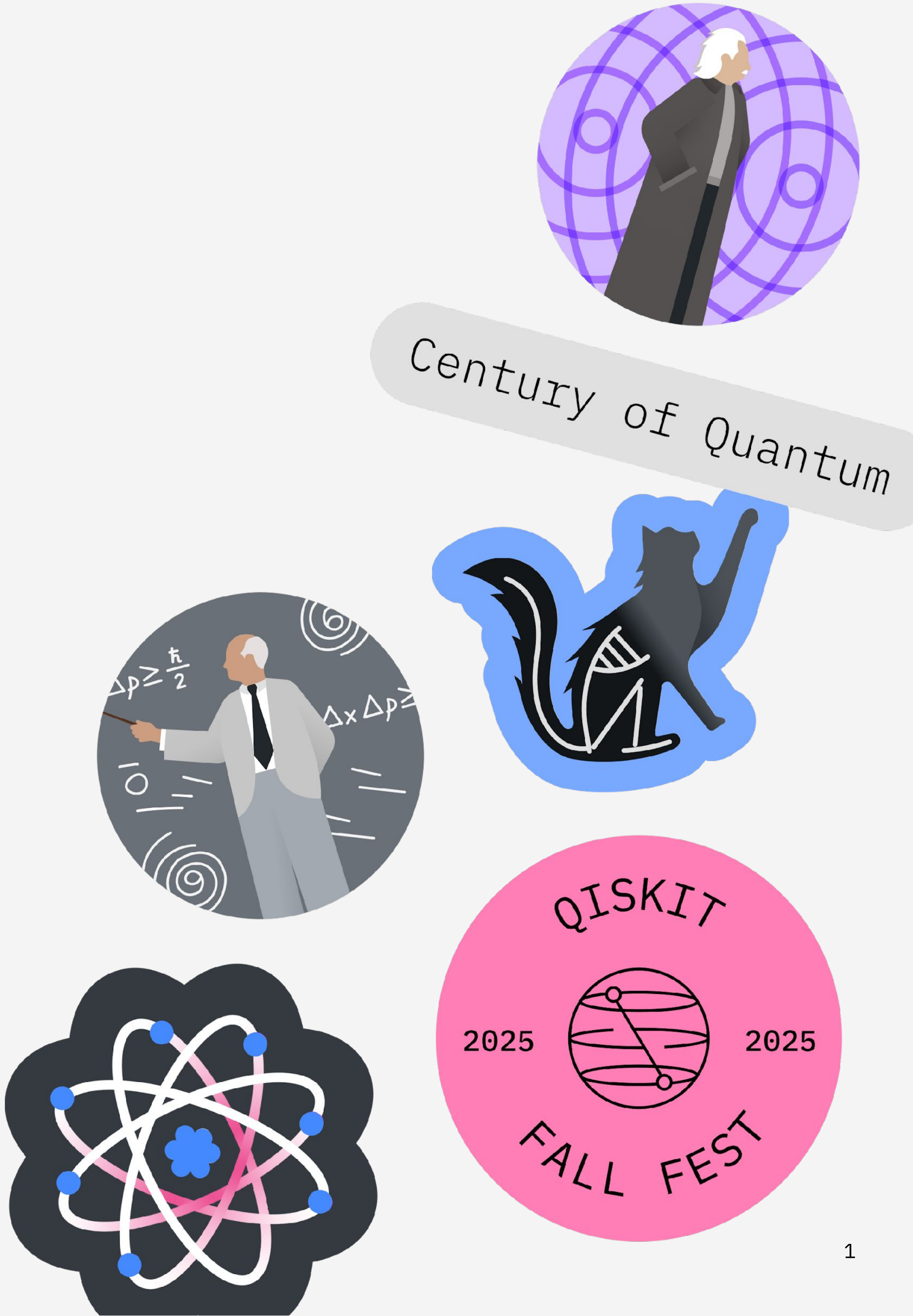
Fundamental Quantum Algorithms:

An Overview of Deutsch's/Deutsch-Jozsa Algorithms



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What are Quantum Algorithms?

