Optimization
Assignment 4
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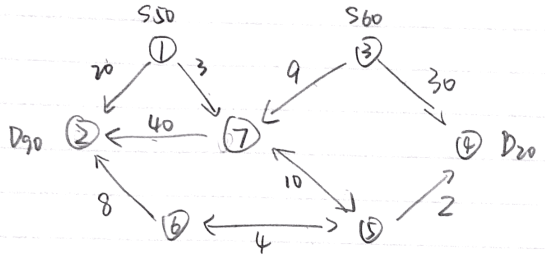
ql299

Q1

(a)

Optimization Hw 4 Queenie Liu ql299

Q1 min-cost network flow



(a) x_{ij}: # of units of product shipped on arc (i,j)

$$\text{Min} \quad 20x_{12} + 3x_{17} + 9x_{37} + 30x_{34} + 2x_{54} + 10x_{75} + 10x_{57} + 4x_{56} + 4x_{65} + 40x_{72} + 8x_{62}$$

$$\text{s.t.} \quad x_{12} + x_{17} = 50$$
$$x_{37} + x_{34} = 60$$
$$x_{72} + x_{75} - x_{37} - x_{17} - x_{57} = 0$$
$$x_{54} + x_{57} + x_{56} - x_{75} - x_{65} = 0$$
$$x_{62} + x_{65} - x_{56} = 0$$
$$-x_{34} - x_{54} = -20$$
$$-x_{12} - x_{72} - x_{62} = -90$$
$$x_{ij} \geq 0, \quad \forall (i,j) \in A, \quad A: \text{arcs in graph}$$

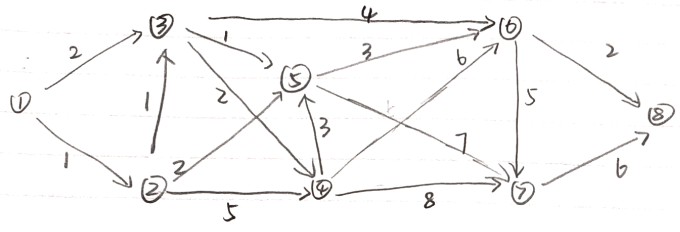
(b)

Q1				
	Coefficient	Variable	Output	
Decision Variable	20	x12	50	
	3	x17	0	
	9	x37	60	
	30	x34	0	
	2	x54	20	
	10	x75	60	
	10	x57	0	
	4	x56	40	
	4	x65	0	
	40	x72	0	
	8	x62	40	
Objective				
Minimize	2660			
Constraint	x12+x17	50	=	50
	X37+X34	60	=	60
	X72+X75-X37-X17-X57	0	=	0
	X54+X57+X56-X75-X65	7.105E-15	=	0
	x62+x65-x56	0	=	0
	x34+x54	20	=	20
	x12+x72+x62	90	=	90

Q2

(a)

Q2 Shortest path from city 1 to city 8



(a) x_{ij} : flow on arc (i,j)

$$\min \; 2x_{13} + x_{12} + x_{23} + 2x_{25} + 5x_{24} + x_{35} + 2x_{34} + 4x_{36} + 3x_{45} + 6x_{46} + 8x_{47} + 3x_{56} + 7x_{57} + 5x_{67} + 2x_{68} + 6x_{78}$$

s.t.

$$x_{13} + x_{12} = 1$$
$$x_{23} + x_{25} + x_{24} - x_{12} = 0$$
$$x_{34} + x_{35} + x_{36} - x_{13} - x_{23} = 0$$
$$x_{45} + x_{46} + x_{47} - x_{34} - x_{24} = 0$$
$$x_{56} + x_{57} - x_{25} - x_{35} - x_{45} = 0$$
$$x_{68} + x_{67} - x_{36} - x_{56} - x_{46} = 0$$
$$x_{78} - x_{67} - x_{57} - x_{47} = 0$$
$$-x_{68} - x_{78} = -1$$
$$x_{ij} \geq 0, \quad \forall (i,j) \in A, \quad A: \text{arcs in graph}$$

(b)

