Satellite laser ranging (SLR)

Part 1: let's get familiar with the topic of SLR.

Quentin GAROT

Advanced geosoftware applications
TU Graz

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OUTLINE

Satellite Laser Ranging
Introduction

Satellite Laser Ranging (SLR) allows the precise measurement of long distances. The principle is the following:

- A device emits a laser pulse toward a satellite equipped with at least one retroreflector.
- A portion of the received light is reflected back toward the ground station.
- By timestamping the emission and reception of the laser pulse, the time round-trip can be determined.
- Given the speed of light, the distance traveled by the pulse can be calculated.

Satellite Laser Ranging
Introduction

However, the conversion from time to distance is not trivial, due to the fact that the atmosphere reduces the speed of light.

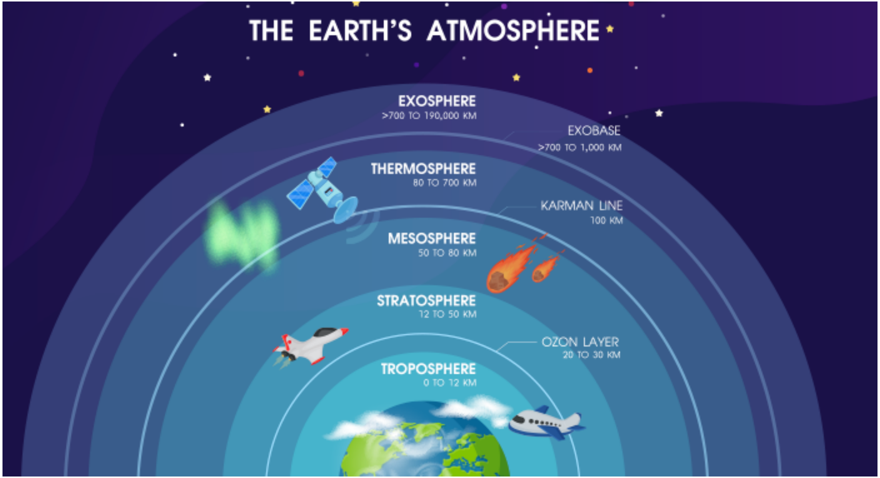
The speed of light varies depending on the composition of the air it passes through. Considering a medium with a refractive index n, the speed of light in the medium is:

$$v = \frac{c}{n}$$

where c is the speed of light in vaccum.

Satellite Laser Ranging

Atmosphere of Earth



Satellite Laser Ranging

Crossing the Troposphere

Goal: determine the time τ_t it takes for light to cross the Troposphere.

Gladstone law

The Gladstone law relates the refractive index n of a gas to its density ρ:

$$\frac{n-1}{\rho} = \mathcal{K}$$

where $\mathcal{K}$ is a constant.

Given the density, the refractive index can be determined, and thus the speed of light can be calculated to compute τ_t.

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Crossing the Troposphere - Compute ρ with isothermal-barotropic model

Assumptions:

- z is altitude
- molecular weight and temperature are constant

Equation:

$$\rho(z) = \rho_0 e^{-z/H}, \text{ with } H = \frac{Mg}{RT}$$

Yet:

$$\frac{n_0 - 1}{\rho_0} = \mathcal{K}$$

So:

$$\frac{n(z) - 1}{\rho(z)} = \frac{n_0 - 1}{\rho_0}, \quad \text{hence:} \quad n(z) = (n_0 - 1)e^{-z/H} + 1$$

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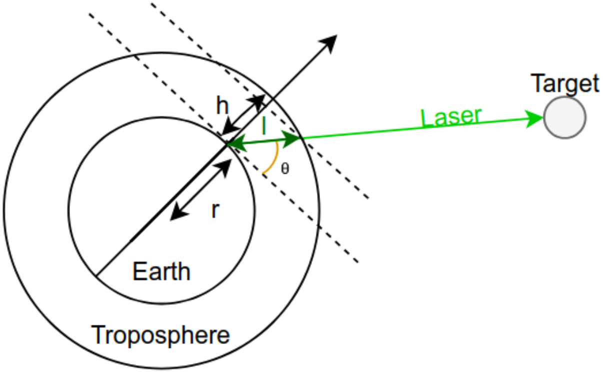
Crossing the Troposphere - Geometry of the problem

