

ORIE 5380, CS 5727: Optimization Methods

Homework Assignment 3

Due October 4, 12:00 pm

Please submit a single PDF document formatted to print and show all your work clearly.

Feel free to scan and submit handwritten work. Do not spend too much time on wordprocessing your answers.

Question 1

(This problem is partially from Hillier and Lieberman.) Consider the linear program

$$\begin{array}{ll}\max & 5x_1 + 8x_2 \\ \text{st} & 4x_1 + 2x_2 \leq 80 \\ & -x_1 + 2x_2 \leq 20 \\ & 4x_1 - x_2 \leq 40 \\ & x_1, x_2 \geq 0.\end{array}$$

a) Solve the linear program above by using the simplex method. Show all systems of equations that the simplex method visits and state the optimal solution you reach.

b) Plot the set of feasible solutions to the linear program above and mark each solution that the simplex method visits on its way to the optimal solution.

Question 2

Consider the linear program

$$\begin{array}{ll}\max & 2x_1 - 6x_2 + 2x_3 \\ \text{st} & -2x_1 - x_2 - x_3 \leq -2 \\ & 2x_1 - x_2 + x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0.\end{array}$$

a) Find a feasible solution to the linear program above by using the phase-1 linear program.

b) If there is a feasible solution to the linear program, then find the optimal solution to the linear program above by starting from the feasible solution you found in Part a.

c) Use Excel's solver to verify that your solution in Part b is indeed the optimal solution.

(There are three more problems on the next page.)

Question 3

Consider the linear program

$$\begin{aligned}
 \max \quad & x_2 \\
 \text{st} \quad & 4x_1 + x_2 \leq 10 \\
 & -x_1 + x_2 \leq -1 \\
 & -x_1 - x_2 \leq -3 \\
 & x_1, x_2 \geq 0.
 \end{aligned}$$

- Find a feasible solution to the linear program above by using the phase-1 linear program.
- If there is a feasible solution to the linear program, then find the optimal solution to the linear program above by starting from the feasible solution you found in Part a.
- Use Excel's solver to verify that your solution in Part b is indeed the optimal solution.

Question 4

Consider the linear program

$$\begin{aligned}
 \max \quad & -2x_1 + 2x_2 - x_3 + 3x_4 \\
 \text{st} \quad & -3x_1 + x_2 + 4x_3 + x_4 \leq 0 \\
 & 3x_1 - x_2 - 3x_3 - 2x_4 \leq 3 \\
 & x_1, x_2, x_3, x_4 \geq 0.
 \end{aligned}$$

Use the simplex method to check whether this linear program is unbounded or has alternative optimal solutions. If the problem has alternative optimal solutions, then state two possible optimal solutions.

Question 5

Consider the linear program

$$\begin{aligned}
 \max \quad & 5x_1 + 7x_2 - 12x_3 - 10x_4 \\
 \text{st} \quad & 2x_1 - 2x_2 - 3x_3 - 2x_4 \leq 6 \\
 & 2x_1 + 5x_2 - 4x_3 - 4x_4 \leq 3 \\
 & x_1, x_2, x_3, x_4 \geq 0.
 \end{aligned}$$

Use the simplex method to check whether this linear program is unbounded or has alternative optimal solutions. If the problem has alternative optimal solutions, then state two possible optimal solutions.