第四章作业

1.数学推导

1.1 前向推导:

$$P_{k+1} = P_k + V_k t$$

$$V_{k+1} = V_k + at$$

1.2 在末速度固定的情况下:

小将 a_i 展开

$$a_i = -\frac{12}{T^3}(P_{if} - V_{i0}T - P_{i0}) + \frac{6}{T^2}(V_{if} - V_{i0})$$

令 $12(P_{i0} - P_{if}) = a_{i1}$

$$= \frac{12(P_{i0} - P_{if})}{T^3} + \frac{6V_{if} - 6V_{i0}}{T^2}$$

令 $6(V_{if} - V_{i0}) = a_{i2}$

$$\beta_i = \frac{6}{T^2}(P_{if} - V_{i0}T - P_{i0}) - \frac{2}{T}(V_{if} - V_{i0})$$

令 $6(P_{if} - P_{i0}) = \beta_{i1}$

$$= \frac{6(P_{if} - P_{i0})}{T^2} - \frac{(4V_{i0} + 2V_{if})}{T}$$

$-(4V_{i0} + 2V_{if}) = \beta_{i2}$

将 alpha i 和 beta i 带入 J

$$J = T + \frac{1}{3}T^3(a_1^2 + a_2^2 + a_3^2) + T^2(a_1\beta_1 + a_2\beta_2 + a_3\beta_3) + T(\beta_1^2 + \beta_2^2 + \beta_3^2)$$

⇒

$$J = T + \frac{1}{T}\left(\frac{1}{3}\sum_{i=1}^{3} a_{i2}^2 + \sum_{i=1}^{3} \beta_{i1} a_{i2} + \sum_{i=1}^{3} \beta_{i2}^2\right)$$
$$+ \frac{1}{T^2}\left(\frac{2}{3}\sum_{i=1}^{3} a_{i1} a_{i2} + \sum_{i=1}^{3} \beta_{i1} a_{i2} + \sum_{i=1}^{3} a_{i1} \cdot \beta_{i2} + 2\sum_{i=1}^{3} \beta_{i1} \cdot \beta_{i2}\right)$$
$$+ \frac{1}{T^3}\left(\frac{1}{3}\sum_{i=1}^{3} a_{i1}^2 + \sum_{i=1}^{3} \beta_{i1} \cdot a_{i1} + \sum_{i=1}^{3} \beta_{i1}^2\right)$$

$$\text{令} J_1 = \left(\frac{1}{3}\sum_{i=1}^{3} a_{i2}^2 + \sum_{i=1}^{3} \beta_{i1} a_{i2} + \sum_{i=1}^{3} \beta_{i2}^2\right)$$
$$J_2 = \left(\frac{2}{3}\sum_{i=1}^{3} a_{i1} a_{i2} + \sum_{i=1}^{3} \beta_{i1} a_{i2} + \sum_{i=1}^{3} a_{i1} \cdot \beta_{i2} + 2\sum_{i=1}^{3} \beta_{i1} + \beta_{i2}\right)$$
$$J_3 = \left(\frac{1}{3}\sum_{i=1}^{3} a_{i1}^2 + \sum_{i=1}^{3} \beta_{i1} a_{i1} + \sum_{i=1}^{3} \beta_{i1}^2\right)$$

对 J 求对 T 的偏导

$$J = J_1 \cdot \frac{1}{T} + J_2 \cdot \frac{1}{T^2} + J_3 \cdot \frac{1}{T^3} + T$$
$$\frac{\partial J}{\partial T} = -J_1 T^{-2} - 2J_2 T^{-3} - 3J_3 T^{-4} + 1 = 0$$
$$T^4 - J_1 T^2 - 2J_2 T - 3J_3 = 0$$

求解 T 的四阶方程(companion matrix):

2.成果