# Descriptive Network Analysis B

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Network Analysis
& Infographics

*Week 5*

US UNIVERSITY OF SUSSEX | BUSINESS SCHOOL | SCIENCE POLICY RESEARCH UNIT

# Learning Outcomes

| | Learning outcome | Assessment mode |
|---|---|---|
| 1 | Explain the concept of network and list the main network indicators | ESS |
| 2 | Describe and apply the major techniques for the collection of network data and their statistical analysis | ESS, GPN + GWS |
| 3 | Identify the main characteristics of networks by means of network measures | ESS, GPN + GWS |
| 4 | Employ network analysis techniques to produce network data-based infographics | GPN + GWS |

Note: ESS: Essay; GPN: Group Presentation; GWS: Group Written Submission

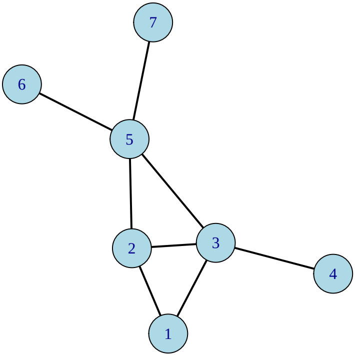
# Network-level measures [recap]

# Network-level measures [recap]

| Measure | Interpretation |
|---|---|
| Diameter | Maximum time/resources for communication, transfer, ... |
| APL | Average time/resources for communication, transfer, ... |
| Density | Connectivity of a network |
| Components | Presence of unconnected groups, bridging opportunities, ... |
| Cutpoints and bridges | Vulnerability/resilience of a network |
| Point/Line connectivity | Vulnerability/resilience of a network |
| Cliques | Highly connected sub-groups, exclusion, ... |
| Inclusiveness | Presence of unconnected nodes, exclusion, ... |
| Reachable pairs | Unconnected nodes or groups, bridging opportunities, ... |
| Transitivity | Social interactions, ‘friends of my friends are my friends’, ... |

# Node-level measures

1. Centrality
   - Degree
   - Closeness
   - Betweenness
   - Centralisation
   - Bonacich's centrality
   - Weighted centrality
2. Brokerage
   - Brokerage roles
   - Effective network size/efficiency
   - Constraint




Note: We will focus on
undirected and unweighted networks

Centrality measures provide an indication of the extent to which a node in a network is connected to the other nodes in the network [Freeman, 1978]

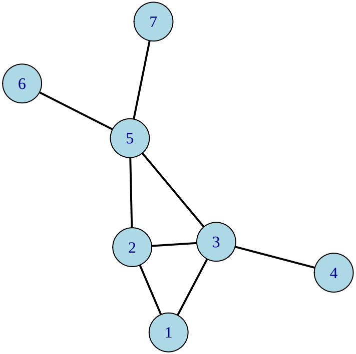
- Degree
- Betweenness
- Closeness

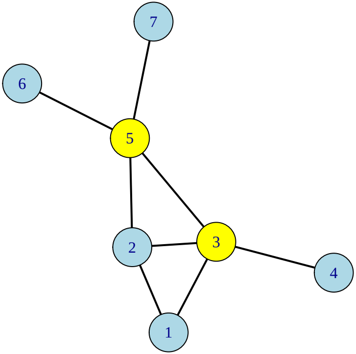
A node's degree is defined as the number of edges that are incident with it or the number of nodes that are adjacent to it

$$C_D(n_i) = \sum_{j=1, i \neq j}^{N-1} x_{ij}$$

$$x_{ij} = \begin{cases} 1, & \text{if nodes } i \text{ and } j \text{ are linked} \\ 0, & \text{otherwise} \end{cases}$$

- $C_D(n_i)$ can range from 0 to $N-1$
- If $C_D(n_i) = 0$, the node $n_i$ is called isolate
- Degree helps to identify most active, popular, influential nodes


7
6
5
2
3
4
1

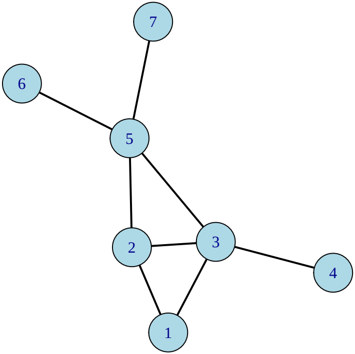



| $n_i$ | $C_D(n_i)$ |
|---|---|
| 1 | 2 |
| 2 | 3 |
| 3 | 4 |
| 4 | 1 |
| 5 | 4 |
| 6 | 1 |
| 7 | 1 |

- Degree centrality is associated with the density of a network

$$\Delta = \frac{\bar{C}_D}{N-1}$$

- $\bar{C}_D$ is the average degree




$$\Delta = \frac{E}{\frac{N(N-1)}{2}} = \frac{8}{\frac{7(7-1)}{2}} = 0.38$$

$$\Delta = \frac{\bar{C}_D}{N-1} = \frac{\frac{(2+3+4+1+4+1+1)}{7}}{7-1} = 0.38$$

