achieve certain accuracy. It is clear that for maneuvering targets, less revisit time and for relax targets more time must be assigned. There are two main methods for this purpose and all other techniques are developed based on them. The first method proposed by Cohen [18] is based on positions residue. The residue is the filter innovation which is directly related to target's maneuver (target's unpredictability). Cheng et al. in [19] use this method in IMM filters.

The second method is adjusting filter's time step from information shortage of target's position proposed by Van Keuk and Blackman [20]. In this way target revisit time is determined so that the predicted error variance is kept under a given threshold. If $P(k+1\mid k)$ is the error covariance matrix of one step prediction in target angular position, then $g(k\mid k+1,T)$ is defined as the square root of the largest element of $P(k+1\mid k)$ for revisit time of T. By increasing the revisit time, the value of g also rises. So, the maximum revisit time can be determined numerically from the following equation:

$$g(k \mid k+1, T) < V_0 \times BW \tag{12}$$

Where $V_0 \leq 1$ is the error coefficient and BW is the half-power beam-width of the antenna. Thus target revisit time, T, is the value for which (12) holds. In Van Keuk's method, revisit time depends on filter's ability to track moving targets. So by the use of a suitable filter, this time can be reduced. IMM filters adapt excellently to maneuvering targets, so Van Keuk's criterion is also developed for IMM filters [21,22]. In a research study [23], a new method based on IMM filters is presented, in which two dynamic models are considered and a constant sample time is assigned for each model and revisit time is obtained as weighted sum of these times. This method which extensively reduces the computational load of the filter, is called Fast Adaptive IMM (FAIMM) since with a single run of the filter, target revisit time is determined.

4. The proposed adaptive revisit time method

In the proposed method, three steady-state filters with different maneuver indices are used in the IMM structure, it is assumed, the first steady state filter has the lowest and the third one has the highest maneuver index, revisit time is calculated from the probability of mode update such that during target tracking, if the first filter has the largest probability of mode update, i.e. low maneuver of the target, then the target revisit time is incremented and vice-versa, if the third filter has the largest probability means high maneuver of the target leading to a decrease in target revisit time.

So by altering the target revisit time, second filter tries to have the maximum value of mode update while the first and third filters distinguish target maneuver level and prevent filter's instability when target maneuver starts. Since in this method, revisit time is acquired with a single execution of the filter, it can be counted as Fast Adaptive IMM filters but the main difference from other FAIMM methods is the way of choosing the revisit time. In this method, there is no constant sampling time assigned to each model like FAIMM filter and instead of calculating the revisit time in every run of the algorithm individually, the revisit time is updated according to its previous value. Hereafter, proposed filter is called Steady-State Kalman filter with Fast Adaptive revisit time and IMM structure (SS-FAIMM).

In SS-FAIMM filter, it is assumed the minimum and maximum assignable time for a target revisit are, respectively, T_{\min} and T_{\max} . The radar tracks each new target with revisit time of T_{\min} and with every execution of filter's algorithm, next revisit time, T(K+1), is calculated from the probability of modes' update, $\mu_i(k)$, and revisit time, T(K), at the time step k according to following steps:

- If $\mu_3(k) > \max(\mu_1(k), \mu_2(k))$ then $T(k+1) = \max(T(k)/\gamma, T_{\min})$
- If $\mu_1(k) > \max(\mu_2(k), \mu_3(k))$ then $T(k+1) = \min(T(k) \times \gamma, T_{\text{max}})$
- Otherwise, T(K+1) = T(K)

Where $\mu_1(k)$, $\mu_2(k)$ and $\mu_3(k)$ are first, second and third filter mode update probabilities in IMM structure, respectively. $\gamma>1$ is the coefficient of revisit time variation that can be determined from the maneuver variations as a design parameter. γ should be chosen such that for a target with new maneuvering level, the correct value of the revisit time is calculated with a single run of the algorithm. Two cases can be assumed; the first case is when the target starts to maneuver so the third filter with highest maneuvering index will have the largest update probability and the algorithm will decrease the revisit time of the target γ times. In second case, the target maneuver is terminated and first filter with lowest maneuvering index has the largest update probability, so the algorithm will increase the revisit time γ times.

