ORIE 5530: Modeling Under Uncertainty

Homework Assignment 1

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Question 1. Let P, H, S be three events (e.g., P is "Your favorite party wins the Presidential election", H is "Your favorite party controls the House of Representatives after the elections", and S is "Your favorite party controls the Senate after the elections"). Find expressions for the events:

(a) only H occurs;

(b) both P and H but not S occur;

(c) at least one event occurs;

(d) at least two events occur;

(e) all three events occur;

(f) none of the events occurs;

(g) at most one event occurs;

(h) at most two events occur.

Question 2. (Basic counting exercise) Consider a box containing three marbles: one red, one green, and one blue. Consider an experiment that consists of taking one marble from the box and then replacing it in the box and drawing a second marble from the box. What is the sample space? If, at all times, each marble in the box is equally likely to be selected, what is the probability of each point in the sample space?

Question 3. (Counting continued) You have N cents and m possible coin denominations $\{S_1, S_2, \ldots, S_m\}$ (say $S_1 = 1$ cent, $S_2 = 2$ cents, etc.). We want to consider how many ways can you break the N cents you have into combinations of coins. For example, say $N = 10$, $S_1 = 1$, $S_2 = 5$ and $S_3 = 10$; then, you have 4 options: (i) one coin of 10 cents, (ii) two coins of 5 cents each, (iii) five coins of 1 cent and one coin of 5 cents, and (iv) ten coins of 1 cent. If you have only $S_1 = 1$ and $S_2 = 5$ (i.e., you are not allowed to use the 10-cent coin) then you have only three options. In this example, 75% of the outcomes do not use the 10-cent coin.

(a) Write a recursion of $\mathcal{C}(N, m)$ for the number of ways to "build" N cents if you have the m denominations $\{S_1, S_2, \ldots, S_m\}$. Code this recursion. Hint: split into combinations that use the denomination S_m and those that do not.

(b) With $S_1 = 1$, $S_2 = 5$, $S_3 = 10$ and $S_4 = 25$, how many ways are there to change $N = 213$ cents?

(c) What fraction of these uses only 1, 5 and 10 cents denominations?

(d) Repeat the same for multiple values of N and plot the results corresponding to the previous two questions (i.e., number of ways to change N using $S_1 = 1$, $S_2 = 5$, $S_3 = 10$, $S_4 = 25$, and fraction of the total using only 1-, 5- and 10-cent coins) as a function of N ranging from 0 to 400.

Question 4. (Conditional probabilities and partially observed outcomes) Consider two boxes A and B. Box A contains two marbles: one red and one white marble. Box B contains three marbles: two red and one white marble. A box is selected at random (i.e., you select box A with probability $1/2$) and a marble is drawn at random from the selected box.

(a) What is the probability that the marble is red?

(b) Say you can see the final outcome (the color of the selected marble), but you do not know from which box it was selected. What is the probability that box A was the one selected given that the marble is white?