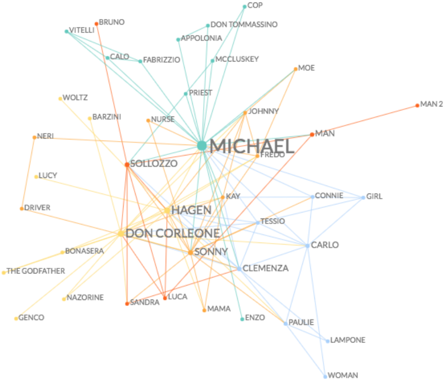
# Node-level measures

## Degree centrality (distribution and scale-free networks)

Interactions between characters in The Godfather (1972)

- $N = 42$
- $E = 104$

```r
library(igraph)
gf <- read_graph("gf1.gml", format = "gml")
```

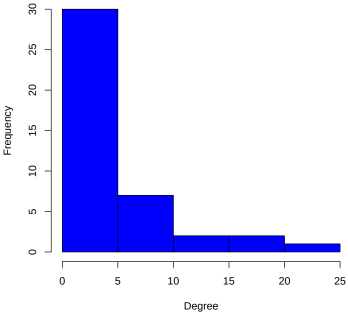



Source: http://moviegalaxies.com

# Node-level measures

## Degree centrality (distribution and scale-free networks)

| $n_i$ | $C_D(n_i)$ | $n_i$ | $C_D(n_i)$ |
|---|---|---|---|
| MICHAEL | 25 | MAMA | 3 |
| DON CORLEONE | 17 | MOE | 3 |
| HAGEN | 17 | VITELLI | 3 |
| SONNY | 15 | NERI | 2 |
| CLEMENZA | 11 | DRIVER | 2 |
| SOLLOZZO | 10 | APPOLONIA | 2 |
| CARLO | 9 | COP | 2 |
| KAY | 8 | DON TOMMASSINO | 2 |
| JOHNNY | 7 | LAMPONE | 2 |
| TESSIO | 7 | NAZORINE | 2 |
| PAULIE | 6 | NURSE | 2 |
| FREDO | 6 | THE GODFATHER | 2 |
| CONNIE | 5 | WOMAN | 2 |
| MAN | 4 | BARZINI | 1 |
| LUCA | 4 | BRUNO | 1 |
| GIRL | 4 | ENZO | 1 |
| SANDRA | 4 | GENCO | 1 |
| BONASERA | 3 | LUCY | 1 |
| MCCLUSKEY | 3 | MAN 2 | 1 |
| CALO | 3 | PRIEST | 1 |
| FABRIZIO | 3 | WOLTZ | 1 |




```r
degree_gf <- cbind(V(gf)$label, degree(gf))
write.csv(degree_gf, "degree_gf.csv")
```

```r
pdf('gf_dist.pdf', width = 6, height = 6)
d_gf <- degree(gf)
hist(d_gf, col = "blue",
     xlab = "Degree",
     ylab = "Frequency",
     main = "")
dev.off()
```

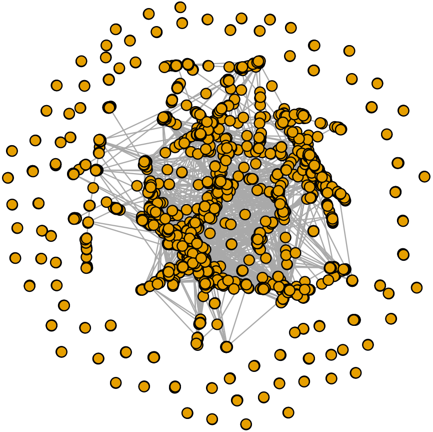
# Node-level measures

## Degree centrality (distribution and scale-free networks)

Yeast data on protein-protein interaction in yeast

- $N = 2617$
- $E = 11855$

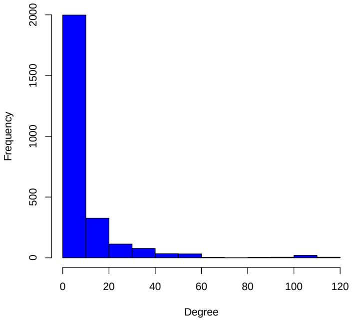
```
library(igraph)
library(igraphdata)
data(yeast)
```

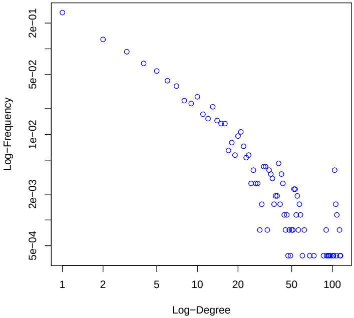


Source: [von Mering et al., 2002]

# Node-level measures

## Degree centrality (distribution and scale-free networks)




```r
pdf('yeast1.pdf', width = 6, height = 6)
d.yeast <- degree(yeast)
hist(d.yeast, col = "blue",
     xlab = "Degree",
     ylab = "Frequency",
     main = "")
dev.off()
```

```r
pdf('yeast2.pdf', width = 6, height = 6)
dd.yeast <- degree.distribution(yeast)
d <- 1:max(d.yeast) - 1
ind <- (dd.yeast != 0)
plot(d[ind], dd.yeast[ind],
     log = "xy", col = "blue",
     xlab = c("Log-Degree"),
     ylab = c("Log-Frequency"),
     main = "")
dev.off()
```

