ORIE 5380 HW2 Queenie Liu

Q1 Max $Z = c_1x_1 + 2x_2$

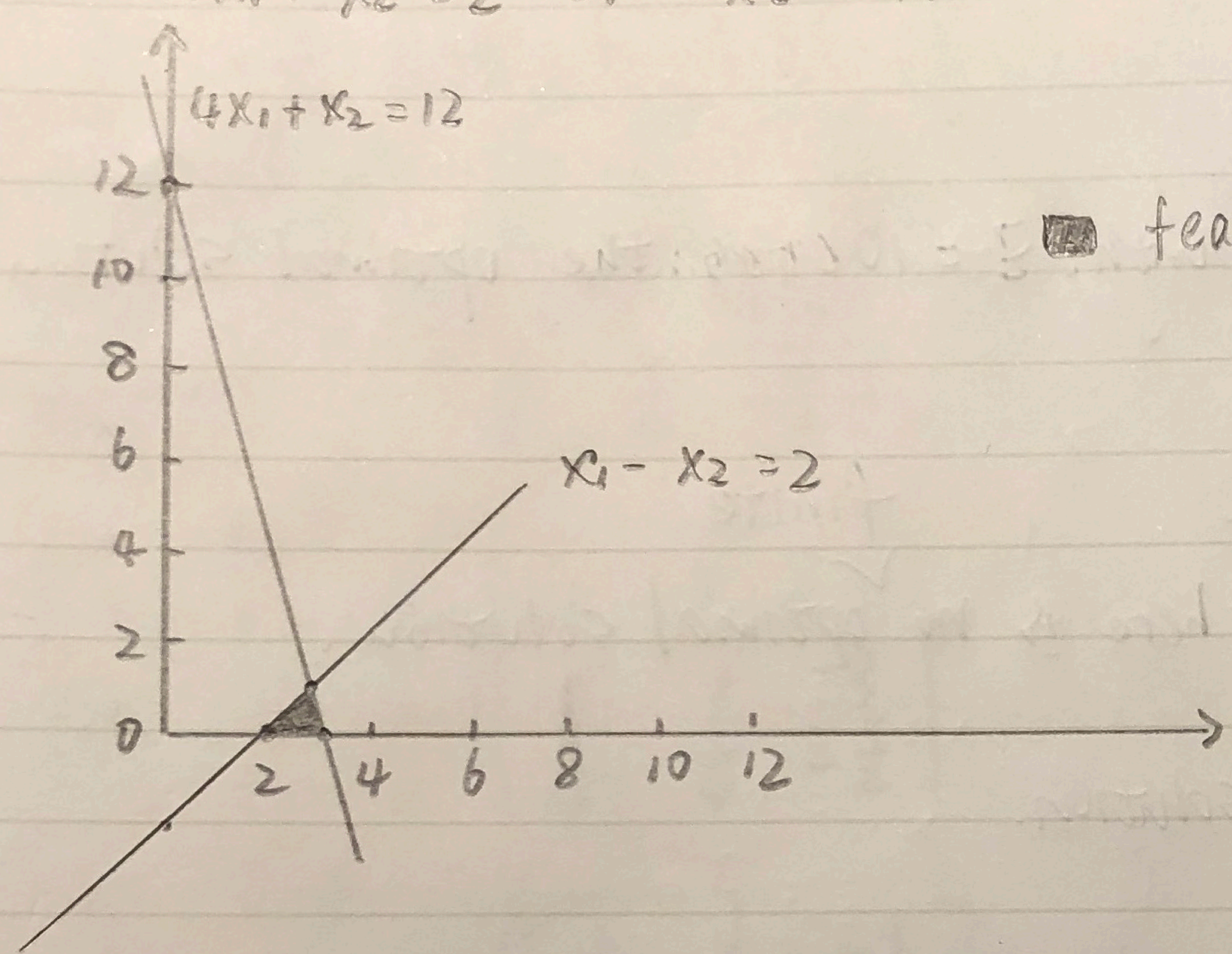
s.t. $4x_1 + x_2 \le 12$

$x_1 - x_2 \ge 2$

$x_1 \ge 0$, $x_2 \ge 0$

a) $4x_1 + x_2 = 12 \Rightarrow x_2 = 12 - 4x_1 \Rightarrow (0, 12) \quad (3, 0)$

