ORIE 5380: Optimization Methods Fall 2019, Project

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Abstract

The paper aims for developing a model that could help the cargo operator of Express Air to develop a weekly cycle to manage its fleet to deliver a certain amount of cargo among three airports, airports A, B, and C. To address this case, we have utilized integer linear programming to construct an optimization model to minimize the total cost of a weekly aircraft movement cycle for Express Air. Finally, the minimum weekly cost we calculated is 17925.

1 Introduction

The cargo business of Express Air operates an aircraft fleet to deliver a certain amount of cargo between each origin-destination airport among three airports, airports A, B, C. Express Air owns 1200 aircraft at its disposal. The number of cargo needs to be delivered each day between each pair of origin-destination airports is mentioned in {Table1}.

Therefore, there are two conditions for each cargo:

1. Be delivered on the same day it arrives into the system.
2. Remain in the same airport and wait to be carried on the next day.

If the amount of cargo being delivered each day does not fulfill the need, it would remain in the same airport to be carried the next day.

Also, there are three situations for each aircraft:

1. Deliver the cargo between each origin-destination airport
2. Remain in the same airport without any actions.
3. Be repositioned to the other two airports based on the demands.

First, each aircraft can only carry one aircraft load – no more, no less. The distances between any origin-destination pair are far enough that it will take a full day for each aircraft to deliver one cargo. If the aircraft is assigned to deliver the cargo from airport A to airport B, then the next day it will be available to be assigned any load within airport B. In addition, instead of carrying the cargo, these aircraft can also remain in the same location or be repositioned to the other two airports if requested.

The cost for the cargo operator of Express air will have two critical components. First, the cost will arise if the cargo didn't be delivered on time and remain in the same airport, which is the holding cost. Second, repositioning the empty aircraft from one airport to another airport will also generate fees. The holding cost of each full aircraft load of cargo is 10 per day, while the empty repositioning cost will depend on the origin and destination airports, which is also demonstrated in {Table2}.

Linear Programming (LP) is a problem-solving approach to help managers make decisions and also a mathematical method to achieve the maximum or minimum value of a linear function, which contains different decision variables, one objective, and various constraints. Thus, based on these restrictions and requirements, we decided to construct a minimizing optimization model to figure out the minimum total cost each week for the cargo operator of Express Air.

2 Data Description

2.1 Demand

{Table1} shows us the number of cargo arriving into the system on each day in a week for each Origin-Destination pair such as A-B and B-C. For example, on each Monday, there will be 100 aircraft loads cargo into the system that are needed to be delivered from airport A to airport B.

Day / Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	100	200	100	400	300
A-C	50	50	50	50	50
B-A	25	25	25	25	25
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	400	200	300	300	400

Table 1: Amounts of cargo (in aircraft loads) arriving into the system on each day that need to be carried between each origin-destination airport

2.2 Cost

The repositioning cost for each aircraft among different airports is as below:

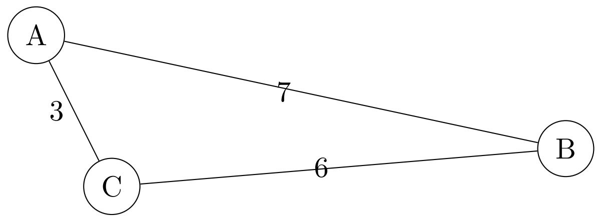

Figure 1: Empty repositioning costs among different airports

As we can see in the {Figure1}, the repositioning cost from airport A to B and airport B to A is 7, the repositioning cost from airport B to C and airport C to B is 6, and the repositioning cost from airport A to C and airport C to A is 3.

Day / Airports-Airports	A	B	C
A		7	3
B	7		6
C	3	6	

Table 2: Empty repositioning costs among different airports

We represent the cost of re positioning in table r, r_{ij} is the reposition cost from airport i to j, and if the airplane stay at airport i, no cost will incur, so r_{ii} is 0.

3 Optimization Model

3.1 Overview

According to the context of this project, there are three airports A, B and C and 5 delivery days in a week which are Mondays, Tuesdays, Wednesdays, Thursdays and Fridays. The holding cost of an aircraft load of cargo is 10. In our model, i, j denotes airports and T denotes delivery days. Also, we have:
x_{ijT} denotes the amount of cargo that is delivered from airport i to airport j on day t; y_{ijT} denotes the number of repositioning aircraft from airport i to airport j on day t; z_{ijT} denotes the amount of cargo that are hold for later delivery from airport i to airport j on day t; r_{ij} denotes the repositioning cost of an aircraft from airport i to j.