- Large networks seems to self-organize into a scale-free state
- The probability that a node interacts with $k$ nodes follows a power law: $P(k) \sim k^{-\gamma}$
- $2 < \gamma < 3$, but also outside this range
- This feature is not predicted by current random network models

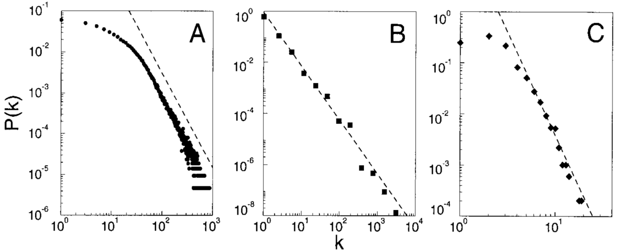
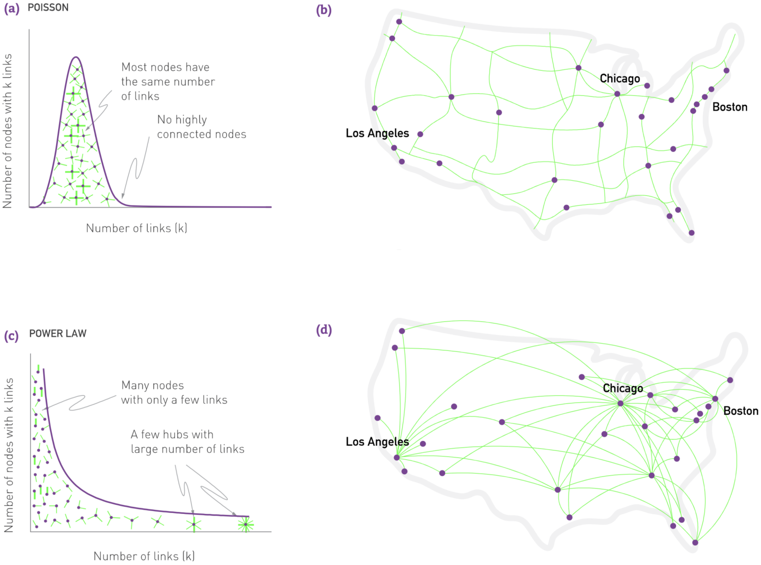



**Fig. 1.** The distribution function of connectivities for various large networks. **(A)** Actor collaboration graph with $N$ = 212,250 vertices and average connectivity $\langle k \rangle$ = 28.78. **(B)** WWW, $N$ = 325,729, $\langle k \rangle$ = 5.46 (6). **(C)** Power grid data, $N$ = 4941, $\langle k \rangle$ = 2.67. The dashed lines have slopes (A) $\gamma_{actor}$ = 2.3, (B) $\gamma_{www}$ = 2.1 and (C) $\gamma_{power}$ = 4.

Source: [Barabasi and Albert, 1999]

# Node-level measures

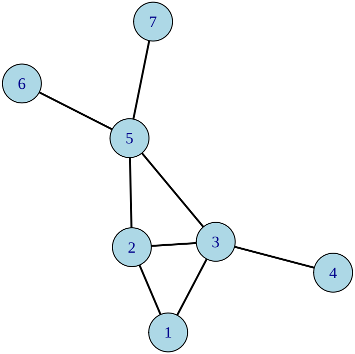
## Degree centrality (distribution and scale-free networks)




Source: Random networks (top, road connections) and scale-free networks (bottom, flight connections) [Barabási, 2016]

# Node-level measures
## Degree centrality (*K*-core)

- A *k*-core is a subgraph in which each node has a degree of at least *k*
- The *k*-core measure identifies nodes that are at the 'core' of the network (especially if the network presents a core-periphery structure)



- A *k*-core is a subgraph in which each node has a degree of at least $k$
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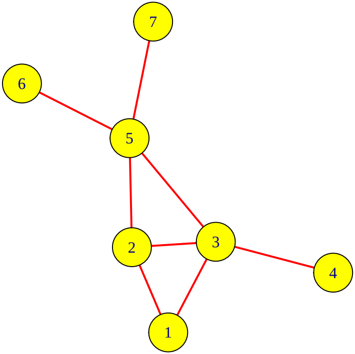
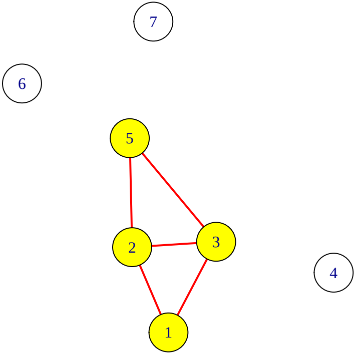


- A $k$-core is a subgraph in which each node has a degree of at least $k$
- The $k$-core measure identifies nodes that are at the 'core' of the network (especially if the network presents a core-periphery structure)