$x_1 - x_2 = 2 \Rightarrow x_2 = x_1 - 2 \Rightarrow (2, 0) \quad (0, -2)$



b) corner point $(2, 0)$, $(3, 0)$, $(2.8, 0.8)$

$\begin{cases} 4x_1 + x_2 = 12 \\ x_1 - x_2 = 2 \end{cases} \Rightarrow \begin{matrix} 5x_1 = 14 \\ x_1 = 2.8 \end{matrix} \Rightarrow x_2 = 0.8 \Rightarrow \begin{cases} x_1 = 2.8 \\ x_2 = 0.8 \end{cases}$

$Z = c_1x_1 + 2x_2$

$\therefore Z = \begin{cases} 2c_1 \\ 3c_1 \\ 2.8c_1 + 1.6 \end{cases}$

Also, slope of obj : $-\frac{c_1}{2}$

slope of constraints : $4, -1$

$\therefore -1 \le \frac{c_1}{2} \le 4$

$-2 \le c_1 \le 8$

$\therefore$ optimal solution	c_1
$(2, 0)$	$c_1 < -2$
points on the line between $(2, 0)$ $(2.8, 0.8)$	$c_1 = -2$
$(2.8, 0.8)$	$-2 < c_1 < 8$
points on the line between $(2.8, 0.8)$ $(3, 0)$	$c_1 = 8$
$(3, 0)$	$c_1 > 8$

