

Space: the final frontier

FW 891

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Purpose

- Background
- Recognizing dependency in simple models
- How can we deal with dependency problems
 - Introduction to spatial random effects

Why do we care about space

- Most of our problems in ecology and mgmt are inherently spatial

Special Feature

Space: The Final Frontier for Ecological Theory¹

Most ecologists realize that the interplay of dispersal, disturbance, and spatial mosaics can profoundly alter the outcome of species interactions. Indeed, an appreciation of the spatial dimension and spatial heterogeneity is well established in natural history. Nonetheless, for many ecologists, “spatial complications” are used as a catch-all for explaining away surprising results, or as a condemnation of ecological theory that is accused of oversimplification because of its neglect of spatial variation. What is missing are serious experiments that explicitly test major hypotheses emerging from recent theoretical explorations of spatial effects. The purposes of this Special Feature are to alert ecologists to the general insights emerging from spatial models, and to show how these concepts apply to the natural world.

Nearby things tend to be more alike

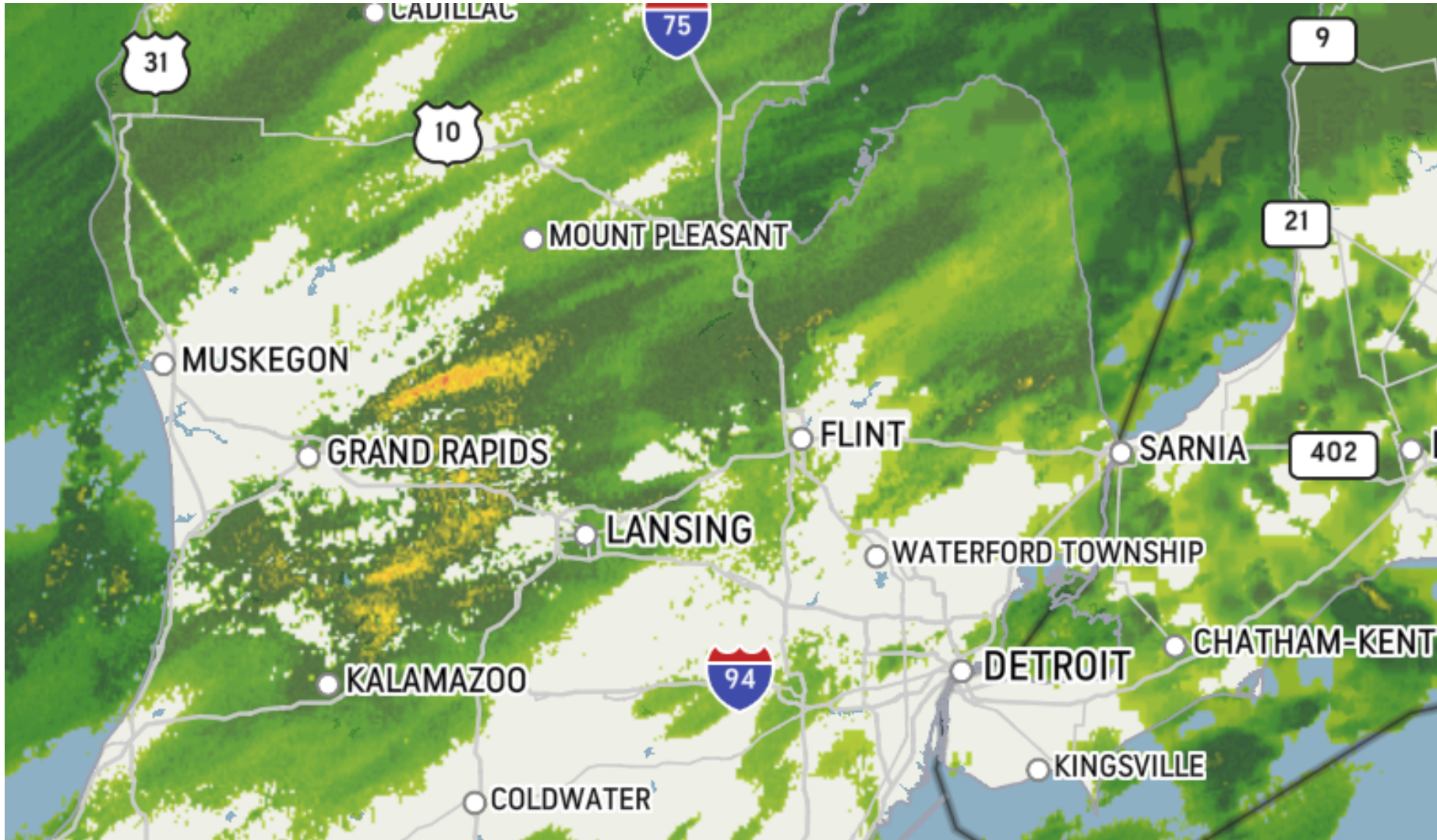


Photo credit: accuweather.com

Paraphrased from Fisher 1935 and Tobler 1970

However...

- It is really hard to estimate spatial (or spatial-temporal) dynamics in most statistical models



Spatial dependency poses issues for traditional experimental design



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- We have hustled spatial dependence to the back door
 - Not rejecting this paradigm of experimental inference

Observational data represent the wilder side of science and statistical inference



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- Causal inference more challenging than in controlled experiments
- Notions of space and time even more relevant?
- This is most of our data!

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Hierarchical modeling as a powerful tool for ecological inference

- Unifying field of statistics
- Many technical benefits
- Helps deal with dependency
 - i.e., Hurlburt's pseudoreplication
- So we are going to use these to address the spatial dependency issue

Recognizing dependency in statistical models

What do we look for when checking statistical models?

- Zuur and Ieno 2016 offered a 10 step approach

Protocol for presenting regression-type analyses

1. State appropriate questions
2. Visualize the experimental design
3. Conduct data exploration
4. Identify the dependency structure in the data
5. Present a statistical model
6. Fit said model
7. “validate” the model
8. Interpret and present output
9. Model visualizations
10. Simulate from the model

The take home point from Zuur and Ieno

“Before starting the analysis carefully consider which observations for your response variable can be dependent. If dependency is present, apply a statistical technique that is able to cope with it”

Thinking about dependency

- Suppose samples are collected by different laboratories
 - Perhaps due to differences in techniques or scientists themselves among laboratories, we likely have among-lab dependency

Thinking about dependency

- Suppose related species are analyzed together in one analysis
 - Perhaps due to similarities between the species we have among-species dependency

Thinking about dependency

- Suppose we take repeated measurements on individual rats
 - Perhaps due to individual differences among rats, the data scientists collect from one rat is more likely to be similar to other data from collected from that rat

