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Joint Probabilistic Data Association Methods Avoiding Track Coalescence

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Abstract:
For the problem of tracking multiple targets the Joint Probabilistic Data Association (JPDA) filter has shown to be very effective in handling clutter and missed detections . The JPDA, however, also tends to coalesce neighbouring tracks. Through comparing JPDA with the Exact Nearest Neighbour PDA (ENNPDA) filter, Fitzgerald has shown that hypotheses pruning is an effective way to prevent track coalescence. The dramatic pruning used for ENNPDA however leads to an undesired sensitivity to clutter and missed detections. In this paper new algorithms are derived which combine the advantages of JPDA and ENNPDA. The effectiveness of the new algorithms is shown through Monte Carlo simulations.

1 Introduction

For the problem of tracking multiple targets the Joint Probabilistic Data Association (JPDA) filter has shown to be very effective in handling clutter and missed detections (Bar-Shalom and Fortmann, [1]). The JPDA, however, also tends to coalesce neighbouring tracks. With the Exact Nearest Neighbour PDA (ENNPDA) filter Fitzgerald [7] has shown that hypotheses pruning is an effective way to prevent track coalescence. The dramatic pruning used for ENNPDA however leads to an undesired sensitivity to clutter and missed detections. The aim of this paper is to combine the advantages of both methods through the development of a method which avoids track coalescence while preserving JPDA's resistance to clutter and missed detections. It continues the work started in [19].

For the development of the new method the following subsequent steps are undertaken in the sequel:

a. Embedding of unassociated measurements from clutter and randomly detected targets into a linear descriptor system with stochastic i.i.d. coefficients. This representation is new and forms the key to our results.
b. Development of the exact Bayesian filter equations for the conditional density of the joint state of the multiple targets. The filter algorithm resulting from a Gaussian approximation of this conditional density is new and will be referred to as Coupled PDA (CPDA).
c. Development of an hypotheses pruning rule which avoids track coalescence. The resulting filter algorithm is new and will be referred to as the track coalescence-avoiding CPDA, or shortly CPDA*.
d. Elimination of the cross-coupling between the individual target states, by which the CPDA* simplifies to the track-coalescence avoiding JPDA, or shortly JPDA*.

The paper is organized as follows. In section 2, we introduce the stochastic model considered. In section 3, we embed the observations into a linear descriptor system with stochastic coefficients. In section 4, we develop the CPDA filter algorithm. In section 5, we develop the CPDA* and JPDA* filter algorithms. In section 6, we compare the newly derived CPDA, CPDA* and JPDA* filters with the JPDA and the ENNPDA filters through Monte-Carlo simulations for a simple multi-target tracking example. Finally, in section 7, we summarize the results and draw conclusions.

Our development significantly differs from other recent JPDA developments such as sub-optimal JPDA weight evaluation (Fitzgerald, [7]; Nagarajan et al., [8]; Zhou and Bose, [9]; Roecker and Phillis, [10]; Roecker, [11]), competitive filter algorithms like Mixture Reduction (Salmond, [3]) and Symmetric Measurement Equation (Kamen, [4]; Kamen and Sastry, [5]; Lee and Kamen, [6]) and multi-resolutional target tracking (Hong [17]).

2 The Stochastic Model

In this section we will describe the target model and the measurement model.

The target model

We consider M targets and we assume that the state of the i-th target is modeled as follows :

$$x^i_{t+1} = a^i\, x^i_t + b^i\, w^i_t\,, \qquad i = 1\,,\ldots,M\,, \tag{1}$$

where x^i_t is the n-vectorial state of the i-th target, a^i and b^i are $(n \times n)$-matrices and w^i_t is a sequence of i.i.d. standard Gaussian variables of dimension n with w^i_t , w^j_t independent for all $i \neq j$ and w^i_t, x^i_0, x^j_0 independent for all $i \neq j$.
Let $x_t \triangleq \mathrm{Col}\{x^1_t,\ldots,x^M_t\}$, $A \triangleq \mathrm{Diag}\{a^1,\ldots,a^M\}$, $B \triangleq \mathrm{Diag}\{b^1,\ldots,b^M\}$, and $w_t \triangleq \mathrm{Col}\{\, w^1_t,\ldots,w^M_t\,\}$. Then we can model the state of our M targets as follows:

$$x_{t+1} = Ax_t + Bw_t \tag{2}$$

The measurement Model

A set of measurements consists of two types of measurements, namely measurements originating from targets and measurements originating from clutter. Firstly we will treat the two types of measurements separately. Subsequently we treat the random insertion of clutter measurements between the target measurements.

Measurements originating from targets

We assume that a potential measurement associated with state x_t^i (which we will denote by z_t^i) is modeled as follows:

$$z_t^i = h^i x_t^i + g^i v_t^i \qquad , i = 1, \ldots, M \tag{3}$$

where z_t^i is an m-vector, h^i is an $(m \times n)$-matrix and g^i is an $(m \times m)$-matrix, and v_t^i is a sequence of i.i.d. standard Gaussian variables of dimension m with v_t^i and v_t^j independent for all $i \neq j$. Moreover v_t^i is independent of x_0^j and w_t^j for all i,j .