3.2 Mathematical Detail

$$
\begin{aligned}
\min \quad & \sum_{T \in weekdays} \sum_{i \in airports} \sum_{j \in airports} r_{ij} * y_{ij} + \sum_{T \in weekdays} \sum_{i \in airports} \sum_{j \in airports, i!=j} 10 * z_{ij} \\
\text{s.t.} \quad & \sum_{j} y_{ji,T-1} + \sum_{j,j!=i} x_{ji,T-1} = \sum_{j} y_{ij,T}, + \sum_{j,j!=i} x_{ij,T} \forall i \in airports, \forall T \in \{2,3,4,5\} \\
& \sum_{j} y_{ji,5} + \sum_{j,j!=i} x_{ji,5} = \sum_{j} y_{ij,1}, + \sum_{j,j!=i} x_{ij,1} \forall i \in airports, \\
& z_{ij,T-1} + D_{ij,T} = x_{ij,T} + z_{ij,T} \forall i \in airports, \forall j \in airports, i! = j, \forall T \in \{2,3,4,5\} \\
& z_{ij,5} + D_{ij,1} = x_{ij,1} + z_{ij,1} \forall i \in airports, \forall j \in airports, i! = j \\
& \sum_{T \in weekdays} x_{ij,T} = \sum_{T \in weekdays} D_{ij,T} \forall T \in weekdays \\
& \sum_{i \in airports} \sum_{j \in airports} x_{i,j,T} + \sum_{i \in airports} \sum_{j \in airports, i!=j} y_{i,j,T} = 1200, \forall T \in weekdays \\
& x_{ijT} \geq 0, y_{ijT} \geq 0, z_{ijT} \geq 0 \forall i,j \in airports, T \in weekdays
\end{aligned}
\tag{1}
$$

It is crucial to keep track of the flow balance of all aircraft and this is our first and second constraint. The number of aircraft that is repositioned from airport j to airport i on day T-1(note when i equals to j, that means the aircraft stays at this airport on day T-1), plus the number of aircraft that delivers cargo from airport j to airport i on day T-1 is the total number of aircraft that is available in use on day T and this is equal to the number of aircraft that is repositioned from airport i to airport j on day T (note when i equals to j, that means the aircraft stay at this airport on day T), plus the number of aircraft that delivers cargo from airport i to airport j on day T. The second constraint is a special case where the number of aircraft that is available at the beginning of Monday (day 1) in an airport is equal to the number of aircraft that arrives the airport or stays at this airport on last Friday (day 5).

We also need to consider the flow balance of cargo that need to be delivered among three airports and this is our third and fourth constraints. The amount of cargo that is hold for later delivery from airport i to airport j on day T-1 plus the amount of cargo that arrived into the system and needs to be delivered form airport i to airport j on day T is equal to the amount of cargo that is actually delivered from airport i to airport j on day T plus the amount of cargo that is hold for later delivery from airport i to j on day T. The fourth constraint is a special case where the amount of cargo that is hold for later delivery from airport i to j on Friday (day 5) plus the amount of cargo arrived into the system that needs to be delivered from airport i to j on Friday (day 5) is equal to the amount of cargo that is actually delivered from airport i to j on the next Monday (day 1) plus the amount of cargo that is hold for later delivery from airport i to j on Monday (day 1).

Our fifth constraint is that, since this is a cycle, the total amount of cargo that is actually delivered from airport i to j through out the week is equal to the total amount of cargo that arrived into the system that needs to be delivered from airport i to j through out the week.

Our sixth constraint is that the total number of aircraft whether move or not on each day is equal to 1200, the total number of aircraft available.

Finally, we have all decision variables are greater than or equal to zero.

4 Result & Analysis

After solving our optimization model by calling Gurobi from Python, we have figured out the most optimizing solution for the cargo operator of Express Air. The results are stated in the following three tables.

The most optimizing minimum total cost each week is 17925.

