

**ORIE 5380, CS 5727: Optimization Methods**  
**Homework Assignment 2**  
**Due September 27, 12:00 pm**

Please submit a single PDF document formatted to print. Show all your work clearly.

**Question 1**

(This problem is from Hillier and Lieberman.)

D 3.4-7. Consider the following problem, where the value of  $c_1$  has not yet been ascertained.

$$\text{Maximize } Z = c_1x_1 + 2x_2,$$

subject to

$$4x_1 + x_2 \leq 12$$

$$x_1 - x_2 \geq 2$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

a) Plot the set of feasible solutions for this linear program.

b) Using the graphical method for solving linear programs, state the optimal solution to the linear program for all possible values of  $c_1$  and indicate the appropriate ranges for  $c_1$  that would render each one of these solutions optimal. (You may want to use the fact that the optimal solution to a linear program always occurs at a corner point of the set of feasible solutions.)

**Question 2**

Consider the linear program

$$\begin{array}{ll} \text{minimize} & x_1 + x_2 \\ \text{st} & -x_1 + x_2 \geq 10 \\ & -2x_1 + x_2 \leq 20 \\ & x_1 \geq 0, x_2 \geq 0. \end{array}$$

a) Plot the set of feasible solutions for this linear program and find the optimal solution by using the graphical method for solving linear programs.

b) Does the linear program have a finite optimal solution when we change the objective function to minimize  $x_1 - x_2$ ? Explain your answer by using the graphical method for solving linear programs.

*(There are three more problems on the next page.)*

### Question 3

Use row operations to compute the inverse of the matrix

$$\begin{bmatrix} 5 & 0 & -2 \\ 1 & -2 & 2 \\ 2 & -1 & 0 \end{bmatrix}$$

Show all your work.

### Question 4

(This problem is from Vanderbei)

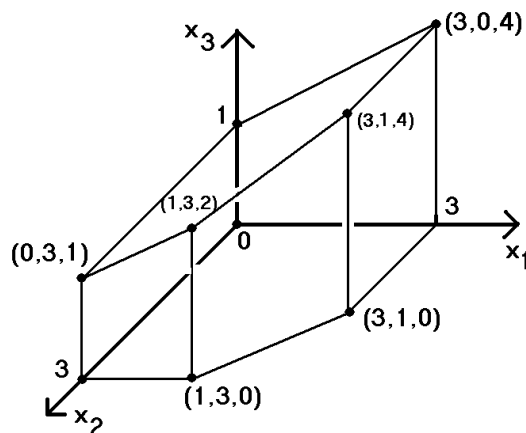
Solve the linear program below by using the simplex method. Show all systems of equations that the simplex method visits and state the optimal solution you reach.

$$\begin{aligned} &\text{maximize} && 6x_1 + 8x_2 + 5x_3 + 9x_4 \\ &\text{subject to} && 2x_1 + x_2 + x_3 + 3x_4 \leq 5 \\ &&& x_1 + 3x_2 + x_3 + 2x_4 \leq 3 \\ &&& x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

### Question 5

Below are a linear program and a graph for its set of feasible solutions.

$$\begin{aligned} &\text{maximize} && x_1 + 2x_2 + x_3 \\ &\text{subject to} && x_1 \leq 3 \\ &&& x_2 \leq 3 \\ &&& x_1 + x_2 \leq 4 \\ &&& -x_1 + x_3 \leq 1 \\ &&& x_1, x_2, x_3 \geq 0. \end{aligned}$$



Use the simplex method to solve the linear program. Show all systems of equations that the simplex method visits and state the optimal solution you reach. Use the figure above to mark *each* solution that the simplex method visits on its way to the optimal solution. (If there is a tie in terms of which variable you want to increase in the simplex method, you can pick one arbitrarily.)