ORIE 5530: Modeling Under Uncertainty

Project Presentation Example 1

Professor Mark S. Squillante

Each of you will be given 4 minutes to present a brief summary of your project on December 6th, 2pm – 6pm. The material that follows, which I will cover in class, is **intended solely** to be an **illustrative example**. Your brief summary presentation, as illustrated in this example, should seek to answer the types of questions provided in the previous project assignement.

Problem. I manage a store that sells the latest fashions. In managing the store, the problem of particular interest to be solved concerns determining the best price and the best initial inventory for a specific fashion product in the coming fashion season.

Initial Model. To start, I developed a discrete-time Markov chain for the fashion product where there is an initial inventory of x units at the start of the fashion season. The fashion season (time horizon) is divided into n periods and one customer arrives in every period. Customers are independent and, given a price p, each customer buys the fashion product with probability $\lambda(p)$ and does not buy the fashion product with probability $1 - \lambda(p)$. Define the random variable $B_i(p)$ to equal 1 if the ith customer buys the fashion product when it is offered at price p, i.e., $\mathbb{P}[B_i(p) = 1] = \lambda(p)$. When a customer buys the product, which happens with probability $\lambda(p)$, the reward received is p and the inventory is reduced by one.

Initial Best Solution. Next, I begin by initializing x and p. I assume that the possible range of values for the initial inventory x is from 0 to 100, and assume that the possible range of values for the price p is from 0 to 10. I further suppose the probability $\lambda(p)$ equals 0 when $p = 10$ and the probabilty $\lambda(p)$ equals 1 when $p = 0$, with the probability values in between being linear in p. I then constructed a DTMC of this pricing inventory problem and solved the DTMC to obtain long-run average rewards $\mathbb{E}_x[\sum_{i=1}^n pB_i(p)]$ for a given pair of parameters x and p. Using this model, I manually searched to obtain the best values of x and p. Sensitivity analysis is performed with respect to changes in x and p within a local region of their best values to gain insights into how the rewards change over this region.

Simulation Model and Solution. Now, I initialized x and p with the best values obtained from above. I then used simulation to solve the above initial model where the probability that each customer buys the fashion product in each period follows a uniform distribution over the interval from $\max\{\lambda(p) - \epsilon, 0\}$ to $\min\{\lambda(p) + \epsilon, 1\}$, for some relatively small value of $\epsilon > 0$. This simulation solution provides the long-run average rewards for a given pair of parameters x and p, and I used the simulation model in the same manner as above to manually search for the best values of x and p. Sensitivity analysis is performed with respect to changes in x, p and ϵ within a local region of their best values to gain insights into how the rewards change over this region. Comparisons are made between the results from simulation and those from the DTMC.

Dynamic Programming Model. Lastly, I constructed an infinite horizon discounted dynamic programming model of the above fashion season product pricing inventory problem with the goal of determining the optimal initial inventory x and the optimal prices p_i for every period $i = 1, \ldots, n$. I return to the original assumptions that the probability $\lambda(p)$ equals 0 when $p = 10$ and the probabilty $\lambda(p)$ equals 1 when $p = 0$, where the probabilities in between are linear in p. Within the infinite horizon discounted dynamic programming model, I solved the corresponding dynamic optimization problem to obtain the optimal initial inventory x and the optimal prices p_i, $i = 1, \ldots, n$. Comparisons are made among these optimal dynamic programming results and those from the above simulation and DTMC models.