With $z_t \triangleq \mathrm{Col}\{z_t^1,...,z_t^M\}$, $H \triangleq \mathrm{Diag}\{h^1,...,h^M\}$, $G \triangleq \mathrm{Diag}\{g^1,...,g^M\}$, and $v_t \triangleq \mathrm{Col}\{v_t^1,...,v_t^M\}$, we obtain:

$$z_t = Hx_t + Gv_t \tag{4}$$

We next introduce a model that takes into account that not all aircraft have to be detected at moment t, which implies that not all potential measurements z_t^i necessarily have to available at moment t. To this end we define the following variables:

Let P_d^i be the detection probability of aircraft i and let $\phi_{i,t} \in \{0,1\}$ be the detection indicator for target i, which assumes the value one with probability $P_d^i > 0$, independently of $\phi_{j,t}$, $j \neq i$. This yields the following detection indicator vector ϕ_t:

$$\phi_t \triangleq \mathrm{Col}\{\phi_{1,t}, \ldots, \phi_{M,t}\} .$$

Thus the number of detected targets is $D_t \triangleq \sum_{i=1}^{M} \phi_{i,t}$. Furthermore we assume that $\{\phi_t\}$ is a sequence of i.i.d. vectors.

In order to link the detection indicator vector with the measurement model, we introduce the following operator Φ: For an arbitrary (0,1)-valued S-vector ψ we define $D(\psi) \triangleq \sum_{i=1}^{S} \psi_i$ and the operator Φ producing $\Phi(\psi)$ as a (0,1)-valued matrix of size $D(\psi) \times S$ of which the i-th row equals the i-th non-zero row of $\mathrm{Diag}\{\psi\}$. Hence by defining

$$\tilde{z}_t \triangleq \underline{\Phi}(\phi_t) z_t \,, \text{ where } \underline{\Phi}(\phi_t) \triangleq \Phi(\phi_t) \otimes I_m \,,$$

we get the vector which contains all measurements originated from targets at moment t in a fixed order. In reality, however, we do not know the order in which the targets are. Hence we introduce the stochastic $D_t \times D_t$ permutation matrix χ_t, which is conditionally independent of $\{\phi_t\}$. We also assume that $\{\chi_t\}$ is a sequence of independent matrices. Hence

$$\tilde{\tilde{z}}_t \triangleq \underline{\chi}_t \tilde{z}_t \,, \text{ where } \underline{\chi}_t \triangleq \chi_t \otimes I_m \,,$$

is a vector which contains all measurements originated from targets at moment t in a random order.

Measurements originating from clutter

Let F_t be the number of false measurements at moment t. We assume that F_t has Poisson distribution:

$$p_{F_t}(F) = \begin{cases} e^{-\lambda V} \dfrac{(\lambda V)^F}{F!}, & F = 0, 1, 2, \ldots \\ = 0 \qquad\qquad , & \text{else,} \end{cases}$$

where λ is the spatial density of false measurements (i.e. the average number per unit volume) and V is the volume of the validation region. Thus λV is the expected number of false measurements in the validation gate.

We assume that the false measurements are uniformly distributed in the validation region, which means that a column-vector v_t^* of F_t i.i.d false measurements is assumed to have the following density:

$$p_{v_t^* \mid F_t}(v^* \mid F) = V^{-F}$$

where V is the volume of the validation region. Furthermore we assume that the process $\{v_t^*\}$ is a sequence of independent vectors, which are independent of $\{x_t\}, \{w_t\}, \{v_t\}$ and $\{\phi_t\}$.

Random insertion of clutter measurements

Let L_t be the total number of measurements at moment t. Thus

$$L_t = D_t + F_t$$

With $\tilde{y}_t \triangleq \mathrm{Col}\{\tilde{\tilde{z}}_t, v_t^*\}$, it follows with the above defined variables that

$$\tilde{y}_t = \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots \\ v_t^* \end{bmatrix} \tag{5}$$

With this, the measurements originating from clutter still have to be randomly inserted between the measurements originating from targets. To do so, we firstly introduce the following target indicator and clutter indicator processes, denoted by ψ_t and ψ_t^* respectively:

Let $\psi_{i,t} \in \{0,1\}$ be a target indicator for measurement i, which assumes the value one if measurement i belongs to a target and zero if measurement i comes from clutter. This yields the following target indicator vector ψ_t of size L_t:

$$\psi_t \triangleq \mathrm{Col}\{\psi_{1,t}, \ldots, \psi_{L_t,t}\} \,,$$

Let $\psi_{i,t}^* \in \{0,1\}$ be a clutter indicator for measurement i, which assumes the value one if measurement i comes from clutter and is zero if measurement i belongs to an aircraft (thus $\psi_{i,t}^* = 1 - \psi_{i,t}$). This yields the following clutter indicator vector ψ_t^* of size L_t:

$$\psi_t^* \triangleq \mathrm{Col}\{\psi_{1,t}^*, \ldots, \psi_{L_t,t}^*\} .$$

In order to link the target and clutter indicator vectors with the measurement model, we make use of the operator Φ introduced before. With this the random insertion of clutter measurements reads as follows:

$$y_t = \left[\underline{\Phi}(\psi_t)^T \;\vdots\; \underline{\Phi}(\psi_t^*)^T\right] \tilde{y}_t \tag{6}$$

Substituting (5) into (6) yields the following model for the observation vector y_t at moment t:

$$y_t = \left[\underline{\Phi}(\psi_t)^T \;\vdots\; \underline{\Phi}(\psi_t^*)^T\right] \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots \\ v_t^* \end{bmatrix} \tag{7}$$

This, together with equation (2), forms a complete characterization of our tracking problem in terms of stochastic difference equations.