## Degree centrality ($K$-core)

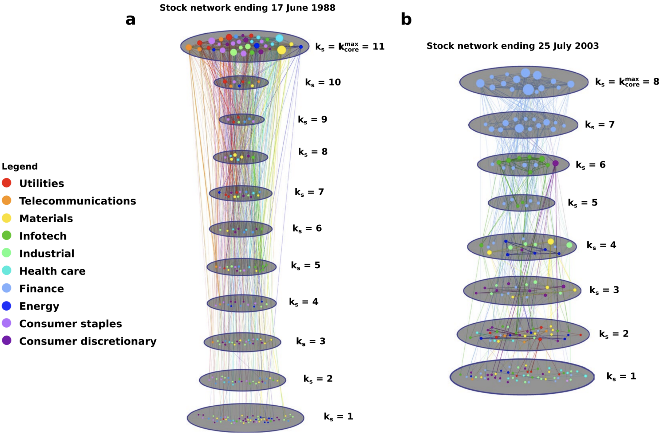



**Figure 6.** K-shell decomposition of stock market networks before and after 1990s deregulation. (**a**) Pre-deregulation; stock network for 100-day period ending 17 June 1988. The various sectors of the economy are all well-represented in each shell, including the maximum k-core, implying that the economic health of the United States is not dependent upon any single industry. (**b**) Post-deregulation; stock network for 100-day period ending 25 July 2003. The innermost cores of the network are dominated by a single industry (finance), meaning that turmoil in the financial sector would have negative effects on the stock market and the U.S. economy as a whole. Each network is color-coded by economic sector and divided into its constituent shells.

Source: [Burleson-Lesser et al., 2020]

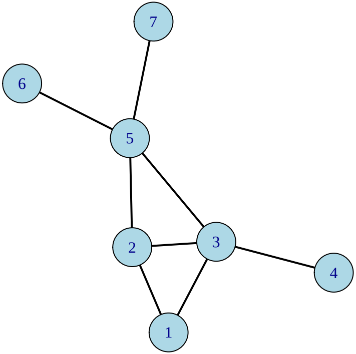
A node's closeness centrality is defined as the inverse of the sum of the geodesic distances of the node from all the nodes of the network

$$C_C(n_i) = \frac{1}{\sum_{j=1, i \neq j}^{N-1} d(n_i, n_j)}$$

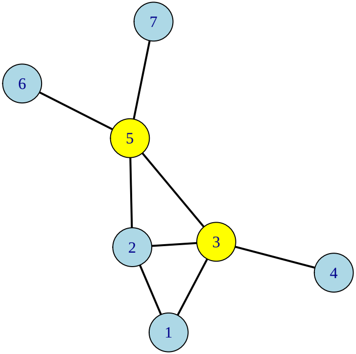
- $d(n_i, n_j)$ is the geodesic distance between nodes $n_i$ and $n_j$
- $C_C(n_i)$ can range from 0 to $\frac{1}{N-1}$
  - If $C_C(n_i) = 0$, one or more nodes are not reachable from $n_i$
  - If $C_C(n_i) = \frac{1}{N-1}$, the node $n_i$ is adjacent to all other nodes
- Closeness helps to identify nodes that are close to most of the nodes in the network
- Closeness is an indicator of speed of information dissemination (how long a node would take to reach the other nodes in the network)

# Node-level measures

## Closeness centrality



# Node-level measures
## Closeness centrality

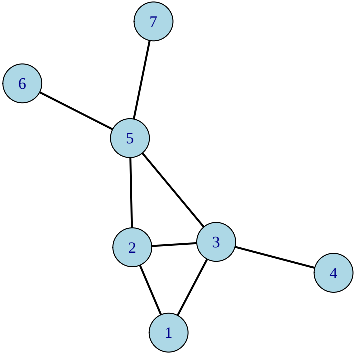


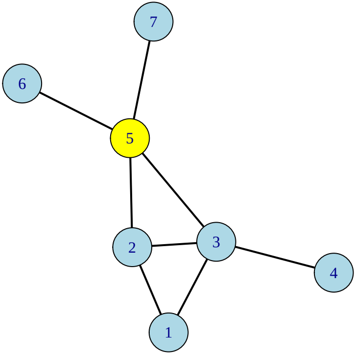

| $n_i$ | $C_C(n_i)$ |
|---|---|
| 1 | 0.08 |
| 2 | 0.11 |
| 3 | 0.12 |
| 4 | 0.07 |
| 5 | 0.12 |
| 6 | 0.07 |
| 7 | 0.07 |

A node's betweenness centrality is the proportion of shortest paths between pairs of nodes that include the node

$$C_B(n_i) = \sum_{j<k \text{ and } j,k\neq i} g_{jk}(n_i)/g_{jk}$$

- $g_{jk}(n_i)$ is the number of shortest paths between nodes $n_j$ and $n_k$ that contain node $n_i$ (assumption of equally likely geodesic linkings)
- $g_{jk}$ is the total number of shortest paths between nodes $n_j$ and $n_k$
- $C_B(n_i) = 0$ can range from 0 to $\frac{(N-1)(N-2)}{2}$
  - If $C_B(n_i) = 0$, the node $n_i$ falls on no shortest paths
  - If $C_B(n_i) = \frac{(N-1)(N-2)}{2}$, the node $n_i$ falls on all shortest paths
- Betweenness centrality helps to identify points where the network is likely break apart (this measure is related to the measure of cutpoints)
- Betweenness centrality is an indicator of which nodes are more likely to be in the communication paths between other nodes: control and power


