Stackberg Model

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1 model

1.1 Original Model

$$\begin{aligned} &min \quad x_i^T Q_i x_i + c_i^T x_i \\ &s.t. \quad A_i x_i \leq d_i \end{aligned} \tag{1}$$

1.2 Lagrange Model

$$L = x_i^T Q_i x_i + c_i^T x_i + u_i^T (A_i x_i - d_i) \tag{2}$$

$$KKT \begin{cases} \frac{\partial L}{\partial x_i} = 2Q_i x_i + c_i^T + A_i^T u_i = 0 \\ -u_i \leq 0 \\ u_i - M_i I_i \leq 0 \\ A_i x_i \leq d_i \\ -A_i x_i + I_i M_i \leq -d_i + E_i M_i \end{cases} \tag{3}$$

the original problem is translated into the following problem

$$\begin{aligned} &min \quad 1^T y_i \\ &s.t. \quad B_i y_i \leq r_i \end{aligned} \tag{4}$$

$$\begin{bmatrix} 2Q_i & A_i^T & 0 \\ -2Q_i & -A_i^T & 0 \\ A_i & 0 & 0 \\ -A_i & 0 & diag(M_i) \\ 0 & diag(E_i) & diag(-M_i) \\ 0 & diag(-E_i) & 0 \end{bmatrix} \begin{bmatrix} x_i \\ u_i \\ I_i \end{bmatrix} \leq \begin{bmatrix} -c_i \\ c_i \\ d_i \\ -d_i + M_i \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

1.3 The Leader Model

$$\begin{aligned} &min \quad f(w) \\ &s.t. \quad Rw \geq r \end{aligned} \tag{6}$$

$$w = \{z, y_1, \cdots y_N\} \tag{7}$$

$$\begin{bmatrix} -U_1 & B_1 & \cdots & \cdots & \cdots \\ -U_2 & \vdots & B_2 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \cdots \\ -U_N & \cdots & \cdots & \cdots & B_N \end{bmatrix} \begin{bmatrix} z \\ y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \leq \begin{bmatrix} r_1^0 \\ r_2^0 \\ \vdots \\ r_n^0 \end{bmatrix} \tag{8}$$

$$r = \begin{bmatrix} r_1^0 \\ r_2^0 \\ \vdots \\ r_n^0 \end{bmatrix} \tag{9}$$

1.4 Example

the model for conventional ship is

$$\begin{aligned} &min \quad \frac{1}{2} b E_{si}^2 - a E_{si} + C_{si} P_{si} \\ &s.t \begin{cases} E_{si} - E_{si}^{max} \leq 0 \\ -E_{si} + E_{si}^{min} \leq 0 \\ -P_{si} \leq 0 \\ P_{si} - P_{si}^{max} \leq 0 \end{cases} \end{aligned} \tag{10}$$

the model can be written in the form of matrix

$$min \quad \begin{bmatrix} E_{ei} E_{si} P_{ei} P_{si} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{b}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{ei} \\ E_{si} \\ P_{ei} \\ P_{si} \end{bmatrix} + \begin{bmatrix} 0 & -a & 0 & C_{si} \end{bmatrix} \begin{bmatrix} E_{ei} \\ E_{si} \\ P_{ei} \\ P_{si} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_{ei} \\ E_{si} \\ P_{ei} \\ P_{si} \end{bmatrix} \leq \begin{bmatrix} E_{si}^{max} \\ E_{si}^{min} \\ 0 \\ P_s^{max} \end{bmatrix} \tag{12}$$

the AES model:

$$
\begin{aligned}
&min \quad \frac{1}{2}dE_{ei}^2 - cE_{ei} + C_{si}P_{si} + C_{ei}P_{ei} \\
&s.t \begin{cases} E_{ei} - E_{ei}^{max} \leq 0 \\ -E_{ei} + E_{ei}^{min} \leq 0 \\ -P_{ei} \leq 0 \\ P_{ei} - P_{ei}^{max} \leq 0 \\ -P_{si} \leq 0 \\ P_{si} - P_{si}^{max} \leq 0 \end{cases}
\end{aligned} \tag{13}
$$

the model can be written in the form of matrix

$$
min \quad \begin{bmatrix} E_{ei} E_{si} P_{ei} P_{si} \end{bmatrix} \begin{bmatrix} \frac{d}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{ei} \\ E_{si} \\ P_{ei} \\ P_{si} \end{bmatrix} + \begin{bmatrix} -c & 0 & C_{ei} & C_{si} \end{bmatrix} \begin{bmatrix} E_{ei} \\ E_{si} \\ P_{ei} \\ P_{si} \end{bmatrix} \tag{14}
$$

$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_{ei} \\ E_{si} \\ P_{ei} \\ P_{si} \end{bmatrix} \leq \begin{bmatrix} E_{ei}^{max} \\ E_{ei}^{min} \\ 0 \\ P_e^{max} \\ 0 \\ P_s^{max} \end{bmatrix} \tag{15}
$$

the leader model:

$$
\begin{aligned}
&min \quad \sum_{t=1}^{T} \left[\sum_{i \in \Omega_C \bigcup \Omega_A} (\lambda_{si} - C_{si}) P_{si} + \sum_{i \in \Omega_A} (\lambda_{ei} - C_{ei}) P_{ei} \right] \\
&s.t. \quad Mw \geq r
\end{aligned} \tag{16}
$$

$$
w = \{z, y_1, \cdots y_N\} \tag{17}
$$

$$
z_t = \begin{bmatrix} C_{et} \\ C_{st} \end{bmatrix} \tag{18}
$$

the model of conventional ship has 4 constrains,so the number of u and I vector is 4:

$$y_t^{conventional} = \begin{bmatrix} E_e \\ E_s \\ P_e \\ P_s \\ u1 \\ u2 \\ u3 \\ u4 \\ I1 \\ I2 \\ I3 \\ I4 \end{bmatrix} \tag{19}$$

the model of AES has 6 constrains,so the number of u and I vector is 6:

$$y_t^{AES} = \begin{bmatrix} E_e \\ E_s \\ P_e \\ P_s \\ u1 \\ u2 \\ u3 \\ u4 \\ u5 \\ u6 \\ I1 \\ I2 \\ I3 \\ I4 \\ I5 \\ I6 \end{bmatrix} \tag{20}$$