4.1 Situation 1: Successful Delivery

Day / Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	290	200	100	400	110
A-C	50	50	50	50	50
B-A	25	25	25	25	25
B-C	25	25	25	25	25
C-A	20	60	40	40	40
C-B	330	270	300	200	400

Table 3: Successful delivery flow between each origin-destination airport

{Table3} shows the successful delivery flow between each pair of origin-destination airports. For example, {"A-B"} {"Monday"} {"290"} means the number of loads of cargo as well as the number of aircraft carrying cargo from airport A to airport B on Monday is 290.

4.2 Situation 2: Cargo Holding

Day / Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	0	0	0	0	190
A-C	0	0	0	0	0
B-A	0	0	0	0	0
B-C	0	0	0	0	0
C-A	20	0	0	0	0
C-B	70	0	0	0	0

Table 4: Amount of cargo holding among airports

{Table4} demonstrates the number of cargo holding in the airport to wait for the next day delivery. For instance, {"A-B"} {"Friday"} {"190"} refers that there are 190 loads of cargo which were not satisfied from airport A to airport B on Friday so that they remained in the airport A to wait to be carried on the next day.

4.3 Situation 3: Empty Reposition

Day / Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-A	0	0	75	0	0
A-B	0	0	0	0	0
A-C	0	0	0	0	0
B-A	205	140	310	95	275
B-B	0	0	110	0	0
B-C	255	430	0	365	275
C-A	0	0	0	0	0
c-B	0	0	0	0	0
C-C	0	0	165	0	0

Table 5: Amount of empty repositioning aircraft

{Table5} implicates the number of empty aircraft to be repositioned based on the demands. For example, {"A-A"} {"Wednesday"} {"75"} means that there will be 75 empty aircraft remaining in the airport A on Wednesday instead of carrying the cargo or being repositioned to other airports. {"B-A"} {"Friday"} {"275"} refers that there will be 275 empty aircraft being repositioned from airport B to airport A on Friday.

5 Discussion

After we were done with the min cost model, we then considered to explore more questions that could be answered by this model. Firstly, we wanted to know the benefits of enlarging the fleet size. We increased the fleet size to different numbers and keep the cargo arrived into the system the same. {Table6} is what we got:

Fleet size	Minimum operation cost
1200	17925
1250	16925
1300	16025
1350	15525
1390	15125
1450	15125
2000	15125

Table 6: Variations of minimum cost with different fleet sizes

We noticed that at the beginning when we tried to enlarge the fleet size, the minimum operation cost decreased and stopped decreasing when the fleet size is larger than 1390 with a minimum cost of 15125. We believed that this is because when we started to enlarge the fleet size, the minimum cost will decrease since there are more aircraft can be used to deliver which lead to the amount of cargo that is hold for later delivery decrease and thus lower the total holding cost. However, when we further increase the fleet size to 1390, all cargo will be delivered to the destination the same day when it arrives into the system, so no holding cost will incur at this time and the cost now is all about repositioning the aircraft. The pattern of repositioning now is fixed and no more aircraft is needed to reduce the total operation cost. Thus, the extra aircraft will just stay where they begin to not induce more repositioning cost.

Then we were interested in the effects of having more cargo to deliver over different airports. We keep the fleet size the same and change the amount of cargo arrived into the system. When we changed the cargo amount arrived of B-A on each day to 100 (before is 25):

Origin-Destination \ Day	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	100	200	100	400	300
A-C	50	50	50	50	50
B-A	100	100	100	100	100
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	400	200	300	300	400

Table 7: New amounts of cargo (in aircraft loads) arriving into the system on each

The minimum cost now is 15300.

However, when we changed the cargo amount arrived of A-B on Monday to 200 (before is 100):

Origin-Destination \ Day	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	200	200	100	400	300
A-C	50	50	50	50	50
B-A	100	100	100	100	100
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	400	200	300	300	400

Table 8: New amounts of cargo (in aircraft loads) arriving into the system on each

The minimum cost now is 20425.

From the above results, we concluded that when the amount of cargo arrived into the system become more balanced (i.e. lower variance) among all origin-destination pairs, the cost will decrease if we can find a solution with our model.This is because in this case, more aircraft will be delivering instead of flying vacant, thus less repositioning cost will incur. When the amount of cargo arrived into the system become more imbalanced (i.e. larger variance) among all origin-destination pairs, the cost will increase if we can find a solution with our model. This is because more aircraft will be flying vacant to be repositioned, thus more repositoning cost will incur.