Module 1 Exercises: Interactions

January 29, 2016

How to do Interactions 1

This set of exercises is long. If you *really* know your stuff you can probably easily complete it in 9 hours, but if you're learning new stuff in here there is no way you'll be able to get every problem done in 9 hours.

So you have two choices. The *bad* choice would be to bang your head against this assignment until you have done every problem. We will not stop you from doing this. But we will discourage you.

The *good* choice would be to spend a good solid 9 hours on it (including the time you're watching videos), but do so in a way that maximizes your learning of the full range of material that's represented here. That might look something like this:

1. Start on this assignment today. 9 hours spread over Thursday, Friday, Saturday, and Sunday is not unreasonable. 9 hours on one day is insane and highly unproductive.

2. Watch the videos on vectors (at 2x speed or 0.5x speed, as suits your needs).

3. Talk to a ninja or an instructor if you are confused.

4. Do a few of the vector problems to make sure you at least have a general handle on it. Choose questions that challenge you, and that cover different aspects of the section.

5. Talk to a ninja or instructor if you are confused.

6. Watch the videos on force models (again, at a speed and skipping level that suits you)

7. Talk to a ninja or instructor if you are confused.

8. Do a few of the force model questions; again, choose questions that challenge you, and that cover different aspects of the section.

9. You know what comes next here.

10. Continue in this mode for the remainder of the assignment.

11. Once you've done a few problems from each section, go back and start over again, doing additional problems.

12. Repeat until you (1) feel like you totally have stuff under control, or (2) have spent a solid 9 hours on the assignment.

13. If you get to 9 hours of good, solid, work, and you've sought help when you felt lost, and you still feel lost after we do in-class activities on Monday, then **please talk to us about it.**

14. *Please do not bang your head on a question. If you are lost on a question, ask for help!*

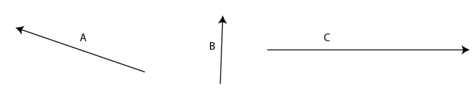
Vectors and Vector Operations: Cartesian

You've probably dealt with vectors some before. Vectors can be thought of both in the physical world (e.g., position vectors), as well as in more abstract spaces, as we'll discuss in future modules. They are a pretty critical building block throughout engineering, math, and science – so you kind of need to become comfortable with thinking in vector space.

There are a set of videos on the website dealing with vectors and vector operations. You should watch these videos, and then do an appropriate number of the exercises below.

1. **Dot and Cross:** What does the dot product of two vectors *tell you* about the two vectors? What about the cross product?

2. **Geometrical Vectors:** The diagram shows three vectors, $\vec{A}$, $\vec{B}$, and $\vec{C}$. All three are in the plane of the page; their magnitudes are (respectively) 2, 1, and 3. For each operation below, either draw the results of the identified operations, or (if appropriate) give a best guess as to the value, or (if appropriate) identify the operation as nonsense.



   (a) $\vec{A} + \vec{B}$
   (b) $\vec{A} - \vec{C}$
   (c) $\vec{A} \cdot \vec{B}$
   (d) $(\vec{A} \times \vec{B}) \times \vec{C}$
   (e) $(\vec{A} \cdot \vec{B}) \times \vec{C}$

3. **Cartesian Vectors:** Let $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$, $\vec{B} = \hat{\imath} - \hat{\jmath}$, and $\vec{C} = -5\hat{\jmath}$. Find the results of identified operations, or (if appropriate) identify the operation as nonsense.

   (a) $|\vec{A} + \vec{B}|$
   (b) $\vec{A} \times \vec{C}$
   (c) $\vec{A} \cdot \vec{B}$
   (d) $(\vec{A} \times \vec{B}) \times \vec{C}$
   (e) $(\vec{A} \times \vec{B}) \cdot \vec{C}$

4. **Vector Construction:** $\vec{A}$ and $\vec{B}$ are two arbitrary, non-parallel vectors in three-dimensional space. Using them, construct the following vectors (i.e., find mathematical expressions for the specified vector in terms of the vectors $\vec{A}$ and $\vec{B}$) :

   (a) The vector $\hat{A}$, which has a length of 1, and points in the direction of $\vec{A}$.

   (b) The vector $\hat{n}$, which has a length of 1, and is perpendicular to both $\vec{A}$ and $\vec{B}$.