In the first case, with a single run of the algorithm, revisit time must be change such that maneuver index of the second filter in the next step, should be equal to maneuvering index of the third filter.

$$if \mu_1(k) > \max(\mu_2(k), \mu_3(k)) \to T(k+1) = T(k)/\gamma$$
 (13)

$$\lambda_2(k+1) = \lambda_3(k) \tag{14}$$

And finally using equations (8) and (13), we have:

$$\lambda_2(k+1) = \frac{\sigma_{\nu_2} \times T^2(k+1)}{\sigma_w} = \frac{\sigma_{\nu_2} \times T^2(k)}{\sigma_w \times \gamma^2} = \frac{\lambda_2(k)}{\gamma^2}$$
 (15)

Where σ_{v_i} is the standard deviation of the *i*-th filter process noise for constant velocity motion. Using (14) and (15), the relation between γ and maneuver indices of the second and third filters is:

$$\gamma = \sqrt{\frac{\lambda_3(k)}{\lambda_2(k)}} \tag{16}$$

With a similar reasoning, in the second case where the first filter has the largest update probability, we have:

$$\gamma = \sqrt{\frac{\lambda_2(k)}{\lambda_1(k)}} \tag{17}$$

Consequently, for all of the three steady-state filters, the relation between γ and maneuvering indices can be formulated as:

$$\gamma = \sqrt{\frac{\lambda_{i+1}}{\lambda_i}} \tag{18}$$

In which, λ_i is the maneuvering index of the *i*-th steady-state filter.

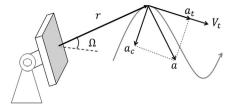


Fig. 1. Geometry of 3D target motion.

4.1. Maximum and minimum target revisit time

Maximum and minimum target revisit time may differ depending on the type of estimation algorithm and targets maneuver. Determination of the maximum target revisit time is of great importance in the case of adaptive tracking of maneuvering targets. For example, if a target is in a constant velocity and level motion for a long time, filter will increase the target revisit time adaptively. If the duration between two successive radar visits is too long, the target can suddenly start to maneuver before next revisit and the radar may lose the target. So by considering the desired target maneuver characteristics for a radar mission and the filter design parameters, maximum revisit time should be chosen such that in case of sudden maneuver of the target, tracking radar doesn't miss it. Constant Angular velocity model is used in the IMM Steady-state filters and it is considered flying target has constant turn motion in the center of the radar [24]. In this constant turn motion, the target tangential acceleration (along-track) is zero ($\alpha_t = 0$) and the normal acceleration (cross-track) is nonzero ($\alpha_c \neq 0$). In Fig. 1, different components of a turn motion is

Constant turn motion can be modeled by following random dynamic [25]:

$$\dot{\Omega} = \frac{V_t}{r} + \upsilon \tag{19}$$

Where $\dot{\Omega}$ is the angular velocity of target with respect to the radar, V_t is the tangential velocity of the target and r is the range of the target from the radar. In (19), variable υ represents the deviation of target's angular velocity from its constant velocity model which has a Gaussian distribution of zero mean and standard deviation of:

$$\sigma_{\rm V} = \frac{\sigma_{\alpha_{\rm t}}}{r} \tag{20}$$

Where σ_{α_t} is the standard deviation of tangential velocity. According to (8) and (20), we have:

$$T = \sqrt{\frac{\sigma_w \lambda}{\sigma_v}} = \sqrt{\frac{\sigma_w \lambda \times r}{\sigma_{\alpha_t}}}$$
 (21)

 λ is the maneuvering index of the target and σ_w is the standard deviation of measurement noise. Finally the maximum value of target revisit time is obtained as:

$$T_{\text{max}} = \sqrt{\frac{\sigma_w \lambda_{\text{max}} \times r}{\sigma_{\alpha_t}}}$$
 (22)

As it's observed from (22), the maximum revisit time depends on target trajectory characteristics such as normal acceleration and range as well as the estimator filter's specification like its maximum maneuvering index. The maximum maneuvering index in the proposed method belongs to the third steady-state filter in IMM. With a similar approach, radar's minimum revisit time is:

$$T_{\min} = \sqrt{\frac{\sigma_w \lambda_{\min} \times r}{\sigma_{\alpha_t}}} \tag{23}$$

It's worth noting that in all the previous methods, minimum and maximum revisit time is considered as constant while in the proposed algorithm, these values are adaptively varying according to the target's range from radar.

4.2. Calculation of steady-state filters gains

In this section, a method to designing three steady-state filters in IMM structured is proposed where the gains and maneuvering indices of these filters are calculated based on desired accuracy and measurement error.

In the IMM structure, the position prediction is a weighted sum of the three steady-state filters estimation running in parallel. Position prediction and its error variance are obtained from (24) and (25) respectively [26].

$$\hat{x}_1(k+1 \mid k) = \sum_{i=1}^{3} \mu_i(k)\hat{x}_1^i(k+1 \mid k)$$
 (24)

$$P(k+1 \mid k)_{11} = \sum_{i=1}^{3} \mu_i(k) \{ P^j(k+1 \mid k)_{11} \}$$

$$+ \left(\hat{x}_1(k+1 \mid k) - \hat{x}_1^i(k+1 \mid k)\right)^2$$
 (25)

In which $\mu_i(k)$ is the probability of mode update (*i*-th filter weight in IMM), $\hat{x}_1(k+1|k)$ and $P^i(k+1|k)_{11}$ are the position prediction and the corresponding variance for the *i*-th filter respectively.