1-8

optimal Objective:
8.0
optimal Solution:
x13 0.0
x12 1.0
x23 1.0
x25 0.0
x24 0.0
x35 1.0
x34 0.0
x36 0.0
x45 0.0
x46 0.0
x47 0.0
x56 1.0
x57 0.0
x67 0.0
x68 1.0
x78 0.0

```
\ Model Q2_18
\ LP format - for model browsing. Use MPS format to capture full model detail.
Minimize
  2 x13 + x12 + x23 + 2 x25 + 5 x24 + x35 + 2 x34 + 4 x36 + 3 x45 + 6 x46
   + 8 x47 + 3 x56 + 7 x57 + 5 x67 + 2 x68 + 6 x78
Subject To
 firstconstraint: x13 + x12 = 1
 secondconstraint: - x12 + x23 + x25 + x24 = 0
 thirdconstraint: - x13 - x23 + x35 + x34 + x36 = 0
 fourthconstraint: - x24 - x34 + x45 + x46 + x47 = 0
 fifthconstraint: - x25 - x35 - x45 + x56 + x57 = 0
 sixthconstraint: - x36 - x46 - x56 + x67 + x68 = 0
 seventhconstraint: - x47 - x57 - x67 + x78 = 0
 eighthconstraint: x68 + x78 = 1
Bounds
End
```

1-6

```
optimal Objective:
6.0
optimal Solution:
x13 0.0
x12 1.0
x23 0.0
x25 1.0
x24 0.0
x35 0.0
x34 0.0
x36 0.0
x45 0.0
x46 0.0
x56 1.0
```

```
\ Model Q2_16
\ LP format – for model browsing. Use MPS format to capture full model detail.
Minimize
  2 x13 + x12 + x23 + 2 x25 + 5 x24 + x35 + 2 x34 + 4 x36 + 3 x45 + 6 x46
   + 3 x56
Subject To
 firstconstraint: x13 + x12 = 1
 secondconstraint: – x12 + x23 + x25 + x24 = 0
 thirdconstraint: – x13 – x23 + x35 + x34 + x36 = 0
 fourthconstraint: – x24 – x34 + x45 + x46 = 0
 fifthconstraint: – x25 – x35 – x45 + x56 = 0
 sixthconstraint: x36 + x46 + x56 = 1
Bounds
End
```

4-8

```
optimal Objective:
8.0
optimal Solution:
x45 0.0
x46 1.0
x47 0.0
x56 0.0
x57 0.0
x67 0.0
x68 1.0
x78 0.0
```

```
\ Model Q2_48
\ LP format – for model browsing. Use MPS format to capture full model detail.
Minimize
  3 x45 + 6 x46 + 8 x47 + 3 x56 + 7 x57 + 5 x67 + 2 x68 + 6 x78
Subject To
 fourthconstraint: x45 + x46 + x47 = 1
 fifthconstraint: – x45 + x56 + x57 = 0
 sixthconstraint: – x46 – x56 + x67 + x68 = 0
 seventhconstraint: – x47 – x57 – x67 + x78 = 0
 eighthconstraint: x68 + x78 = 1
Bounds
End
```