Forces: Ideas and Models

Whether we're discussing a static situation in which stuff doesn't move, or a dynamic situation in which stuff does move, we still need to think about the ways in which the stuff we are concerned with (the *system* interacts with the rest of the universe. There are a set of videos on the website that deal with the idea of forces and models for forces. Watch these videos, and then do an appropriate number of the exercises below.

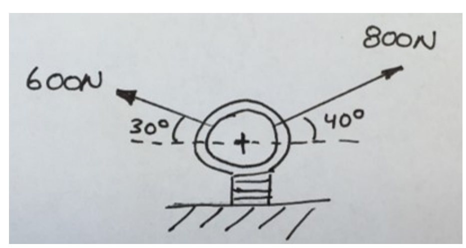
5. Consider a sprinter who is running (and accelerating) over a level surface. What fundamental forces are acting on the sprinter (or parts of the sprinter)? How do these translate into phenomenological forces? How large might these different forces be? What force is responsible for accelerating the runner forward?

6. "I am pushing a coffee cup across my desk. Since it is in motion, I know that the magnitude of the frictional force acting on the cup is given by $\mu_k N$, where μ_k is the coefficient of kinetic friction and N is the normal force the table exerts on the cup." True or false? Discuss, and use a free body diagram to explain.

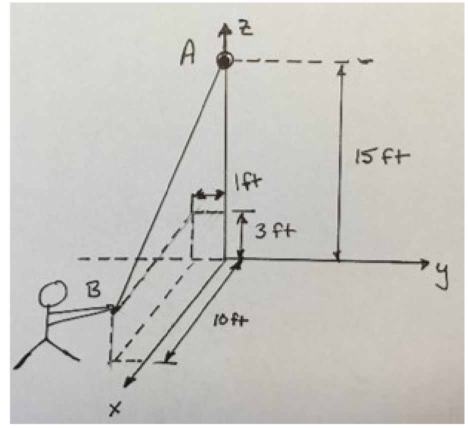
7. "I am pushing on a coffee cup that is sitting on my desk. Since it is not moving, I know that the magnitude of the frictional force acting on the cup is given by $\mu_s N$, where μ_s is the coefficient of static friction and N is the normal force the table exerts on the cup." True or false? Discuss, and use a free body diagram to explain.

8. Your physicist friend tells you, "Newton's third law says that for every force there is an equal and opposite force. For example, when an object is sitting on the ground, the force exerted by gravity ($-mg\hat{j}$), is exactly offset by the normal force ($+mg\hat{j}$)." How do you respond to your friend's claim? Is she right? Wrong? Why?

9. Two forces are applied to an eye hook. They lie in the plane of the page with the orientation and magnitudes as shown. Determine



the orientation and magnitude of the net or resultant force acting on the hook.

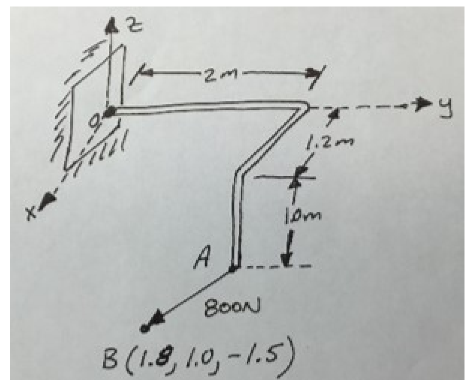


10. A man is pulling a rope running between points A and B with a force of 70 lbs. Describe this force as a vector in terms of components aligned with the x-y-z coordinate system shown.

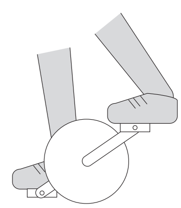
Moments: Ideas and Calculating

Moments (which you may previously have heard referred to as "torques" – same thing – are sort of the rotational analog to forces: just as forces cause linear acceleration, moments cause angular acceleration. You can think about moments both as a result of applying forces (when you apply a force to an object, you are also applying a moment to the object); you can also think about externally applied moments that do not have associated net external forces (e.g., when a motor applies a torque to a wheel).

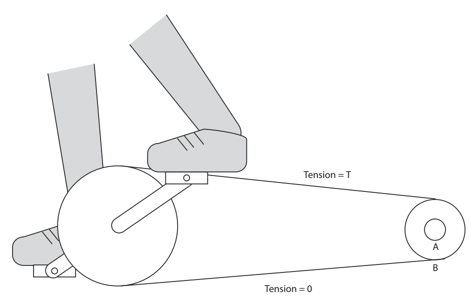
There are a set of videos about the ideas around moments and how to calculate them (yay, cross products). Watch these, and then do an appropriate number of the problems below.



