HW3_Q1

November 14, 2019

1 Q1

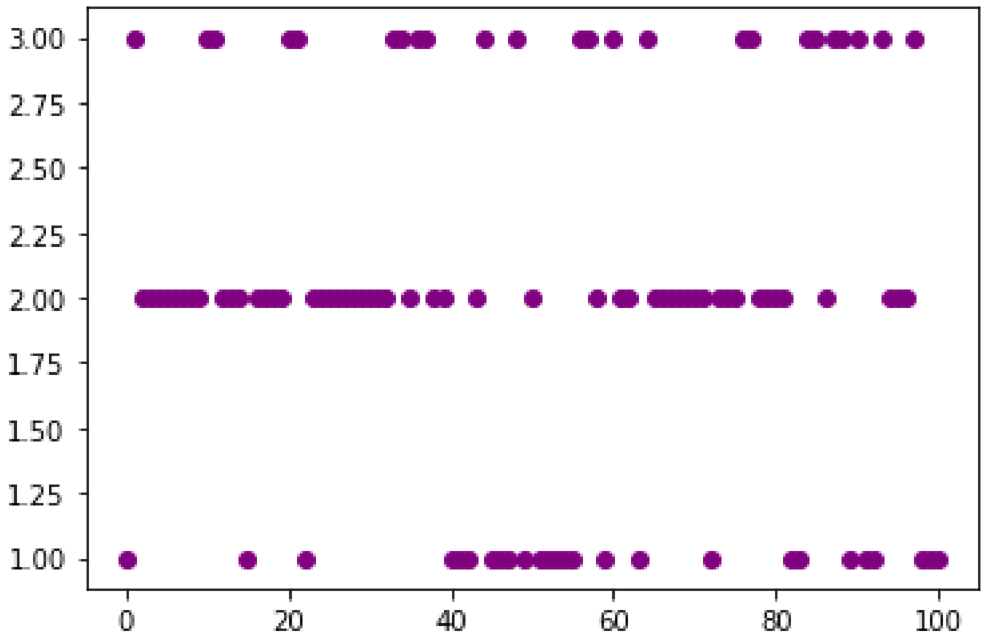
[1]:
```
import numpy as np
```

[23]:
```
mk = np.array([[0.4,0.38,0.22],[0.12,0.7,0.18],[0.2,0.5,0.3]])
```

[27]:
```
def mk_sim(mk,iteration = 100):
    iteration = 100
    results = []
    x_i = 0 #suppose state 1 is 0th row
    i = 0
    results.append(1)
    while(i < iteration):
        x_i = np.random.choice([0,1,2],p = mk[x_i])
        results.append(x_i+1)
        i += 1
    return results
```

(a)

[28]:
```
import matplotlib.pyplot as plt
result = mk_sim(mk)
plt.scatter(np.arange(101),result,color='purple', marker='o')
plt.show()
```



[29]:
```
results = mk_sim(mk,1000)
print(results)
```

```
[1, 2, 2, 2, 3, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 1, 3, 1, 1, 2, 2, 2, 3, 3, 1, 1,
2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 3, 3, 1, 2, 2, 2, 1, 3, 2, 3, 3, 3, 3, 3, 2, 2,
2, 2, 1, 1, 2, 3, 3, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2,
1, 1, 1, 2, 3, 1, 1, 3, 2, 2, 2, 2, 2, 2, 3, 2, 2, 3, 3, 2, 1]
```

(b)

[30]:
```
def result_1(x):
    return sum(x)/len(x)
result_1(result)
```

[30]: `1.9801980198019802`

[31]:
```
def result_2(x):
    beta = 0.9
    su = 0.0
    for i in range(len(x)):
        su += (beta**i)*x[i]**2
    return su
result_2(result)
```

[31]: `45.4098699631307`

(c)

[32]:
```
Exp1 = []
Exp2 = []
for i in range(4000):
    re = mk_sim(mk,1000)
    Exp1.append(result_1(re))
    Exp2.append(result_2(re))
print(np.mean(np.array(Exp1)))
print(np.mean(np.array(Exp2)))
```

```
2.0098490099009902
40.84200902122871
```

(d)

[35]:
```
R = np.linalg.inv(np.identity(3) - 0.9*mk) @ np.array([1,2,3])**2
print(R[0])
```

```
40.778184356869474
```

The result is very similar to the similation result.

[ ]:
```
```

Modeling HW3 Queenie Liu ql299

2. Conditional Expectation

(a) N: # of games A wins

$$P(A) = P(A|N=0)P(N=0) + P(A|N=1)P(N=1) + P(A|N=2)P(N=2)$$
$$= \sum_{i=0}^{2} P(A|N=i)P(N=i)$$
$$= 0 + P(A)\,2p(1-p) + p^2 = \frac{p^2}{1-2p(1-p)}$$

(b) X: # of games played

$$E(X) = E[X|S=0]P(S=0) + E[X|S=1]P(S=1) + E[X|S=2]P(S=2)$$
$$= \sum_{i=0}^{2} E[X|S=i]P(S=i)$$
$$= 2(1-p)^2 + (2+E[X])\cdot 2p(1-p) + 2p^2$$
$$= 2 + E[X]\cdot 2p(1-p) = \frac{2}{1-2p(1-p)}$$

3. Conditional Expectation

$$E[T_i] = E[i-1 \to i] = \frac{1}{2} + \frac{1}{2} + E[i-2 \to i]\cdot\frac{1}{2} = 1 + \frac{1}{2}E[i-2 \to i]$$
$$= 1 + \frac{1}{2}\left[E[T_{i-1}] + E[T_i]\right]$$

$E[T_i] = 2 + E[T_{i-1}]$, $i > 1$

Since $E[T_1] = 1$, $E[T_2 = 3] \cdots E[T_i] = 2i-1$

Thus $\Rightarrow$ $E[0 \to n] = \dfrac{(1+2n-1)n}{2} = \boxed{n^2}$