Classical Algorithms:

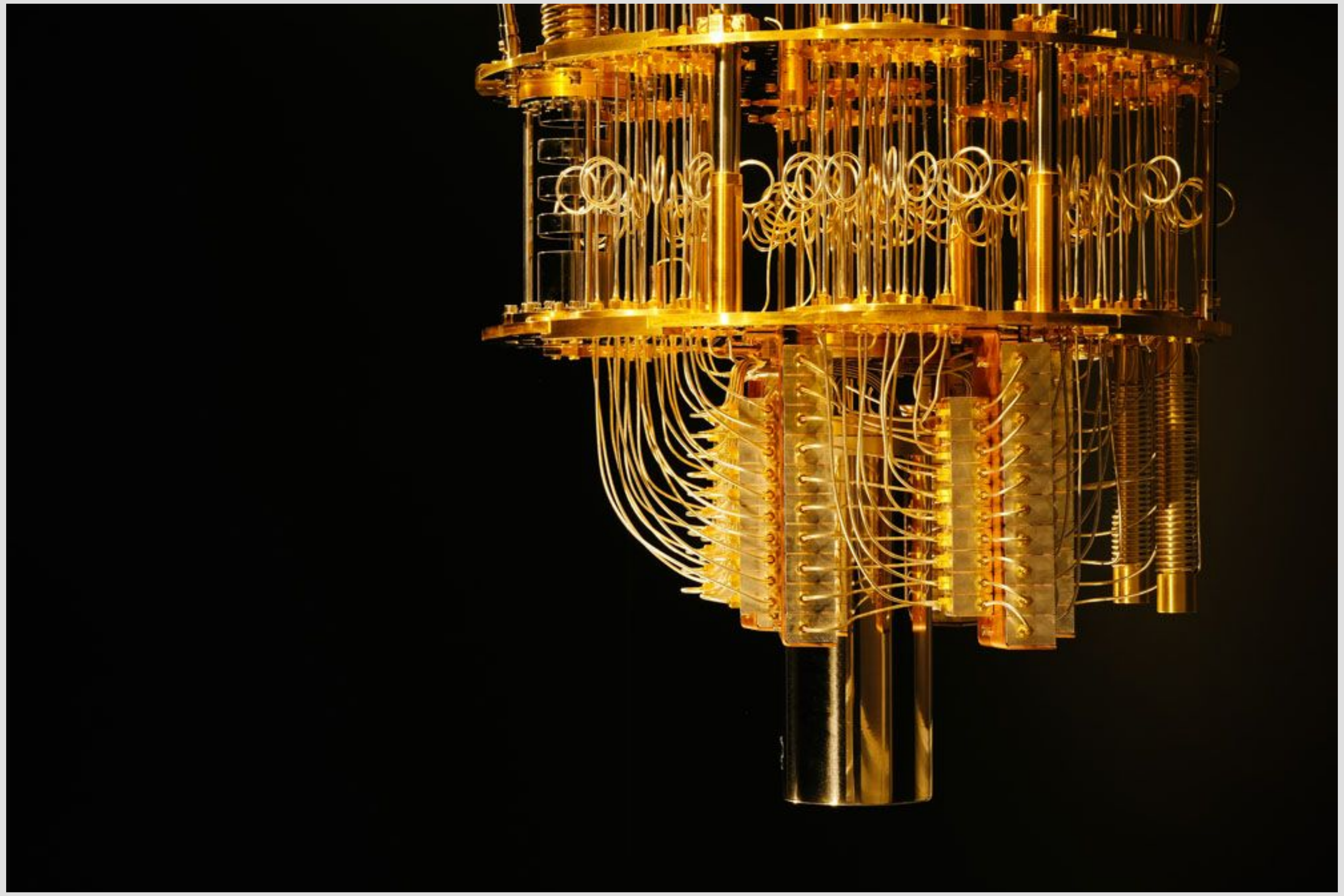
- A step-by-step procedure used to solve a problem
- Performed on a classical computer

- Relationship between algorithms and computer hardware

What are Quantum Algorithms?

Quantum Algorithms:

- A step-by-step procedure with a quantum instruction set
- Performed on a quantum computer
- Utilize features of quantum computation such as superposition or entanglement



What are Quantum Algorithms?

