Modeling HW3 Queenie Lin ql299

2. Conditional Expectation

(a) N: # of games A wins

$$P(A) = P(A|N=0)P(N=0) + P(A|N=1)P(N=1) + P(A|N=2)P(N=2)$$
$$= \sum_{i=0}^{2} P(A|N=i)P(N=i)$$
$$= 0 + P(A)\,2p(1-p) + p^2 = \frac{p^2}{1-2p(1-p)}$$

(b) X: # of games played

$$E(X) = E[X|S=0]P(S=0) + E[X|S=1]P(S=1) + E[X|S=2]P(S=2)$$
$$= \sum_{i=0}^{2} E[X|S=i]P(S=i)$$
$$= 2(1-p)^2 + (2+E[X])\cdot 2p(1-p) + 2p^2$$
$$= 2 + E[X]\cdot 2p(1-p) = \frac{2}{1-2p(1-p)}$$

3. Conditional Expectation

$$E[T_i] = E[i-1 \to i] = \frac{1}{2} + \frac{1}{2} + E[i-2 \to i]\cdot\frac{1}{2} = 1 + \frac{1}{2}E[i-2 \to i]$$
$$= 1 + \frac{1}{2}[E[T_{i-1}] + E[T_i]]$$
$$E[T_i] = 2 + E[T_{i-1}], \quad i > 1$$

Since $E[T_1] = 1$, $E[T_2 = 3] \cdots E[T_i] = 2i-1$

Thus $\Rightarrow$ $E[0 \to n] = \frac{(1+2n-1)n}{2} = \boxed{n^2}$