Quantum Game Theory

Project Description:

Quantum computing is a new type of qubit enabled computing paradigm based on quantum properties such as superposition, interface and entanglement for data processing and other tasks. It can be used to work on problems traditional supercomputers would not be able to handle efficiently. Classical game theory is a process of modeling that is widely used in AI applications. The extension of this theory to the quantum field is the quantum game theory. It can be a promising tool for overcoming critical problems in quantum communication and the implementation of quantum artificial intelligence. This project began with learning quantum computing and game theory, then followed by development of a system that applies quantum computing to game theory and analyzes their performance.

Introduction/Scope:

Game theory is a field of math used to analyze strategies of two or more complying participants. Game theory was limited to a classical approach, but the development of quantum mechanics and quantum computing provided an alternative approach. This approach allows players to play the classical games using quantum moves which produces outcomes that classical game mechanics would render impossible. This method is known as quantum game theory, which utilizes properties of quantum mechanics. The way to implement those properties in quantum computing is by using quantum bits, better known as qubits. They represent information similar to the fundamental bit of a conventional computer. However, single qubits can represent two normal bits of information by simultaneously representing a 0 or 1. Therefore, two qubits can represent four normal bits of information. By following this logic, x number of qubits can be equated to 2^x bits. This goes to show the advantage quantum computing has in the computing field.

The purpose of this project is to investigate the potential of quantum computing over classical computing in game theory. This can be done through developing a system that applies quantum properties such as superposition and entanglement to traditional games. The games that are being analyzed include coinflip, prisoner's dilemma, survival of the fittest, Monty hall, and tic tac toe. The results will demonstrate the performance difference between that of classical and quantum computing in these selected games and to demonstrate the advantages of quantum computing within the aspects of these games. Through this comparison, we hope to gain insight into the strengths and limitations of quantum computing and classical in game theory.

Analysis of Process:

Our team started by learning basic concepts of quantum computing and game theory, how quantum mechanics can be applied to game theory. Our research was done using the resources provided to us by our sponsor, as well as additional research using the internet. After understanding the basic concepts of quantum game theory, our team invested time in learning the tools that simulate quantum circuits. This includes the various libraries CIRQ, sympy, numpy, and matplotlib.pyplot. Google colab was the environment we used for coding and testing. The process of learning these tools was reading through documentation and testing different functions on google colab. After our team was familiar with the basics of our tools we applied our skills by coding and testing quantum games. We started with the simplest of games, the quantum coin flip. This included both researching how to quantize the coin flip game as well as coding a working circuit. From there we continued to research different games and how to apply quantum mechanics to them, and implemented that research with CIRQ and colab. After completing each quantum game we would do extensive testing to determine and verify how these quantum games differ from their classical counterparts. These differences would then be documented in the colab with the working code.

Results/Findings:

Quantum Coin Flip:

From our research and testing of the game, quantum coin flip, it was found that in a game with multiple players if one player has access to quantum strategies and one player doesn't, the player that does will always have an advantage. In this game the player with quantum strategies is guaranteed to win every single game. However, if both players had quantum strategies the game would not differ, outcome wise, from the classical game

Prisoner's Dilemma:

The quantum version of the prisoner's dilemma showed us that through entanglement both players can have quantum strategies in a game, and be different, outcome wise, from the classical game. The classical version of the prisoner's dilemma has a Nash equilibrium, no matter what the other player chooses to do, the 2^{nd} will choose the same option as it is always optimal. In the case of the prisoner's dilemma, the prisoner will always choose to snitch because it is optimal. However, the quantum version of this game has no Nash equilibrium, depending on what prisoner 1 one chooses to do, prisoner 2 has a optimal choice to counter prisoner 1's choice

Prisoner's Dilemma Expanded:

This was to see if the Benjamin & Hayden gate used to entangle the prisoner's dilemma was expandable and we find that it was infinitely expandable. However, we also found that having more players completely changes the strategies that players need to employ to win. Also, we found a way to cheat similarly to the quantum coin flip. If a player has access to gates that affect multiple qubits like the CNOT gate that can have an advantage to the players that do not.

Quantum Monty Hall:

There were two Quantum Monty Hall tests, this was to test different ways to quantize the game. In the classical Monty Hall game the outcome is if the player chooses to stay with their initial choice they have a 1/3 chance of winning but if they choose to change their initial choice after one of the doors was removed they have a 2/3 chance of winning. In the first application of the quantum Monty Hall game the doors were placed in a superposition and the player would measure a door to decide what door they wanted to open. In this version of the game the outcome was somewhat similar to the classical game with a ½ chance of winning if they choose to stay with their initial choice and a 2/3 chance of winning if they choose to change their choice

Quantum Monty Hall 2:

In the second version of the Monty Hall game the gates were all entangled. This changes the outcome so that no matter what choice the player made they had a 50/50 chance of winning

Conclusion:

This project investigated the potential of quantum computing in game theory through analyzing the performance of selected games such as coinflip, prisoner's dilemma, survival of the fittest, Monty hall, and tic tac toe, the project demonstrated the advantages of quantum computing in game theory, particularly in the aspects of entanglement and superposition. The findings of the project showed that quantum strategies can provide players with a significant advantage over classical strategies in certain games. The project also highlighted the limitations of quantum computing in game theory, in that not every game is changed by applying quantum. Overall, this project provides insights into the strengths and limitations of quantum computing in game theory