What dependency is not

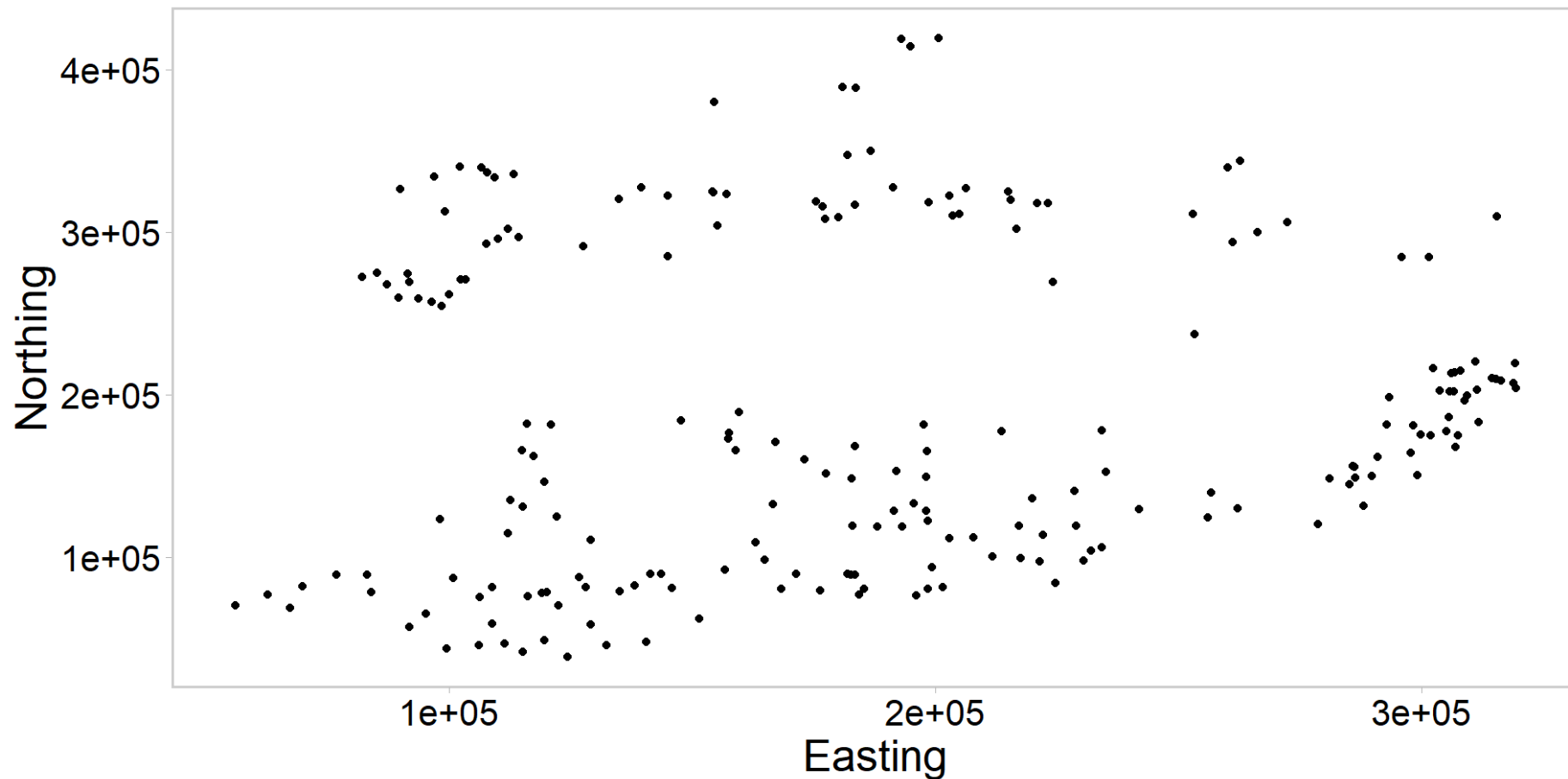
- Two sites have high bird counts, and they weren't sampled by the same person or lab
- However, both sites have a low cover measurement, which is the cause of the high bird counts



Visualize experimental design: spatial dependency

```
1 library(tidyverse)
2 library(ggqfc)
3 data <- read.table(file = "data/ph_dat.txt", header = TRUE, dec = ".")
4 data %>%
5   ggplot(aes(x = Easting, y = Northing)) +
6   geom_point(shape = 16) + theme_qfc()
```

