HW 8 Optimization Queenie Liu ql299

Q1

$$\begin{aligned}\max\quad & 3x_1 + 4x_2 \\ \text{s.t}\quad & 2x_1 + x_2 \le 240 \\ & x_1 + 2x_2 \le 180 \\ & x_2 \le 60 \\ & x_1, x_2 \ge 0\end{aligned}$$

(a)
$$\begin{aligned} y_1(2x_1 + x_2) &\le 240y_1 \\ y_2(x_1+2x_2) &\le 180y_2 \\ y_3 x_2 &\le 60y_3 \end{aligned} \Rightarrow \begin{aligned}\min\quad & 240y_1 + 180y_2 + 60y_3 \\ \text{s.t}\quad & 2y_1 + y_2 \ge 3 \\ & y_1 + 2y_2 + y_3 \ge 4 \\ & y_1, y_2, y_3 \ge 0\end{aligned}$$

(b) For primal,

$(x_1, x_2) = (120, 0) \Rightarrow 3x_1 + 4x_2 = 360$

$(x_1, x_2) = (100, 20) \Rightarrow 3x_1 + 4x_2 = 380$

$(x_1, x_2) = (60, 60) \Rightarrow 3x_1 + 4x_2 = 420$

For dual

$(y_1, y_2, y_3) = (1, 1, 1) \Rightarrow obj = 480$

$(y_1, y_2, y_3) = (1, 1, 2) \Rightarrow obj = 540$

$(y_1, y_2, y_3) = (2, 2, 2) \Rightarrow obj = 960$

I observe that the obj value of the dual is always larger than that of the primal.

(c)
$$\begin{array}{lllll|l} 3x_1 & +4x_2 & & & & = Z \\ \hline 2x_1 & +x_2 & +w_1 & & & = 240 \\ x_1 & +2x_2 & & +w_2 & & = 180 \\ & x_2 & & & +w_3 & = 60 \end{array}$$

$x_1, x_2, w_1, w_2, w_3 \ge 0$

$w_1 = 240,\ w_2 = 180,\ w_3 = 60,\ x_1 = x_2 = Z = 0$

$\min(240, 90, 60) = 60$

$$\Rightarrow \begin{array}{llll|l} 3x_1 & & & -4w_3 & = Z - 240 \\ \hline 2x_1 & +w_1 & & -w_3 & = 180 \\ x_1 & & +w_2 & -2w_3 & = 60 \\ & x_2 & & +w_3 & = 60 \end{array}$$

$w_1 = 180,\ w_2 = 60,\ x_2 = 60,\ x_1 = w_3 = 0,\ Z = 240$

$\min(90, 60) = 60$

$$\rightarrow \begin{array}{lll|l} & -3w_2 & +2w_3 & = Z - 400 \\ \hline w_1 & -2w_2 & +3w_3 & = 60 \\ x_1 & +w_2 & -2w_3 & = 60 \\ x_2 & & +w_3 & = 60 \end{array}$$

$w_1 = 60,\ x_1 = 60,\ x_2 = 60,\ w_2 = w_3 = 0,\ Z = 400$

$\min(20, 00) = 20$

$$\rightarrow \begin{array}{llll|l} & -\frac{2}{3}w_1 & -\frac{5}{3}w_2 & & = Z - 460 \\ \hline & \frac{1}{3}w_1 & -\frac{2}{3}w_2 & +w_3 & = 20 \\ x_1 & +\frac{2}{3}w_1 & -\frac{1}{3}w_2 & & = 100 \\ x_2 & -\frac{1}{3}w_1 & +\frac{2}{3}w_2 & & = 40 \end{array}$$

$x_1 = 100,\ x_2 = 40,\ w_1 = w_2 = 0,\ w_3 = 20,\ \underline{Z = 460}$

↳ Optimal solution Optimal Objective

1d) max $-240y_1 - 180y_2 - 60y_3$

s.t. $-2y_1 - y_2 \le -3$

$-y_1 - 2y_2 - y_3 \le -4$

$y_1, y_2, y_3 \ge 0$

min $u$

s.t. $-y_1 - y_2 \le -3 + u$

$-y_1 - 2y_2 - y_3 \le -4 + u$

$y_1, y_2, u \ge 0$

$$
\begin{array}{l}
u = z \\
\hline
-2y_1 - y_2 + w_1 - u = -3 \\
-y_1 - 2y_2 - y_3 + w_2 - u = -4
\end{array}
\Rightarrow
\begin{array}{l}
-y_1 - 2y_2 - y_3 + w_2 = z - 4 \\
\hline
-y_1 + y_2 + y_3 + w_1 - w_2 = 1 \\
y_1 + 2y_2 + y_3 - w_2 + u = 4
\end{array}
$$

$$
\Rightarrow
\begin{array}{l}
-3y_1 + y_3 + 2w_1 - w_2 = z - 2 \\
\hline
-y_1 + y_2 + y_3 + w_1 - w_2 = 1 \\
3y_1 - y_3 - 2w_1 + w_2 + u = 2
\end{array}
\Rightarrow
\begin{array}{l}
u = z \\
\hline
y_2 + \frac{2}{3}y_3 + \frac{1}{3}w_1 - \frac{2}{3}w_2 + \frac{1}{3}u = \frac{5}{3} \\
y_1 - \frac{1}{3}y_3 - \frac{2}{3}w_1 + \frac{1}{3}w_2 + \frac{1}{3}u = \frac{2}{3}
\end{array}
$$

$y_1 = \frac{1}{3}$, $y_2 = \frac{5}{3}$, $u = z = 0$ => feasible

$$
\begin{array}{l}
-240y_1 - 180y_2 - 60y_3 = z \\
\hline
y_2 + \frac{2}{3}y_3 + \frac{1}{3}w_1 - \frac{2}{3}w_2 = \frac{5}{3} \\
y_1 - \frac{1}{3}y_3 - \frac{2}{3}w_1 + \frac{1}{3}w_2 = \frac{2}{3}
\end{array}
\Rightarrow
\begin{array}{l}
-20y_3 - 100w_1 - 40w_2 = z + 460 \\
\hline
y_2 + \frac{2}{3}y_3 + \frac{1}{3}w_1 - \frac{2}{3}w_2 = \frac{5}{3} \\
y_1 - \frac{1}{3}y_3 - \frac{2}{3}w_1 + \frac{1}{3}w_2 = \frac{2}{3}
\end{array}
$$

$y_2 = \frac{5}{3}$, $y_1 = \frac{2}{3}$, $z = -460$

=> for min, $z = 460$, $y_1 = \frac{2}{3}$, $y_2 = \frac{5}{3}$

1e)