Time-delocalised quantum subsystems and operations: **on the existence of processes with indefinite causal structure in quantum mechanics**

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Based on

arXiv:1801.07594

Observation

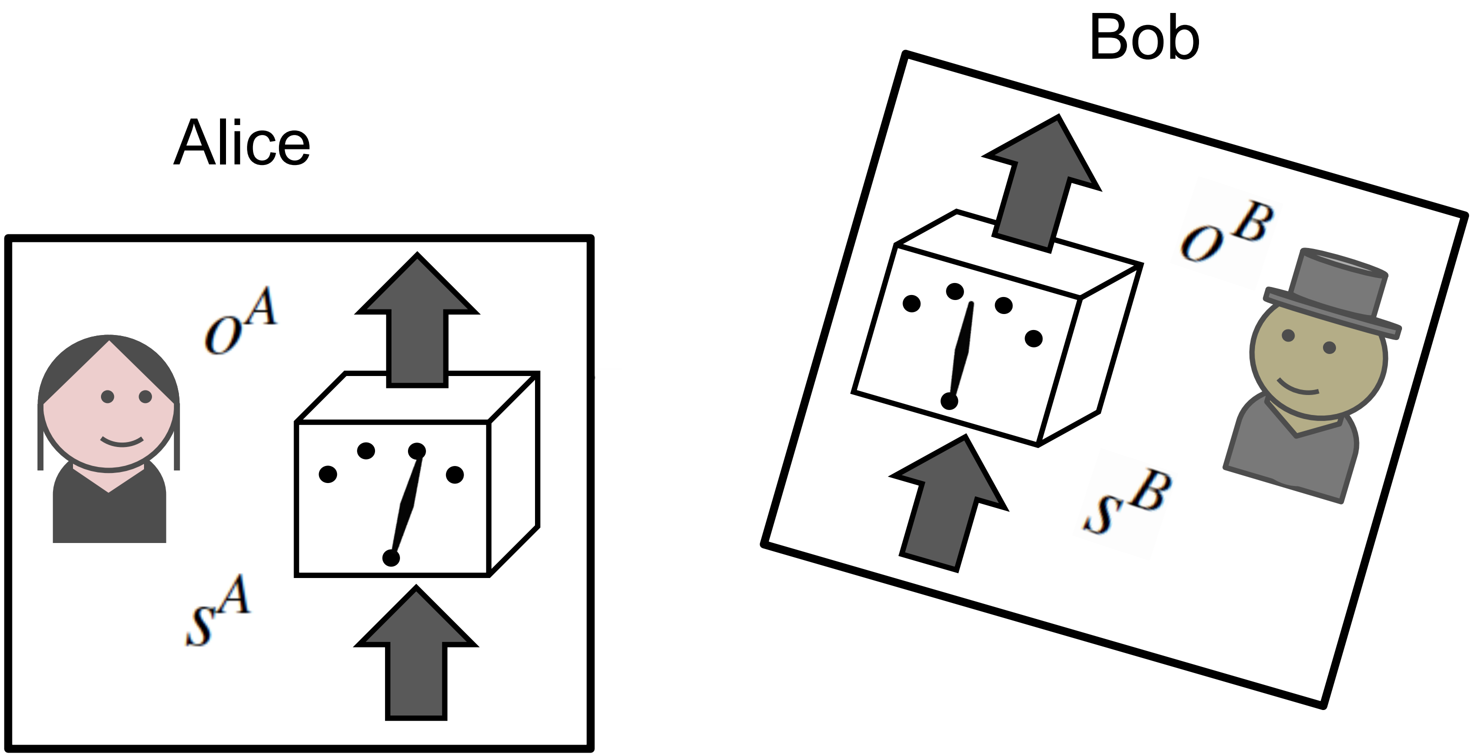
In standard QM, quantum operations are assumed to always take place composed in acyclic circuits that respect the causal structure of space-time (definite causal order).

Questions

Is this a fundamental restriction or an artifact of our formulation of QM?

Could the time and causal order of operations be indefinite similarly to other variables in QM? (Could be relevant for quantum gravity).

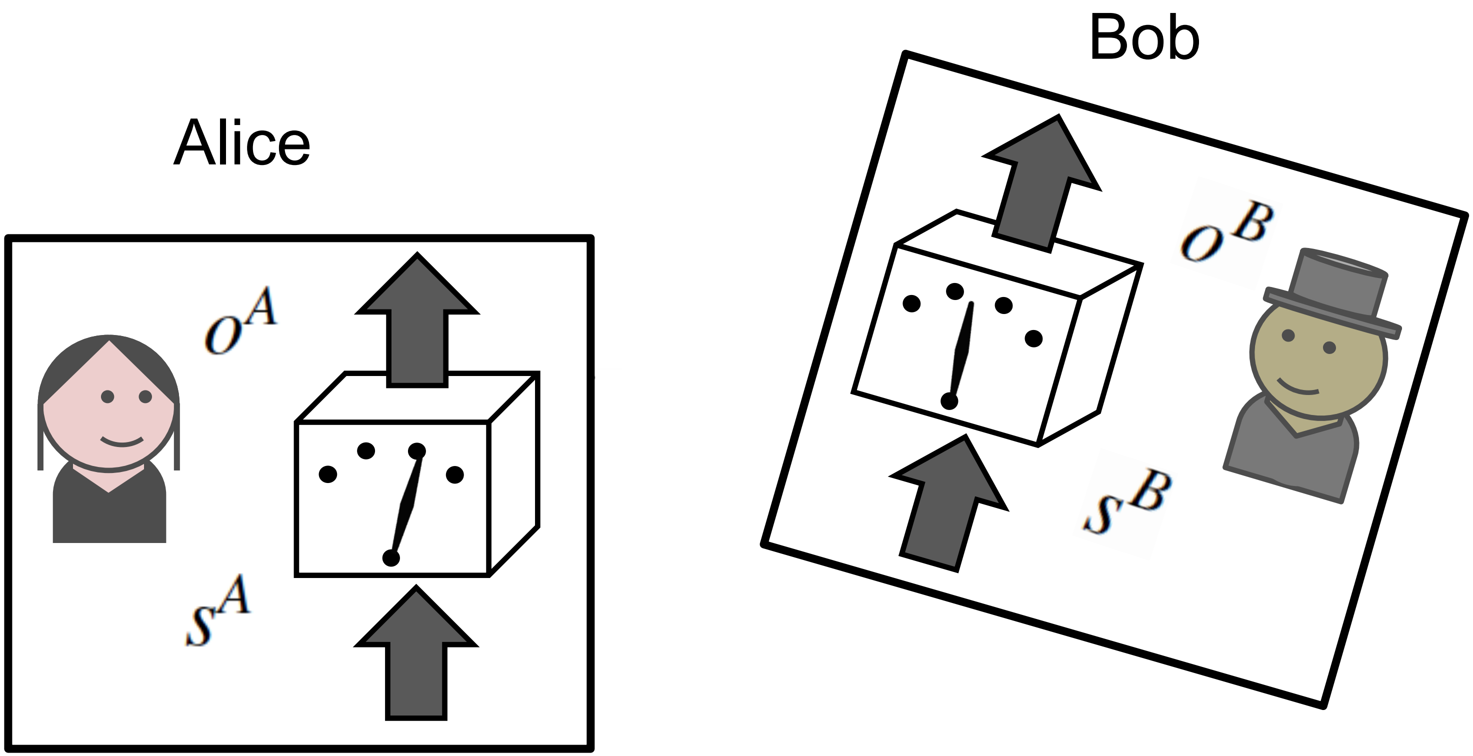
The process matrix framework



No assumption of global causal order.