In IMM filter, maximum error occurs when the update probability of the filter with high maneuver dynamic model has the largest possible weight and this happens when the target starts to maneuver [27]. In the proposed filter, three steady-state filters are used where each of them has constant velocity model with different maneuver indices. When the update probability of the third model is at maximum possible value ($\mu_3(k) = 1$), IMM filter's estimation error also maximizes. Therefore referring (25), we have:

$$if \mu_3(k) = 1 \rightarrow P(k+1 \mid k)_{11} = P^3(k+1 \mid k)_{11} = P_{\text{max}}$$
 (26)

In the proposed method, $P_{\rm max}$ is a design parameter and is determined according to Van Keuk's approach from (12). After assigning the maximum prediction error, the gain of steady-state filters are attainable in such a way that the prediction error variance remains constant. In the steady-state filters, the prediction error variance is obtained as [26]:

$$P^{i}(k+1 \mid k)_{11} = \frac{\alpha_{i} \times \sigma_{w}^{2}}{1-\alpha_{i}}$$
 (27)

Where σ_w^2 is the measurement noise variance and α_i is first coefficient of the *i*-th filter's gain. Using (26) and (27), the first coefficient of the third filter's gain, α_3 , is:

$$\alpha_3 = \frac{P_{\text{max}}}{P_{\text{max}} + \sigma_w^2} \tag{28}$$

Finally, the second coefficient of the filter's gain, β_3 , and its maneuvering index, λ_3 , are acquired from α_3 , using (14) and (15).

$$\beta_3 = -2\alpha_3 + 4 - 4\sqrt{1 - \alpha_3} \tag{29}$$

$$\lambda_3 = \beta_3 / \sqrt{1 - \alpha_3} \tag{30}$$

According (18), the maneuvering indices of other steady-state filters in IMM structure, are obtained as:

$$\lambda_2 = \lambda_3 / \gamma^2 \tag{31}$$

$$\lambda_1 = \lambda_3 / \gamma^4 \tag{32}$$

After calculating maneuvering indices of all steady-state filters, their gains are obtained from (6) and (7). In general, the process of steady-state filter's design can be summarized in four steps:

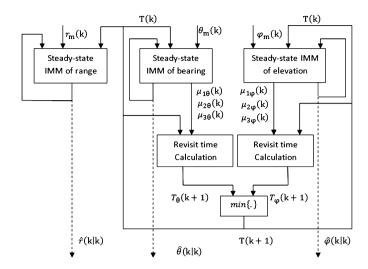


Fig. 2. One cycle diagram of three-dimensional SS-FAIMM.

- A) Assigning the maximum error of the filter, P_{max} , and the sampling time variation coefficient, λ , as design parameters.
- B) Calculating the maneuvering indices of steady-state filters from (30), (31) and (32).
- C) Obtaining steady-state filter's gain from maneuvering indices by (6) and (7).
- D) Determination of maximum and minimum revisit time by (22) and (23).

4.3. Three-dimensional SS-FAIMM

Steady-state filters in SS-FAIMM is directly defined in the radar coordinates, most often spherical coordinates, so there is no need for coordinate conversion. Since the tracking radar measurements in range, r_m , bearing, θ_m , and elevation, φ_m , are statistically independent, the estimation filter is uncoupled in any direction of spherical coordinates, in other world, as shown in Fig. 2 the estimation filter can be implemented by three one-dimensional steady-state IMM filters running in parallel.

According to Van Keuk's criterion in (12), the maximum revisit time is calculated based on maximum angular prediction error, $g(k \mid k+1,T)$, so in three-dimensional SS_FAIMM, the revisit times of bearing and elevation filters are, separately, calculated (Fig. 2) and the final revisit time is:

$$T(k+1) = \min\{T_{\theta}(k+1), T_{\omega}(k+1)\}$$
(33)

Where $T_{\theta}(k+1)$ and $T_{\varphi}(k+1)$ are the revisited times calculated based on the proposed algorithm for bearing and elevation angle, respectively.