3 Embedding into a descriptor system with stochastic coefficients

Since $\left[\underline{\Phi}(\psi_t)^T \;\vdots\; \underline{\Phi}(\psi_t^*)^T\right]$ is a permutation matrix, its inverse satisfies:

$$\left[\underline{\Phi}(\psi_t)^T \;\vdots\; \underline{\Phi}(\psi_t^*)^T\right]^T = \begin{bmatrix} \underline{\Phi}(\psi_t) \\ \cdots \\ \underline{\Phi}(\psi_t^*) \end{bmatrix} \tag{8}$$

Pre-multiplying (7) by this inverse yields

$$\begin{bmatrix} \underline{\Phi}(\psi_t) \\ \cdots \\ \underline{\Phi}(\psi_t^*) \end{bmatrix} y_t = \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots \\ v_t^* \end{bmatrix} \tag{9}$$

From (9), it follows that

$$\underline{\Phi}(\psi_t) y_t = \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \qquad \text{if } D_t > 0 \tag{10}$$

Substitution of (4) into (10) yields:

$$\underline{\Phi}(\psi_t) y_t = \underline{\chi}_t \underline{\Phi}(\phi_t) H x_t + \underline{\chi}_t \underline{\Phi}(\phi_t) G v_t \quad \text{if } D_t > 0 \tag{11}$$

Notice that (11) is a linear Gaussian descriptor system with stochastic i.i.d. coefficients $\underline{\Phi}(\psi_t)$ and $\underline{\chi}_t \underline{\Phi}(\phi_t)$.

Since χ_t has an inverse, (11) can be transformed into

$$\underline{\chi}_t^T \Phi(\psi_t) \mathrm{y}_t = \underline{\Phi}(\phi_t) H x_t + \underline{\Phi}(\phi_t) G v_t \qquad \text{if } D_t > 0 \qquad (12)$$

Next we introduce an auxiliary indicator process $\widetilde{\chi}_t$ as follows:

$$\widetilde{\chi}_t \triangleq \chi_t^T \Phi(\psi_t) \qquad \text{if } D_t > 0 .$$

With this we get a simplified version of (12):

$$\widetilde{\underline{\chi}}_t \mathrm{y}_t = \underline{\Phi}(\phi_t) H x_t + \underline{\Phi}(\phi_t) G v_t = \underline{\Phi}(\phi_t) z_t = \widetilde{z}_t \text{ if } D_t > 0 \qquad (13)$$

4 Development of the Coupled PDA (CPDA) filter

Let Y_t denote the σ-algebra of measurements y_t up to and including moment t. In this section we develop Bayesian characterizations of the conditional density $p_{x_t \mid Y_t}(x)$.

From (13) follows that all relevant associations and permutations can be covered by $(\phi_t, \widetilde{\chi}_t)$-hypotheses. Hence, through defining the weights

$$\beta_t(\phi, \widetilde{\chi}) \triangleq \mathrm{Prob}\{\phi_t = \phi, \widetilde{\chi}_t = \widetilde{\chi} \mid Y_t\},$$

the law of total probability yields:

$$p_{x_t \mid Y_t}(x) = \sum_{\widetilde{\chi}, \phi} \beta_t(\phi, \widetilde{\chi}) \, p_{x_t \mid \phi_t, \widetilde{\chi}_t, Y_t}(x \mid \phi, \widetilde{\chi}) \qquad (14)$$

And thus our problem is to characterize the terms in the latter summation. This problem is solved in two steps, the first of which is the following Proposition.

Proposition

For any $\phi \in \{0,1\}^M$, such that $D(\phi) \triangleq \sum_{i=1}^{M} \phi_i \le L_t$, and any $\widetilde{\chi}_t$ matrix realization $\widetilde{\chi}$ of size $D(\phi) \times L_t$, the following holds true:

$$p_{x_t \mid \phi_t, \widetilde{\chi}_t, Y_t}(x \mid \phi, \widetilde{\chi}) = \frac{p_{\widetilde{z}_t \mid x_t, \phi_t}(\widetilde{\underline{\chi}} \mathrm{y}_t \mid x, \phi) \cdot p_{x_t \mid Y_{t-1}}(x)}{F_t(\phi, \widetilde{\chi})} \qquad (15)$$

$$\beta_t(\phi, \widetilde{\chi}) = F_t(\phi, \widetilde{\chi}) \prod_{i=1}^{M} [\, \lambda(1 - P_{\mathrm{d}}^i)/P_{\mathrm{d}}^i \,]^{1-\phi_i} / c_t \qquad (16)$$

where $\widetilde{\underline{\chi}} \triangleq \widetilde{\chi} \otimes I_m$, and where $F_t(\phi, \widetilde{\chi})$ and c_t are such that they normalize $p_{x_t \mid \phi_t, \widetilde{\chi}_t, Y_t}(x \mid \phi, \widetilde{\chi})$ and $\beta_t(\phi, \widetilde{\chi})$ respectively.