O. O., F. Costa, and Č. Brukner, Nat. Commun. 3, 1092 (2012).

The process matrix framework

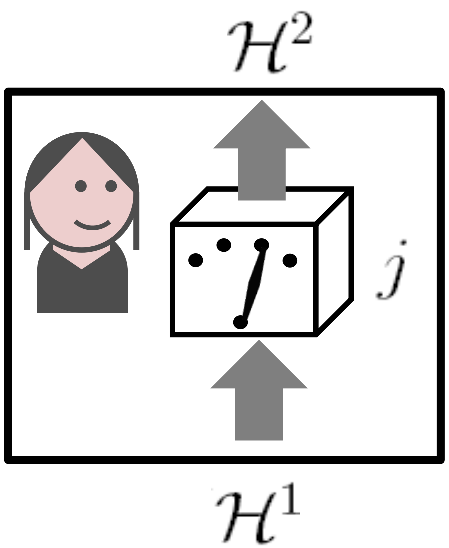


Joint probabilities $\longrightarrow$

$$p(o^A, o^B, \ldots | s^A, s^B, \ldots)$$

Quantum processes

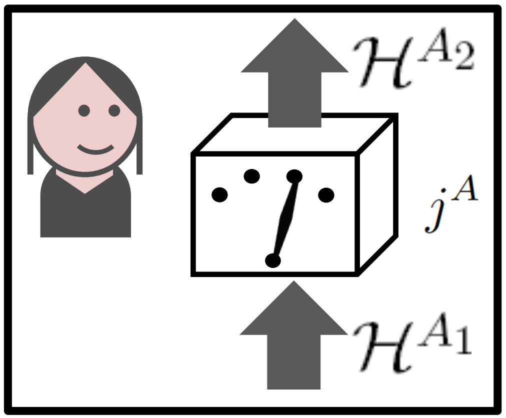
Local descriptions agree with quantum mechanics



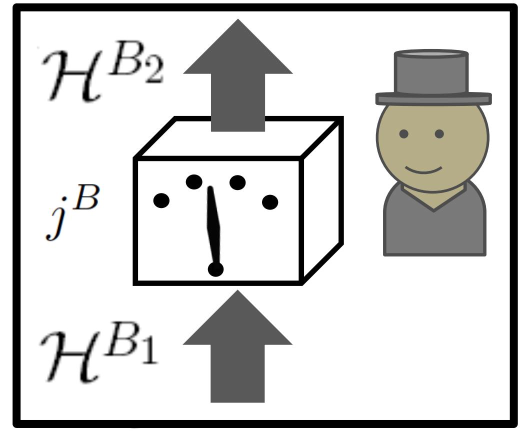
Operations → quantum instruments

[Each outcome associated with a CP maps $\mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2)$]

Quantum processes



$$\mathcal{M}^A_{j^A} : \mathcal{L}(\mathcal{H}^{A_1}) \to \mathcal{L}(\mathcal{H}^{A_2})$$



$$\mathcal{M}^B_{j^B} : \mathcal{L}(\mathcal{H}^{B_1}) \to \mathcal{L}(\mathcal{H}^{B_2})$$

Assumption 1: The probabilities are functions of the local CP maps,

$$P(\mathcal{M}^A_{j^A}, \mathcal{M}^B_{j^B}, \cdots)$$

Local validity of QM $\Rightarrow$ $P(\mathcal{M}^A, \mathcal{M}^B, \cdots)$ is **linear** in $\mathcal{M}^A$ $\mathcal{M}^B$ ...

Choi-Jamiołkowski isomorphism

CP maps		Positive semidefinite operators
$\mathcal{M}: \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$	$\longleftrightarrow$	$M^{12} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$

Choi-Jamiołkowski isomorphism

CP maps

Positive semidefinite operators

$$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2) \quad \longleftrightarrow \quad M^{12} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$$

For a version of the isomorphism with a **physical interpretation** (based on time reversal), see O.O and N. J. Cerf, NJP 18, 073037 (2016).

The process matrix

Representation

$$P(\mathcal{M}^{A}_{j^A}, \mathcal{M}^{B}_{j^B}, \cdots) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2}_{j^A} \otimes M^{B_1 B_2}_{j^B} \otimes \cdots\right)\right]$$

Process matrix

The process matrix

Representation

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Process matrix

Similar to Born‘s rule but can describe signalling.

The process matrix

Assumption 2: The parties can share entangled input ancillas.

Conditions on W:

1. Non-negative probabilities: $W^{A_1A_2B_1B_2\cdots} \geq 0$

2. Probabilities sum up to 1:

$$\mathrm{Tr}\left[W^{A_1A_2B_1B_2\cdots}\left(M^{A_1A_2} \otimes M^{B_1B_2} \otimes \cdots\right)\right] = 1$$

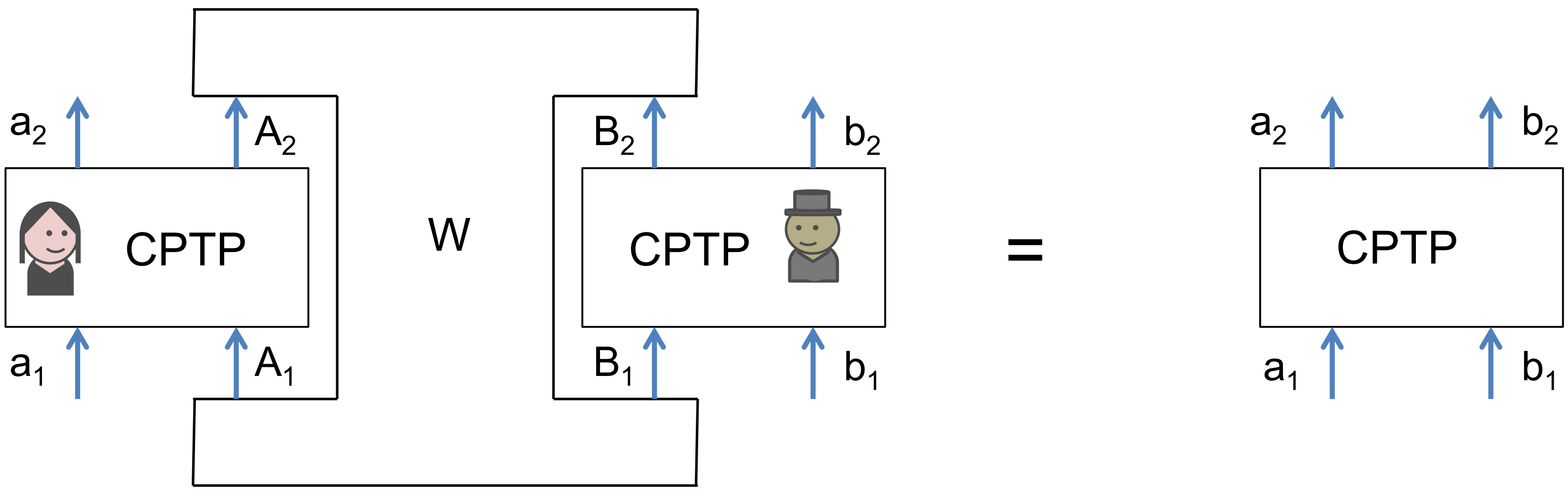
on all CPTP maps $M^{A_1A_2}$ $M^{B_1B_2}$..

Note: $M^{A_1A_2}$ is CPTP iff $M^{A_1A_2} \geq 0, \ \mathrm{Tr}_{A_2} M^{A_1A_2} = \mathbb{1}^{A_1}$.

