ORIE 5380 HW3 Queenie Liu ql299

Q1 max $5x_1 + 8x_2$

st $4x_1 + 2x_2 \leq 80$

$-x_1 + 2x_2 \leq 20$

$4x_1 - x_2 \leq 40$

$x_1, x_2 \geq 0$

a) Add slack variables s_1, s_2, s_3

$4x_1 + 2x_2 + s_1 = 80$

$-x_1 + 2x_2 + s_2 = 20$

$4x_1 - x_2 + s_3 = 40$

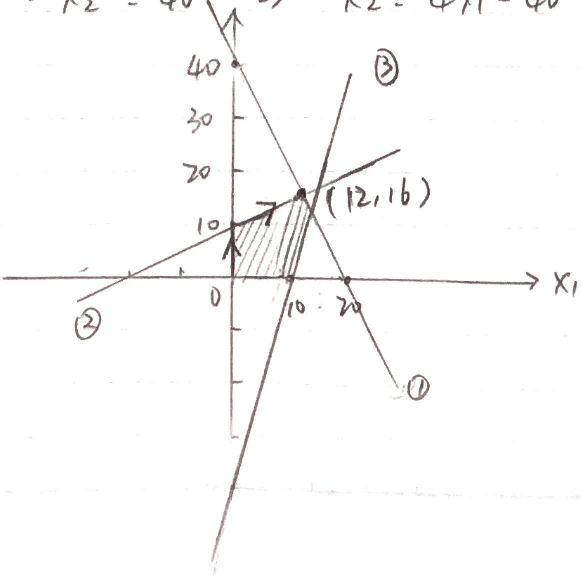
Z	x_1	x_2	s_1	s_2	s_3	RHS
1	5	8	0	0	0	0
0	4	2	1	0	0	80
0	-1	$\boxed{2}$	0	1	0	20
0	4	-1	0	0	1	40
1	-9	0	0	4	0	80
0	$\boxed{5}$	0	1	-1	0	60
0	$-1/2$	1	0	$\frac{1}{2}$	0	10
0	$7/2$	0	0	$\frac{1}{2}$	1	50
1	0	0	$9/5$	$11/5$	0	188
0	1	0	$1/5$	$-1/5$	0	12
0	0	1	$1/10$	$2/5$	0	16
0	0	0	$-7/10$	$6/5$	1	8

∴ Optimal solution: $Z = 188$, $(x_1, x_2, s_1, s_2, s_3) = (12, 16, 0, 0, 8)$

b) ① $4x_1 + 2x_2 = 80 \Rightarrow x_2 = 40 - 2x_1 \Rightarrow (0, 40), (20, 0)$

② $-x_1 + 2x_2 = 20 \Rightarrow x_2 = 10 + \frac{1}{2}x_1 \Rightarrow (0, 10), (-20, 0)$

③ $4x_1 - x_2 = 40 \Rightarrow x_2 = 4x_1 - 40 \Rightarrow (0, -40), (10, 0)$



Q2 $\max \ 2x_1 - 6x_2 + 2x_3$

s.t $-2x_1 - x_2 - x_3 \le -2$

$2x_1 - x_2 + x_3 \le 1$

$x_1, x_2, x_3 \ge 0$

a) $\max \ 2x_1 - 6x_2 + 2x_3$

$-2x_1 - x_2 - x_3 + s_1 = -2$

$2x_1 - x_2 + x_3 + s_2 = 1$

$x \ge 0, \ s \ge 0$

$\Rightarrow \min \ y = z$ $\qquad z=0$

$-2x_1 - x_2 - x_3 + s_1 - y = -2$ $\qquad s_1 = -2, \ s_2 = 1 \quad x_1 = x_2 = x_3 = y = 0$

$2x_1 - x_2 + x_3 + s_2 - y = 1$ $\qquad$ not feasible

$-2x_1 - x_2 - x_3 + s_1 = z - 2$ $\qquad s_2 = 2, \ y = 3 \qquad z = 2$

$2x_1 + x_2 + x_3 - s_1 + y = 2$ $\qquad x_1 = x_2 = x_3 = s_1 = 0$

$4x_1 + 2x_3 - s_1 + s_2 = 3$ $\qquad$ feasible.