5. Evaluation of proposed filter's performance

For evaluating the performance of SS-FAIMM filter and comparing it with common methods, Monte Carlo simulation with 1000 runs is executed. In this simulation, the half-power beam-width of radar's antenna is assumed as 3 deg. The standard deviation of angular measurements in azimuth and elevation are taken equal to 0.3 deg. Also filter's maximum error, $P_{\rm max}$, is assumed as 1 deg² and time variation coefficient, γ , as 1.5. The gains of steady-state filters for tracking the target in azimuth and elevation angles are shown in Table 1.

5.1. Target trajectories

In this simulation, the trajectories of 6 flying targets are modeled which are introduced in [28] as the benchmark targets for

Table 1 Steady-state filters coefficients.

-				
Constant-gain filter	Position gain (α)	Velocity gain (eta)	Maneuver index (λ)	Revisit time variation coefficient (γ)
1st filter 2nd filter	0.688 0.821	0.390 0.665	0.698 1.571	1.5
3rd filter	0.917	1.016	3.535	1.5

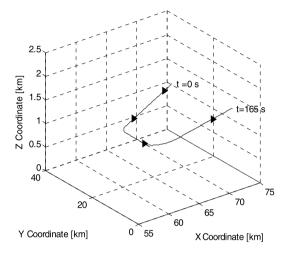


Fig. 3. Trajectory of target 1.

evaluation of tracking filters [28–30]. These targets have maximum horizontal acceleration of 7 g and vertical acceleration of 2 g and are counted as maneuvering targets. The range of these targets varies from 20 km to 120 km while their elevation angle respecting radar varies from 2 to 7 degrees and azimuth angle from +60 to -60 degrees, which are suitable for phased-array radar with single face planar array antenna.

The first target trajectory is shown in Fig. 3 represent a large aircraft, such as a cargo aircraft. Target 1 initializes at a range of 80 km and flies at a constant speed of 290 m/s at an altitude of 1.26 km. After one minute, the aircraft perform a 2 g turn and continues on the new course for a period of 30 s, where a 3 g turn is made and the aircraft flies away from the radar to final range of 70 km.

The second target trajectory is shown in Fig. 4 and represents a smaller, more maneuverable aircraft, such as a small commercial jet. Target 2 initializes at a range of 63 km and speed of 305 m/s and altitude of 4.57 km, the target perform a 2.5 g turn through 90 deg of course change and after that, the target descends gradually to an altitude of 3.05 km. After a 4 g rolling out, a straight and level flight is performed at a constant speed of 305 m/s. The trajectory ends near a range of 28 km.

The third and fourth targets are shown in Fig. 5 and 6 respectively and represent medium bombers with good maneuverability. Target 3 has initial speed of 457 m/s and flies straight and level for the first 30 s. A 4 g turn is then performed through a 45 deg course change. Straight and level flight is continued for the next 30 s, a second 4 g turn through a 90 deg course change to straight and level flight is performed while the aircraft decelerated to a speed of 274 m/s. Target 4 flies at a constant speed of 251 m/s and an altitude of 2.29 km for the first 30 s, a 4 g turn is performed through a course change of 45 deg. After another 30 s, a second 6 g turn is performed. The aircraft climbs to an altitude of 4.57 km. Following that, straight and level flight is maintained for the end of the trajectory.

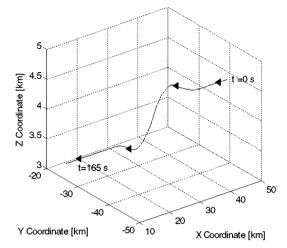


Fig. 4. Trajectory of target 2.

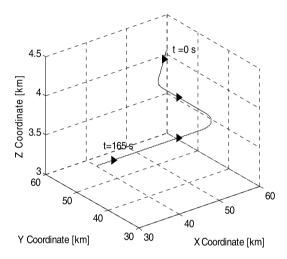


Fig. 5. Trajectory of target 3.

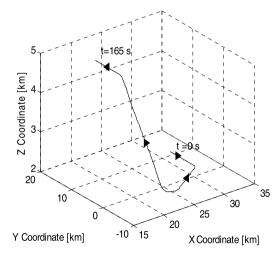


Fig. 6. Trajectory of target 4.

Targets 5 and 6 are shown in Fig. 7 and 8 and represent fighter aircrafts. Target 5 is initialized in thrusting acceleration at an altitude of 1.5 km. After a period of 30 s, a 5 g turn is performed. This turn is followed 20 s later by a 7 g turn. Following second turn, straight and level flight is performed for 30 s upon which a 6 g turn is performed concurrently with a pitch up and a climb. After

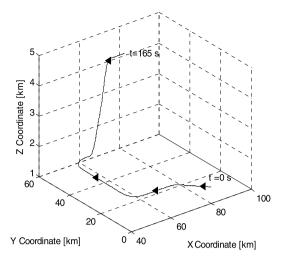


Fig. 7. Trajectory of target 5.