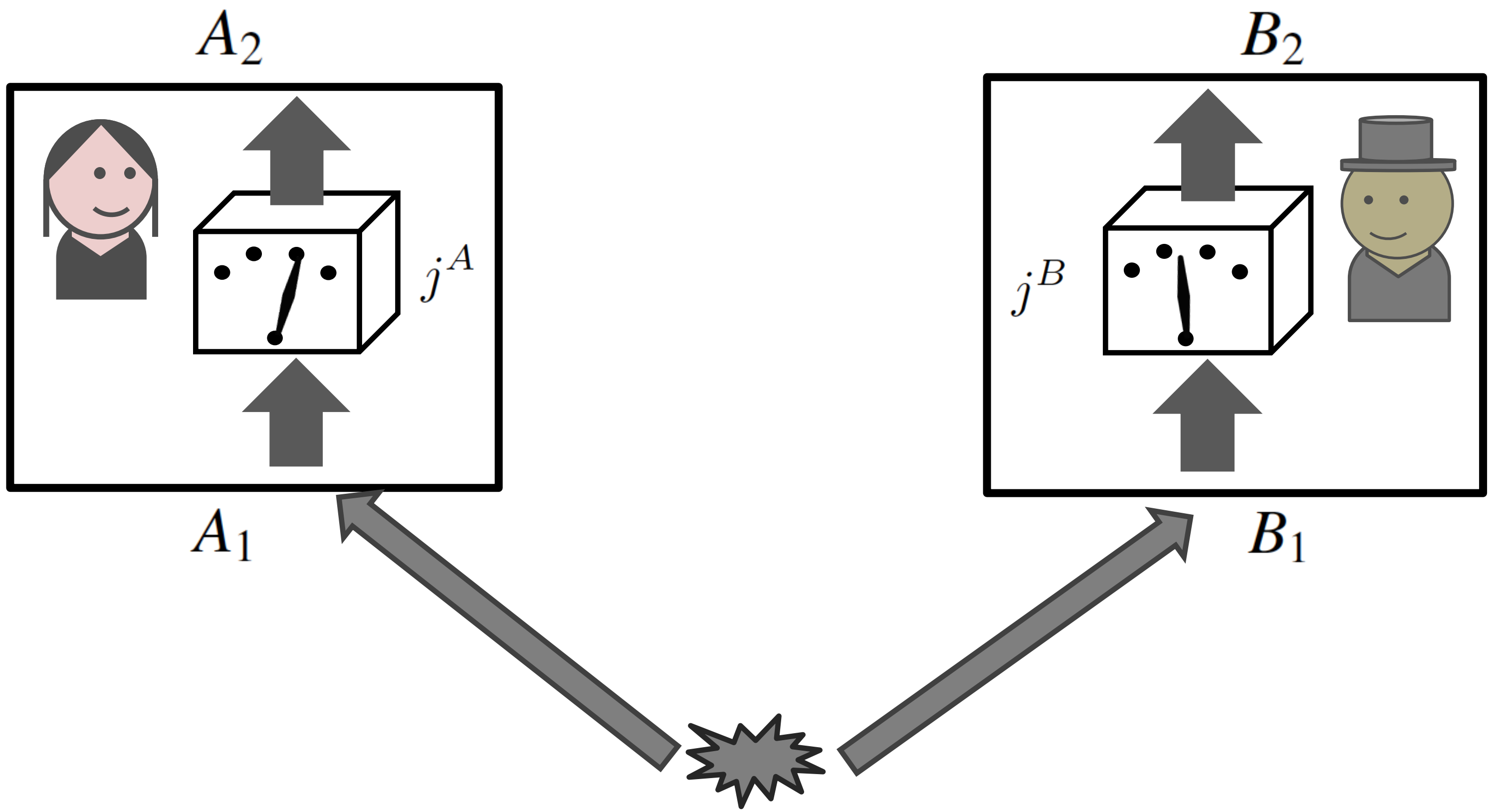
The process matrix

An equivalent formulation as a second-order operation:

[Quantum supermaps, Chiribella, D'Ariano, and Perinotti, EPL 83, 30004 (2008)]