$-x_2 + \frac{1}{2}s_1 + \frac{1}{2}s_2 = z - \frac{1}{2}$ $\qquad x_1 = \frac{3}{4}$

$x_2 - \frac{1}{2}s_1 - \frac{1}{2}s_2 + y = \frac{1}{2}$ $\qquad y = \frac{1}{2}, \quad x_2 = x_3 = s_1 = s_2 = 0$

$x_1 + \frac{1}{2}x_3 - \frac{1}{4}s_1 + \frac{1}{4}s_2 = 3/4$ $\qquad z = \frac{1}{2}$

$y = z$ $\qquad$ feasible sol:

$x_2 - \frac{1}{2}s_1 - \frac{1}{2}s_2 + y = \frac{1}{2}$ $\qquad (x_1, x_2, x_3) = (\frac{3}{4}, \frac{1}{2}, 0)$ with $z = 0$

$x_1 + \frac{1}{2}x_3 - \frac{1}{4}s_1 + \frac{1}{4}s_2 = 3/4$

b) $2x_1 - 6x_2 + 2x_3 = z$

$x_2 - \frac{1}{2}s_1 - \frac{1}{2}s_2 = \frac{1}{2}$

$x_1 + \frac{1}{2}x_3 - \frac{1}{4}s_1 + \frac{1}{4}s_2 = \frac{3}{4}$

$-6x_2 + x_3 + \frac{1}{2}s_1 - \frac{1}{2}s_2 = z - \frac{3}{2}$

$x_2 - \frac{1}{2}s_1 - \frac{1}{2}s_2 = \frac{1}{2}$

$x_1 + \frac{1}{2}x_3 - \frac{1}{4}s_1 + \frac{1}{4}s_2 = \frac{3}{4}$

$x_3 - \frac{5}{2}s_1 - \frac{7}{2}s_2 = z + \frac{3}{2}$

$x_2 - \frac{1}{2}s_1 - \frac{1}{2}s_2 = \frac{1}{2}$

$x_1 + \frac{1}{2}x_3 - \frac{1}{4}s_1 + \frac{1}{4}s_2 = \frac{3}{4}$

$s_1 = s_2 = 0$

$-2s_1 - 4s_2 = z$

$x_2 - \frac{1}{2}s_1 - \frac{1}{2}s_2 = \frac{1}{2}$

$2x_1 + x_3 - \frac{1}{2}s_1 + \frac{1}{2}s_2 = \frac{3}{2}$

$\therefore$ optimal sol.