Visualize experimental design: spatial dependency

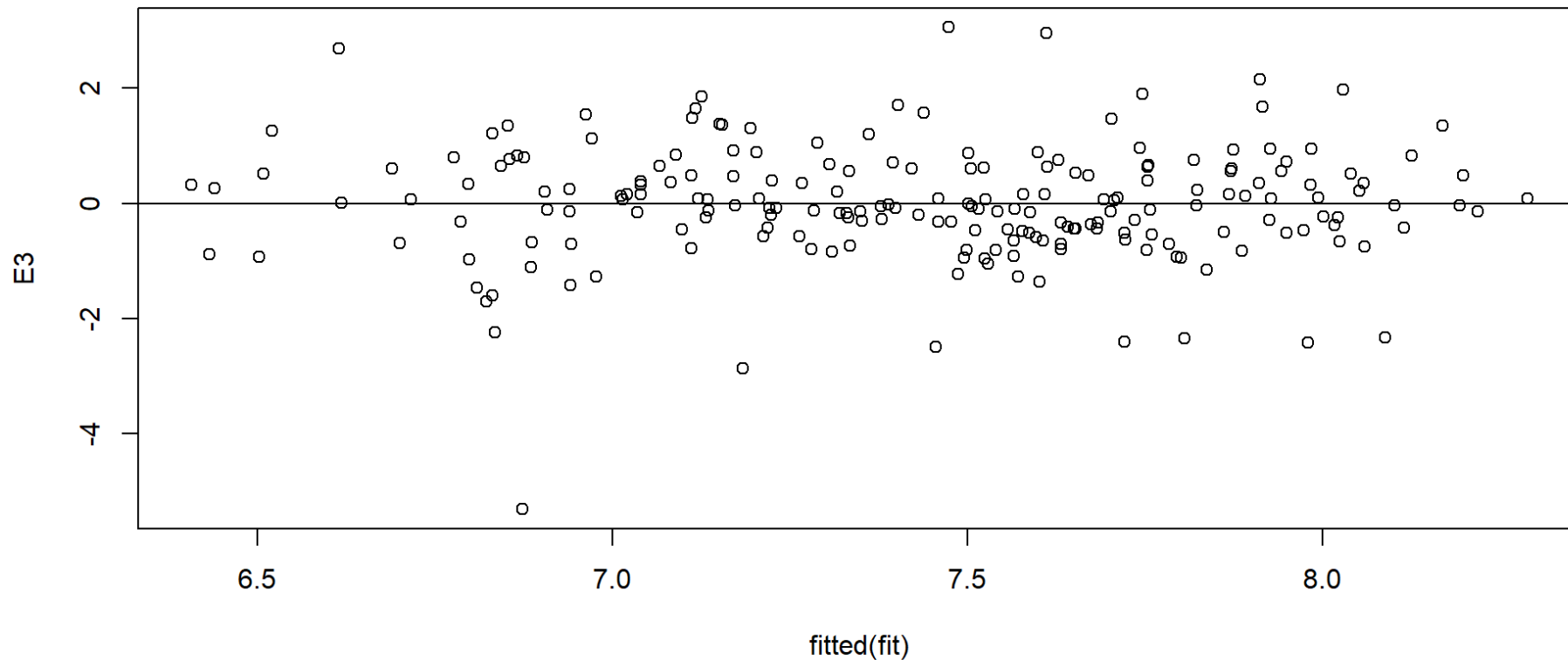


Running an analysis

```
1 data$fForested <- factor(data$Forested,  
2                           levels = c(1, 2),  
3                           labels = c("Yes", "No"))  
4 data$LOGAltitude <- log10(data$Altitude)  
5  
6 # Model selected using some stepwise AIC approach  
7 fit <- lm(pH ~ LOGAltitude + SDI + fForested + LOGAltitude:fForested,  
8          data = data)  
9 E3 <- rstandard(fit) # could also use resid(), but this is better way
```

