NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

2017/2018 **QF4102 Financial Modelling** **Semester I**

QF4102 Assignment 3

A3.1 Consider the **transformed** Black-Scholes PDE model:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\sigma^2}{2}\dfrac{\partial^2 u}{\partial x^2} + \left(r - q - \dfrac{\sigma^2}{2}\right)\dfrac{\partial u}{\partial x} - ru = 0, \quad x \in (-\infty, \infty), \quad t \in [0, T), \\ u(x, T) = \varphi(x). \end{cases}$$

(i) Derive the $O[\Delta t + (\Delta x)^2]$ fully implicit scheme for a European vanilla call option. Assume that $x \in (-\infty, \infty)$ is truncated to $x \in [-x_{max}, x_{max}]$, with $x_{max} = I\,dx$ for a sufficiently large integer I. Write down the system of linear equations and its matrix form, as well as the boundary conditions.

(ii) Implement a Matlab function for the fully implicit scheme derived in (i). Use the following function heading in your m-file:

```
function OptVal=FD_ids_call_trans(S0, X, r, q, T, sigma, I, N, xmax)
```

Test your implementation with a European call option which has a strike price of \$5 and time to maturity of 1 year. The underlier has a current price of \$5.25, volatility of 30% and dividend yield of 10%. The risk free rate is 3%. Use a grid with values of x lying in the truncated domain $[-5, 5]$. For $N = 1500$ and I taking values from 100 to 1500 in increments of 100, obtain option value estimates (with linear interpolation) for the given option. Plot these estimates versus I. Comment.

(iii) Write another Matlab function to implement the same fully-implicit scheme but for pricing an American vanilla call option. Use the following function heading in your m-file:

```
function OptVal=FD_ids_Acall_trans(S0, X, r, q, T, sigma, I, N, xmax, omega, eps)
```

In your code, use the **PSOR** algorithm with over-relaxation parameter `omega` $= 1.3$ and convergence tolerance `eps` $= 1 \times 10^{-6}$. Test your implementation with an American vanilla call option with the same option and grid parameters as in (ii). For $N = 1500$ and I taking values from 100 to 1500 in increments of 100, obtain option value estimates (with linear interpolation) for the given option. Plot these estimates versus I. Comment.

Important: Besides the two .m files for the required Matlab functions, also prepare a Matlab script file (named `A3p1.m`) to contain all Matlab statements used to generate results in (ii) and (iii). This file will be executed during the grading of the assignment. If you use separate m-file(s) to implement the PSOR algorithm, then such file(s) have to submitted as well.

A3.2 A digital (or binary) call option on an asset has the payoff function at expiry T given by

$$\varphi(S_T) = \begin{cases} 1 & S_T > X, \\ 0 & S_T \le X \end{cases}$$

where S_T is the terminal asset price and X the strike price. The value of this European-styled option at time 0 has the closed form formula:

$$\exp(-rT)N(x)$$

where r is the risk-free rate, and

$$x = \frac{\ln(S_0/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

(i) Use a Matlab m-file to implement a function which returns the value of the above digital call option. Use the following function heading in your m-file:

```
function OptVal=BS_DigitalCall(S0, X, r, q,  T, sigma)
```

Consider a 3-asset digital call option with the payoff function

$$\varphi(S_{1T}, S_{2T}, S_{3T}) = \begin{cases} 1 & \max(S_{1T}, S_{2T}, S_{3T}) > X, \\ 0 & \max(S_{1T}, S_{2T}, S_{3T}) \le X \end{cases}$$

where S_{1T}, S_{2T} and S_{3T} are the terminal prices of three correlated assets S_1, S_2 and S_3, with $T = 0.75$ year. You are given the following parameters for the three assets:

i	$S_{i,0}$	volatility σ_i	dividend q_i
1	\$9.5	0.35	0.01
2	\$10.2	0.21	0.04
3	\$8.8	0.18	0

The risk free rate is 0.05, and assets are correlated with the following correlation coefficients:

	Asset 1	Asset 2	Asset 3
Asset 1	1	0.88	-0.17
Asset 2	0.88	1	0.34
Asset 3	-0.17	0.34	1

(ii) Use a Matlab m-file to implement a function which returns the Monte-Carlo estimate to the value of this 3-asset digital call option. Use the following function heading in your m-file:

```
function MC_noCV=MC_3AssetMD(S0, X, sigma, C, r, q, T, no_samples)
```

where `S0`, `sigma`, `q` are vectors (each with 3 elements) for initial prices, volatilities and dividend yields of the 3 assets respectively, and `C` is the 3×3 correlation matrix.

(iii) Use the matlab function in (ii) to obtain option value estimates and standard errors for X =\$8.5, \$9.5, \$10.5 respectively. For each X, use a total of 30 simulation runs each with 100 price-path bundles.

(iv) Repeat (iii) with 1,000, 10,000 and 100,000 price-path bundles. Tabulate your results and comment.

(v) Write another Matlab function by modifying the above function to incorporate a control variate (a basket of three single-asset digital calls) for the purpose of variance reduction. Demonstrate numerically the effectiveness of this control variate method for the same set of option/grid parameter considered in (iii) and (iv). Tabulate your results and comment.

Important: Besides the three .m files for the required Matlab functions, also prepare a Matlab script file (named `A3p2.m`) to contain all Matlab statements used to generate results in (iii), (iv) and (v). This file will be executed during the grading of the assignment.

Due date, requirement, guidelines and regulations

(i) The due date/time for the Matlab programmes and brief report is **2359hr on 19 November, 2017**. No late submission will be accepted.

(ii) Work on the assignment problems should commence soonest possible as programming and debugging can be time consuming.

(iii) Use **Matlab** for all programming tasks.
Please add suitable amount of comments to your codes and test your codes thoroughly. The first line of each Matlab m-file should have a comment line containing the names of the group members.

(v) Prepare your report in the Windows Word format or the PDF format with a description of your work done plus supporting figures and tables etc, as well as all necessary analysis and comments.

(vi) The .doc/.pdf and all .m files should all be archived in a single Zip/Rar file. Name your .zip/.rar file with your group index (such as Gxx_Assignment.zip or Gxx_Assignment.rar where Gxx is your assigned group index), and submit it online to the IVLE workbin set up for this purpose. Only one such archive file from each group will be used in the grading process.

(vi) This assignment counts 10% towards the final assessment score of this module.

(vii) Plagiarism (copying work from fellow students, groups or others) **would not be tolerated and all parties involved would be penalized severely**.
Please refer to `http://emodule.nus.edu.sg/ac/` for more information on NUS's disciplinary process on plagiarism.

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