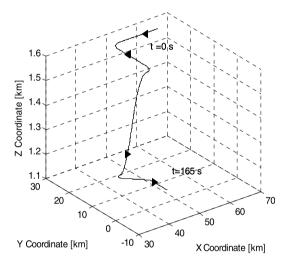


Fig. 8. Trajectory of target 6.

an altitude of 4.45 km is reached, non-acceleration flight is flown for the completion of the trajectory. Target 6 starts at a speed of 426 m/s and an altitude of 1.55 km. Constant speed are maintained for a period of 30 s upon which a 7 g turn is performed. The new course is maintained for another 30 s. A 6 g turn is performed and then the aircraft decreases its altitude. After a final altitude of 1.15 km is obtained and a time span of 30 s, another 6 g turn is commanded. After 30 s, a 7 g turn is performed, upon completion of the turn, straight and level flight is maintained for the end of the trajectory.

5.2. Comparison filters

Two IMM filters with adaptive revisit time are considered for performance comparison with the proposed filter. In these two filters, coordinate conversion from spherical to Cartesian is performed on measurement data and then the converted measurements are applied to the linear filter. The use of coordinate conversion is because that target's dynamic in Cartesian coordinate is usually expressed in linear equations while measurement data by radar including target's range and position angle is in spherical coordinate. Thus the most common method is using the coordinate transform and applying linear filter in Cartesian coordinate [31,32]. In this paper, decorrelated unbiased measurement conversion is used which is presented by Barshalom et al. in a research paper [33].

The first filter, uses Van Keuk's method for revisit time determination, is called converted measurement Adaptive IMM (CM-AIMM) and the second with fast adaptive revisit time and low computational load is called converted measurement Fast adaptive IMM (CM-AIMM) introduced in [23].

Three dynamic models are used in CM-AIMM. The first and second dynamic are constant velocity model with white noise acceleration which differ in process noise level and the third dynamic is white noise jerk model that is a constant acceleration model [26]. Maximum acceleration of the targets in the simulation is 70 m/s² and maximum acceleration rate is 60 m/s³ so, the standard deviation of the process noise for each model is selected as [8,16]:

$$\sigma_{v_1} = 2 \text{ m/s}^2 \tag{34}$$

$$\sigma_{v_2} = 70 \text{ m/s}^2$$
 (35)

$$\sigma_{v_3} = \min\{30T, 60\} \tag{36}$$

In CM-AMM, a pre-determined set of sampling times is considered and the largest one in the set that satisfies Van Keuk's criterion (inequality of (14)) is chosen as the revisit time. The set of sampling times in this simulation is $\{0.1,1,2,2.5,3,5\}$. The threshold of the filter prediction error in Van Keuk's criterion is set to one quarter of the antenna beam-width.

For the simulation of CM-FAIMM, two dynamic models are used. The first Dynamic model is constant velocity (non-maneuvering model) and the second is white noise jerk model with the process noise standard deviations as:

$$\sigma_{v_1} = 2 \text{ m/s}^2 \tag{37}$$

$$\sigma_{v_2} = \min\{30T, 60\} \tag{38}$$

Where T is revisit time, in CM_FAIMM, the adaptive revisit time at time step k is determined as:

$$T(k) = \mu_1(k) \times T_1 + \mu_2(k) \times T_2 \tag{39}$$

where $\mu_1(k)$ and $\mu_2(k)$ are mode update probabilities of the first and second dynamic model, respectively, T_1 and T_2 are corresponding fixed sampling time assigned to these two models, in this simulation, T_1 and T_2 are, respectively, assigned as 5 s and 0.1 s.

5.3. Results and discussions

Comparison criteria in this simulation are the average numbers of target visit (Dwells), average sampling intervals, the probability of tracking loss, calculations performed in each visit or Mega Floating Point Operations per dwell (MFLOPs/Dwell), average amount of calculations in a second (MFLOPs/s), filter's angle estimation error (RMSE) and prediction error (prediction RMSE). These results are depicted in Tables 2 to 4, for SS-FAIMM, CM-AIMM and CM-FAIMM filters

The accuracy of the filters in estimation of target's angle is obtained as Root Mean Square of angular Error (RMSE) from (40):

$$RMSE(\theta) = \left(\frac{1}{N_t N_m} \sum_{k=1}^{N_t} \left(\sum_{i=1}^{N_m} \left[\theta(k) - \hat{\theta}^i(k \mid k) \right]^2 \right) \right)$$
(40)

In which N_m is the number of Monte Carlo simulation runs, N_t is the total steps of target tracking and $\hat{\theta}^i(k \mid k)$ indicates the estimation result in *i*-th run of the simulation and *k*-th time step.