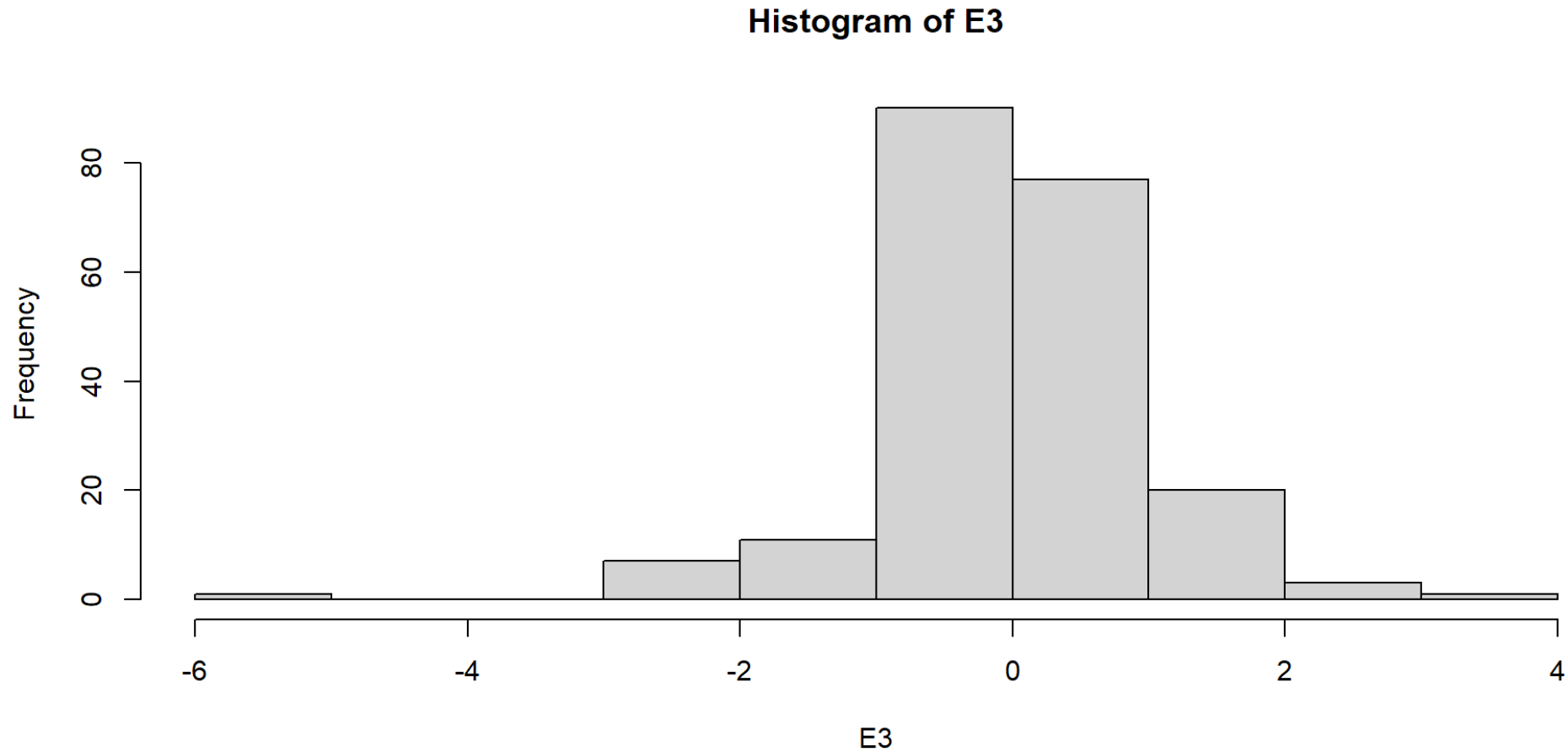
Validating the model

```
1 plot(x = fitted(fit), y = E3)
