Optimization HW9 Queenie Lin ql299

Q1 $\max\ 4x_1 + 3x_2$

s.t. $x_1 + 4x_2 \le 140$

$6x_1 + 3x_2 \le 180$

$x_2 \le 30$

$x_1, x_2 \ge 0$

(a) $\min\ 140y_1 + 180y_2 + 30y_3$

$y_1 + 6y_2 \ge 4$

$4y_1 + 3y_2 + y_3 \ge 3$

$y_1, y_2, y_3 \ge 0$

(b)

$$\begin{array}{l}
4x_1 + 3x_2 \qquad\qquad = z \\
\hline
x_1 + 4x_2 + \boxed{w_1} \qquad = 140 \\
6x_1 + 3x_2 \qquad + \boxed{w_2} \quad = 180 \\
\qquad x_2 \qquad\qquad + \boxed{w_3} = 30
\end{array}$$

$x_1 = x_2 = 0$

$w_1 = 140,\ w_2 = 180,\ w_3 = 30$

$z = 0$

$\min\{140, 30\} = 30$

$$\begin{array}{l}
x_2 \qquad - \frac{2}{3}w_2 = z - 120 \\
\hline
\frac{7}{2}x_2 + \boxed{w_1} - \frac{1}{6}w_2 = 110 \\
\boxed{x_1} + \frac{1}{2}x_2 \qquad + \frac{1}{6}w_2 = 30 \\
\qquad x_2 \qquad\qquad + \boxed{w_3} = 30
\end{array}$$

$x_2 = w_2 = 0$

$x_1 = 30,\ w_1 = 110,\ w_3 = 30$

$z = 120$

$\min\{\frac{220}{7}, 60, 30\} = 30$

$$\begin{array}{l}
-\frac{2}{3}w_2 - w_3 = z - 150 \\
\hline
\boxed{w_1} - \frac{1}{6}w_2 - \frac{7}{2}w_3 = 5 \\
\boxed{x_1} \qquad + \frac{1}{6}w_2 - \frac{1}{2}w_3 = 15 \\
\qquad \boxed{x_2} \qquad\qquad + w_3 = 30
\end{array}$$

Optimal Sol.

$$\boxed{\begin{array}{c} x_1 = 15,\ x_2 = 30 \\ w_1 = 5,\ w_2 = w_3 = 0 \\ z = 150 \end{array}}$$

(c) Optimal Sol to the dual is

$z_1 = z_2 = 0,\ y_1 = 0,\ y_2 = \frac{2}{3},\ y_3 = 1$

1(d) from part (c)

$$z_1 = z_2 = 0, \quad y_1 = 0, \quad y_2 = \frac{2}{3}, \quad y_3 = 1$$

check with part (a)

$$y_1 + 6y_2 = 4 \geq 4 \quad \checkmark$$
$$4y_1 + 3y_2 + y_3 = 3 \geq 3 \quad \checkmark$$
$$y_1 = 0 \geq 0 \quad \checkmark$$
$$y_2 = \frac{2}{3} \geq 0 \quad \checkmark$$
$$y_3 = 1 \geq 0 \quad \checkmark$$

obj $140y_1 + 180y_2 + 30y_3 = 150$

which is the same as the obj value in part (b)

$\therefore$ The solution in part (c) is indeed the optimal solution to the dual.

1(e) $6x_1 + 3x_2 \leq 180 + \epsilon$

$140y_1 + 180y_2 + 30y_3 \Rightarrow 140y_1 + (180+\epsilon)y_2 + 30y_3$

$\therefore$ The obj value will increase by $y_2 \cdot \epsilon = \frac{2}{3}\epsilon$

(f) when changed the second constraint by ϵ

$\Rightarrow 6x_1 + 3x_2 \leq 180 + \epsilon$

$$\Rightarrow \quad -\frac{2}{3}w_2 - w_3 = z - 150$$

$$\begin{array}{llll}
w_1 & -\frac{1}{6}w_2 - \frac{7}{2}w_3 = 5 & -\frac{1}{6}\epsilon \\
x_1 & +\frac{1}{6}w_2 - \frac{1}{2}w_3 = 15 & +\frac{1}{6}\epsilon \\
x_2 & +w_3 = 30 &
\end{array}$$

$\therefore$ when $\frac{1}{6}\epsilon \geq 5$, the solution is not feasible

$\therefore \epsilon \leq 30$

(g) $\epsilon = 6$

$$\begin{array}{ll}
\max & 4x_1 + 3x_2 \\
\text{s.t} & x_1 + 4x_2 \leq 140 \\
& 6x_1 + 3x_2 \leq 186 \\
& x_2 \leq 30 \\
& x_1, x_2 \geq 0
\end{array}$$

(h) $\epsilon = 36$

$$\begin{array}{ll}
\max & 4x_1 + 3x_2 \\
\text{s.t} & x_1 + 4x_2 \leq 140 \\
& 6x_1 + 3x_2 \leq 216 \\
& x_2 \leq 30 \\
& x_1, x_2 \geq 0
\end{array}$$

(g)

The new optimal solution is 154 which matches my answer to part e.

variables	x1	x2	
	16	30	
obj	154		
constraint	136	<=	140
	186	<=	186
	30	<=	30

(h)

The new optimal solution is 173.71429, which increased by 23.71429. This results from the ∈ is no longer smaller than what part f stated, which is 30.

variables	x1	x2	
	21.142857	29.714286	
obj	173.71429		
constraint	140	<=	140
	216	<=	216
	29.714286	<=	30

(i) $x_1 = 15$, $x_2 = 30$, $w_1 = 5$, $w_2 = w_3 = 0$, $Z = 150$

by complimentary slackness

$x_1 \cdot z_1 = 0 \Rightarrow z_1 = 0$

$x_2 \cdot z_2 = 0 \Rightarrow z_2 = 0$

$w_1 \cdot y_1 = 0 \Rightarrow y_1 = 0$

dual : $\min\ 140y_1 + 180y_2 + 30y_3$

$$y_1 + 6y_2 \geq 4$$
$$4y_1 + 3y_2 + y_3 \geq 3$$
$$y_1, y_2, y_3 \geq 0$$

$\therefore\ 6y_2 = 4 \Rightarrow y_2 = \frac{2}{3}$

$2 + y_3 = 3 \Rightarrow y_3 = 1$

$\therefore$ optimal sol :

$y_1 = 0$, $y_2 = \frac{2}{3}$, $y_3 = 1$

$z_1 = z_2 = 0$