Average amount of floating point operation (FLOPS) in every dwell on the target represents the calculations needed by the algorithm in every run; According to the filter's structure these calculations can be independent of the type of target maneuver. Average

Table 2 SS-FAIMM filter performance.

Target	Target 1	Target 2	Target 3	Target 4	Target 5	Target 6
Dwells	42.8	52.25	48.68	63.7	47.12	50.23
Ave. sampling Interval (s)	3.79	3.12	3.35	2.62	3.45	3.25
Track loss (%)	0.0	0.0	0.0	0.0	0.0	0.4
MFlops/DwelL	2.57	2.57	2.57	2.57	2.57	2.57
MFlops/S	0.68	0.82	0.78	0.98	0.74	0.79
Azimuth RMSE (deg)	0.186	0.191	0.198	0.192	0.217	0.204
Elevation RMSE (deg)	0.192	0.195	0.208	0.188	0.199	0.191
Prediction Azimuth RMSE (deg)	0.381	0.413	0.415	0.373	0.474	0.457
Prediction Elevation RMSE (deg)	0.430	0.433	0.409	0.415	0.445	0.415

Table 3 CM-AIMM filter performance.

Target	Target 1	Target 2	Target 3	Target 4	Target 5	Target 6
Dwells	47.17	54.41	53.51	48.54	52.02	52.67
Ave. sampling Interval (s)	3.46	3.00	3.04	3.36	3.11	3.07
Track loss (%)	0	1	0	2.5	1	2
MFlops/DwelL	10.94	12.45	12.19	12.35	11.86	12.03
MFlops/S	3.16	4.15	4.01	3.69	3.81	3.92
Azimuth RMSE (deg)	0.202	0.211	0.206	0.228	0.212	0.213
Elevation RMSE (deg)	0.195	0.206	0.203	0.207	0.1990	0.1985
Prediction Azimuth RMSE (deg)	0.348	0.404	0.382	0.489	0.427	0.428
Prediction Elevation RMSE (deg)	0.319	0.347	0.337	0.354	0.329	0.325

Table 4 CM-FAIMM filter performance.

Target	Target 1	Target 2	Target 3	Target 4	Target 5	Target 6
Dwells	49.6	56.7	54.32	64.2	58.12	59.42
Ave. sampling Interval (s)	3.23	2. 82	2.91	3.02	2.98.	2.95
Track loss (%)	0	1.2	0.5	2	1	1.7
MFlops/DwelL	3.96	3.96	3.96	3.96	3.96	3.96
MFlops/S	1.22	1.4	1.36	1.31	1.32	1.34
Azimuth RMSE (deg)	0.212	0.203	0.227	0.216	0.223	0.224
Elevation RMSE (deg)	0.201	0.198	0.223	0.198	0.2014	0.205
Prediction Azimuth RMSE (deg)	0.332	0.434	0.453	0.476	0.451	0.462
Prediction Elevation RMSE (deg)	0.324	0.361	0.375	0.412	0.362	0.329

Table 5Average performance of the filters against maneuvering targets.

Filter name	SS-FAIMM	CM-FAIMM	CM-AIMM
Dwells	50.79	57.06	51.39
Ave. sampling Interval (s)	3.26	2.98	3.17
Tracking loss (%)	0.07	1.06	1.08
MFlops/DwelL	2.57	3.96	11.97
MFlops/S	0.798	1.32	3.79
Azimuth RMSE (deg)	0.198	0.217	0.211
Elevation RMSE (deg)	0.196	0.2044	0.202
Prediction Azimuth RMSE (deg)	0.419	0.434	0.413
Prediction Elevation RMSE (deg)	0.425	0.361	0.335

FLOPS per second depicts the average computational load of filter for any target. For obtaining the number of floating point operations in MATLAB software, LightSpeed tool is used [34].

It can be seen from Table 2 to 4 that maximum value of the tracking loss probability for SS-FAIMM filter is 0.4 and for CM-AIMM and CM-FAIMM is 2.5 and 2 percent respectively. In this aspect the proposed filter performs at least 5 times better than the others. For better comparison, average performance of the filters with respect to maneuvering targets is shown in Table 5.

The results show that SS-FAIMM with average sampling time of 3.26 s has the best performance in target revisit time as well as better estimation accuracy than the other two. Also the proposed filter has the lowest tracking loss probability thus it's deduced that SS-FAIMM with its adaptive policy can increase the target revisit time without an increment in tracking loss probability. The average computational load of the proposed filter is 21 percent of

CM-AIMM's and 60 percent of CM-FAIMM's. The proposed filter is weaker in elevation angle prediction; however the prediction value of SS-FAIMM doesn't violate the design parameter.