$(x_1, x_2, x_3) = (0, \frac{1}{2}, \frac{3}{2})$

with $z = 0$

Q2

Objective	0	

x1	x2	x3
0	0.5	1.5

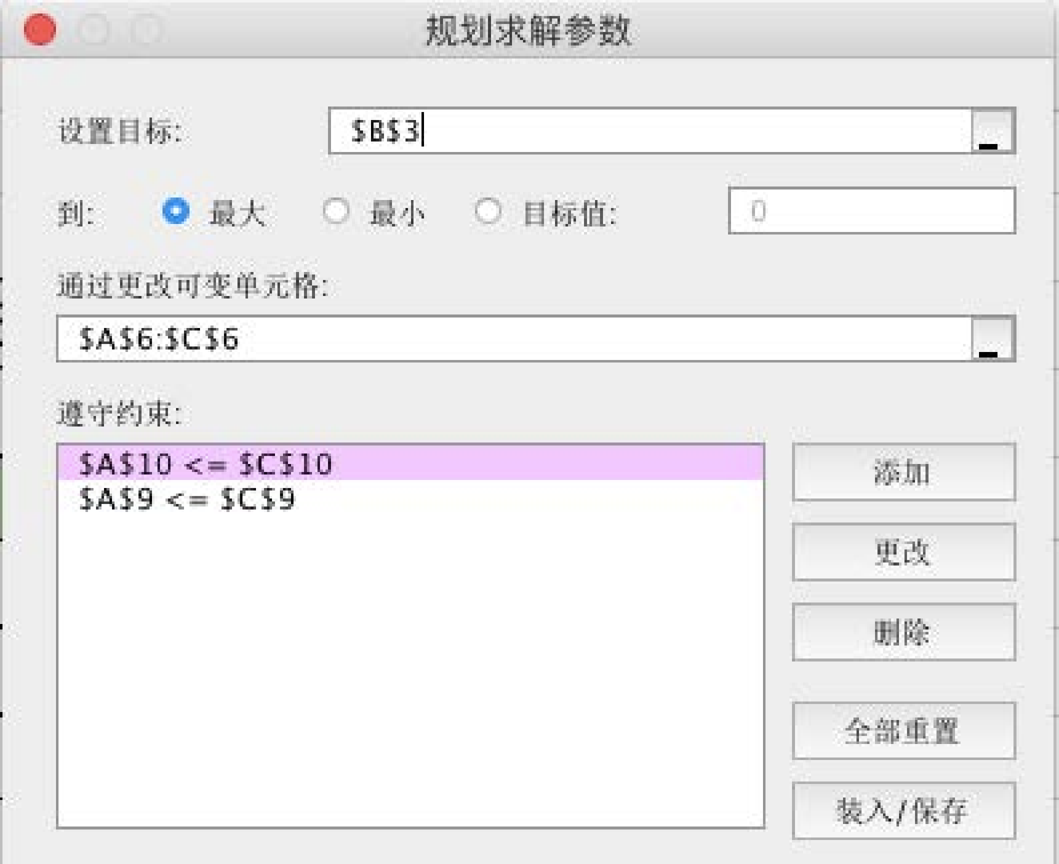
Constraints		
-2	<=	-2
1	<=	1

Q2

Objective	=2*A6-6*B6+2*C6	

x1	x2	x3
0	0.5	1.5

Constraints		
=-2*A6-B6-C6	<=	-2
=2*A6-B6+C6	<=	1


规划求解参数
设置目标:
$B$3
到:
最大
最小
目标值:
0
通过更改可变单元格:
$A$6:$C$6
遵守约束:
$A$10 <= $C$10
$A$9 <= $C$9
添加
更改
删除
全部重置
装入/保存

Q3 max x_2

s.t $4x_1 + x_2 \le 10$

$-x_1 + x_2 \le -1$

$-x_1 - x_2 \le -3$

a) $x_1, x_2 \ge 0$

$\Rightarrow$ max x_2

s.t
$$\begin{aligned} 4x_1 + x_2 + s_1 &= 10 \\ -x_1 + x_2 + s_2 &= -1 \\ -x_1 - x_2 + s_3 &= -3 \end{aligned}$$

$s_1 = 10$, $s_2 = -1$, $s_3 = -3$, $x_1 = x_2 = 0$ not feasible.

$\Rightarrow$ min y $= z$
$$\begin{aligned} 4x_1 + x_2 + s_1 - y &= 10 \\ -x_1 + x_2 + s_2 - y &= -1 \\ -x_1 - x_2 + s_3 - y &= -3 \end{aligned}$$

$$-x_1 - x_2 + s_3 = z - 3$$

$$\begin{aligned} 5x_1 + 2x_2 + s_1 - s_3 &= 13 \\ 2x_2 + s_2 - s_3 &= 2 \\ x_1 + x_2 - s_3 + y &= 3 \end{aligned}$$

$s_1 = 13$, $s_2 = 2$, $y = 3$

$z = 3$, $x_1 = x_2 = 0$

x_1 min $(\frac{13}{5}, 3) = \frac{13}{5}$

$$-\tfrac{3}{5}x_2 + \tfrac{1}{5}s_1 + \tfrac{4}{5}s_3 = z - \tfrac{2}{5}$$

$$\begin{aligned} x_1 + \tfrac{2}{5}x_2 + \tfrac{1}{5}s_1 - \tfrac{1}{5}s_3 &= \tfrac{13}{5} \\ 2x_2 + s_2 - s_3 &= 2 \\ \tfrac{3}{5}x_2 - \tfrac{1}{5}s_1 - \tfrac{4}{5}s_3 + y &= \tfrac{2}{5} \end{aligned}$$

x_2 min $(\frac{2}{3}, 1, \frac{13}{2}) = \frac{2}{3}$

$$y = z$$

$$\begin{aligned} x_1 + \tfrac{1}{3}s_1 + \tfrac{1}{3}s_3 - \tfrac{2}{3}y &= \tfrac{7}{3} \\ \tfrac{2}{3}s_1 + s_2 + \tfrac{5}{3}s_3 - \tfrac{10}{3}y &= \tfrac{2}{3} \\ x_2 - \tfrac{1}{3}s_1 - \tfrac{4}{3}s_3 + \tfrac{5}{3}y &= \tfrac{2}{3} \end{aligned}$$

$x_1 = \frac{7}{3}$, $x_2 = \frac{2}{3}$, $s_2 = \frac{2}{3}$ $\Rightarrow$ feasible solution

$s_3 = s_4 = y = 0$, $z = 0$

b)

$$x_2 = z$$

$$x_1 + \frac{1}{3}s_1 + \frac{1}{3}s_3 = \frac{7}{3}$$
$$\frac{2}{3}s_1 + s_2 + \frac{5}{3}s_3 = \frac{2}{3}$$
$$x_2 - \frac{1}{3}s_1 - \frac{4}{3}s_3 = \frac{2}{3}$$

$$\frac{1}{3}s_1 + \frac{4}{3}s_3 = z - \frac{2}{3}$$

$$x_1 + \frac{1}{3}s_1 + \frac{1}{3}s_3 = \frac{7}{3}$$
$$\frac{2}{3}s_1 + s_2 + \frac{5}{3}s_3 = \frac{2}{3}$$
$$x_2 - \frac{1}{3}s_1 - \frac{4}{3}s_3 = \frac{2}{3}$$

s_3 $\min(-1, \frac{2}{5}) = \frac{2}{5}$

$$-\frac{1}{5}s_1 - \frac{4}{5}s_2 = z - \frac{6}{5}$$

$$x_1 - \frac{2}{15}s_1 - \frac{1}{5}s_2 = \frac{11}{5}$$
$$\frac{2}{5}s_1 + \frac{3}{5}s_2 + s_3 = \frac{2}{5}$$
$$x_2 + \frac{1}{5}s_1 + \frac{4}{5}s_2 = \frac{6}{5}$$