2-6

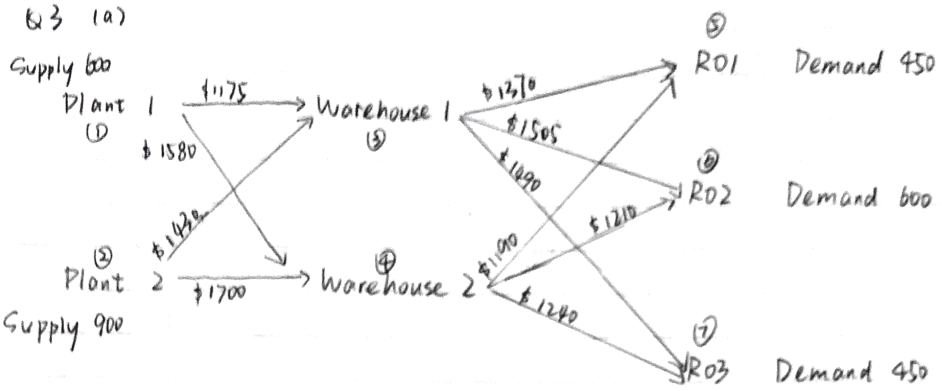
```
optimal Objective:
5.0
optimal Solution:
x23 1.0
x25 0.0
x24 0.0
x35 0.0
x34 0.0
x36 1.0
x45 0.0
x46 0.0
x56 0.0
```

```
\ Model Q2_26
\ LP format – for model browsing. Use MPS format to capture full model detail.
Minimize
  x23 + 2 x25 + 5 x24 + x35 + 2 x34 + 4 x36 + 3 x45 + 6 x46 + 3 x56
Subject To
 secondconstraint: x23 + x25 + x24 = 1
 thirdconstraint: – x23 + x35 + x34 + x36 = 0
 fourthconstraint: – x24 – x34 + x45 + x46 = 0
 fifthconstraint: – x25 – x35 – x45 + x56 = 0
 sixthconstraint: x36 + x46 + x56 = 1
Bounds
End
```

Q3

(a)

Q3 (a)



From the diagram above,

Plant 1 & Plant 2 are supply nodes;

RO1 & RO2 & RO3 are demand nodes;

Warehouse 1 & Warehouse 2 are transhipment nodes.

(b) min-cost shipping. → min-cost network flow

X_{ij} : # of units shipped on arc (i, j)

Nodes: ① => Plant 1 ⑤ => RO1

② => Plant 2 ⑥ => RO2

③ => Warehouse 1 ⑦ => RO3

④ => Ware house 2

$$\begin{aligned}\text{Min}\quad & 1175X_{13} + 1580X_{14} + 1430X_{23} + 1700X_{24} + 1370X_{35} + 1505X_{36} + 1490X_{37} \\ & + 1190X_{45} + 1210X_{46} + 1240X_{47}\end{aligned}$$

$$\begin{aligned}\text{S.t}\quad & X_{13} \le 375,\ X_{14} \le 450 \qquad X_{35}+X_{36}+X_{37} - X_{13} - X_{23} = 0 \\ & X_{13} + X_{14} = 600 \qquad X_{45}+X_{46}+X_{47} - X_{14} - X_{24} = 0 \\ & X_{23} \le 525,\ X_{24} \le 600 \\ & X_{23} + X_{24} = 900 \\ & X_{35} \le 300,\ X_{45} \le 375 \\ & -X_{35} - X_{45} = -450 \\ & X_{36} \le 450,\ X_{46} \le 450 \\ & -X_{36} - X_{46} = -600 \qquad A: \text{arcs in graph} \\ & X_{37} \le 300,\ X_{47} \le 225 \\ & -X_{37} - X_{47} = -450 \qquad X_{ij} \ge 0,\ \forall (i,j) \in A\end{aligned}$$

(b)

optimal Objective:
4217625.0
optimal Solution:
x13 375.0
x14 225.0
x23 375.0
x24 525.0
x35 300.0
x36 150.0
x37 300.0
x45 150.0
x46 450.0
x47 150.0

```
\ Model Q3
\ LP format - for model browsing. Use MPS format to capture full model detail.
Minimize
  1175 x13 + 1580 x14 + 1430 x23 + 1700 x24 + 1370 x35 + 1505 x36 + 1490 x37
   + 1190 x45 + 1210 x46 + 1240 x47
Subject To
 firstconstraint: x13 + x14 = 600
 secondconstraint: x23 + x24 = 900
 thirdconstraint: x35 + x45 = 450
 fourthconstraint: x36 + x46 = 600
 fifthconstraint: x37 + x47 = 450
 sixthconstraint: - x13 - x23 + x35 + x36 + x37 = 0
 seventhconstraint: - x14 - x24 + x45 + x46 + x47 = 0
Bounds
 x13 <= 375
 x14 <= 450
 x23 <= 525
 x24 <= 600
 x35 <= 300
 x36 <= 450
 x37 <= 300
 x45 <= 375
 x46 <= 450
 x47 <= 225
End
```