7
6
5
2
3
4
1




| $n_i$ | $C_B(n_i)$ |
|---|---|
| 1 | 0.00 |
| 2 | 1.50 |
| 3 | 6.50 |
| 4 | 0.00 |
| 5 | 9.00 |
| 6 | 0.00 |
| 7 | 0.00 |

- Centrality measures are dependent on the size of a network
- Comparison between network of different size can be misleading
- The standard normalisation approach account for the size of the network

$$C_D^{'}(n_i) = \frac{C_D(n_i)}{N-1}$$

$$C_C^{'}(n_i) = (N-1)C_C(n_i)$$

$$C_B^{'}(n_i) = \frac{C_B(n_i)}{(N-1)(N-2)/2}$$

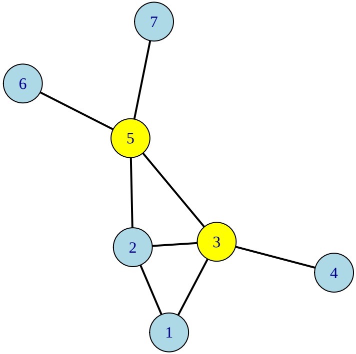
- Centrality measures are dependent on the size of a network
- Comparison between network of different size can be misleading
- The standard normalisation approach account for the size of the network

$$C'_D(n_i) = \frac{C_D(n_i)}{N-1}$$

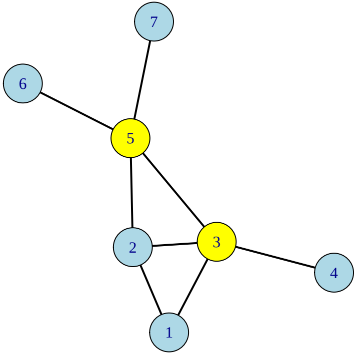
$$C'_C(n_i) = (N-1)C_C(n_i)$$

$$C'_B(n_i) = \frac{C_B(n_i)}{(N-1)(N-2)/2}$$

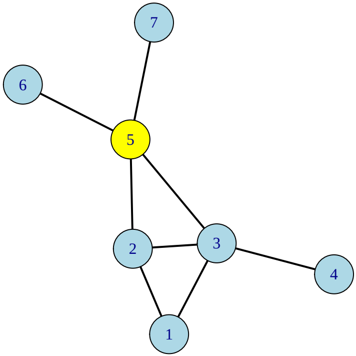
- This does not completely address the size effect issue (e.g. friendship network of 1000 nodes vs. friendship network of 10 nodes)




| $n_i$ | $C_D(n_i)$ | $C'_D(n_i)$ |
|---|---|---|
| 1 | 2 | $\frac{2}{7-1} = 0.33$ |
| 2 | 3 | $\frac{3}{7-1} = 0.50$ |
| 3 | 4 | $\frac{4}{7-1} = 0.67$ |
| 4 | 1 | $\frac{1}{7-1} = 0.17$ |
| 5 | 4 | $\frac{4}{7-1} = 0.67$ |
| 6 | 1 | $\frac{1}{7-1} = 0.17$ |
| 7 | 1 | $\frac{1}{7-1} = 0.17$ |



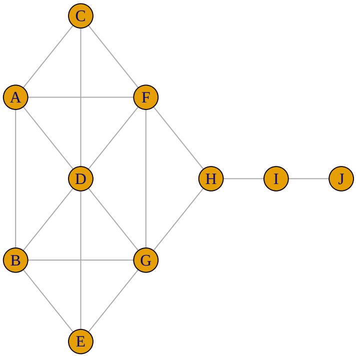

| $n_i$ | $C_C(n_i)$ | $C_C^{'}(n_i)$ |
|---|---|---|
| 1 | 0.08 | $(7-1)*0.08 = 0.50$ |
| 2 | 0.11 | $(7-1)*0.11 = 0.67$ |
| 3 | 0.12 | $(7-1)*0.12 = 0.75$ |
| 4 | 0.07 | $(7-1)*0.07 = 0.46$ |
| 5 | 0.12 | $(7-1)*0.12 = 0.75$ |
| 6 | 0.07 | $(7-1)*0.07 = 0.46$ |
| 7 | 0.07 | $(7-1)*0.07 = 0.46$ |




| $n_i$ | $C_B(n_i)$ | $C_B'(n_i)$ |
|---|---|---|
| 1 | 0.00 | $\frac{0.00}{\frac{(7-1)*(7-2)}{2}} = 0.00$ |
| 2 | 1.50 | $\frac{1.50}{\frac{(7-1)*(7-2)}{2}} = 0.10$ |
| 3 | 6.50 | $\frac{6.50}{\frac{(7-1)*(7-2)}{2}} = 0.43$ |
| 4 | 0.00 | $\frac{0.00}{\frac{(7-1)*(7-2)}{2}} = 0.00$ |
| 5 | 9.00 | $\frac{9.00}{\frac{(7-1)*(7-2)}{2}} = 0.60$ |
| 6 | 0.00 | $\frac{0.00}{\frac{(7-1)*(7-2)}{2}} = 0.00$ |
| 7 | 0.00 | $\frac{0.00}{\frac{(7-1)*(7-2)}{2}} = 0.00$ |