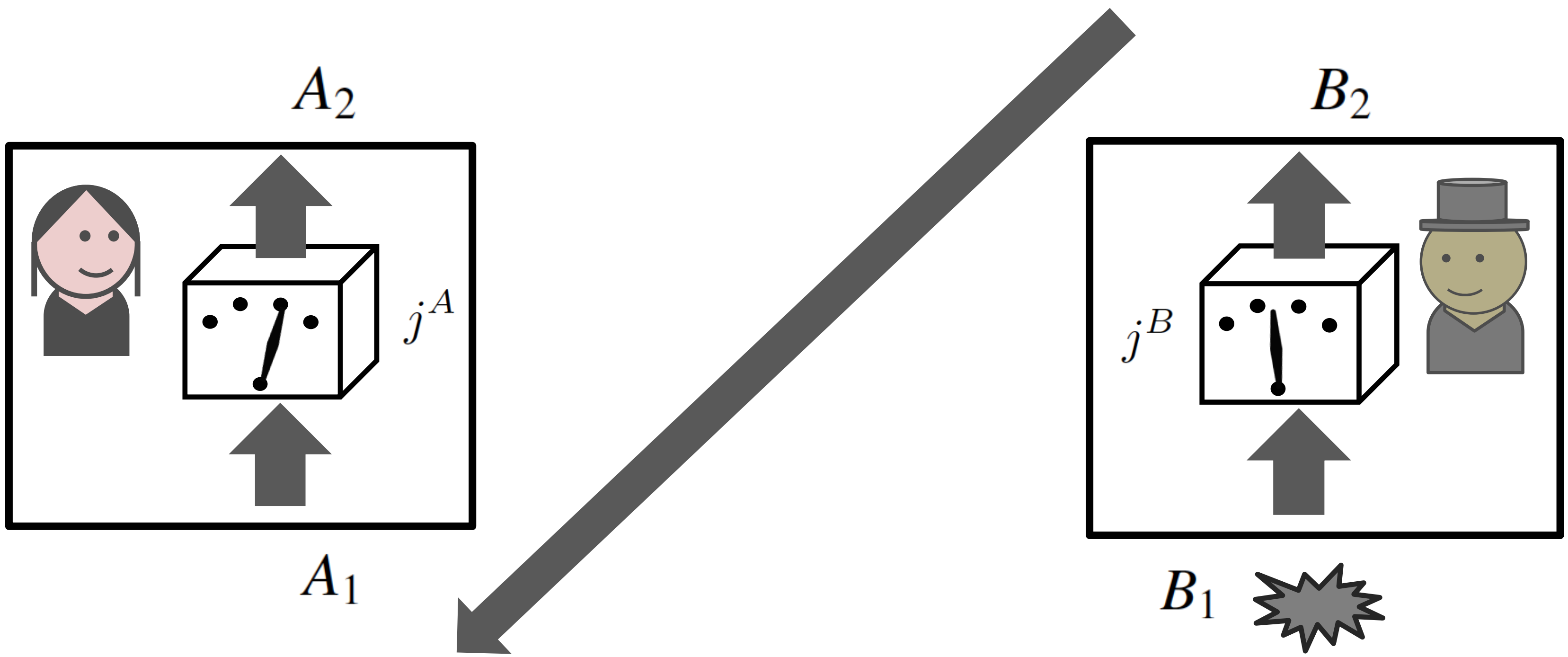


Example: bipartite state



$$W^{A_1 A_2 B_1 B_2} = \rho^{A_1 B_1} \otimes \mathbb{1}^{A_2 B_2}$$

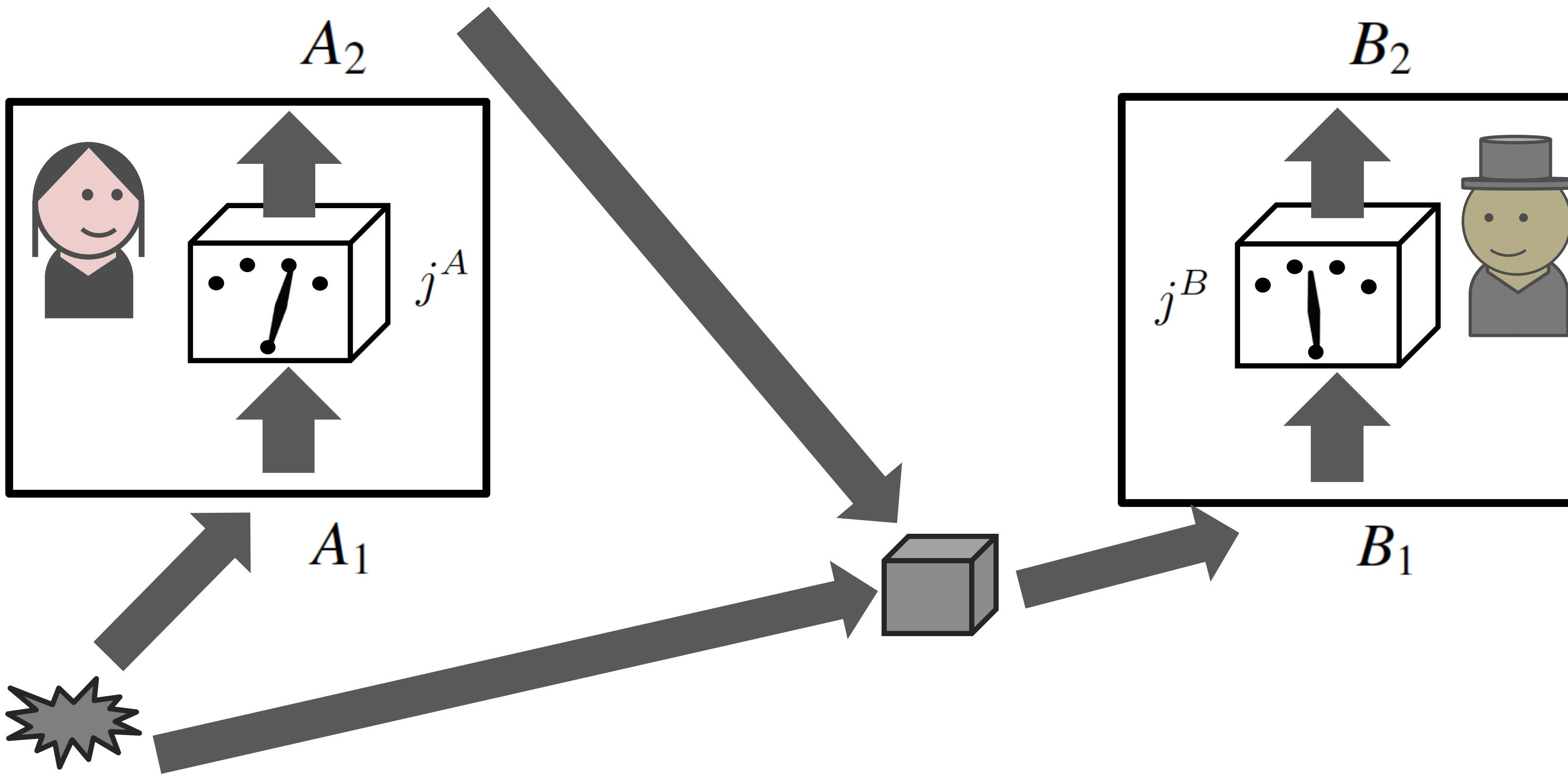
Example: channel B→A



$$W^{A_1A_2B_1B_2} = \mathbb{1}^{A_2} \otimes (C^{A_1B_2})^T \otimes \rho^{B_1}$$

Example: channel with memory A→ B

(The most general possibility compatible with no signalling from B to A!)



$$W^{A_1 A_2 B_1 B_2} = W^{A_1 A_2 B_1} \otimes \mathbb{1}^{B_2}$$

Bipartite processes with causal realization

$W^{A \npreceq B}$ – no signalling from A to B

$W^{B \npreceq A}$ – no signalling from B to A

More generally, we may conceive probabilistic mixtures of fixed-order processes:

$$W_{cs}^{A_1 A_2 B_1 B_2} = q W^{A \npreceq B} + (1-q) W^{B \npreceq A}$$

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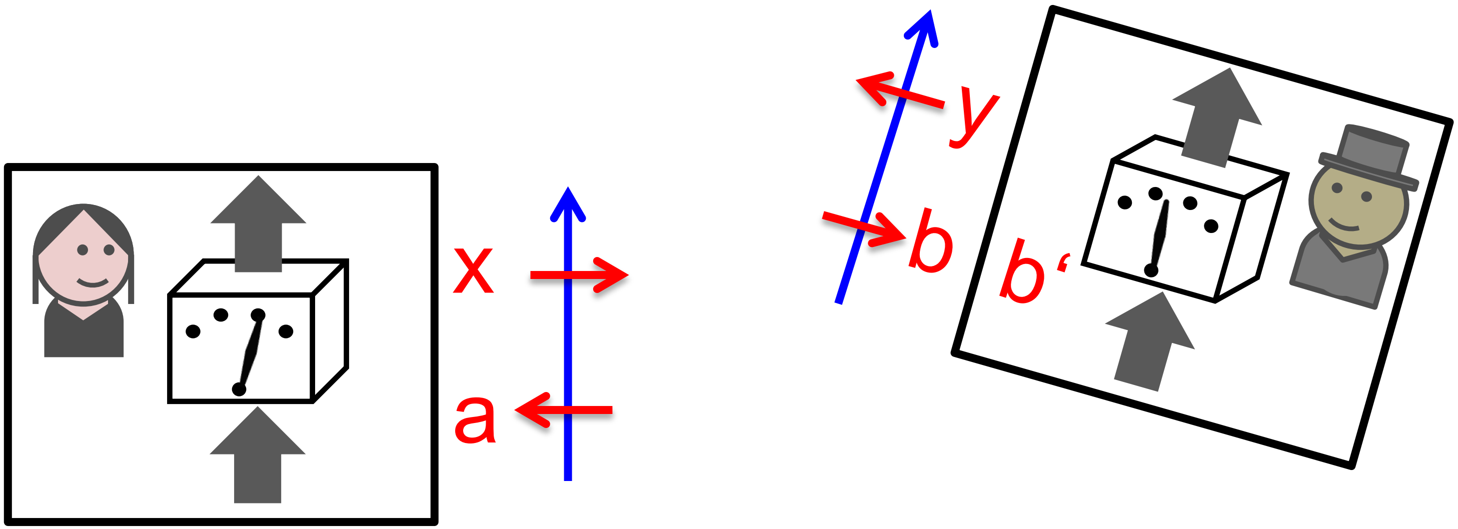
$$W_{cs}^{A_1A_2B_1B_2} = qW^{A\not\preceq B} + (1-q)W^{B\not\preceq A}$$

causally separable process

Not all processes are causally separable!

A causal game

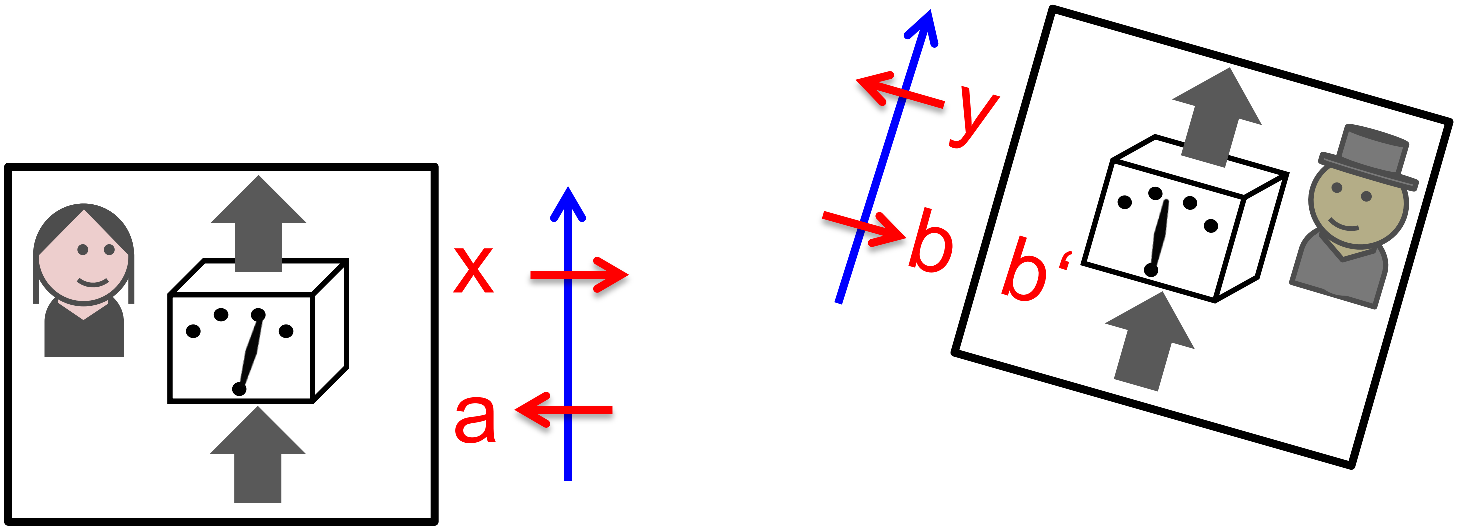


Their goal is to maximize:

$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)]$$

O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

A causal inequality



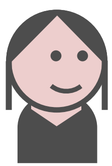
Definite causal order →

$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)] \leq \frac{3}{4}$$

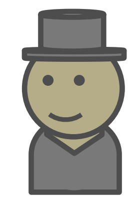
O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

A causally nonseparable process

Can violate the inequality with $p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$.