11. A multi-section beam is attached to a wall. What is the moment about point O created by the 800 N force action on point A as shown?



12. Imagine that you are riding a bicycle (without cleats or toe clips). At what point during a pedaling cycle is the moment on the crank generated by your feet on the pedals greatest (assume you are calculating the moment about the center point of the crank)? When is it least? What is the direction of the moment? Think about this both in 2D and in 3D: how do your answers differ, and why?



13. (Think in 2D for this one): As you step on the pedal, the moment you apply to the crank is offset a moment produced by tension in the chain – thus, the tension in the chain is proportional to the moment you apply to the crank. This tension, in turn, applies a moment to a cog attached to the rear wheel. Which cog, A or B, will yield the highest moment on the rear wheel? What will the direction of this moment?

14. Why are multi-speed bikes desirable? Given the answer to above, it seems like a pretty simple design would yield very high moment on the rear wheel. Isn't this always desirable, since this means you can exert more force against the ground, and hence accelerate faster?

15. What might a 1000 N-m torque wrench look like? Why? Propose a nominal design (size, configuration, etc.).

Free Body Diagrams

A key piece of abstracting mechanical systems is the ability to choose and draw an appropriate free body diagram for your system. As with most modeling work, you need to make choices in representing the system: what forces will you include, and what forces will you ignore? Will you think about distributed forces or equivalent forces?

There are a set of videos on creating FBDs. Watch these and then proceed to do an appropriate number of the problems below.

16. Recall the multi-section beam in the previous section (question 15). Draw a free body diagram of multi-section beam in the previous section. Under what conditions would it be reasonable to neglect the weight of the beam? Why?

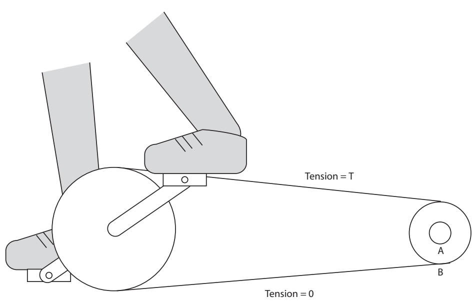
17. The picture at the right shows a person trying to remove a nut with a wrench. Draw a free body diagram for

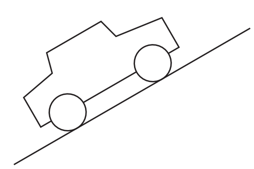
    (a) The wrench only

    (b) The nut only

    Then, using your FBD?s, explain how the wrench is used to remove the nut.

18. Consider a bike that is stationary with a rider standing on the pedals (see below). Draw a complete FBD for the crankset. Sketch your vectors such that your FBD appears to satisfy the equilibrium condition.





19. The diagram at right shows a front-wheel drive car on a hill. The car is in drive, but is not moving: the brakes are engaged.

    Draw free-body diagrams for the following systems:

    (a) System = the entire car.

    (b) System = a rear wheel, hub, and rotor of the car. Assume a disc brake, as shown at right. Note that when the brake is engaged, the caliper pinches the rotor between the brake pads.

(c) System = a front wheel, hub, and rotor of the car. Assume a disc brake, as shown at right. Note that when the brake is engaged, the caliper pinches the rotor between the brake pads. Remember that it's a front-wheel drive car in drive!

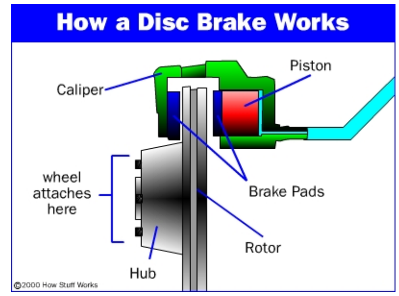


Figure 1: Cross section of disc brake, from `howstuffworks.com`

20. A pin joint or pin connector is a mathematical idealization of the interaction between two connected bodies. The two bodies can rotate freely relative to one another about an axis passing through the pin joint. It does not allow relative motion between the bodies. Wheels rotating on axels or bearing are often modeled as pin joints. See the .pdf document on the website which describes pin joints and various other supports.

    Find a real connection or support that can be modeled as a pin joint and take a photo of it. Draw a free body diagram of the pin between the two bodies. What idealizations must be made so that this representation can be made?

21. The diagram at the right shows a rope routed over a pulley wheel that is suspended from a bracket by a pin connector. Draw a free body diagram the wheel of the pulley.