Q2 $\min \ X_1 + X_2$

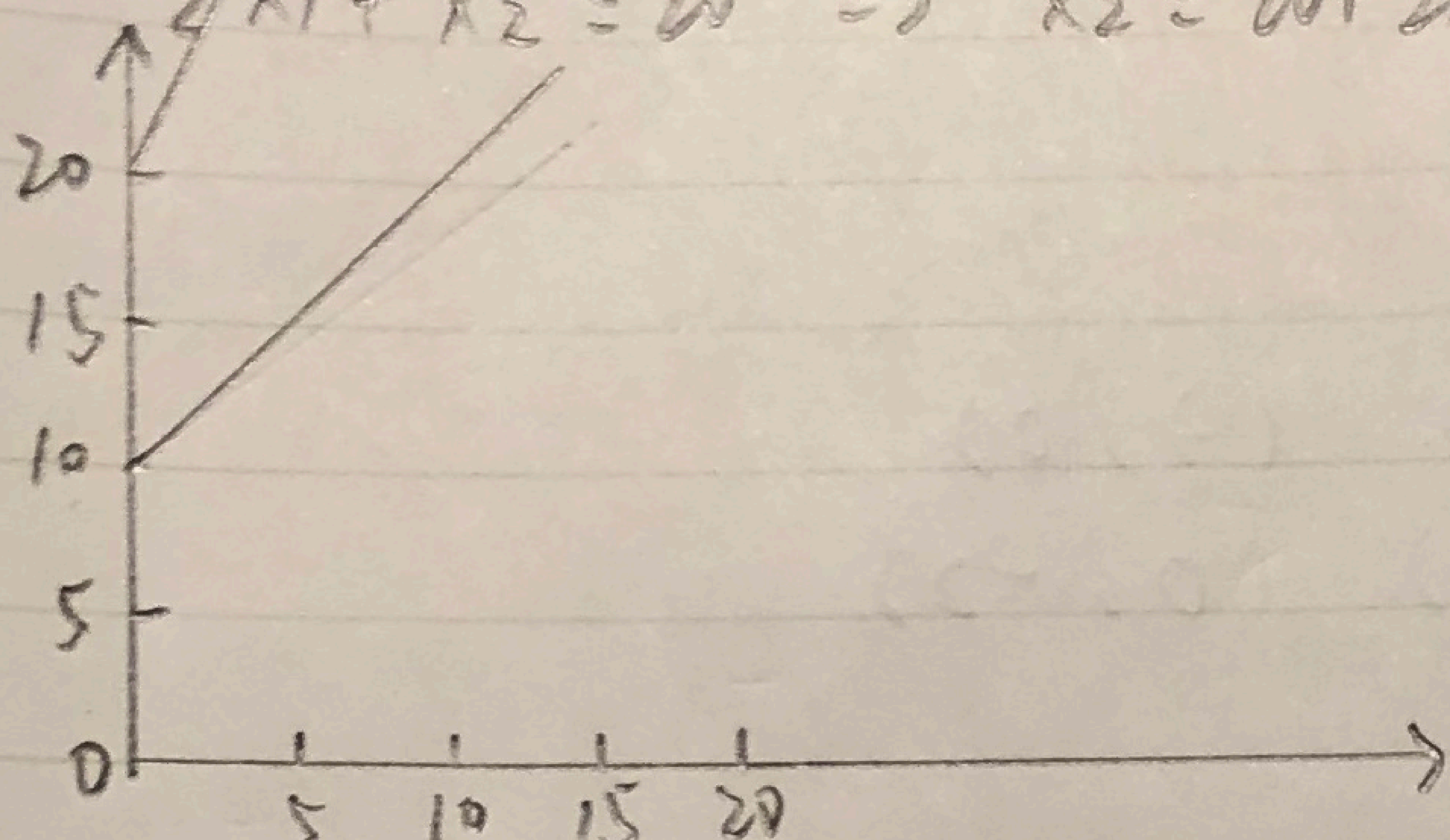
s.t. $-X_1 + X_2 \geq 10$

$-2X_1 + X_2 \leq 20$

$X_1 \geq 0, \ X_2 \geq 0$

a) $-X_1 + X_2 = 10 \Rightarrow X_2 = 10 + X_1 \Rightarrow (0, 10), \ (-10, 0)$

$-2X_1 + X_2 = 20 \Rightarrow X_2 = 20 + 2X_1 \Rightarrow (0, 20), \ (-10, 0)$



According to the graph, $X_1 = 0$, $X_2 = 10$ with $Z = 10$ is the optimal solution

b) $\min \ X_1 - X_2 \Rightarrow \max \ X_2 - X_1$

Since X_2 can be infinitely large, there is no finite optimal solution.

$\therefore$

There is only infinite optimal solution.

Q3 $\begin{bmatrix} 5 & 0 & -2 \\ 1 & -2 & 2 \\ 2 & -1 & 0 \end{bmatrix}$

$R_2 = R_2 - \frac{1}{5} R_1$

$$\left[\begin{array}{ccc|ccc} 5 & 0 & -2 & 1 & 0 & 0 \\ 1 & -2 & 2 & 0 & 1 & 0 \\ 2 & -1 & 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & -2 & \frac{12}{5} & -\frac{1}{5} & 1 & 0 \\ 2 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = R_3 - \frac{2}{5} R_1$

$$= \left[\begin{array}{ccc|ccc} 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & -2 & \frac{12}{5} & -\frac{1}{5} & 1 & 0 \\ 0 & -1 & \frac{4}{5} & -\frac{2}{5} & 0 & 1 \end{array}\right]$$

$R_3 = R_3 - \frac{1}{2} R_2$

$$= \left[\begin{array}{ccc|ccc} 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & -2 & \frac{12}{5} & -\frac{1}{5} & 1 & 0 \\ 0 & 0 & -\frac{2}{5} & -\frac{3}{10} & -\frac{1}{2} & 1 \end{array}\right]$$

$R_3 = -\frac{5}{2} R_3$

$$= \left[\begin{array}{ccc|ccc} 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & -2 & \frac{12}{5} & -\frac{1}{5} & 1 & 0 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{5}{4} & -\frac{5}{2} \end{array}\right]$$

$R_2 = R_2 - \frac{12}{5} R_3$

$$= \left[\begin{array}{ccc|ccc} 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & -2 & -2 & 6 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{5}{4} & -\frac{5}{2} \end{array}\right]$$

$R_2 = -\frac{1}{2} R_2$

$$= \left[\begin{array}{ccc|ccc} 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{5}{4} & -\frac{5}{2} \end{array}\right]$$

$R_1 = R_1 + 2R_3$

$$= \left[\begin{array}{ccc|ccc} 5 & 0 & 0 & \frac{5}{2} & \frac{5}{2} & -5 \\ 0 & 1 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{5}{4} & -\frac{5}{2} \end{array}\right]$$