General Quantum Algorithm Design:

1. Encode classical input into a quantum state

2. Perform operations to transform that state

3. Obtain a quantum state which encodes the solution

4. Perform measurement

Deutsch's Algorithm

- Developed by David Deutsch in 1985
- Example problem for which a quantum computer could solve something more efficiently than a classical computer
- Utilizes quantum parallelism and interference

David Deutsch
Photo: Hanna-Katrina Jędrosz

Deutsch’s Algorithm: Objective

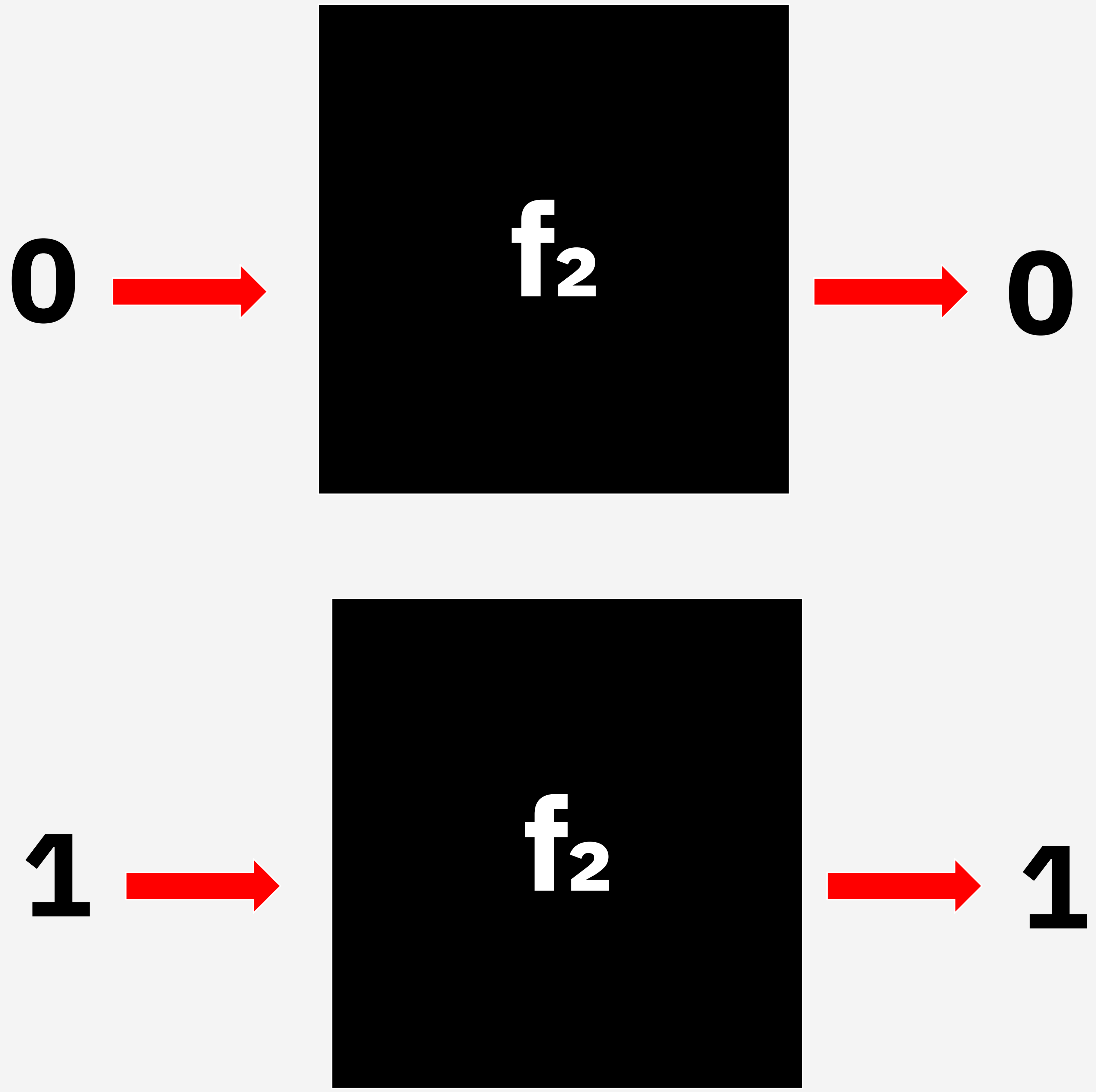
- Determine whether an unknown function is **Constant** or **Balanced**.
- input bit, x = {0, 1}
- output bit, f(x) = {0, 1}