$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$



two-level
systems

The operations of Alice and Bob do not occur in a definite order.

O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

Other causal inequalities and violations

Bipartite inequalities:

Simplest inequalities:
Branciard, Araujo, Feix, Costa, Brukner, NJP 18, 013008 (2016)

Biased version of the original inequality:
Bhattacharya and Banik, arXiv:1509.02721 (2015)

Multiparite inequalities:

Violation with perfect signaling:
Baumeler and Wolf, Proc. ISIT 2014, 526-530 (2014)

Violation by classical local operations:
Baumeler, Feix, and Wolf, PRA 90, 042106 (2014)
Baumeler and Wolf, NJP 18, 013036 (2016)

Simplest tripartite polytope:
Abbott, Giarmatzi, Costa, Branciard, PRA 94, 032131 (2016)

Can such processes be realized physically?

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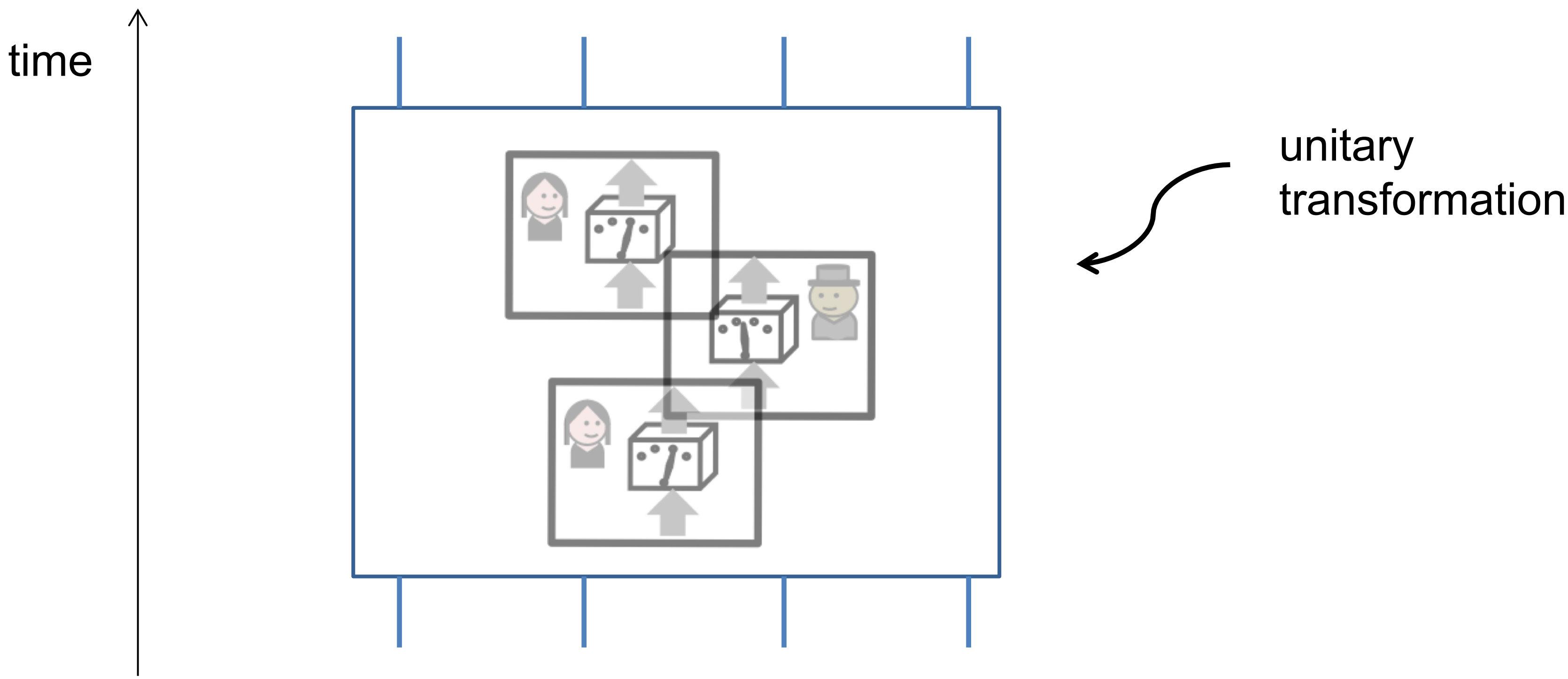
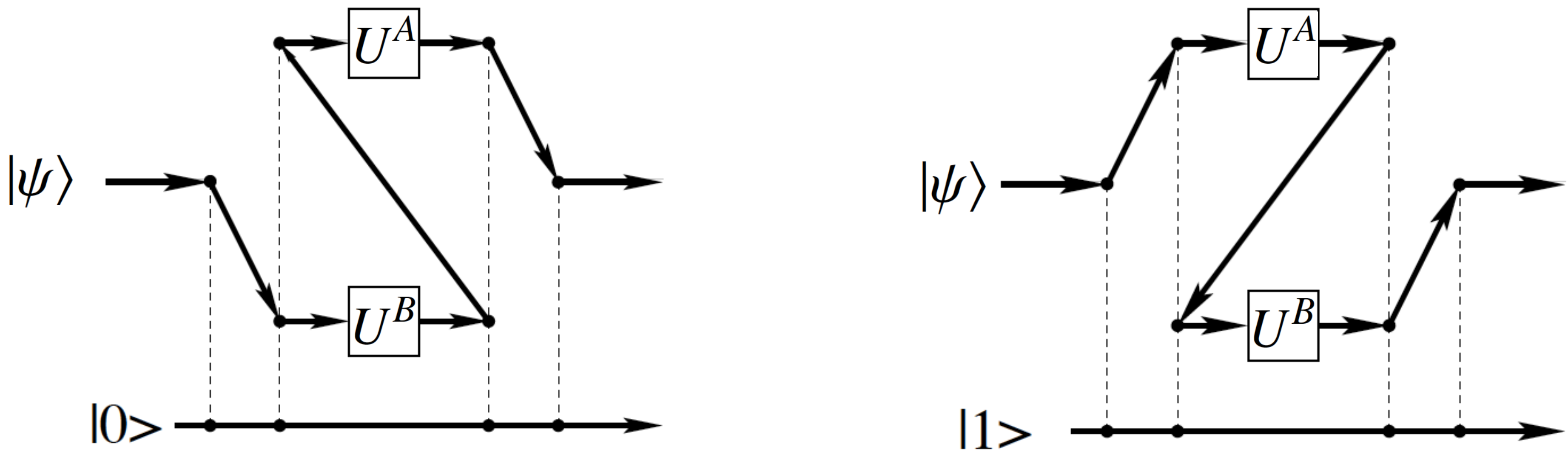
Not *a priori* impossible!



From the outside the experiment may still agree with standard unitary evolution in time.