6. Conclusion

In this paper a new IMM filter based on steady-state filters is proposed with adaptive algorithm for target revisit time determination. The process of determining the filter's gain coefficients from parameters such as filter maximum prediction error and maximum acceleration of targets is described. The performance of the proposed filter is compared with two other IMM filters through simulating flying targets with standard maneuvers. The results show the new filter, is better than the two other compared filters regarding estimation accuracy and target tracking loss, while it needs the least number of target visits and has minimum computational load. Therefore, using the proposed filter leads to more efficient consumption of the phased-array radar resources.

Acknowledgments

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.dsp.2017.06.007.

References

- [1] M. Efe, D.P. Atherton, A tracking algorithm for both highly maneuvering and non-maneuvering targets, in: Proceedings of the 36th IEEE Conference on Decision and Control, 1997, vol. 4, 1997, pp. 3150–3155.
- [2] T. Kirubarajan, Y. Bar-Shalom, E. Daeipour, Adaptive beam pointing control of a phased array radar in the presence of ECM and false alarms using IMMPDAF, Proc. Am. Control Conf. 4 (1995) 2616–2620.
- [3] E. Daeipour, Y. Bar-Shalom, X. Li, Adaptive beam pointing control of a phased array radar using an IMM estimator, Am. Control Conf. 2 (1994) 2093–2097.
- [4] M. Efe, D. Atherton, Adaptive beam pointing control of a phased array radar using the AIMM algorithm, in: IEE Colloquium on Target Tracking and Data Fusion, vol. 1, 1996, pp. 1–8.
- [5] P. Sarunic, R. Evans, Adaptive update rate tracking using IMM nearest neighbour algorithm incorporating rapid re-looks, IEE Proc. Radar Sonar Navig. 144 (1997) 195–204.
- [6] G. Van Keuk, Adaptive computer controlled target tracking with a phased array radar, in: International Radar Conference, vol. 1, 1975, pp. 429–434.
- [7] S.N. Salinger, D. Wangsness, Target-handling capacity of a phased-array tracking radar, IEEE Trans. Aerosp. Electron. Syst. 1 (1972) 43–50.
- [8] G. Watson, W. Blair, Tracking performance of a phased array radar with revisit time controlled using the IMM algorithm, in: Record of the 1994 IEEE National Radar Conference, 1994, pp. 160–165.
- [9] H.-J. Shin, S.-M. Hong, D.-H. Hong, Adaptive-update-rate target tracking for phased-array radar, IEE Proc. Radar Sonar Navig., 142 (1995) 137–143.
- [10] S. Ahmeda, I. Harrison, M. Woolfson, Adaptive probabilistic data-association algorithm for tracking in cluttered environment, IEE Proc. Radar Sonar Navig. 143 (1996) 17–22.
- [11] H. Benoudnine, M. Keche, A. Ouamri, M. Woolfson, I. Harrison, AlMMJPDAF algorithm for tracking multiple manoeuvring targets in a cluttered environment, Model. Identif. Control 1 (1999) 29–32.
- [12] D. Mao, X. Anke, P. Dongliang, G. Yunfei, An improved IMMJPDA algorithm for tracking multiple maneuvering targets in clutter, in: The Sixth World Congress on Intelligent Control and Automation, vol. 1, 2006, pp. 4317–4320.
- [13] H.J. Lin, D. Atherton, An investigation of the SFIMM algorithm for tracking manoeuvring targets, in: Proceedings of the 32nd IEEE Conference on Decision and Control, vol. 1, 1993, pp. 930–935.
- [14] P.S. Maybeck, K.P. Hentz, Investigation of moving-bank multiple model adaptive algorithms, J. Guid. Control Dyn. 10 (1987) 90–96.
- [15] X.-R. Li, Y. Bar-Shalom, Multiple-model estimation with variable structure, IEEE Trans. Autom. Control 41 (1996) 478–493.
- [16] Y. Bar-Shalom, X.R. Li, T. Kirubarajan, Estimation with Applications to Tracking and Navigation: Theory Algorithms and Software, John Wiley & Sons, 2004.
- [17] F. Crouse, A general solution to optimal fixed-gain (α - β - γ etc.) filters, IEEE Signal Process. Lett. 22 (2015) 901–904.
- [18] S. Cohen, Adaptive variable update rate algorithm for tracking targets with a phased array radar, IEE Proc. F, Commun. Radar Signal Process. 133 (1986) 277–280.
- [19] C. Ting, H. Zi-Shu, T. Ting, An IMM-based adaptive-update-rate target tracking algorithm for phased-array radar, in: International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS), vol. 1, 2007, pp. 854-857.
- [20] G. van Keuk, S.S. Blackman, On phased-array radar tracking and parameter control, IEEE Trans. Aerosp. Electron. Syst. 29 (1993) 186–194.
- [21] Y. Bar-Shalom, X.-R. Li, Multitarget-Multisensor Tracking: Principles and Techniques, vol. 19, YBS, Storrs, Conn., 1995.
- [22] H. Benoudnine, M. Keche, A. Ouamri, M. Woolfson, New efficient schemes for adaptive selection of the update time in the IMMJPDAF, IEEE Trans. Aerosp. Electron. Syst. 48 (2012) 197–214.
- [23] H. Benoudnine, M. Keche, A. Ouamri, M. Woolfson, Fast adaptive update rate for phased array radar using IMM target tracking algorithm, in: IEEE International Symposium on Signal Processing and Information Technology, vol. 1, 2006, pp. 277–282.
- [24] L. Zhu, X. Cheng, High manoeuvre target tracking in coordinated turns, IET Radar Sonar Navig. 9 (8) (2015) 1078–1087.