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
0	0	0	1	1
1	0	1	0	1

Deutsch’s Algorithm: Classical Approach

- Requires 2 queries

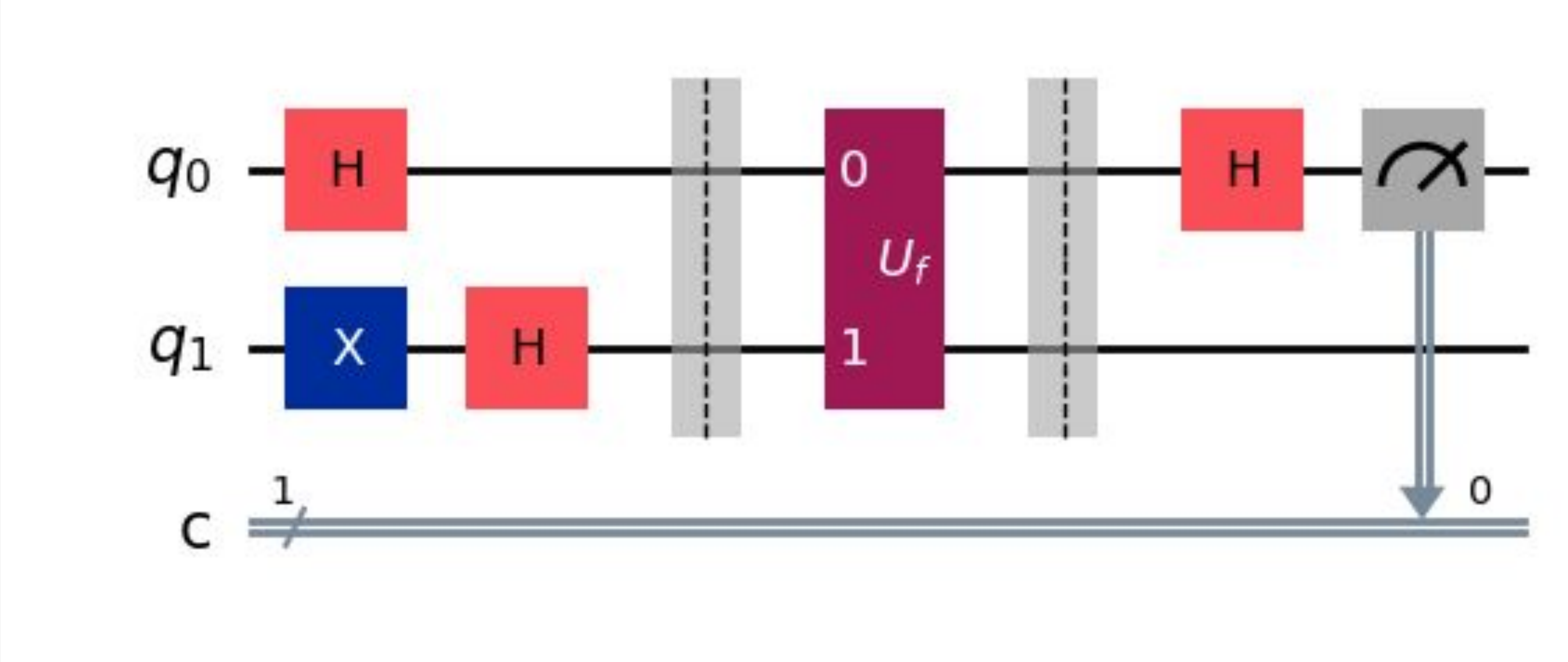
x	$f_2(x)$
0	0
1	1



Balanced!