2 abline(h = 0, v = 0)
```



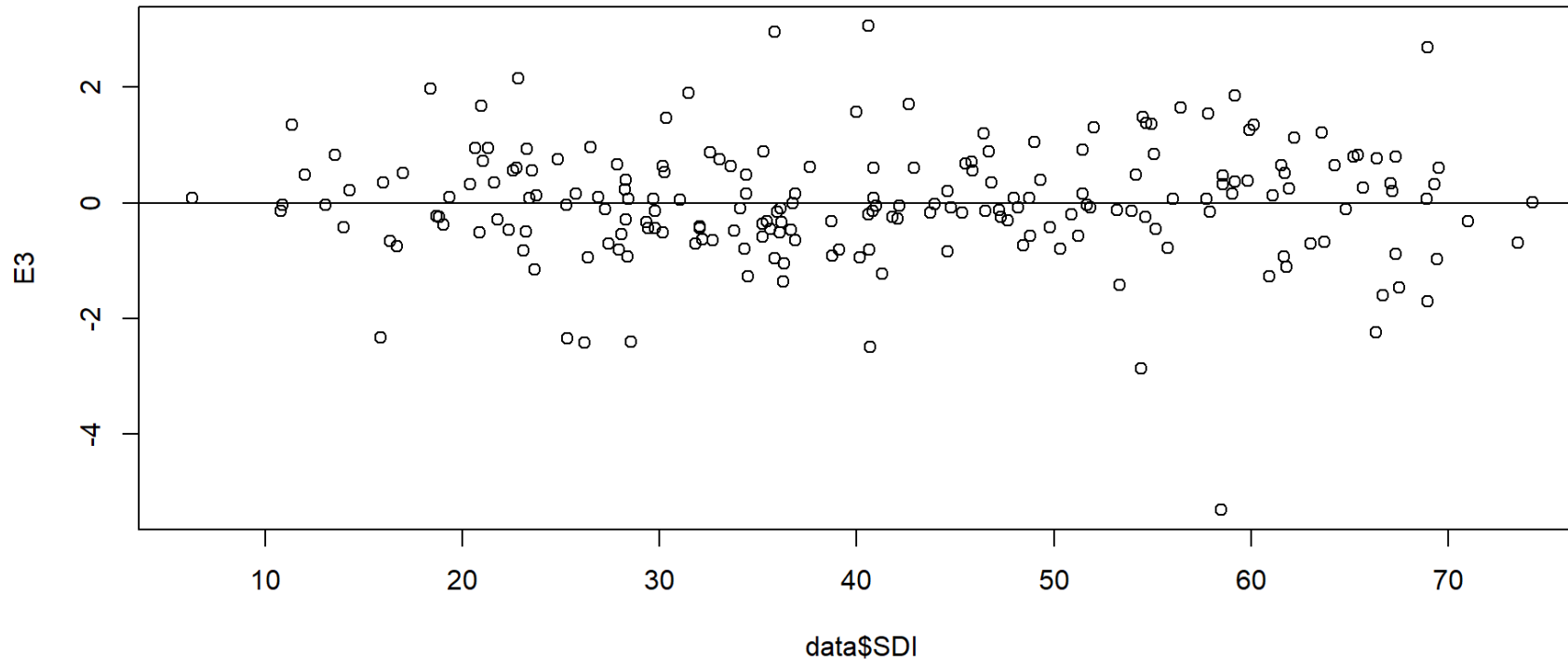
Validating the model

```
1 hist(E3)
```