- [25] X.R. Li, V.P. Jilkov, Survey of maneuvering target tracking. Part I. Dynamic models, IEEE Trans. Aerosp. Electron. Syst. 39 (2003) 1333–1364.
- [26] Y. Bar-Shalom, L.X. Rong, K. Thiagalingam, Estimation with Applications to Tracking and Navigation: Theory Algorithms and Software, John Wiley & Sons, 2004
- [27] M. Silbert, S. Sarkani, T. Mazzuchi, Comparing the state estimates of a Kalman filter to a Perfect IMM against a maneuvering target, in: Proceedings of the 14th International Conference on Information Fusion (FUSION), vol. 1, 2011, pp. 1–5.
- [28] W. Blair, G. Watson, T. Kirubarajan, Y. Bar-Shalom, Benchmark for radar allocation and tracking in ECM, IEEE Trans. Aerosp. Electron. Syst. 34 (4) (1998) 1097–1114.
- [29] W. Blair, G. Watson, G. Gentry, S. Hoffman, Benchmark problem for beam pointing control of phased array radar against maneuvering targets in the presence of ECM and false alarms, Proc. Am. Control Conf. 34 (4) (1995) 2601–2605.
- [30] W. Koch, Tracking and Sensor Data Fusion (in English), Mathematical Engineering, vol. 1, Springer, Berlin, Heidelberg, 2014, pp. 157–185.
- [31] Steven V. Bordonaro, Peter Willett, Yaakov Bar-Shalom, Performance analysis of the converted range rate and position linear Kalman filter, in: Asilomar Conference on Signals, Systems and Computers, vol. 1, 2013, pp. 1751–1755.
- [32] S. Xiaoquan, Z. Yiyu, Y. Bar-Shalom, Unbiased converted measurements for tracking, IEEE Trans. Aerosp. Electron. Syst. 34 (3) (1998) 1023–1027.
- [33] S. Bordonaro, P. Willett, Y. Bar-Shalom, Decorrelated unbiased converted measurement Kalman filter, IEEE Trans. Aerosp. Electron. Syst. 50 (2) (2014) 1431–1444.
- [34] T. Minka, Lightspeed matlab toolbox, https://github.com/tminka/lightspeed, 2017



Farhad Masoumi-Ganjgah was born in 1986, Givi, Iran. He received his Bachelor of Science in Electrical Engineering with major of Telecommunications in 2009 and his Master of Science in Electrical Engineering with major of Communication Systems in 2011. He's now Ph.D. candidate of Communication Systems Engineering at Malek Ashtar University of Technology (MUT), Tehran, Iran. His research interests include Detection and Estimation, Signal Processing, Positioning

and Navigation.



Reza Fatemi-Mofrad was born in Tehran, Iran, in 1974. He received the B.S. degree in electronics engineering from Sharif University of Technology, Tehran, Iran in 1998, M.S. degree in telecommunications engineering from Tehran University in 2001 and Ph.D. degree in telecommunications engineering from K. N. Toosi University of Technology, Tehran, Iran in 2011. His research interests include radar systems design, simulation and data processing. Since 2011, he has

been with the electrical and electronic engineering department of MUT, Tehran, Iran.



Nader Ghadimi was born in Shiraz, Iran, in 1956. He received the B.S. degree in electronics engineering from Shiraz University of Technology, Shiraz, Iran in 1981, M.S. and Ph.D. degrees in telecommunications engineering from Sharif University of Technology in 1990 and 2001 respectively. His research interests include signal processing and radar systems. He is currently with MUT, Tehran, Iran.