∴ optimal solution:

$x_1 = \frac{11}{5}$, $x_2 = \frac{6}{5}$, $s_3 = \frac{2}{5}$, $s_1 = s_2 = 0$

with $z = \frac{6}{5}$

Q3

Objective	1.2

x1	x2
2.2000003	1.2

Constraints		
10.000001	<=	10
-1	<=	-1
-3.4	<=	-3

Q3

Objective	=B6

x1	x2
2.20000025	1.2

Constraints		
=4*A6+B6	<=	10
=-A6+B6	<=	-1
=-A6-B6	<=	-3


规划求解参数
设置目标:
$B$3
到:
最大
最小
目标值:
0
通过更改可变单元格:
$A$6:$B$6
遵守约束:
$A$10 <= $C$10
$A$11 <= $C$11
$A$9 <= $C$9
添加
更改
删除
全部重置
装入/保存

Q4 $\max\ -2x_1 + 2x_2 - x_3 + 3x_4$

s.t. $-3x_1 + x_2 + 4x_3 + x_4 \le 0$

$3x_1 - x_2 - 3x_3 - 2x_4 \le 3$

$x_1, x_2, x_3, x_4 \ge 0$

$\Rightarrow -3x_1 + x_2 + 4x_3 + x_4 + s_1 = 0$

$3x_1 - x_2 - 3x_3 - 2x_4 + s_2 = 3$

z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
1	-2	2	-1	3	0	0	0
0	-3	1	4	1	1	0	0
0	3	-1	-3	-2	0	1	3
1	4	0	-9	1	-2	0	0
0	-3	1	4	1	1	0	0
0	0	0	1	-1	1	1	3

$x_1 = x_3 = x_4 = s_1 = 0$

$x_2 = 0$ $\qquad z = 0$

$s_2 = 3$

x_1 (non-basic) has a positive coefficient in obj and non-positive in constraints

To increase x_1; in first constraint,

x_1 has a negative coefficient $\Rightarrow$ x_1 can increase as much as we want

in 2nd constraint,

x_1 has a zero coefficient $\Rightarrow$ x_1 can increase as much as we want

$\therefore$ The linear program is unbounded.

Q5 max $5x_1 + 7x_2 - 12x_3 - 10x_4$

s.t. $2x_1 - 2x_2 - 3x_3 - 2x_4 \le 6$

$2x_1 + 5x_2 - 4x_3 - 4x_4 \le 3$

$x_1, x_2, x_3, x_4 \ge 0$

$2x_1 - 2x_2 - 3x_3 - 2x_4 + s_1 = 6$

$2x_1 + 5x_2 - 4x_3 - 4x_4 + s_2 = 3$

Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
1	5	7	-12	-10	0	0	0
0	2	-2	-3	-2	1	0	6
0	2	$\boxed{5}$	-4	-4	0	1	3
1	-11/5	0	32/5	22/5	$\boxed{0}$	7/5	21/5
0	14/5	0	-23/5	-18/5	1	2/5	36/5
0	$\boxed{2/5}$	1	-4/5	-4/5	0	1/5	3/5

$x_2 = \frac{3}{5}$

				x_4			
1	0	11/2	2	$\boxed{0}$	0	$\boxed{0}$	15/2
0	0	-7	1	2	1	-1	3
0	1	5/2	-2	-2	0	1/2	3/2

$s_1 = 3$ $\quad s_2 = 0$

$x_1 = \frac{3}{2}$ $\quad x_2 = x_3 = x_4 = 0$

Since there exists a non-basic variable (x_4) whose coefficient in the obj function row is 0, then we have multiple solution to the linear program

Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
1	0	11/2	2	0	0	0	15/2
0	0	-7/2	1/2	1	1/2	-1/2	3/2
0	1	9/2	-1	0	1	-1/2	9/2

$Z = \frac{15}{2}$

$x_1 = \frac{9}{2}$

$x_4 = \frac{3}{2}$ $\quad s_1 = s_2 = 0$

$x_2 = x_3 = 0$

∴ There exists multiple optimal solution

∴ Two possible optimal solution

Sol 1: $Z = \frac{15}{2}$
$x_1 = \frac{3}{2}$
$x_2 = x_3 = x_4 = 0$

Sol 2: $Z = \frac{15}{2}$
$x_1 = \frac{9}{2}$, $x_4 = \frac{3}{2}$
$x_2 = x_3 = 0$