The quantum SWITCH

The order of operations depends on a variable in a quantum superposition:

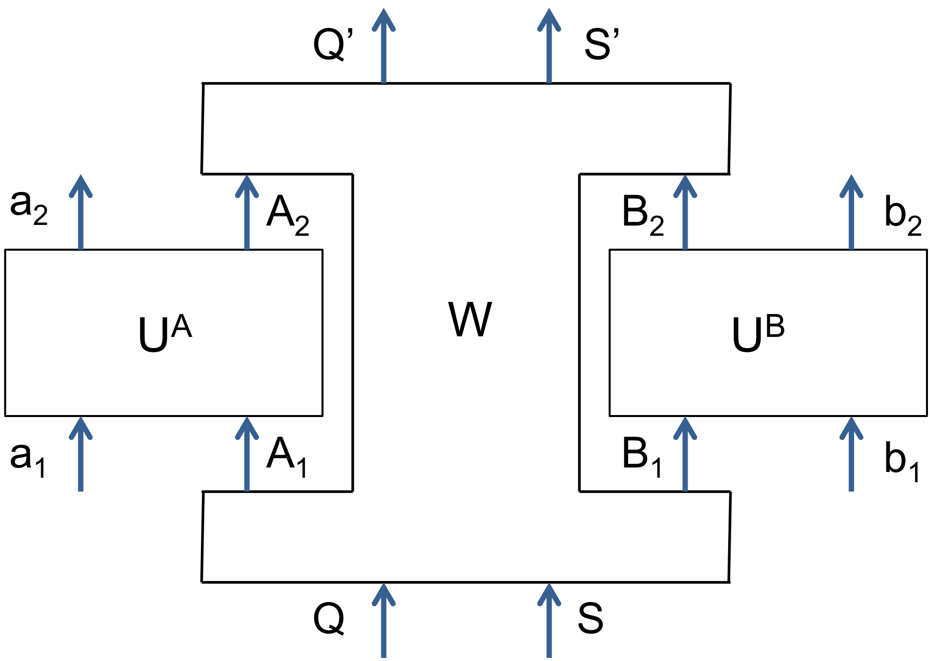


$$(\alpha|0\rangle + \beta|1\rangle)|\psi\rangle \to \alpha|0\rangle U^A U^B|\psi\rangle + \beta|1\rangle U^B U^A|\psi\rangle$$

Chiribella, D'Ariano, Perinotti, and Valiron, PRA 88, 022318 (2013), arXiv:0912.0195 (2009)

The quantum SWITCH

A supermap:



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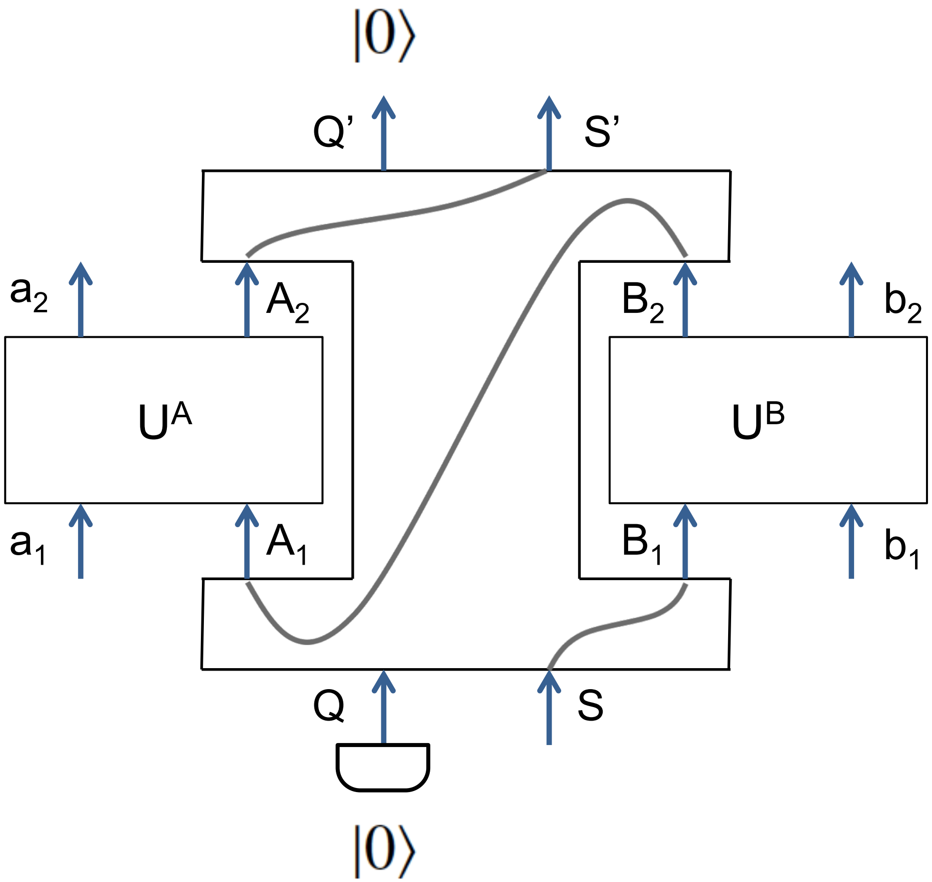

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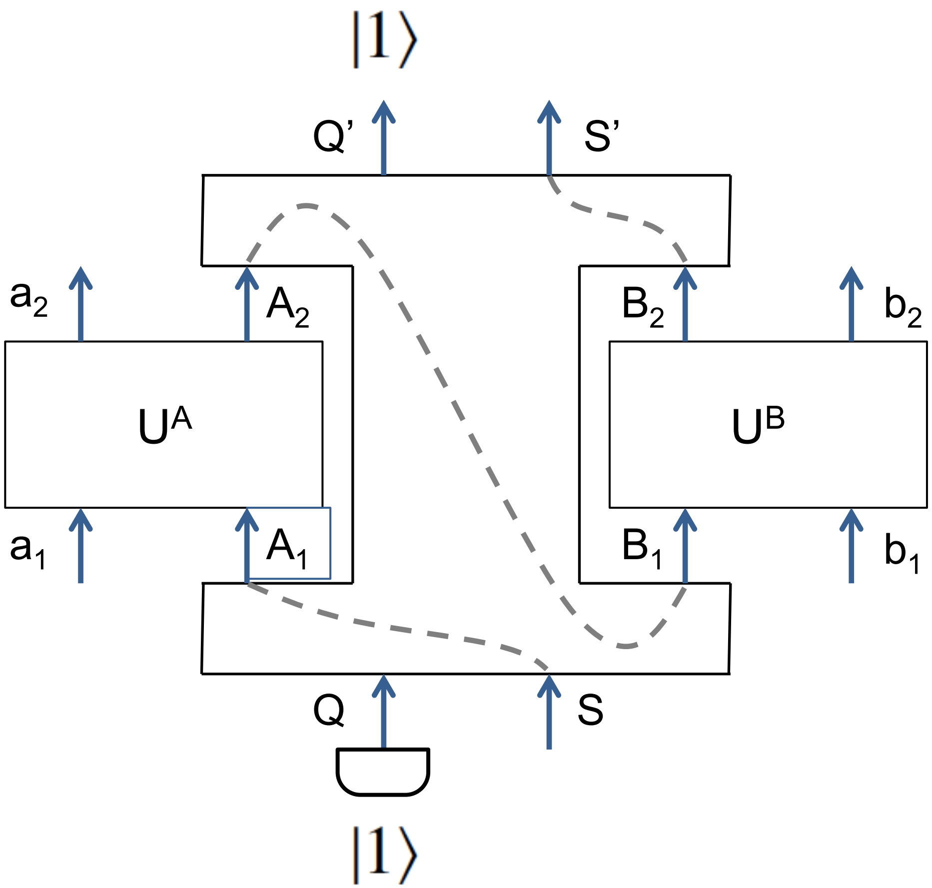