Deutsch’s Algorithm: Quantum Approach

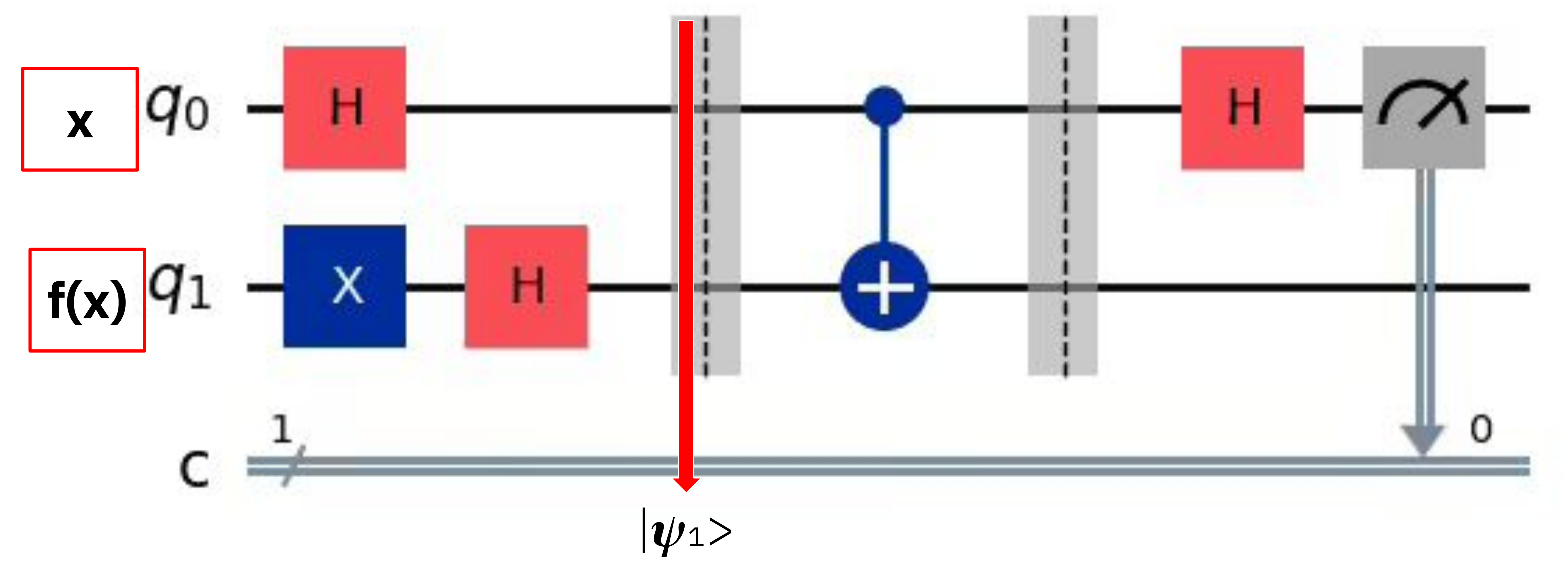
- Requires 1 query



$$\begin{cases} 0 & \text{if constant} \\ 1 & \text{if balanced} \end{cases}$$

x	$f_2(x)$
0	0
1	1

Deutsch's Algorithm: Circuit Explanation



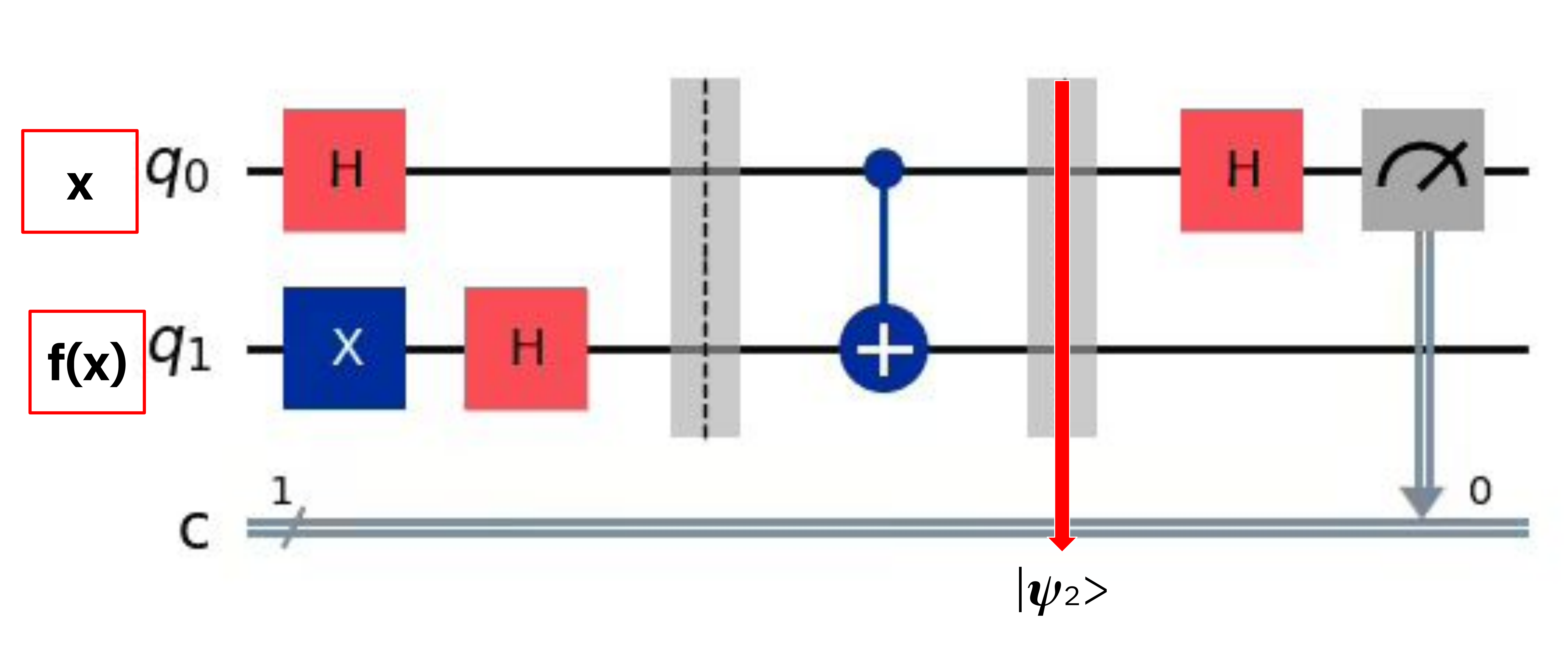
x	$f_2(x)$
0	0
1	1

$$q_0 : \frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$$

$$q_1 : \frac{1}{\sqrt{2}}\Big(|0\rangle - |1\rangle\Big)$$

$$|\psi_1\rangle = \frac{1}{2}\Big((|0\rangle - |1\rangle)\,|0\rangle + (|0\rangle - |1\rangle)\,|1\rangle\Big)$$