$R_1 = \frac{1}{5} R_1$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 1 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{5}{4} & -\frac{5}{2} \end{array}\right]$$

$\therefore$ Inverse : $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ 1 & 1 & -3 \\ \frac{3}{4} & \frac{5}{4} & -\frac{5}{2} \end{bmatrix}$

Q4 Max $6x_1 + 8x_2 + 5x_3 + 9x_4$

$$2x_1 + x_2 + x_3 + 3x_4 \le 5$$
$$x_1 + 3x_2 + x_3 + 2x_4 \le 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

$$2x_1 + x_2 + x_3 + 3x_4 + s_1 = 5$$
$$x_1 + 3x_2 + x_3 + 2x_4 + s_2 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Slack variable $s_1, s_2 \geq 0$

Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
1	6	8	5	9	0	0	0
0	2	1	1	3	1	0	5
0	1	3	1	$\boxed{2}$	0	1	3

1	$-\frac{3}{2}$	$\frac{11}{2}$	$-\frac{1}{2}$	0	0	$-\frac{9}{2}$	$\frac{27}{2}$
0	$\boxed{\frac{1}{2}}$	$-\frac{7}{2}$	$-\frac{1}{2}$	0	1	$-\frac{3}{2}$	$\frac{1}{2}$
0	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{3}{2}$

1	0	-5	-2	0	3	0	15
0	1	-7	-1	0	2	-3	1
0	0	5	1	1	-1	2	1

1	0	0	-1	1	-2	2	16
0	1	0	$\frac{2}{5}$	$\frac{7}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	$\frac{12}{5}$
0	0	1	$\boxed{\frac{1}{5}}$	$\frac{1}{5}$	$-\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

1	0	5	0	2	1	4	17
0	1	-2	0	1	1	-1	2
0	0	5	1	1	-1	2	1

$x_1 = 2$ $x_2 = 0$ $s_1 = 0$

$x_3 = 1$ $x_4 = 0$ $s_2 = 0$

$\therefore$ Optimal solution : $x_1 = 2$, $x_2 = 0$, $x_3 = 1$, $x_4 = 0$ with $Z = 17$

Q5 Max $x_1 + 2x_2 + x_3$

$$
\begin{aligned}
x_1 \quad & \leq 3 \\
x_2 \quad & \leq 3 \\
x_1 + x_2 \quad & \leq 4 \\
-x_1 + x_3 \quad & \leq 1 \\
x_1, x_2, x_3 & \geq 0
\end{aligned}
$$

$$
\begin{aligned}
x_1 + 2x_2 + x_3 &= 8 \\
x_1 + s_1 &= 3 \\
x_2 + s_2 &= 3 \\
x_1 + x_2 + s_3 &= 4 \\
-x_1 + x_3 + s_4 &= 1
\end{aligned}
$$

(0, 0, 0)

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	RHS
1	1	2	1	0	0	0	0	0
0	1	0	0	1	0	0	0	3
0	0	$\boxed{1}$	0	0	1	0	0	3
0	1	1	0	0	0	1	0	4
0	-1	0	1	0	0	0	1	1

(0, 3, 0)

1	-1	0	-1	0	-2	0	0	6
0	$\boxed{1}$	0	0	1	0	0	0	3
0	0	1	0	0	$\boxed{1}$	0	0	3
0	$\boxed{1}$	0	0	0	$\boxed{-1}$	1	0	1
0	-1	0	1	0	0	0	1	1

(1, 3, 0)

1	0	0	-1	0	1	1	0	7
0	0	0	0	1	1	-1	0	2
0	0	1	0	0	1	0	0	3
0	1	0	0	0	-1	-1	0	1
0	0	0	$\boxed{1}$	0	-1	0	1	2

(1, 3, 2)

