ORIE 5380, CS 5727: Optimization Methods
Homework Assignment 7

Due November 8, 12:00 pm

Please submit a single PDF document formatted to print and show all your work clearly. Feel free to scan and submit handwritten work. Do not spend too much time on wordprocessing your answers.

Question 1

This problem is open-ended. You will solve an optimization model via Gurobi that will address a real world problem.

We are organizing a conference. There are 71 papers that have been submitted to the conference. We have 21 referees to review these papers. Each paper needs to be reviewed by 3 referees. So, we need a total of $71 \times 3 = 213$ reviews, which means that each referee needs to review $213/21 = 10.14$ papers on average. Since reviewing fractional papers is not possible, a reasonable solution could have 18 referees reviewing 10 papers each, and 3 referees reviewing 11 papers each so that we can collect a total of 213 reviews for 71 papers.

We want to figure out which papers should be assigned to which referees. Not all referees are experts in the subject matter of all papers. Before doing the assignment, we sent the abstracts of the papers to the referees and asked the referees to tell us how comfortable they are reviewing each paper. The answers from the referees are in one of the 4 categories: "yes, I can definitely review this paper," "I can maybe review this paper," "no, I do not want to review this paper," "I have a conflict of interest with this paper, so it is unethical for me to review this paper." Ideally, we would like the referees to be assigned to papers for which they said "yes, I can definitely review this paper" or "I can maybe review this paper." If it is unavoidable, then a referee may be assigned to a paper for which she said "no, I do not want to review this paper." However, we definitely do not want to a referee to be assigned to a paper for which she said "I have a conflict of interest with this paper, so it is unethical for me to review this paper."

The attached spreadsheet gives the answers we collected from the referees. The rows correspond to the papers and the columns correspond to the referees. The entries indicate the answer of each referee for each paper. (This is real data by the way, reflecting the preferences of actual referees for papers in a real conference.)

a) Formulate a linear or integer program that finds a "good" assignment of referees to papers. It is up to you how to define "good" and how to come up with the cost parameters you need. Clearly state your decision variables, objective and constraints in proper mathematical notation. There is a way to formulate the problem as a min-cost network flow problem, possibly with upper bounds on the amount of flow on each arc.

You can talk to your classmates about their formulation in this part. Please keep the collaboration at the level of a corridor discussion and do not sketch models on paper together. Make sure to be by yourself when you are coming up with your ultimate formulation.

b) Write a program to solve your linear or integer programming model in Part a in Gurobi.

To present your solution, in the attached spreadsheet, keep the color in cell (i, j) only if paper i is assigned to referee j. Otherwise, remove the fill color from the cell. In this way, we can quickly see whether each referee got a paper that she is interested in refereeing.

Submit your program to solve the model in Gurobi and the spreadsheet in PDF format colored in the fashion described in the previous sentence. For formatting the colors in your spreadsheet, consider using conditional formatting in Excel, which can be found under the format menu. Make sure your code is completely your own work. In Gurobi, as you are adding a decision variable into the model, you can use the variable type `GRB.BINARY` and `GRB.INTEGER` to construct binary and integer decision variables, but there is a way to formulate the problem as a min-cost network flow problem with upper bounds on the amount of flow on each arc, in which case, you may get integer solutions for "free." Figuring out this min-cost network flow formulation is not necessary. Any formulation that does the job is enough.

Question 2

In this problem, you will show that the dual of the dual gives us the primal problem back. Consider the linear program

$$\begin{aligned}
\max \quad & 5\,x_1 + 9\,x_2 + 4\,x_3 \\
\text{st} \quad & 3\,x_1 + 4\,x_2 + 5\,x_3 \leq 7 \\
& x_1 + 2\,x_2 + x_3 \leq 3 \\
& x_1 + 2\,x_3 \leq 1 \\
& x_1 + x_2 \leq 1 \\
& x_1, x_2, x_3, x_4 \geq 0.
\end{aligned}$$

a) Write the dual of this linear program.

b) Write the dual of the dual. (Hint: We know how to write the dual of a linear program that maximizes its objective function with less-than-or-equal-to constraints. The dual in Part a minimizes its objective function and it has greater-than-or-equal-to constraints. So, rearrange the dual so that it maximizes its objective function and it has less-than-or-equal-to constraints. After rearranging, you can write the dual of the dual.)

c) Write the dual of the dual in Part b as a linear program, where we maximize the objective function with less-than-or-equal to constraints. Compare this form of the dual of the dual with the primal linear program given at the very beginning.