Proof: see [18].

Our next step is given by the following Theorem.

Theorem 1

Let $p_{x_t \mid Y_{t-1}}(x)$ be Gaussian with mean $\bar{x}_t$ and covariance $\bar{P}_t$ and let $F_t(\phi, \widetilde{\chi})$ be defined by the Proposition. Then $F_t(0, \widetilde{\chi}) = 1$, while for $\phi \neq 0$:

$$F_t(\phi, \widetilde{\chi}) = \exp\{ -\tfrac{1}{2} \mu_t^T(\phi, \widetilde{\chi}) Q_t(\phi)^{-1} \mu_t(\phi, \widetilde{\chi}) \} \cdot \cdot [\, (2\pi)^{mD(\phi)} \mathrm{Det}\{Q_t(\phi)\} \,]^{-\frac{1}{2}} \qquad (17)$$

where: $\mu_t(\phi, \widetilde{\chi}) \triangleq \widetilde{\underline{\chi}} \mathrm{y}_t - \underline{\Phi}(\phi) H \bar{x}_t$

$$Q_t(\phi) \triangleq \underline{\Phi}(\phi)(H \bar{P}_t H^T + G G^T) \underline{\Phi}(\phi)^T$$

Moreover $p_{x_t \mid Y_t}(x)$ is a Gaussian mixture, while its overall mean $\hat{x}_t$ and its overall covariance $\hat{P}_t$ satisfy:

$$\hat{x}_t = \bar{x}_t + \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \mu_t(\phi, \widetilde{\chi}) \Big) \qquad (18)$$

$$\begin{aligned} \hat{P}_t = \bar{P}_t - & \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \underline{\Phi}(\phi) H \bar{P}_t \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \Big) + \\ & + \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \mu_t(\phi, \widetilde{\chi}) \mu_t^T(\phi, \widetilde{\chi}) \Big) K_t^T(\phi) + \\ & - \Bigg(\sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \mu_t(\phi, \widetilde{\chi}) \Big) \Bigg) \cdot \\ & \cdot \Bigg(\sum_{\substack{\phi' \\ \phi' \neq 0}} K_t(\phi') \Big(\sum_{\widetilde{\chi}'} \beta_t(\phi', \widetilde{\chi}') \mu_t(\phi', \widetilde{\chi}') \Big) \Bigg)^T \end{aligned} \qquad (19)$$

with: $K_t(\phi) \triangleq \bar{P}_t H^T \underline{\Phi}(\phi)^T Q_t(\phi)^{-1}$ if $\phi \neq 0$, and $K_t(0) \triangleq 0$

Proof: see [18].

Theorem 1 implies that we get a recursive algorithm if the conditional density $p_{x_t \mid Y_{t-1}}(x)$ is approximated by a Gaussian shape. We refer to this recursive algorithm as the CPDA filter. It consists of evaluating the following three subsequent steps:

CPDA step 1: Prediction:

$$\bar{x}_t = A \hat{x}_{t-1} \qquad (20)$$

$$\bar{P}_t = A \hat{P}_{t-1} A^T + B B^T \qquad (21)$$

CPDA step 2: Evaluation of the detection/permutation hypotheses:

$$\beta_t(\phi, \widetilde{\chi}) = F_t(\phi, \widetilde{\chi}) \prod_{i=1}^{M} [\lambda(1 - P_{\mathrm{d}}^i)/P_{\mathrm{d}}^i]^{1-\phi_i} / c_t \qquad (22.a)$$

$$F_t(\phi, \widetilde{\chi}) \cong \exp\{ -\tfrac{1}{2} \mu_t^T(\phi, \widetilde{\chi}) Q_t(\phi)^{-1} \mu_t(\phi, \widetilde{\chi}) \} \cdot \cdot [\, (2\pi)^{mD(\phi)} \mathrm{Det}\{Q_t(\phi)\} \,]^{-\frac{1}{2}} \qquad (22.b)$$

with c_t a normalizing constant and

$$\mu_t(\phi, \widetilde{\chi}) = \widetilde{\underline{\chi}} \mathrm{y}_t - \underline{\Phi}(\phi) H \bar{x}_t \qquad (23.a)$$

$$Q_t(\phi) = \underline{\Phi}(\phi)(H \bar{P}_t H^T + G G^T) \underline{\Phi}(\phi)^T \qquad (23.b)$$