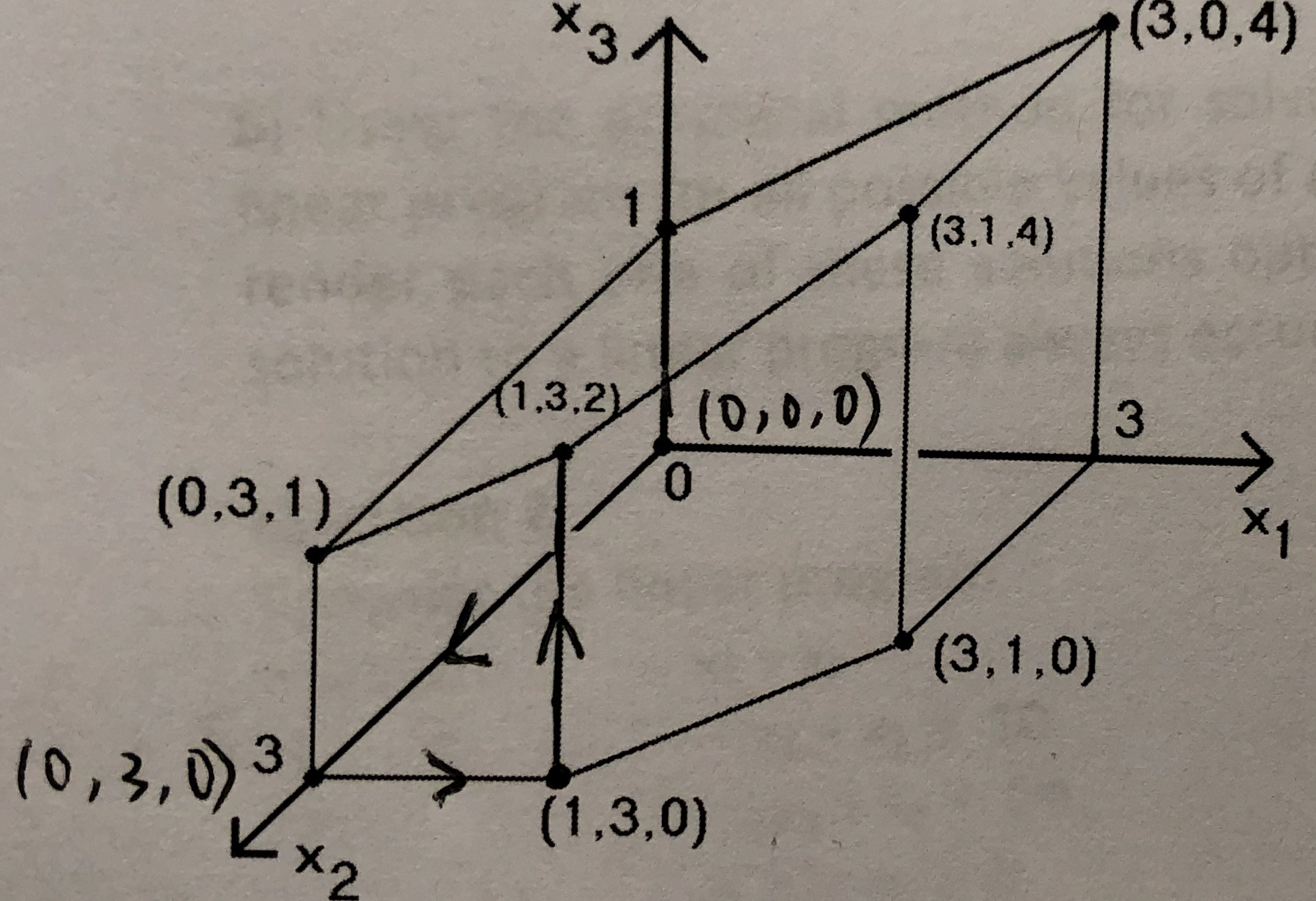
1	0	0	0	0	0	1	1	9
0	0	0	0	1	1	-1	0	2
0	0	1	0	0	1	0	0	3
0	1	0	0	0	-1	1	0	1
0	0	0	1	0	-1	0	1	2

(1, 3, 2)

$x_2 = 3$
$x_1 = 1$
$x_3 = 2$

∴ optimal solution: $x_1 = 1$, $x_2 = 3$, $x_3 = 2$ with $Z = 9$

$$x_1 \quad , \quad x_2 \quad , \quad x_3 \geq 0.$$



(0, 0, 0)

(0, 3, 0)

(1, 3, 0)

(1, 3, 2)