Notation:

- r: radius of Earth
- h: troposphere altitude
- θ: laser elevation angle
- $l(\theta)$: distance traveled by light in the troposphere



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Crossing the Troposphere - Geometry of the problem

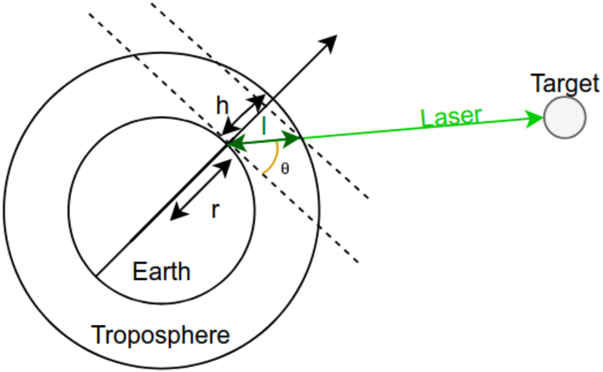
Distance traveled by light in the troposphere:

$$l(\theta) = -r\sin(\theta) + \sqrt{(r+h)^2 - r^2\cos^2(\theta)}$$

Altitude z of a point M located in the dark green section:

$$z = s\sin(\theta)$$

with s the distance between M and the ground station.



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Crossing the Troposphere - Final expression of τ_t

The time τ_t is given by:

$$\int_0^{l(\theta)} \frac{n(z(s))}{c} ds = \frac{1}{c}\int_0^{l(\theta)} ((n_0-1)e^{-z(s)/H}+1)ds = \frac{1}{c}\int_0^{l(\theta)} ((n_0-1)e^{-s\sin(\theta)/H}+1)ds$$

One integrates:

$$\frac{1}{c}\left((n_0-1)\left[-\frac{H}{\sin(\theta)}e^{-s\sin(\theta)/H}\right]_0^{l(\theta)} + l(\theta)\right) = \frac{1}{c}\left(\frac{(n_0-1)H}{\sin(\theta)}\left(1-e^{-l(\theta)\sin(\theta)}\right) + l(\theta)\right)$$

Hence:

$$\tau_t = \frac{1}{c}\left(\frac{(n_0-1)H}{\sin(\theta)}\left(1-e^{-l(\theta)\sin(\theta)}\right) + l(\theta)\right)$$

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Crossing the Ionosphere

Goal: determine the time τ_i it takes for light to cross the Ionosphere.

The ionosphere consists of a layer of ionized plasma located between $h_1 = 60$ and $h_2 = 1000$ km in altitude.

- Laser communication in space: $f = 200$ THz
- Plasma oscillation: $f_p = \frac{1}{2\pi}\sqrt{\frac{n^* e^2}{m\epsilon_0}} = 9.0$ MHz
- Speed of light:

$$v = \frac{c}{\sqrt{1 - \frac{f_p^2}{f^2}}}$$

Satellite Laser Ranging

Crossing the Ionosphere - Final expression of τ_t

Distance traveled by light in the ionosphere: *(same formula as before)*

$$\tilde{l}(\theta) = -(r + h_1)\sin(\theta) + \sqrt{(r + h_1 + h_2)^2 - (r + h_1)^2 \cos^2(\theta)}$$

Hence:

$$\tau_i = \frac{\tilde{l}(\theta)}{v}$$

Now that we know the times it takes for light to cross the Troposphere and Ionosphere, we are able to determine the distance error committed if we do not take into account the different speed of light.

Satellite Laser Ranging
Optical path length

Definition: Optical Path Length (OPL)

Let A and B two points. The OPL $\mathcal{L}_{AB}$ is defined as the distance that light would have traveled in vacuum during the time it takes to cross from A to B in the given medium.

This definition allows to know the errors made on the distance if we consider the speed of light (in vacuum) everywhere on the atmosphere:

$$\epsilon_t = c\tau_t - l(\theta) \text{ and } \epsilon_i = c\tau_i - \tilde{l}(\theta)$$

Laser ranging data
Introduction

- **SLR:** consists in measuring the time round-trip of the laser light, from the station to a retroreflector equipped satellite.
- **International Laser Ranging Service (ILRS)**: provides global SLR ranging data.
- **Data correction**: includes several factors like atmospheric parameters and ground target calibration values.
- **ILRS data**: two kind of data formats, *full-rate* and *normal points*.