CPDA step 3: Measurement update equations:

$$\hat{x}_t \cong \bar{x}_t + \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \mu_t(\phi, \widetilde{\chi}) \Big) \qquad (24)$$

$$\begin{aligned} \hat{P}_t \cong \bar{P}_t - & \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \underline{\Phi}(\phi) H \bar{P}_t \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \Big) + \\ & + \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \mu_t(\phi, \widetilde{\chi}) \mu_t^T(\phi, \widetilde{\chi}) \Big) K_t^T(\phi) + \\ & - \Bigg(\sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi) \Big(\sum_{\widetilde{\chi}} \beta_t(\phi, \widetilde{\chi}) \mu_t(\phi, \widetilde{\chi}) \Big) \Bigg) \cdot \\ & \cdot \Bigg(\sum_{\substack{\phi' \\ \phi' \neq 0}} K_t(\phi') \Big(\sum_{\widetilde{\chi}'} \beta_t(\phi', \widetilde{\chi}') \mu_t(\phi', \widetilde{\chi}') \Big) \Bigg)^T \end{aligned} \qquad (25)$$

with

$$K_t(\phi) = \bar{P}_t H^T \underline{\Phi}(\phi)^T Q_t(\phi)^{-1} \qquad \text{if } \phi \neq 0 \qquad (26)$$

5 Track-coalescence-avoiding CPDA and JPDA filter algorithms

The ENNPDA filter equations (Fitzgerald, [7]) can be obtained by pruning all less likely hypotheses prior to measurement updating. Obviously, ENNPDA's *resistance to track coalescence* is due to this pruning, while its sensitivity to missed detections and clutter is also due to this pruning. Obviously the latter does not occur if $\lambda = 0$ and $P^i_{\mathrm{d}} = 1$ for all i, since in that case $L_t = D_t = M$ and $\Phi(\psi_t) = \Phi(\phi_t) = I_M$. In that case ENNPDA can be obtained by pruning all less likely χ_t-hypotheses. Hence track-coalescence appears to be avoided by *pruning χ_t-hypotheses only. This leads to* the following track-coalescence-avoiding hypotheses pruning: preserve CPDA's insensitivity to missed detections and clutter through evaluating all (ϕ_t, ψ_t)-hypotheses, and prune per (ϕ_t, ψ_t)-hypothesis all less-likely χ_t-hypotheses. This approach yields a track-coalescence-avoiding Coupled PDA filter, to which we refer as CPDA*:

CPDA* step 1: Prediction:
Equivalent to CPDA step 1: equations (20-21)

CPDA* step 2: Evaluation of the detection/permutation hypotheses:
Equivalent to CPDA step 2: equation (22-23)

CPDA* step 3: Track-coalescence hypotheses pruning
First evaluate for every (ϕ, ψ):

$$\hat{\chi}_t(\phi,\psi) \triangleq \underset{\chi}{\mathrm{Argmax}}\ \beta_t(\phi, \chi^T\Phi(\psi))$$

Next collect all $\hat{\chi}_t(\phi,\psi)$ hypotheses:

$$\hat{\beta}_t(\phi,\psi) \triangleq \beta_t(\phi, \hat{\chi}^T_t(\phi,\psi)\Phi(\psi))/\hat{c}_t$$

where $\hat{c}_t$ is a re-normalizing constant.

CPDA* step 4: Measurement update equations:

$$\hat{x}_t \cong \bar{x}_t + \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi)\Big(\sum_{\psi}\hat{\beta}_t(\phi,\psi)\mu_t(\phi,\hat{\chi}^T_t(\phi,\psi)\Phi(\psi))\Big) \quad (27)$$

$$\begin{aligned}\hat{P}_t \cong\ & \bar{P}_t - \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi)\Phi(\phi)H\bar{P}_t\Big(\sum_{\psi}\hat{\beta}_t(\phi,\psi)\Big) + \quad (28)\\ & + \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi)\Big(\sum_{\psi}\hat{\beta}_t(\phi,\psi)\mu_t(\phi,\hat{\chi}^T_t(\phi,\psi)\Phi(\psi))\cdot \\ & \cdot \mu^T_t(\phi,\hat{\chi}^T_t(\phi,\psi)\Phi(\psi))\Big)K^T_t(\phi) + \\ & -\Big(\sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi)\Big(\sum_{\psi}\hat{\beta}_t(\phi,\psi)\mu_t(\phi,\hat{\chi}^T_t(\phi,\psi)\Phi(\psi))\Big)\Big)\cdot \\ & \cdot\Big(\sum_{\substack{\phi' \\ \phi' \neq 0}} K_t(\phi')\Big(\sum_{\psi'}\hat{\beta}_t(\phi',\psi')\mu_t(\phi',\hat{\chi}^T_t(\phi',\psi')\Phi(\psi'))\Big)\Big)^T\end{aligned}$$

with $\mu_t(.)$ and $K_t(.)$ given by (23.a) and (26).