Deutsch's Algorithm: Circuit Explanation



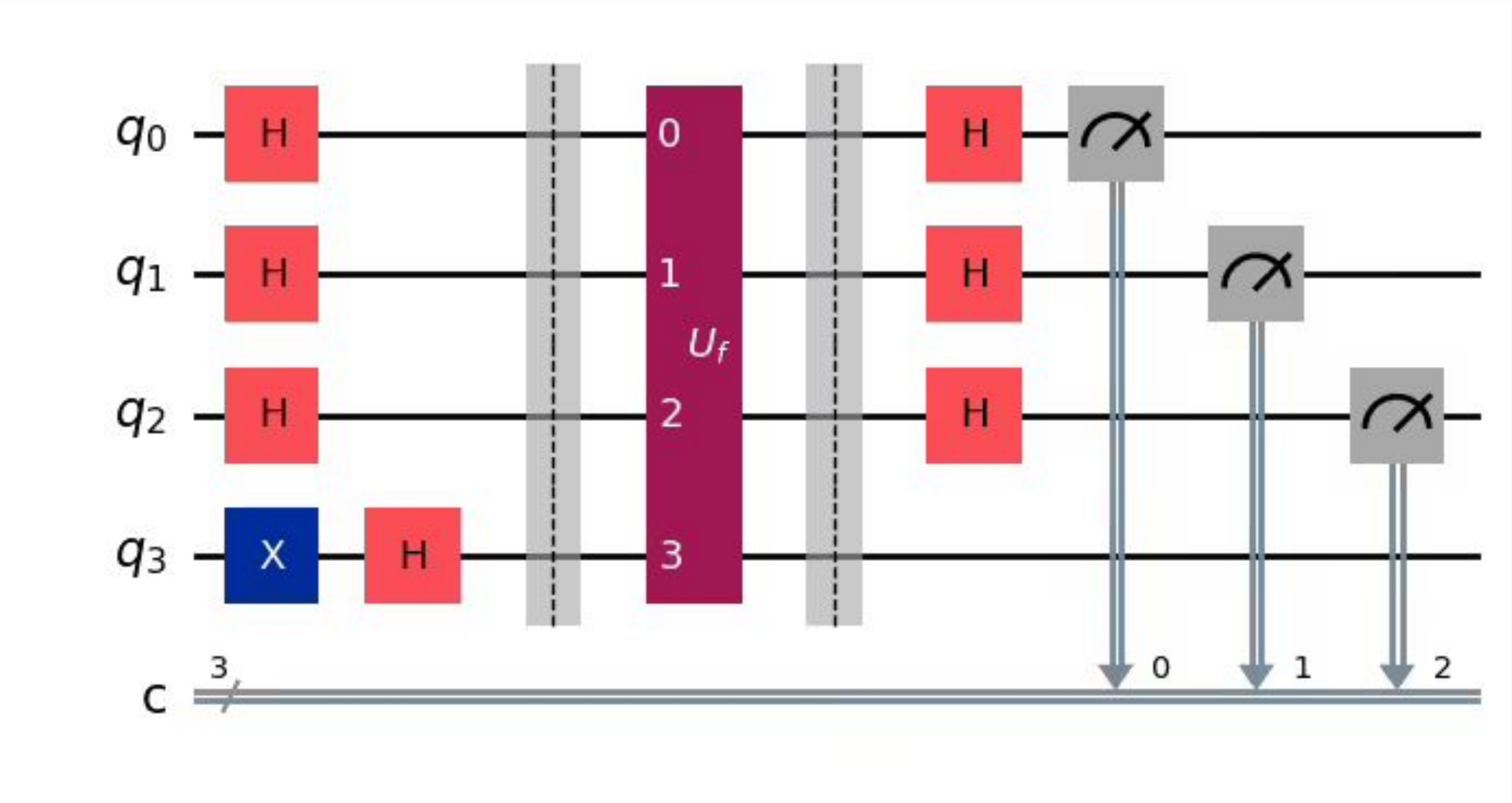
x	$f_2(x)$
0	0
1	1

$$|\psi_2\rangle = \frac{1}{2}\Big((|0\rangle - |1\rangle)\,|0\rangle \boxed{-} (|0\rangle - |1\rangle)\,|1\rangle\Big)$$

$$\begin{cases} 0 & \text{if constant} \\ 1 & \text{if balanced} \end{cases}$$

Deutsch-Jozsa Algorithm

- Published in 1992
- Multi-bit extension of Deutsch's Algorithm
- Pictured: Deutsch-Jozsa for a function with 3 input bits
- **$O(2^n) \rightarrow O(1)$**



