**ORIE 5530: Modeling Under Uncertainty**

# Homework Assignment 2

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1. Simulation Optimization.

   A taxi driver in the city of New York wants to decide how much time he should work per day to maximize his income. Assume that the average money per ride is $p$ dollars. The number of trips he could fulfill depends, of course, on how much time he works. If he works for $t$ hours, the number of dispatches $D$ he gets is a Poisson random variable with mean $\mu(t) = \log(1+t)$. In other words (as expected), the longer he works the more (on average) rides he gets.

   Also, the cost (fuel, car depreciation, etc.) of working increases as the working time $t$ increases. Let us assume here that there is a cost of $c$ dollars for working per hour (half an hour then costs $c/2$, etc.). If he chooses to work 3 hours, the total operating cost is $3c$.

   (a) Let $p = 12$, $c = 3$. Assume the driver only works for an integer number of hours, and can not work more than 10 hours a day. Design and code a simulation based algorithm to find the amount of time he should work in order to maximize his expected net income.

   Of course, once you have such a code you could have much more general specifications (cost need not be linear, dispatches could follow an arbitrary structure, etc.). The reason for this specific stucture we used is that now we can compare your simualtion result to

   (b) Analytically derive the optimal amount of time a driver should work if he wishes to maximize his/her *expected* income? How does this compare to your answer to part (a)?

2. Conditional probability. An urn contains three white, six red, and five black balls. Six of these balls are randomly selected from the urn. Let $W$, $B$ and $R$, denote respectively the number of white, black and red balls selected.

   (a) Compute the probability that $P[W = 2, B = 3]$? Hint: we are returning here to *counting*. Since all outcomes are equally likely, this is about couting outcomes with the required property and dividing by the total number of outcomes.

   (b) Compute the conditional probability mass function of $W$ given that $B = 3$.

   (c) Assume the balls are not drawn simutaneously, but drawn one by one, and each time a ball is picked, another ball with the same color is placed into the urn to replace it. Compute the conditional probability mass function of $W$ given that $B = 3$ again.

   Hint: What has changed here relative to (a) is that now we are returning a ball after we see it. The answer will require you to generalize a bit the idea of the binomial distribution but this should not be difficult. Just remember how we motivated the expression for the binomial.

3. Assume a retailer chain has 2 branches. Let $X_1$ and $X_2$ be the number of customers arriving 10:00-11:00am on Mondays to each of the branches. Assume $X_i \sim Poisson(\lambda_i)$ for $i = 1, 2$ and that the demand to one is independent of demand to the other.

   (a) Derive the probability density function of the total number of customers of the 2 branches. What distribution does this random variable have? Show your work. What is the mean and variance of this random variable?

(b) Starting the other way.

Suppose that you do not know the distribution of demand to each store but you do know that the total demand, $Y$, (the sum of demand to both stores) is a Poisson random variable with rate $\lambda_1 + \lambda_2$. Suppose that when a customer has to choose which store to go to (for example, the grocery store in northern Manhattan or the one in southern Manhattan) the customer chooses with probabilty $\lambda_1/(\lambda_1 + \lambda_2)$ to go to store 1 and with the remaining probability to store 2.

This way, the demand to store 1 is the number of customers that chose store 1 over store 2 (among the total number of customers).

What is $P[X_1 = i | Y = n]$?

What is now the distribution of the demand to store 1? That is, what is $P[X_1 = i]$ ? To store 2?

(c) Now suppose, more realistically, that demand is positively correlated and the correlation coefficient is 0.8. What is the mean and variance of the total demand $X_1 + X_2$?