The computational complexity of CPDA* is similar to those of CPDA. For $M > 1$ they obviously are much complexer than the JPDA algorithm. If $\lambda = 0$ and $P^i_{\mathrm{d}} = 1$ for all i, then the number of scalar computations for one filter cycle is of the order $\mathcal{O}(M^2n^3 + M^3n^2m + M!M^2m^2)$ for the CPDA/CPDA* and of the order $\mathcal{O}(Mn^3 + MN^2m + M! + M^2m^2)$ for the JPDA.
Our next step is to develop a JPDA* filter from the CPDA* filter in a similar way as the JPDA filter follows from the CPDA filter. To do so, we firstly prove the following Theorem:

Theorem 2
Let $p_{x_t \mid Y_{t-1}}(x)$ be Gaussian with mean $\bar{x}_t = \mathrm{Col}\{\bar{x}^1_t,\ldots,\bar{x}^M_t\}$ and covariance $\bar{P}_t = \mathrm{Diag}\{\bar{P}^1_t,\ldots,\bar{P}^M_t\}$, then $\beta_t(\phi,\tilde{\chi})$ of the Proposition satisfies:

$$\beta_t(\phi,\tilde{\chi}) = \Big(\prod_{i=1}^{M} f^i_t(\phi,\tilde{\chi})\,[\,\lambda(1-P^i_{\mathrm{d}})/P^i_{\mathrm{d}}\,]^{1-\phi_i}\Big)/c_t \quad (29)$$

with:

$$f^i_t(\phi,\tilde{\chi}) = \exp\{-\tfrac{1}{2}\sum_{k=1}^{L_t}([\Phi(\phi)]_{*i}{}^T\,\tilde{\chi}_{*k}\,\mu^{ikT}_t[Q^i_t]^{-1}\mu^{ik}_t\,)\,\}\cdot \cdot[\,(2\pi)^m\,\mathrm{Det}\{Q^i_t\}\,]^{-\frac{1}{2}\phi_i} \quad (30)$$

where: $\mu^{ik}_t \triangleq y^k_t - h^i\bar{x}^i_t$

$Q^i_t \triangleq h^i\bar{P}^i_t h^{iT} + g^i g^{iT}$

while $[\Phi(\phi)]_{*i}$ and $\tilde{\chi}_{*i}$ are the i-th columns of $\Phi(\phi)$ and $\tilde{\chi}$ respectively. Moreover $p_{x^i_t \mid Y_t}(x^i)$, $i \in \{1,\ldots,M\}$, is a Gaussian mixture, while its overall mean $\hat{x}^i_t$ and its overall covariance $\hat{P}^i_t$ satisfy:

$$\hat{x}^i_t = \bar{x}^i_t + W^i_t\Big(\sum_{k=1}^{L_t}\beta^{ik}_t\mu^{ik}_t\Big) \quad (31)$$

$$\begin{aligned}\hat{P}^i_t = \bar{P}^i_t - W^i_t h^i\bar{P}^i_t\Big(\sum_{k=1}^{L_t}\beta^{ik}_t\Big) + W^i_t\Big(\sum_{k=1}^{L_t}\beta^{ik}_t\mu^{ik}_t\mu^{ikT}_t\Big)W^{iT}_t + \\ - W^i_t\Big(\sum_{k=1}^{L_t}\beta^{ik}_t\mu^{ik}_t\Big)\Big(\sum_{k=1}^{L_t}\beta^{ik}_t\mu^{ik}_t\Big)^T W^{iT}_t \quad (32)\end{aligned}$$

with: $W^i_t \triangleq \bar{P}^i_t h^{iT}[Q^i_t]^{-1}$

$$\beta^{ik}_t \triangleq \mathrm{Prob}\{[\Phi(\phi)]_{*i,t}{}^T\tilde{\chi}_{*k,t} = 1 \mid Y_t\} = \sum_{\substack{\phi,\tilde{\chi} \\ \phi \neq 0}} \beta_t(\phi,\tilde{\chi})[\Phi(\phi)]_{*i}{}^T\tilde{\chi}_{*k}$$

Proof: see [18].

Through combining the latter Theorem with the CPDA* steps, we can specify the JPDA* filter algorithm.

JPDA* step 1: Prediction for all $i \in \{1,\ldots,M\}$:

$$\bar{x}^i_t = a^i\hat{x}^i_{t-1} \quad (33)$$

$$\bar{P}^i_t = a^i\hat{P}^i_{t-1}a^{iT} + b^i b^{iT} \quad (34)$$

JPDA* step 2: Evaluation of the detection/evaluation hypotheses:

$$\beta_t(\phi,\tilde{\chi}) = \prod_{i=1}^{M} f^i_t(\phi,\tilde{\chi})[\lambda(1-P^i_{\mathrm{d}})/P^i_{\mathrm{d}}]^{1-\phi_i}/c_t \quad (35)$$

$$f^i_t(\phi,\tilde{\chi}) \cong \exp\{-\tfrac{1}{2}\sum_{k=1}^{L_t}([\Phi(\phi)]_{*i}{}^T\tilde{\chi}_{*k}\mu^{ikT}_t[Q^i_t]^{-1}\mu^{ik}_t)\}\cdot \cdot[(2\pi)^m\mathrm{Det}\{Q^i_t\}]^{-\frac{1}{2}\phi_i} \quad (36)$$

with

$$\mu^{ik}_t = y^k_t - h^i\bar{x}^i_t \quad (37.a)$$

$$Q^i_t = h^i\bar{P}^i_t h^{iT} + g^i g^{iT} \quad (37.b)$$

JPDA* step 3: Track-coalescence hypotheses pruning
Equivalent to CPDA* step 3.