The quantum SWITCH

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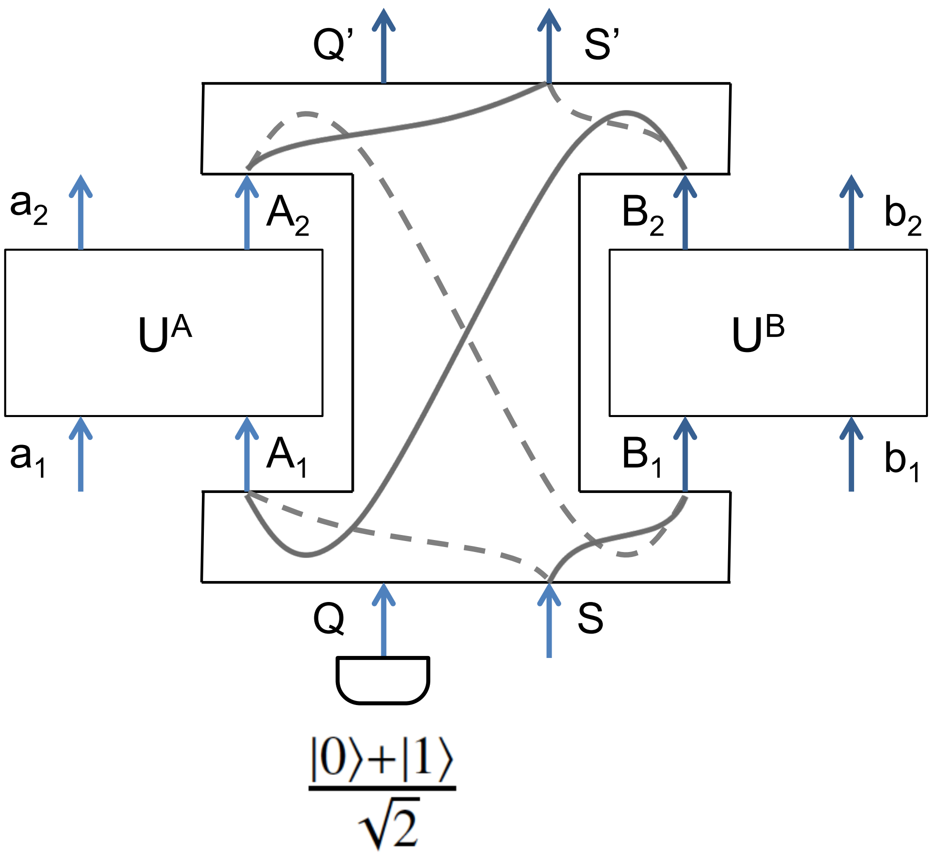
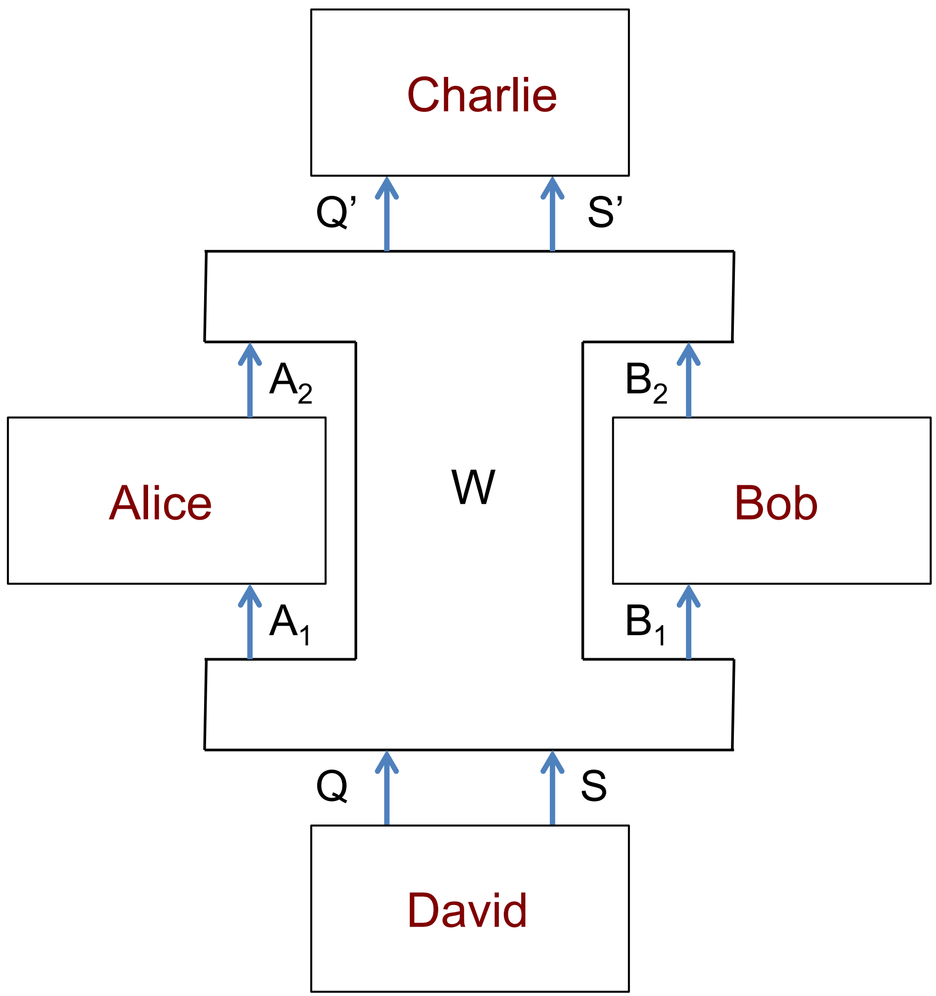


The quantum SWITCH

A process matrix:



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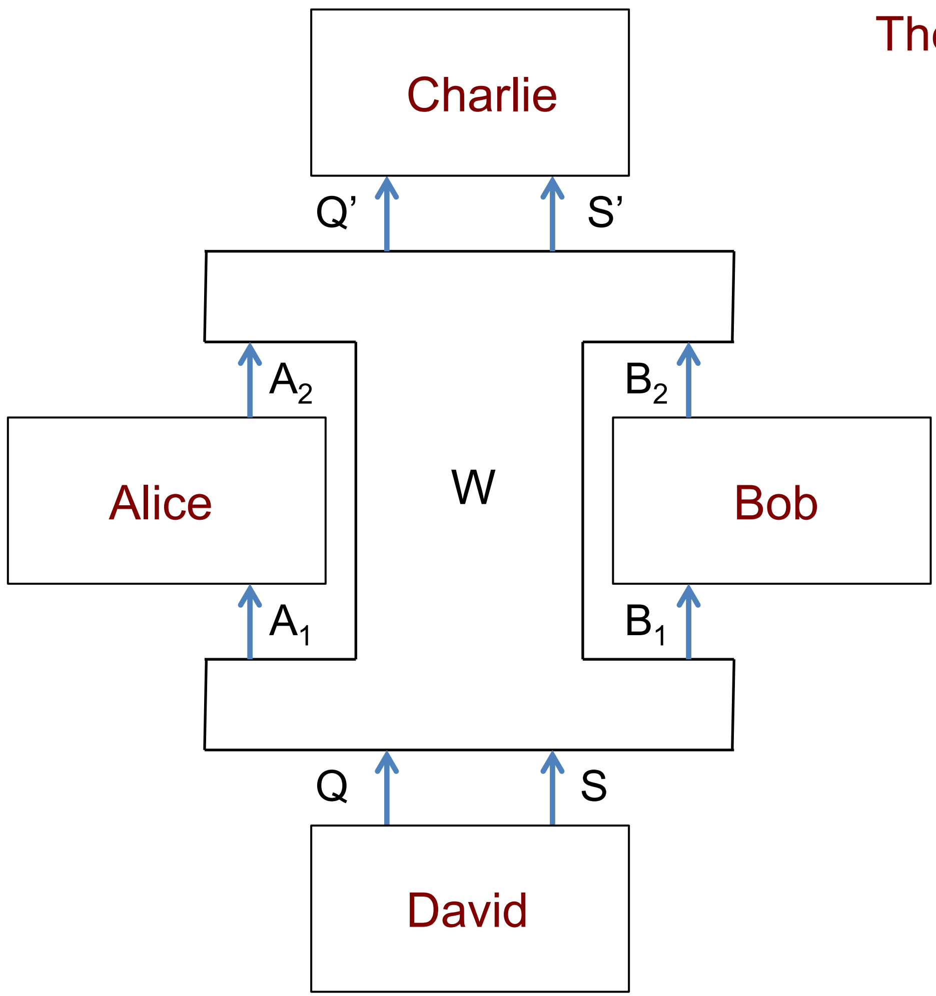

The process matrix is **not causally separable**:

W= |W><W|

(not a probabilistic mixture of different process matrices)

But it allows signaling from Alice to Bob and from Bob to Alice.

