ORIE 5380, CS 5727: Optimization Methods
Homework Assignment 1
Due September 20, 12:00 pm

Please submit a single PDF document formatted to print. If you are doing any work in Excel, then please put screenshots of your spreadsheet in this PDF document. Make sure that your screenshots show the formulas in your spreadsheet. Note that CTRL and ~ key combination (Windows) or COMMAND and ~ key combination (OS X) displays the formulas in an Excel spreadsheet. Show all your work clearly.

Question 1
A farmer needs to prepare 90 pounds of daily feed for his pigs. The feed is prepared as a mixture of corn and soybean meal, each material having the following compositions:

	Pounds per Pound of Material		
Material	Calcium	Protein	Fiber
Corn	0.001	0.09	0.02
Soybean	0.002	0.6	0.06

That is, for example, there is 0.001 pounds of calcium in one pound of corn. The dietary requirements for the pigs are

- At most 1% calcium;
- At least 30% protein;
- At most 5% fiber.

Costs of corn and soybean meal are respectively 0.2 and 0.6 $/pound.

a) Write the problem of determining the minimum-cost feed as a linear program problem. (Writing a linear program means clearly specifying the decision variables and writing the objective and the constraints as a function of the decision variables.)
b) Find the optimal solution by using Excel's solver. Report the optimal values of the decision variables.

Question 2
An employment agency must provide the following numbers of laborers over the next 5 months:

Month	1	2	3	4	5
Number of laborers	110	130	70	165	50

Since the demand for laborers fluctuates and the monthly salaries tend to decline when the laborers are hired for longer terms, it may be more economical to keep more laborers than needed during some months of the planning horizon. The company estimates that the cost of recruiting and maintaining the laborers is a function of their length of stay with the company. The following table summarizes these estimates.

Length of employment (# months)	1	2	3	4	5
Cost / laborer	110	140	170	230	250

The company is interested in the recruiting schedule to minimize the total cost.

a) Ignoring the fact that the numbers of laborers cannot take fractional values, write the problem as a linear program.
b) Find the optimal solution by using Excel's solver. Report the optimal values of the decision variables (Hint: Think about the number of laborers to be recruited for different horizons – 1, 2, 3, 4, 5 months – in each month of the planning horizon. That is, use decision variables of the form x_{ij}, which correspond to the number of people hired at the beginning of month i for j months.)

Question 3
Consider the linear program

$$\begin{aligned} \max \quad & 10\,x_1 + 20\,x_2 \\ \text{st} \quad & -x_1 + 2\,x_2 \leq 15 \\ & x_1 + x_2 \leq 12 \\ & 6\,x_1 + 3\,x_2 \leq 60 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

a) Plot the set of feasible solutions for this linear program.
b) By using the graphical method, find the optimal solution to this linear program. Report the optimal values of the decision variables and the optimal value of the objective function.

Question 4
(This problem is from Hillier and Lieberman)

3.4-12.* The Medequip Company produces precision medical diagnostic equipment at two factories. Three medical centers have placed orders for this month's production output. The table below shows what the cost would be for shipping each unit from each factory to each of these customers. Also shown are the number of units that will be produced at each factory and the number of units ordered by each customer.

	Unit Shipping Cost			
From \ To	Customer 1	Customer 2	Customer 3	Output
Factory 1	$600	$800	$700	400 units
Factory 2	$400	$900	$600	500 units
Order size	300 units	200 units	400 units	

A decision now needs to be made about the shipping plan for how many units to ship from each factory to each customer.

a) Formulate a linear program to decide how many units to ship from each factory to each customer to minimize the total shipment cost. (Hint: Similar to the ad example that we did in class, use decision variables that capture the number of units shipped from each factory to each customer.)
b) Find the optimal solution by using Excel's solver. Report the optimal values of the decision variables and the optimal solution.