JPDA* step 4: Measurement update equations for all $i \in \{1,...,M\}$:

$$\hat{x}_t^i \cong \bar{x}_t^i + W_t^i \Big(\sum_{k=1}^{L_t} \hat{\beta}_t^{ik} \mu_t^{ik} \Big) \tag{38}$$

$$\hat{P}_t^i \cong \bar{P}_t^i - W_t^i h^i \bar{P}_t^i \Big(\sum_{k=1}^{L_t} \hat{\beta}_t^{ik} \Big) + W_t^i \Big(\sum_{k=1}^{L_t} \hat{\beta}_t^{ik} \mu_t^{ik} \mu_t^{ikT} \Big) W_t^{iT} + - W_t^i \Big(\sum_{k=1}^{L_t} \hat{\beta}_t^{ik} \mu_t^{ik} \Big) \Big(\sum_{k'=1}^{L_t} \hat{\beta}_t^{ik'} \mu_t^{ik'} \Big)^T W_t^{iT} \tag{39}$$

with

$$W_t^i = \bar{P}_t^i h^{iT} [Q_t^i]^{-1} \tag{40.a}$$

$$\hat{\beta}_t^{ik} = \sum_{\substack{\phi, \psi \\ \phi \neq 0}} \hat{\beta}_t(\phi, \psi) [\Phi(\phi)]_{*i}^T [\hat{\chi}_t^T(\phi, \psi) \Phi(\psi)]_{*k} \quad , \tag{40.b}$$

with $[.]_{*k}$ the k-th column of $[.]$.

It should be noticed that the JPDA* filter simplifies to the well known JPDA through omitting step 3. This also implies that the numerical complexity of JPDA* is similar to that of JPDA.

6 Monte Carlo simulations

In this section some comparisons are made between the ENNPDA, the JPDA, the CPDA, the JPDA* and the CPDA* filters. In case of a single target, the latter four are all equal to the ordinary PDA. In case of a single target and no clutter (i.e. $\lambda = 0$), the ENNPDA too is equivalent to PDA, otherwise it differs. In case of multiple targets (i.e. $M > 1$), all five filters differ, unless $\lambda = 0$ and $P_d^i = 1$ for all i, in which case JPDA*=ENNPDA. In order to compare the performance of these filters in multiple target situations, Monte Carlo simulations have been performed with respect to track continuity. The simulations are based on simple crossing target scenarios, similar to the scenarios used by Fitzgerald in [7]: two targets modeled as constant velocity objects which move towards each other and cross over at a certain moment in time. Each simulation which starts with perfect estimates is run for 50 scans, with the crossover point at scan 10.

For each filter, the underlying model of the potential target measurements is given by (2) and (4)

$$x_{t+1} = Ax_t + Bw_t$$
$$z_t = Hx_t + Gv_t$$

where $x_t = [x_{1t}^1, x_{2t}^1, x_{1t}^2, x_{2t}^2]^T$ is the combined state of both targets at moment t, $z_t = [z_t^1, z_t^2]$ is the combined measurement vector of both targets, $w_t = [w_t^1, w_t^2]^T$ and $v_t = [v_t^1, v_t^2]^T$ are independent standard white Gaussian noise vectors. Furthermore

$$A = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \sigma_a \cdot \begin{bmatrix} \frac{1}{2}T_s^2 & 0 & 0 & 0 \\ T_s & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}T_s^2 & 0 \\ 0 & 0 & T_s & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad G = \sigma_m \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where σ_a represents the standard deviation of the target acceleration and σ_m represents the standard deviation of the measurement error. For the tracking filters we used $\sigma_a = 1$, $\sigma_m = 10$ and $T_s = 10$. Following Kalata in [16], this yields the following tracking index $\Lambda \triangleq T_s^2 \sigma_a / \sigma_m = 10$. To simulate Fitzgerald's crossing target scenarios with 'constant velocity-objects' we used $\sigma_a = 0$, however.

During our simulations, we counted a tracking run successful if

$$|\, \hat{x}_{1T}^1 - x_{1T}^1 \,| \le 9\sigma_m \quad \text{and} \quad |\, \hat{x}_{1T}^2 - x_{1T}^2 \,| \le 9\sigma_m$$

with $T \triangleq 50T_s$. Otherwise a tracking run is counted unsuccessful.

The first series of Monte Carlo simulations is performed without clutter measurements ($\lambda = 0$) and with detection probability $P_d^i = 1$, $i = 1, 2$. As stated before, under these conditions, the JPDA* is equal to the ENNPDA. Fitzgerald showed in [7] that under these conditions, the JPDA is decidedly worse than the ENNPDA, and thus decidedly worse than the JPDA*. As expected, we obtained similar results as Fitzgerald, shown in figure 1. Each circled point in figure 1 shows the success rate resulting from 100 Monte Carlo runs of the crossing trajectories; for all tracking mechanisms the same random number streams were used to make the comparisons more meaningful.