Deutsch’s Algorithm: Takeaways

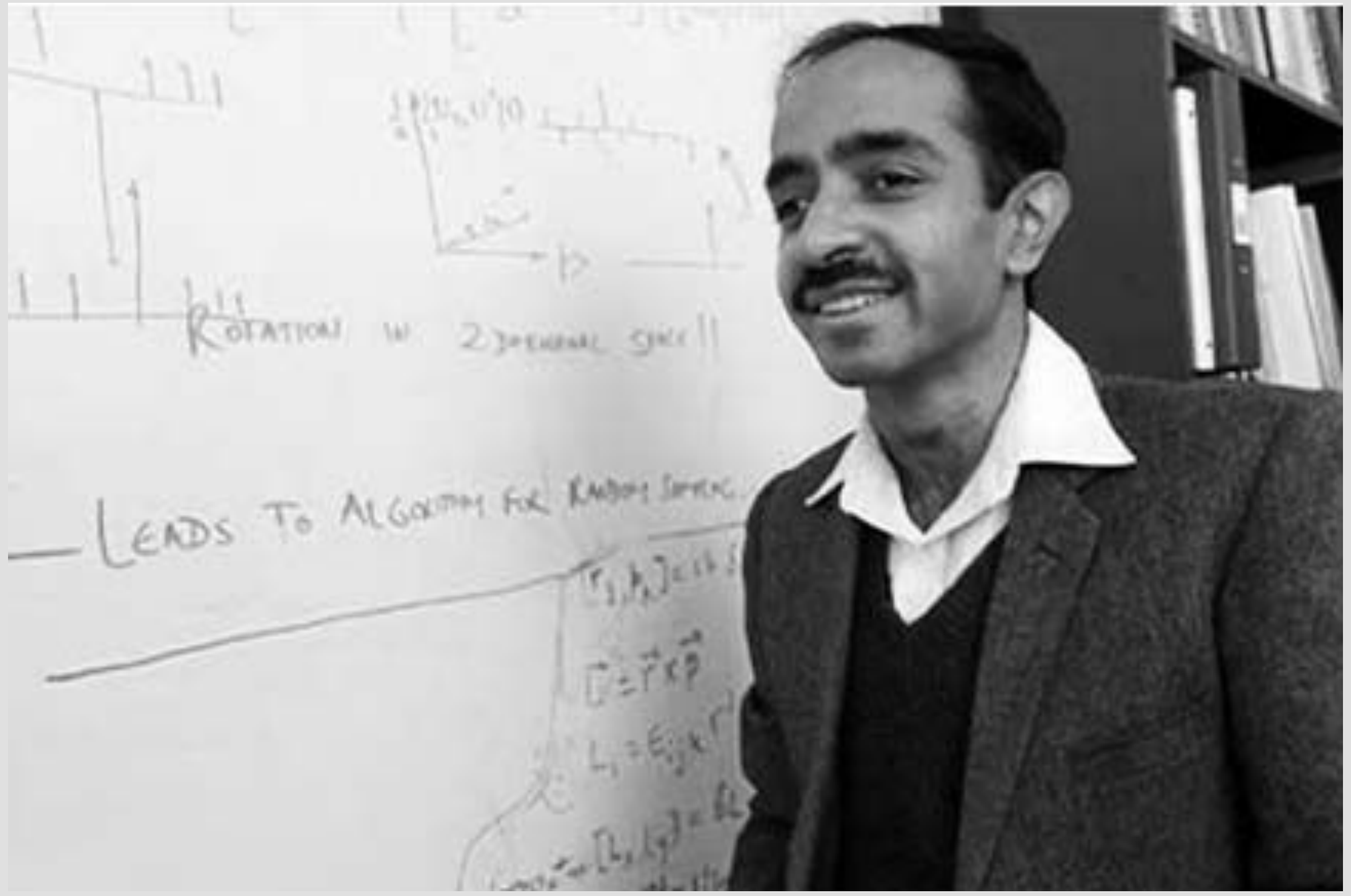
- Quantum algorithms take advantage of creative and often subtle quantum phenomenon
- The problem it solves is of little practical interest
- Are there more interesting problems to solve using quantum algorithms?

What problems suit QCs?

- Quantum Simulation
- Quantum algorithms based on the Fourier Transform
- Quantum Search Algorithms

Grover’s Algorithm (1996): Practical Algorithm

- **Goal:** Find a specific item in an unsorted search space faster than brute force.
- **Quantum Complexity:** $O(\sqrt{N})$ time
- A quadratic speedup over classical algorithms that require $O(N)$ queries



Lov Grover

IBM Quantum Learning Materials

- Deutsch's/Deutsch-Jozsa Algorithms:

  https://quantum.cloud.ibm.com/learning/en/modules/computer-science/deutsch-jozsa

- Grover's Algorithm:

  https://quantum.cloud.ibm.com/learning/en/modules/computer-science/grovers

Thank you!