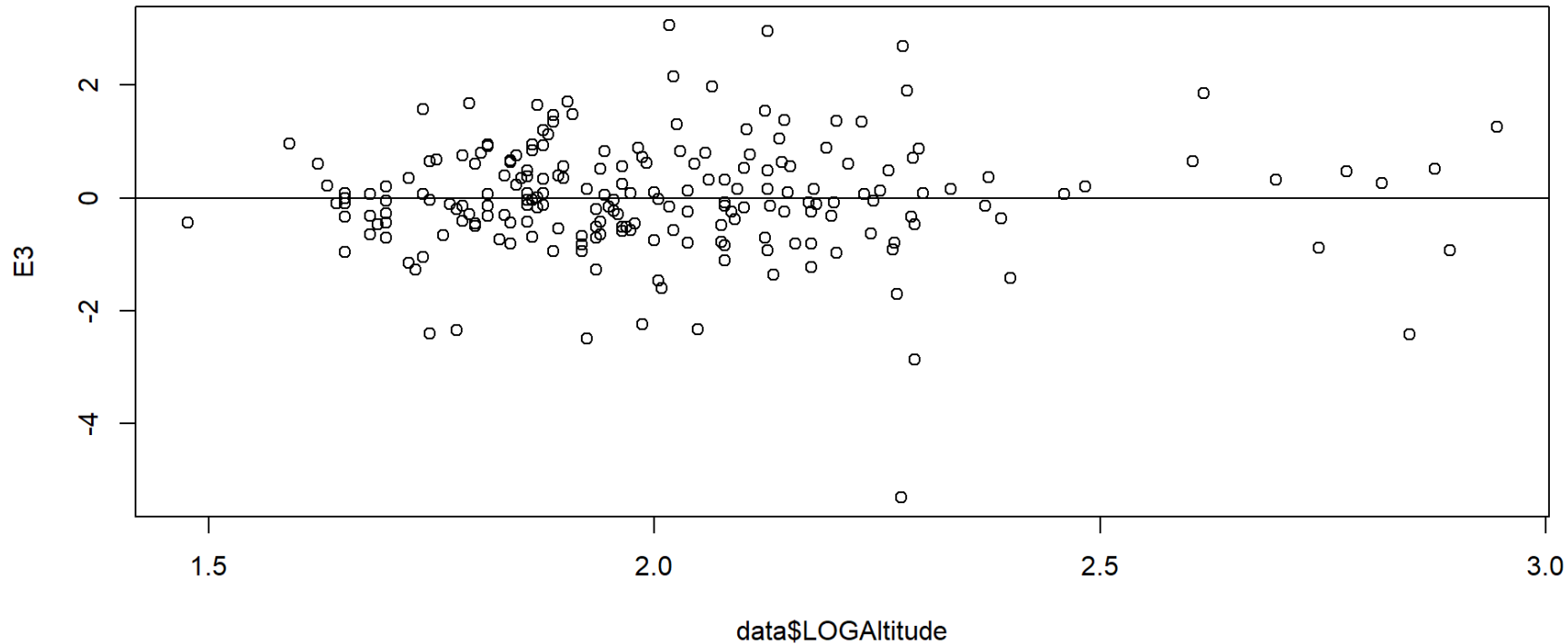
Validating the model

```
1 # Independence due to model misfit  
2 plot(x = data$SDI, y = E3) # residuals vs. predictors  
3 abline(h = 0, v = 0)
```



