HW2 Queenie

November 6, 2019

[1]:
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import numpy as np
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[6]:
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for n in range (1,11):
    l = np.random.poisson(np.log(1+n),10000)
    cost = [(12*x -3*n) for x in l]
    average = sum(cost)/10000
    print(n, average)
```

```
1 5.2872
2 7.1124
3 7.776
4 7.3752
5 6.7248
6 5.352
7 4.0392
8 2.352
9 0.462
10 -1.134
```

1. (b) $f(t) = P\log(1+t) - tc$

$f'(t) = \frac{P}{1+t} - c = 0$

$\begin{cases} P = 12 \\ c = 3 \end{cases} \Rightarrow f'(t) = 0 = \frac{12}{1+t} - 3 \Rightarrow t = 3$

$\therefore$ The result is the same.

2 (a) 3 white 6 red 5 Black

$P(W=2, B=3) = \frac{\binom{3}{2}\binom{5}{3}\binom{6}{1}}{\binom{14}{6}} = \frac{60}{1001}$

(b) $P(B=3) = \frac{\binom{5}{3}\binom{9}{3}}{\binom{14}{6}}$

$P(W=x \mid B=3) = \frac{P((W=x)\cap(B=3))}{P(B=3)} = \frac{\binom{3}{x}\binom{6}{3-x}}{\binom{9}{3}}$

$x=0 \Rightarrow P(W=0 \mid B=3) = \frac{\binom{6}{3}}{\binom{9}{3}} = \frac{5}{21}$

$x=1 \Rightarrow P(W=1 \mid B=3) = \frac{\binom{3}{1}\binom{6}{2}}{\binom{9}{3}} = \frac{15}{28}$

$x=2 \Rightarrow P(W=2 \mid B=3) = \frac{\binom{3}{2}\binom{6}{1}}{\binom{9}{3}} = \frac{3}{14}$

$x=3 \Rightarrow P(W=3 \mid B=3) = \frac{\binom{3}{3}\binom{6}{0}}{\binom{9}{3}} = \frac{1}{84}$

(c) $P(B=3) = \left(\frac{5}{14}\right)^3\left(\frac{9}{14}\right)^3\binom{6}{3}$

$P((W=x)\cap(B=3)) = \left(\frac{3}{14}\right)^x\left(\frac{5}{14}\right)^3\left(\frac{6}{14}\right)^{6-3-x}\binom{6}{3}\binom{3}{x}$

$\therefore P(W=x \mid B=3) = \frac{\left(\frac{3}{14}\right)^x\left(\frac{6}{14}\right)^{3-x}\binom{3}{x}}{\left(\frac{9}{14}\right)^3}$

$x=0 \Rightarrow P(W=0 \mid B=3) = \frac{8}{27}$

$x=1 \Rightarrow P(W=1 \mid B=3) = \frac{4}{9}$

$x=2 \Rightarrow P(W=2 \mid B=3) = \frac{2}{9}$

$x=3 \Rightarrow P(W=3 \mid B=3) = \frac{1}{27}$

3 (a) $P(X+Y=Z) = \sum_{i=0}^{Z} P(X+Y=Z, X=i)$

$$= \sum_{i=0}^{Z} P(X=i, Y=Z-i)$$

$$= \sum_{i=0}^{Z} \frac{e^{-\lambda_1}\lambda_1^{i}}{i!} \frac{e^{-\lambda_2}\lambda_2^{Z-i}}{(Z-i)!}$$

$$= e^{-(\lambda_1+\lambda_2)} \sum \frac{\lambda_1^{i} \lambda_2^{Z-i}}{i!(Z-i)!}$$

$$= \frac{e^{-\lambda}}{Z!} \sum_{i=0}^{Z} \binom{Z}{i} \lambda_1^{i} \lambda_2^{Z-i}$$

$$= \frac{e^{-\lambda}}{Z!} (\lambda_1+\lambda_2)^{Z}$$

$\therefore (X_1+X_2) \sim P(\lambda_1+\lambda_2)$

with $E[X_1+X_2] = \lambda_1+\lambda_2$

$Var[X_1+X_2] = \lambda_1+\lambda_2$

(b) $P(X_1=i \mid Y=n)$

$$P(Y=n) = \frac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{n}}{n!} = \binom{n}{i}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{i}\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-i}$$

$$\therefore P(X_1=i) = \sum_{n=0}^{\infty} (X=i \mid Y=n) = \sum_{n=i}^{\infty} \binom{n}{i}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{i}\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-i} \cdot \frac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{n}}{n!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{i!}(\lambda_1)^{i} e^{\lambda_2} = \frac{e^{-\lambda_1}}{i!}\lambda_1^{i}$$

$\therefore X_1 \sim P(\lambda_1)$

As the same, $X_2 \sim P(\lambda_2)$

(c) $E[X+Y] = \lambda_1+\lambda_2$

$Var(X+Y) = \lambda_1+\lambda_2+\underline{2Cov(X,Y)}$

$\because Cov(X,Y) = 0.8\sqrt{(\lambda_1+\lambda_2)}$

$\therefore Var(X+Y) = \lambda_1+\lambda_2+1.6\sqrt{\lambda_1+\lambda_2}$