# Node-level measures

## Centrality comparison




Source: Krackhardt's fictionary social network (igraphdata) [Krackhardt, 1990]

| $n_i$ | $C_D(n_i)$ | $C_C(n_i)$ | $C_B(n_i)$ |
|---|---|---|---|
| A | 4 | 0.06 | 0.83 |
| B | 4 | 0.06 | 0.83 |
| C | 3 | 0.06 | 0.00 |
| D | 6 | 0.07 | 3.67 |
| E | 3 | 0.06 | 0.00 |
| F | 5 | 0.07 | 8.33 |
| G | 5 | 0.07 | 8.33 |
| H | 3 | 0.07 | 14.00 |
| I | 2 | 0.05 | 8.00 |
| J | 1 | 0.03 | 0.00 |

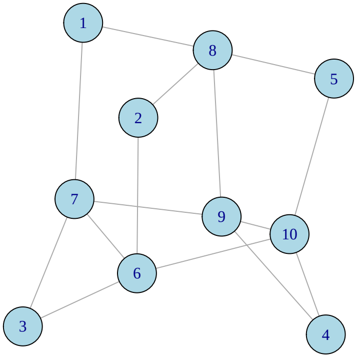
- The distribution of centrality can provide some indication of the extent to which a network is centralized
- [Freeman, 1978] proposed the below measure of centralization

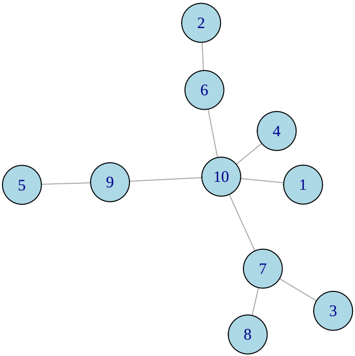
$$C_X = \frac{\sum_{i=1}^{N}[C_X(p^*) - C_X(n_i)]}{max \sum_{i=1}^{N}[C_X(p^*) - C_X(n_i)]}$$

- $C_X(n_i)$, value of centrality for node $i$
- $C_X(p^*)$, largest value of centrality
- The denominator can be theoretically calculated
  - Degree: $(N-1)(N-2)$
  - Closeness: $(N^2 - 3N + 2)/(2N - 2)$
  - Betweenness: $N^3 - 4N^2 + 5N - 2$

# Node-level measures

## Centralization




- $C_D = 0.14$
- $C_C = 0.25$
- $C_B = 0.14$

- $C_D = 0.44$
- $C_C = 0.52$
- $C_B = 0.72$

- The vector of Bonacich's centrality (power centrality) scores is defined as [Bonacich, 1987]:

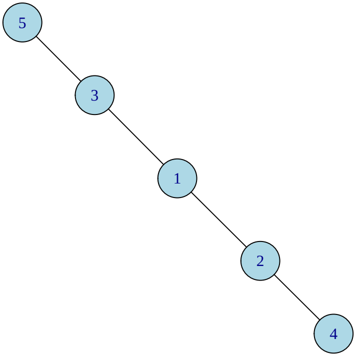
$$c_i(\alpha, \beta) = \sum_j (\alpha + \beta c_j) A_{ij}$$

- $\alpha$ is a scaling vector used to normalise the centrality values (convergence)
- $\beta$ is a parameter that defines the weight of the centrality of $i$'s neighbour nodes
- $A$ is the adjacency matrix

- $\beta = 0$: a node's centrality does not depend from the centrality of other nodes
- $\beta > 0$: a node's centrality positively depends on the centrality of its neighbours
- $\beta < 0$: a node's centrality negatively depends on the centrality of its neighbours (e.g. being link to a central node reduces the node's centrality or influence)
- Heuristic: $\beta$ at $3/4$ of the reciprocal of the largest eigenvalue of the adjacency matrix