Validating the model

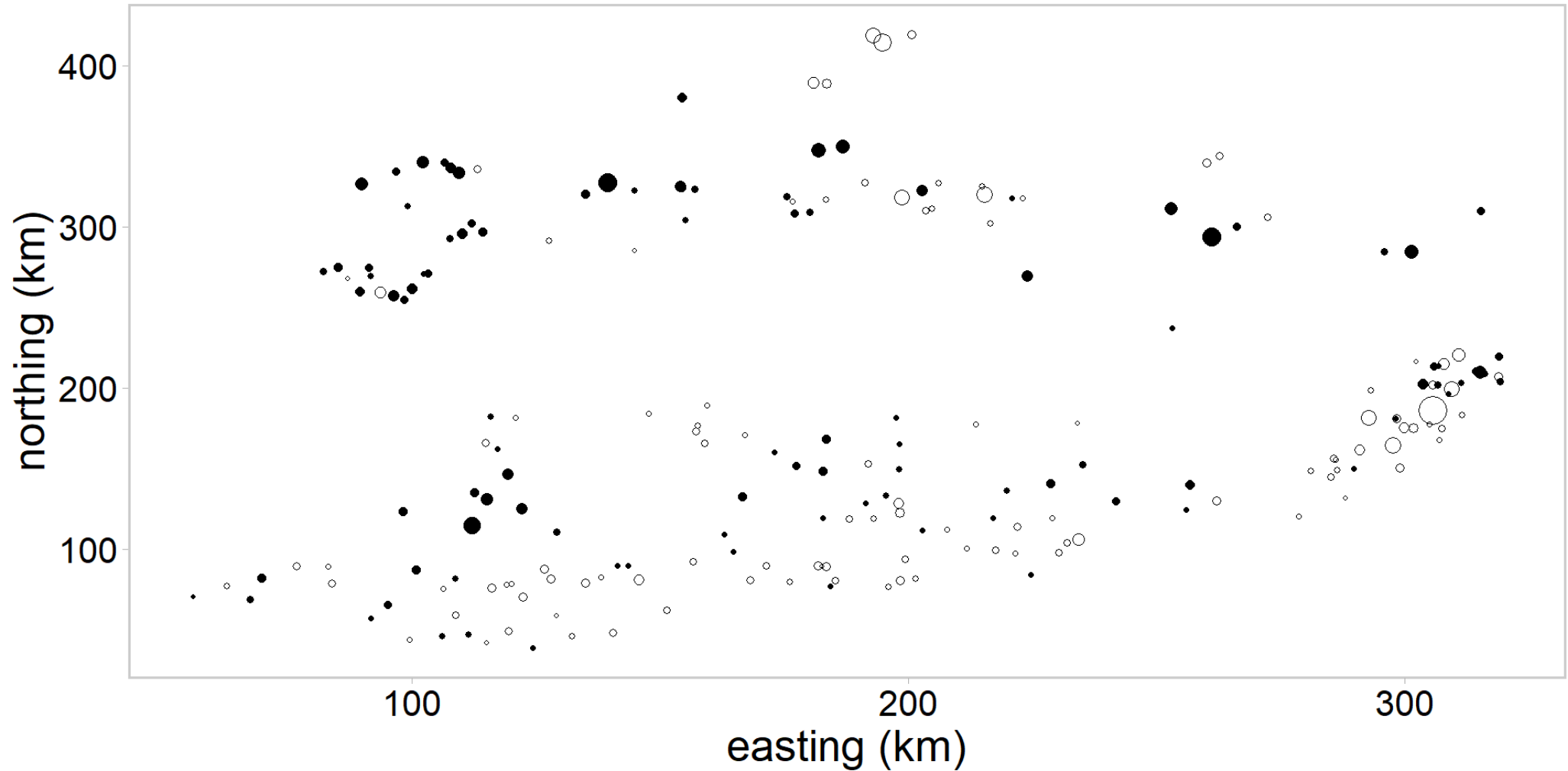
```
1 # Independence due to model misfit
2 plot(x = data$LOGAltitude, y = E3) # residuals vs. predictors
3 abline(h = 0, v = 0)
```