Laser ranging data

Full-rate data - Introduction

- ***Full-rate***: original observations with corrections taking into account each individual measure.
- **History**: the prime SLR product in the 1970s and early to mid 1980s. Then with the introduction of *Normal points*, *Full-rate* format was no longer recommended.
- **Current usage**: After all, in early 2018 the ILRS further stated the importance of *full-rate* data. All stations are requested to submit them regularly to the data centers. *Full-rate* become more revealing for more accuracy.
- **Applied to**: satellite spin-rate determinations, center-of-mass algorithm development, etc.
- **Transmission procedure**: one daily file per satellite; this file contains all passes recorded during the previous 24 hours.

Laser ranging data

Full-rate data - Extract from ILRS full rate data format (V3)

Bytes	Description
1-7	ILRS Satellite Identification Number
8-24	Date (year, day, time of day)
...	...
46-57	Laser Range - in units of two way time with 1 picosecond granularity
...	...
65-68	Wavelength of the laser
...	...
81-85	Tropospheric refraction correction

Laser ranging data

Normal points data - Introduction

- **3rd generation lasers**: capable of firing at 10 Hz in raw mode. Over a large time interval, the amount of collected data is very high.
- ***Normal points***: condensed range observations generated from a large number of *full-rate* observations collected over a certain time interval called *bin*.
- ***Bin* criteria**: different *bin* sizes for different orbital characteristics:

Satellite altitude (km)	*Bin* size (seconds)	Example satellites
< 500	5	GRACE
550 - 800	15	Sentinel-3
800 - 2,000	30	Starlette, LARES
...	...	...
$> 15{,}000$	300	Etalon, GNSS

Laser ranging data

Normal points data - Introduction

- **End criteria**: stations can close the *normal points* recording before the end of the *bin* if they have at least 1000 *full-rate* points OR if the standard error of the mean does not exceed 1 mm.
- ***Normal points* data format**: composed of three parts:
  - Header record
  - Data record
  - Sampled Engineering Data Record

Laser ranging data

Consolidated Data Format (CRD) - Introduction

As of March 2012 the ILRS uses the Consolidated Data Format (CRD) for both *normal point* and *full-rate* data.

Main advantages:

- Flexibility: the data files can be simple and compact.
- Building block structure with multiple record type: stations can include or omit certain records types as needed.
- Inclusion of all data types: *full rate* and *normal point* can be managed in a single file.

Laser ranging data

Consolidated Data Format (CRD) - Sections

There are 3 separate sections to the CRD data format:

- **The header section**: contains data on topics such as station, target, and start time.
- **The configuration section**: contains static data that represents station specific configuration while collecting the data stored in the file.
- **The data section**: contains non-static data like laser transmit and receive times, and other highly dynamic information.

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Consolidated Data Format (CRD) - Sections

Example 1: the header section contains the `target` record. It contains:

- Target name (e.g. "ajisai")
- Target dynamics (e.g. 1 for Earth orbit, 2 for lunar orbit, etc.)
- ...

It also includes the `session` record which describe information relating to the period over which the data is collected:

- Data type (e.g. 0 for full rate, 1 for normal point)
- Starting date (year, month, day, hour, minute, and second)
- Ending date
- Some correction indicators (e.g. tropospheric refraction)
- Range type indicator (e.g. two-way ranging)
- ...

Laser ranging data

Consolidated Data Format (CRD) - Sections

Example 2: the configuration section contains the `laser configuration` record. It contains:

- Laser type
- Wavelength
- Nominal fire rate (Hz)
- Beam divergence (full angle, arcseconds)
- ...

Laser ranging data

Consolidated Data Format (CRD) - Sections

Example 3: the data section contains the `range` record for *full rate*. It contains single-shot measurement data.

- Time-of-flight in seconds
- ...

It also includes `range` record for *normal point*.

- Time-of-flight in seconds
- Normal point window length (seconds)
- Number of raw ranges compressed into the normal point
- ...

Conclusion

- Principle of SLR
- Do not consider the speed of light as constant in order to get accurate measurements
- International Laser Ranging Service, official provider of SLR data

References

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