# Node-level measures

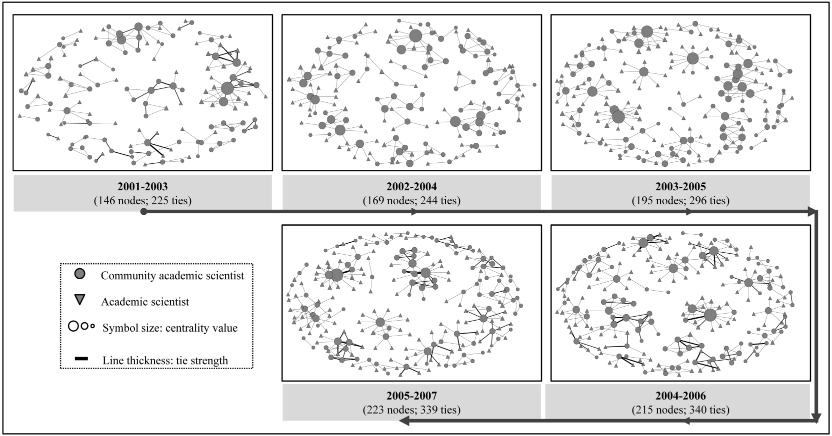
## Bonacich's centrality (example)




| $n_i$ | $c_i(\alpha, \beta)$ | | | | |
|---|---|---|---|---|---|
| | $\beta = -0.50$ | $\beta = -0.25$ | $\beta = 0.00$ | $\beta = 0.25$ | $\beta = 0.50$ |
| 1 | 0.00 | 1.04 | 1.20 | 1.25 | 1.28 |
| 2 | 1.58 | 1.30 | 1.20 | 1.15 | 1.12 |
| 3 | 1.58 | 1.30 | 1.20 | 1.15 | 1.12 |
| 4 | 0.00 | 0.52 | 0.60 | 0.63 | 0.64 |
| 5 | 0.00 | 0.52 | 0.60 | 0.63 | 0.64 |

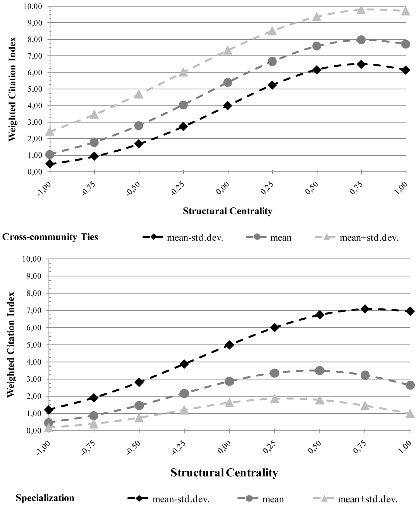
# Node-level measures

## Bonacich's centrality (example)




Source: [Rotolo and Messeni Petruzzelli, 2013]

# Node-level measures
## Bonacich's centrality (example)



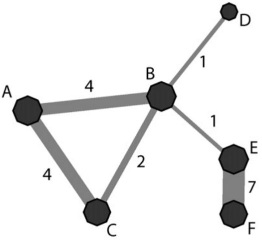

Source: [Rotolo and Messeni Petruzzelli, 2013]

- The information about the strength of ties may be provide additional information about the characteristics of the relationships between nodes
- Social networks
  - duration/intensity of the relationship
  - trust
  - exchange of knowledge
  - ...
- Non-social networks
  - number of goods/people moving between locations
  - number of synapsis in a neural network
  - citations between scientific journals
  - ...

# Node-level measures
## Weighted degree centrality

| Measure | Defintion | Observation |
|---|---|---|
| Weighted degree [Barrat et al., 2004] | $C_D^w(n_i) = \sum_{j=1, i \neq j}^{N-1} w_{ij}$<br><br>- $w_{ij}$, weight of the tie between $n_i$ and $n_j$ | Number of ties is not considered |
| Weighted degree [Opsahl et al., 2010] | $C_D^{w\alpha}(n_i) = C_D(n_i)^{(1-\alpha)} C_D^w(n_i)^{\alpha}$<br><br>- $0 < \alpha < 1$, more importance to degree<br>- $\alpha = 1$, $C_D^w(n_i)$<br>- $\alpha > 1$, less importance to degree | Number of ties and their weight are both considered |

# Node-level measures
## Weighted degree centrality

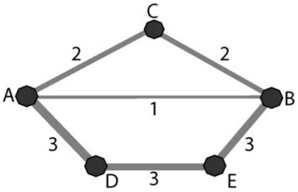



| Node | $C_D$ | $C_D^w$ | $C_D^{w\alpha}$ when $\alpha$= | | | |
|---|---|---|---|---|---|---|
| | | | 0 | 0.5 | 1 | 1.5 |
| A | 2 | 8 | 2 | 4 | 8 | 16 |
| B | 4 | 8 | 4 | 5.7 | 8 | 11.3 |
| C | 2 | 6 | 2 | 3.5 | 6 | 10.4 |
| D | 1 | 1 | 1 | 1 | 1 | 1 |
| E | 2 | 8 | 2 | 4 | 8 | 16 |
| F | 1 | 7 | 1 | 2.6 | 7 | 18.5 |

Source: [Opsahl et al., 2010]

| Measure | Defintion | Observation |
|---|---|---|
| Weighted closeness [Newman, 2001] | $C_C^w(n_i) = \left[\sum_{j=1, i \neq j}^{N-1} d^w(n_i, n_j)\right]^{-1}$<br><br>- $d^w(n_i, n_j) = min(w_{ih}^{-1} + \ldots + w_{hj}^{-1})$ | Number of intermediary nodes is not considered |
| Weighted closeness [Opsahl et al., 2010] | $C_C^{w\alpha}(n_i) = \left[\sum_{j=1, i \neq j}^{N-1} d^{w\alpha}(n_i, n_j)\right]^{-1}$<br><br>- $d^{w\alpha}(n_i, n_j) = min(w_{ih}^{-\alpha} + \ldots + w_{hj}^{-\alpha})$<br>- $\alpha = 0$, unweighted closeness<br>- $\alpha = 1$, weighted closeness by [Newman, 2001]<br>- $0 < \alpha < 1$, shortest distance depending on number of nodes<br>- $\alpha > 1$, shortest distance depending on the weight of the ties | Number of intermediary nodes and their weight are both considered |