Can we find spatial patterns?

```
1 # Can we find spatial patterns in the residuals?
2 data$E3 <- resid(fit)
3 data$my_cex <- 3 * abs(data$E3) / max(data$E3) + 0.75
4 data$sign <- as.numeric(data$E3 >= 0) + 1
5 data$my_pch <- c(1, 16)[data$sign]
6
7 # Convert Easting / Northing to km
8 data$easting_km <- data$Easting/1000
9 data$northing_km <- data$Northing/1000
10
11 data %>%
12   ggplot(aes(x = easting_km, y = northing_km)) +
13   geom_point(size = data$my_cex, shape = data$my_pch) +
14   ylab("northing (km)") + xlab("easting (km)")
```

Can we find spatial patterns?



A point about the types of spatial data

- Point process vs. areal data

Introduction to spatial random effects

Introduction to spatial random effects

- Spatial effects can be incorporated many ways
 - Predictors of model's mean response
 - Models of covariance (e.g., kriging)
 - splines
 - hierarchical intercepts indexed by region
 - Gaussian Processes, etc.
- Recent extensions include modeling spatial deviations as random effects using Gaussian Random Fields

Gaussian Random Fields (GRFs)

- GRFs represent a two-dimensional version of Gaussian processes and define the expected value, variance, and covariance of random draws from a Multivariate Normal Distribution (MVN)
- **Gaussian processes** in this context are analogous to kriging
- Simply put: interpolation technique that allows us to model the covariance among sites according to some function

Spatial Gaussian Processes

- We have a few options with the kernel function
 - Exponential, squared-exponential, matern, etc.
- We will start with an exponential kernel function

The exponential covariance function

$$\epsilon_s \sim \text{MVN}(0, \Sigma)$$

where Σ is a covariance matrix with:

$$\Sigma_{i,j} = \sigma_{gp}^2 \exp(-\text{distances}_{i,j} / \theta_{gp}), \text{ if } i \neq j$$

$$\Sigma_{i,j} = \sigma_{gp}^2, \text{ if } i = j$$

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- ϵ_s is a spatially explicit random effect
- θ_{gp} controls how quickly the correlation decays between locations
- σ_{gp}^2 is the marginal variability of the spatial function at all locations
- $\text{distances}_{i,j}$ is a matrix describing Euclidean distance between sample locations

An spatially explicit model

- pH measured at a n sites
- Want to estimate average pH and its uncertainty

$$y_s \sim N(\mu_s, \sigma_{obs}^2)$$

[likelihood]

$$\mu_s = \beta_0 + \varepsilon_s$$

[mean prediction]

$$\varepsilon_s \sim \text{MVN}(0, \Sigma)$$

[spatial random effects]

where

$$\Sigma_{i,j} = \sigma_{gp}^2 \exp(-\text{distances}_{i,j} / \theta_{gp}), \text{ if } i \neq j$$

[exponential kernel]

$$\Sigma_{i,j} = \sigma_{gp}^2, \text{ if } i = j$$

[marginal spatial variance]

- note the s

**To the R and Stan code to
implement this model**

Recap

- Talked about the importance of space in applied ecological settings
- Discussed a bunch of ways to recognize dependency in applied datasets
- Showed one way (GRFs and GPs) to incorporate spatial effects into hierarchical models

Recap



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