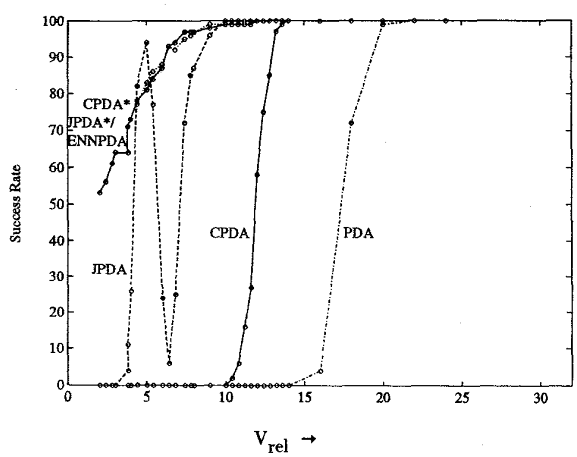

Figure 1: Success Rate for $\Lambda = 10$, $\lambda = 0$ and $P_d^i = 1$, $i = 1, 2$

Figure 1 clearly shows the superior performance of the ENNPDA, the JPDA* and the CPDA* above the other filters, especially at low closing velocities. Remarkable is that, during our simulation period of 50 scans, the CPDA only shows intention to separate the coalescing tracks at relative velocities above 10. For the PDA, this critical relative velocity is even higher.

In order to compare the different methods in a cluttered environment, with a clutter density $\lambda = 0.001$, for each filter we used a gating mechanism to select the measurements to be associated.

Define: $\bar{\bar{P}}_t^i \triangleq h^i \bar{P}_t^i h^{iT} + g^i g^{iT}$

$$R(i) \triangleq \min(f\sqrt{\bar{\bar{P}}_t^i}, \Omega)$$

$$\text{Gate}(i) \triangleq [h^i \bar{x}_t^i - R(i), h^i \bar{x}_t^i + R(i)], \quad i = 1, 2$$

If $\text{abs}(h^i \bar{x}_t^i - h^j \bar{x}_t^j) < \min\Big(f(\sqrt{\bar{\bar{P}}_t^i} + \sqrt{\bar{\bar{P}}_t^j}\ ), \Omega\Big)$, then both gates are merged into one 'large' gate and both tracks will be updated according to the measurements in this 'large' gate. If the gates do not overlap, the tracks will be updated separately, each according to the measurements in its own gate. Obviously the latter only occurs if the tracks are separated far away from each other. During our simulations, we used $f = 6$ and $\Omega = 1000$.

To prevent problems which may occur when the gating mechanism of a tracker doesn't select any plot at all, the detection probability for each tracker is set to $P_d^i = 0.99999$, $i = 1,2$. As in figure 1, each circled point in figure 2 shows the success rate of 'both tracks o.k.' resulting from 100 Monte Carlo runs of the crossing trajectories; for all filters the same random number streams were used to make the comparisons more meaningful.

Figure 2 clearly shows the superior performance of JPDA* and CPDA* above the other filters, especially at low closing velocities. At low closing velocities, JPDA, CPDA and PDA all suffer from track coalescence while JPDA*, CPDA* and ENNPDA don't suffer from track coalescence and show better performance. At higher relative velocity ($V_{rel} \geq 18$) however, the success rate of JPDA*, JPDA, CPDA and CPDA* all seem to converge to the same percentage, far above the success rate of ENNPDA. At very high relative velocity ($V_{rel} \geq 26$), even ordinary PDA shows better performance than ENNPDA. This can be explained by the fact that ENNPDA does not know well enough how to handle clutter. The loss of a target by ENNPDA at high relative velocity is not due to the crossover, but simply due to miss-associations caused by clutter. If ENNPDA makes two or more consecutive wrong decisions, a target is often permanently lost.

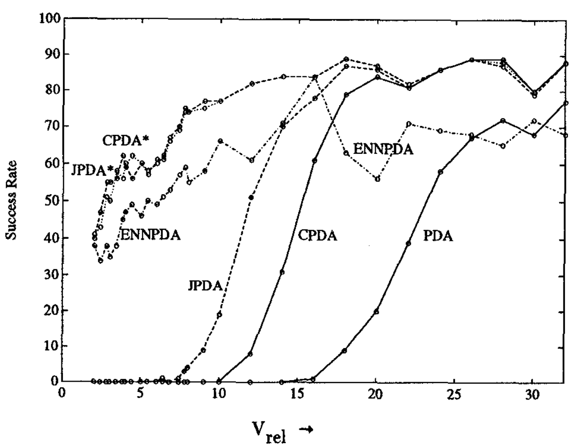

Figure 2: Success rate for $\Lambda = 10$, $\lambda = 0.001$ and $P_d^i = 0.99999$

7 Concluding remarks

For the problem of tracking multiple targets we developed an embedding of multitarget measurements into a linear Gaussian descriptor system with stochastic i.i.d. coefficients. On the basis of this embedding a new filter has been developed, JPDA*, the numerical complexity of which is comparable to that of JPDA, while it combines the advantages of both JPDA and ENNPDA.

Comparison of JPDA* with JPDA and ENNPDA on Fitzgerald's Monte Carlo scenarios shows that JPDA* outperforms both JPDA and ENNPDA.

The pruning method, based on our linear descriptor system, clearly showed its capability to improve the performance of JPDA methods by overcoming the coalescence effect, while preserving JPDA's insensitivity to clutter measurements.

In order to extend the JPDA* approach to the situation of suddenly maneuvering targets, our follow-on research is directed to combining the JPDA* approach with the IMM algorithm [12], and to situations of sensor resolution problems [13].

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