# Node-level measures
## Weighted closeness centrality




| Path | $d(A,B)$ | $d^w(A,B)$ | $d^{w\alpha}(A,B)$ when $\alpha$= | | | |
|---|---|---|---|---|---|---|
| | | | 0 | 0.5 | 1 | 1.5 |
| $\{A,B\}$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $\{A,C,B\}$ | 2 | 1 | 2 | 1.4 | 1 | 0.7 |
| $\{A,D,E,B\}$ | 3 | 1 | 3 | 1.8 | 1 | 0.5 |

Source: [Opsahl et al., 2010]

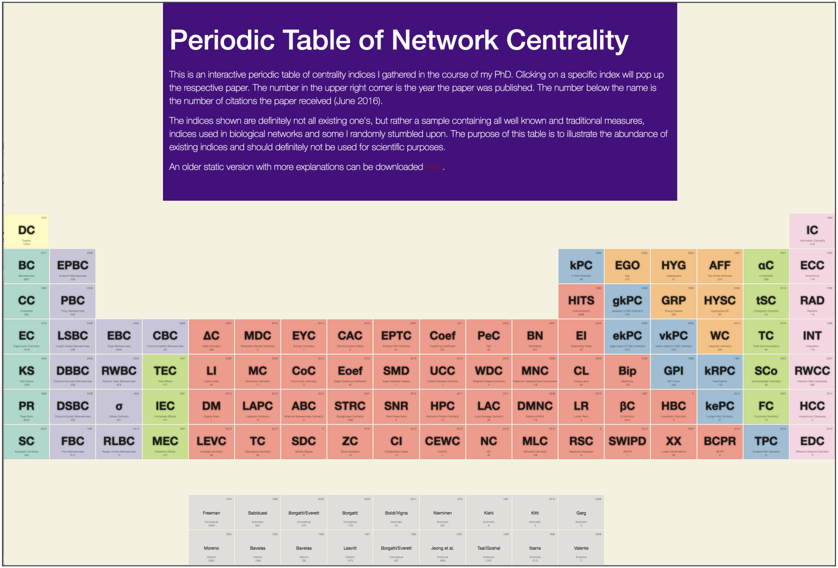
# Node-level measures
## Weighted betweenness centrality

| Measure | Defintion | Observation |
|---|---|---|
| Weighted betweenness [Brandes, 2008] | $C_B^w(n_i) = \sum_{j<k \text{ and } j,k \neq i} g_{jk}^w(n_i)/g_{jk}^w$<br><br>- $g_{jk}^w(n_i)$ shortest paths between nodes $n_j$ and $k$ with $n_i$<br>- $d^w(n_i, n_j) = min(w_{ih}^{-1} + \ldots + w_{hj}^{-1})$ | Number of intermediary nodes is not considered |
| Weighted betweenness [Opsahl et al., 2010] | $C_B^{w\alpha}(n_i) = \sum_{j<k \text{ and } j,k \neq i} g_{jk}^{w\alpha}(n_i)/g_{jk}^{w\alpha}$<br><br>- $g_{jk}^{w\alpha}(n_i)$ shortest paths between nodes $n_j$ and $k$ with $n_i$<br>- $d^{w\alpha}(n_i, n_j) = min(w_{ih}^{-\alpha} + \ldots + w_{hj}^{-\alpha})$<br>- $\alpha = 0$, unweighted betweenness<br>- $\alpha = 1$, weighted betweenness by [Brandes, 2008]<br>- $0 < \alpha < 1$, shortest distance depending on number of nodes<br>- $\alpha > 1$, shortest distance depending on the weight of the ties | Number of intermediary nodes and their weight are both considered |

| Centrality measure | Interpretation |
|---|---|
| Degree | How many nodes can a node reach directly?<br>*information flow, popularity, influence* |
| Closeness | How fast can a node reach every node in the network?<br>*speed, diffusion, efficiency* |
| Betweenness | How likely is a node to be part of the most direct route between two nodes in the network?<br>*control, fragmentation, brokerage* |
| Bonacich's centrality | How well is an actor connected to other well-connected actors in the network?<br>*power, comprehensive view of the network* |
| Weighted centrality | Use of the information about the strength of the ties (and distribution of these in the case of Opsahl's centrality) |

# Node-level measures

## Centrality measures




Source: `http://www.schochastics.net/sna/periodic.html`

# Questions

Next time ...

# Next time ...

- **Seminar: Descriptive network analysis B**
  - Assessment of node-level measures (centrality measures)
- **Lecture: Descriptive network analysis C**
  - Node-level measures (brokerage measures)

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