Software tools for Maximum Likelihood Estimation

More Examples

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Jim Bence

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Evolving list of examples (check again)

- Age comp data
  - multinomial and Dirichlet-multinomial
- Multivariate normal data
- Correlated random effects for vonB model
- Other undocumented examples

Age comp example

- Constant recruitment and survival (necessary in example because the only type of data being used is age comps).
- Initially we assume we have good prior estimates of recruitment and assume they are known.
  - Alternatives
    - Estimate selectivity for each age forced to increase monotonically
    - Estimate selectivity via a logistic function
- multiple samples of age composition. Initially assumed to be multinomial, then Dirichlet multinomial

Model

$$N_a = R\exp(-Z(a-r)), r \leq a \leq \max$$

$$C_{i,a} = q_i S_a N_a$$

$$p_a = \frac{S_a N_a}{\sum_{j=r}^{\max} S_j N_j}$$

$$\underline{n}_i \sim \mathrm{multinom}\left(n_i, \underline{p}_a\right), n_i = \sum_{a=r}^{a=\max} n_{i,a}$$

Setting effective sample size

- Multinomial apps often use data as proportions and “effective sample size” (ESS)
- In such applications the negative log likelihood was written in terms of proportions and ESS
- In RTMB emulate by providing dmultinom product of proportions and ESS as the data (don’t set size!)
  - The NLL returned will depend on the ESS but ESS cannot be estimated (nothing new for multinomial here)
  - Works because the internal dmultinom calcs allow non-integers.

Dirichlet-Multinomial

- Compound distribution, p vector comes from dirichlet then used as parameter of of multinomial.
- Used to introduce overdispersion relative to multinomial
- Using linear form where ESS proportional to sample size

$$\underline{n}_i \sim \mathrm{DirMult}(n_i, \underline{\alpha}_i), \alpha_{i,a} > 0$$

$$E\left(n_{i,a}\right) = n_i \frac{\alpha_{i,a}}{\sum_j \alpha_{i,j}}, ESS_i = \sum_a \alpha_{j,a}$$

$$\alpha_{i,a} = \theta n_i p_a, ESS_i = \theta n_i$$

Multivariate Normal Distribution with unstructured var-cov matrix

- Parameterize so that standard deviations and correlations can be calculated and converted into a variance-covariance matrix
- Estimate the standard deviations on log-scale
- Not sufficient to restrict correlations to -1 to 1 as some combinations of correlations are not consistent with feasible correlation matrices
  - e.g., if *a* and *b* are highly negatively correlated, you can't have *c* highly positively correlated with a and highly negatively correlated with *b*

Converting SDs and correlation matrix to a var-cov matrix

- Typical element of correlation matrix P is $\rho_{i,j} = \sigma^2_{i,j}/(\sigma_{i,i}\sigma_{j,j})$
  - $\sigma^2_{i,j}$ is a typical element of the variance-covariance matrix, with $\sigma^2_{i,j} = \rho_{i,j}\sigma_{i,i}\sigma_{j,j}$
- Matrix algebra to convert correlation matrix and SDs to var-cov matrix

$$\Sigma = \mathrm{diag}(\sigma_{1,1}, \cdots, \sigma_{k,k})\mathrm{P}\,\mathrm{diag}(\sigma_{1,1}, \cdots, \sigma_{k,k})$$

Motivation for being able to deal with unstructured MVN

- You might actually want to assume some data are multivariate normal (e.g., some catch-at-age assessments assume log of catch-at-age MVN). But often we impose structure on correlations.
- You might want to allow different random effects to be correlated. E.g., we might expect the vonB function parameters for a pond will be either positively or negatively correlated with one another.
- Illustrate with another very simple example

Simple application of unstructured multivariate normal

- We have a set of multivariate observations assumed to come from a multivariate normal distribution
- We estimate the mean vector and parameters that determine the variance-covariance matrix
  - Illustrate how to estimate parameters on real number line that determine a “legal” correlation matrix
  - Illustrate how to get the variance-covariance matrix from correlation matrix and vectors of standard deviations
  - See “unstructured” example R script

Correlated random effects for vonB model

- We have already seen how to make a vonB parameter a random effect. We could have made more than one parameter random at the same time using dnorm
  - This assumes the random effects are independent but often that is not plausible [when fish ultimately get big (Linf) they might approach asymptotic size more slowly (K)].
- We can implement this by having the vonB parameters follow a multivariate normal distribution
- Larger and more realistic example data set, reparameterized the vonB (use L2 rather than t0)

New growth data set

- 20 different ponds
- Each observation gives the pond ID and the length and age for an individual fish

Reparameterized vonB

- This is not changing the underlying growth function just how it is parameterized
- Standard parameterization has been criticized for t0 being hard to interpret and for correlation with other parameters
- Use L2 (length at age 2) instead. Chose age within range of observed data

$$L_a = \widetilde{L}_2 + \left(L_\infty - \widetilde{L}_2\right) \exp(-K(a-2))$$

The multivariate random effects model

$$\underline{\omega}_i = \left[\log L\infty_i, \log K_i, \log \widetilde{L2}_i\right]^T \sim MVN\left(\left[\overline{\log L\infty_i}, \overline{\mathrm{l}}\right.\right.$$

$$L_{i,j} \sim N\left(\mathrm{vonb}(\underline{\omega}_i, a_{i,j}), \sigma^2_{L,i,j}\right)$$

$$\sigma_{L,i,j} = \exp[\mathrm{int} + \mathrm{slp} * \mathrm{vonb}(\underline{\omega}_i, a_{i,j})]\,\mathrm{vonb}($$