NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

2016/2017 **QF4102 Financial Modelling** **Semester I**

QF4102 Assignment 3

A3.1 Consider the **transformed** Black-Scholes PDE formulation:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\sigma^2}{2}\dfrac{\partial^2 u}{\partial x^2} + \left(r - q - \dfrac{\sigma^2}{2}\right)\dfrac{\partial u}{\partial x} - ru = 0, \quad x \in (-\infty, \infty), \quad t \in [0, T) \\ u(x, T) = \varphi(x) \end{cases}$$

(i) Derive the Crank-Nicolson finite difference scheme for the above PDE formulation for a European vanilla put option.

(ii) Implement a Matlab function for the Crank-Nicolson scheme derived in (i). Test your implementation with a European put option which has a strike price of \$1 and time to maturity of 1 year. Current asset price is \$0.95, volatility of asset return is 40%, dividend yield is 2% and risk free rate is 5%. Use a grid with values of x lying in the truncated domain $[-5, 2]$. Obtain option value estimates for $\Delta t = \Delta x = 0.2, 0.1, 0.05, 0.025$. Use linear interpolation where appropriate.

(iii) Estimate the convergence order of your numerical results obtained in (ii).

(iv) Write another Matlab function to implement the same Crank-Nicolson scheme but for pricing an American vanilla put option. In your code, use the **PSOR algorithm** with $\omega = 1.3$. Test your implementation with an American put option which has a strike price of \$1 and time to maturity of 0.25 year. Current asset price is \$1.15, volatility of asset return is 35%, dividend yield is 2% and risk free rate is 5%. Use a grid with values of x lying in the truncated domain $[-5, 2]$. Obtain option value estimates for $\Delta t = \Delta x = 0.2, 0.1, 0.05, 0.025$. Use linear interpolation where appropriate.

A3.2 Use a Matlab m-file to implement a function which obtains a Monte-Carlo estimate for the option value of a 3-asset option with the payoff function

$$(X - \min(S_{1T}, S_{2T}, S_{3T}))^+$$

where S_{1T}, S_{2T} and S_{3T} are the terminal prices of three correlated assets S_1, S_2 and S_3, with $T = 0.75$ year. You are given the following parameters for the three assets:

i	$S_{i,0}$	volatility σ_i	dividend q_i
1	\$11.5	0.25	0.01
2	\$14.5	0.31	0.04
3	\$9.5	0.48	0

The risk free rate is 0.05, and assets are correlated with the following correlation coefficients:

	Asset 1	Asset 2	Asset 3
Asset 1	1	0.78	−0.27
Asset 2	0.78	1	0.32
Asset 3	−0.27	0.32	1

(i) Use the Matlab function to obtain option value estimates and standard errors of the given option with $X = \$12, \$13, \$14$ respectively. For each X, use a total of 30 simulation runs each with 100 price-path bundles.

(ii) Repeat (i) with 1,000, 10,000 and 100,000 price-path bundles.

(iii) Write another Matlab function by modifying the above function to incorporate a control variate (a basket of three single-asset vanilla puts) for the purpose of effective variance reduction. Demonstrate numerically the effectiveness of this control variate method for the same set of option parameter considered in (i) and (ii).

Tabulate your results and comment.

Due date, requirement, guidelines and regulations

(i) The due date/time for the Matlab programmes and brief report is **2359hr on 13 November, 2016**. No late submission will be accepted.

(ii) Work on the assignment problems should commence soonest possible as programming and debugging can be time consuming.

(iii) Use **Matlab** for all programming tasks.
Please add suitable amount of comments to your codes and test your codes thoroughly. The first line of each Matlab m-file should have a comment line containing the names of the group members.

(v) Prepare your report in the Windows Word format or the PDF format with a description of your work done plus supporting figures and tables etc, as well as all necessary analysis and comments.

(vi) The .doc/.pdf and all .m files should all be archived in a single Zip/Rar file. Name your .zip/.rar file with your group index (such as Gxx_Assignment.zip or Gxx_Assignment.rar where Gxx is your assigned group index), and submit it online to the IVLE workbin set up for this purpose. Only one such archive file from each group will be used in the grading process.

(vi) This assignment counts 10% towards the final assessment score of this module.

(vii) Plagiarism (copying work from fellow students, groups or others) **would not be tolerated and all parties involved would be penalized severely**.
Please refer to `http://emodule.nus.edu.sg/ac/` for more information on NUS's disciplinary process on plagiarism.

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