ORIE 5380, CS 5727: Optimization Methods
Homework Assignment 9

Due November 22, 12:00 pm

Please submit a single PDF document formatted to print and show all your work clearly. Feel free to scan and submit handwritten work. Do not spend too much time on wordprocessing your answers.

Question 1
Consider the linear program

$$\begin{aligned} \max \quad & 4\,x_1 + 3\,x_2 \\ \text{st} \quad & x_1 + 4\,x_2 \le 140 \\ & 6\,x_1 + 3\,x_2 \le 180 \\ & x_2 \le 30 \\ & x_1, x_2 \ge 0. \end{aligned}$$

a) Write the dual of this problem.

b) Use the simplex method to obtain the optimal solution to the problem above.

c) By using the last system of equations that the simplex method reached in Part b, state the optimal solution to the dual problem.

d) Use the implication of weak duality to argue that the optimal solution to the dual problem that you stated in Part c is indeed the optimal solution to the dual problem. (That is, check that the primal and dual solutions you have are feasible for their respective problems and they provide the same objective values.)

e) Assume that we increase the right side of the second constraint in the linear program above by a small amount ϵ. How much does the optimal objective value change?

f) In this part, you will answer the question of how small is small. Assume that you found out in Part e that if we increase the right side of the second constraint by ϵ, then the optimal objective value changes by $\delta\,\epsilon$, where δ is a number you came up with in Part e. Find the largest value of ϵ so that if we increase the right side of the second constraint by ϵ, then the optimal objective value of the problem changes by $\delta\,\epsilon$.

g) Go ahead and increase the right side of the second constraint in the linear program above by $\epsilon = 6$ and solve the linear program again. How much does the optimal objective value of the linear program change? Does this answer match your answer to Part e? (You can use Excel's solver or Gurobi here.)

(There are two more parts to this problem on the next page.)

h) Now, increase the right side of the second constraint in the linear program above by $\epsilon = 36$ and solve the linear program again. How much does the optimal objective value of the linear program change? Explain your answer in the light of your finding in Part f. (You can use Excel's solver or Gurobi here as well.)

i) Assume that we only know an optimal solution to the problem above, as given by Part b. Use complementary slackness to give an optimal solution to the dual.