Oreshkov and Giarmatzi, NJP 18, 093020 (2016)

Araujo et al., NJP 17, 102001 (2015)

The quantum SWITCH

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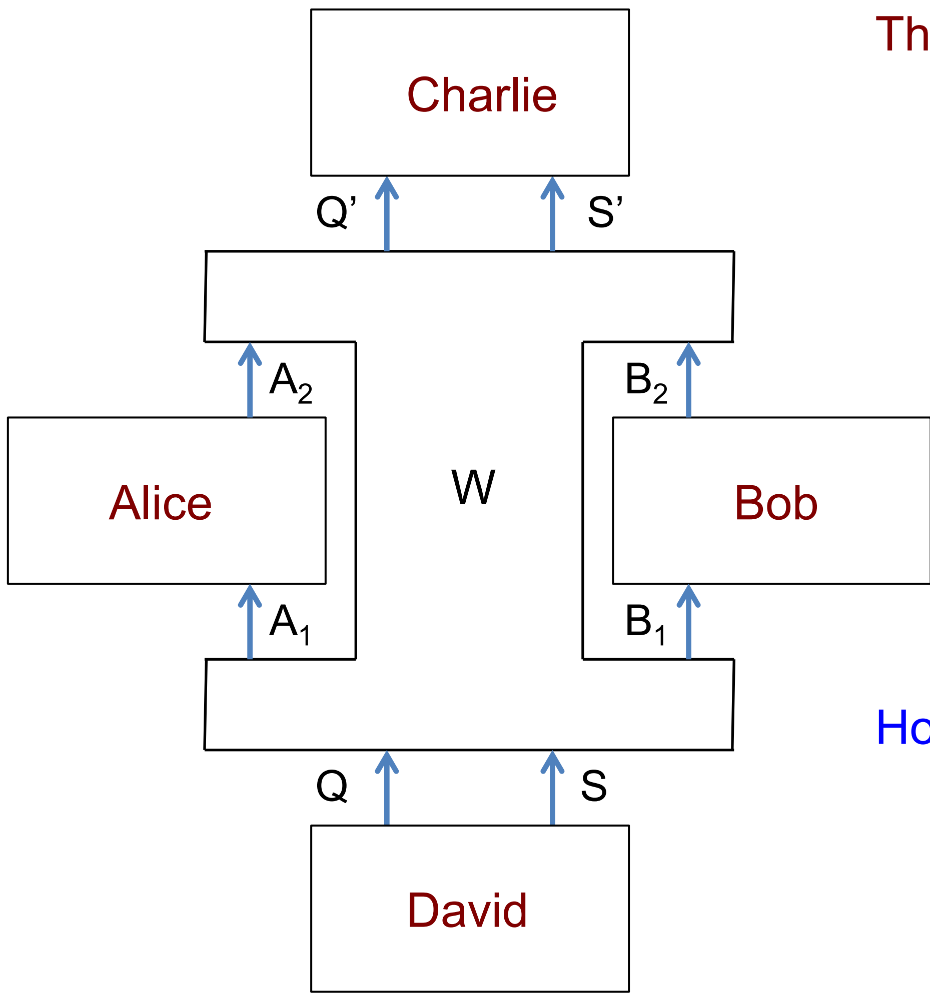

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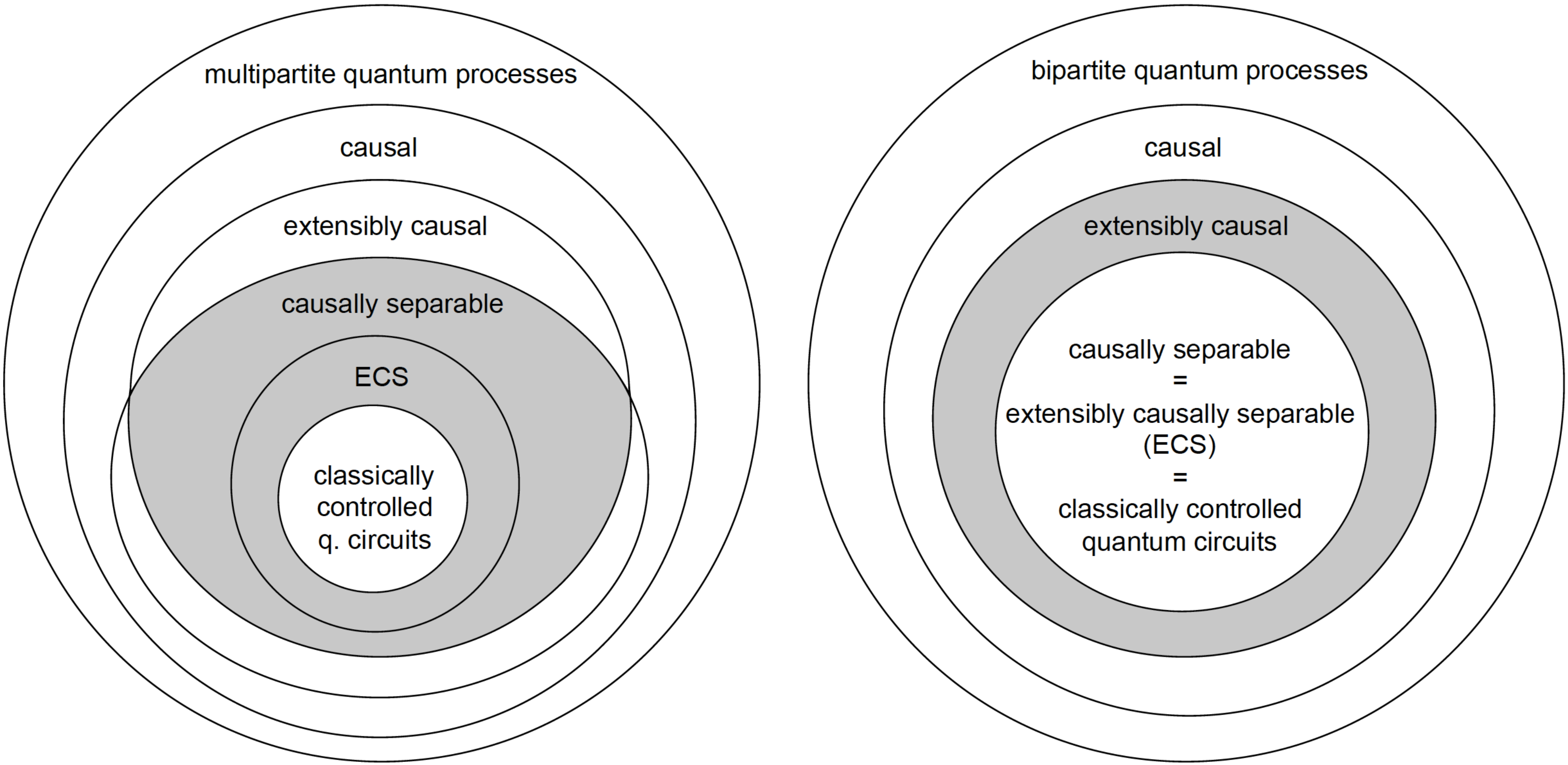
But it allows signaling from Alice to Bob and from Bob to Alice.

However, it cannot violate causal inequalities!

Oreshkov and Giarmatzi, NJP 18, 093020 (2016)

Araujo et al., NJP 17, 102001 (2015)

Causality versus causal separability

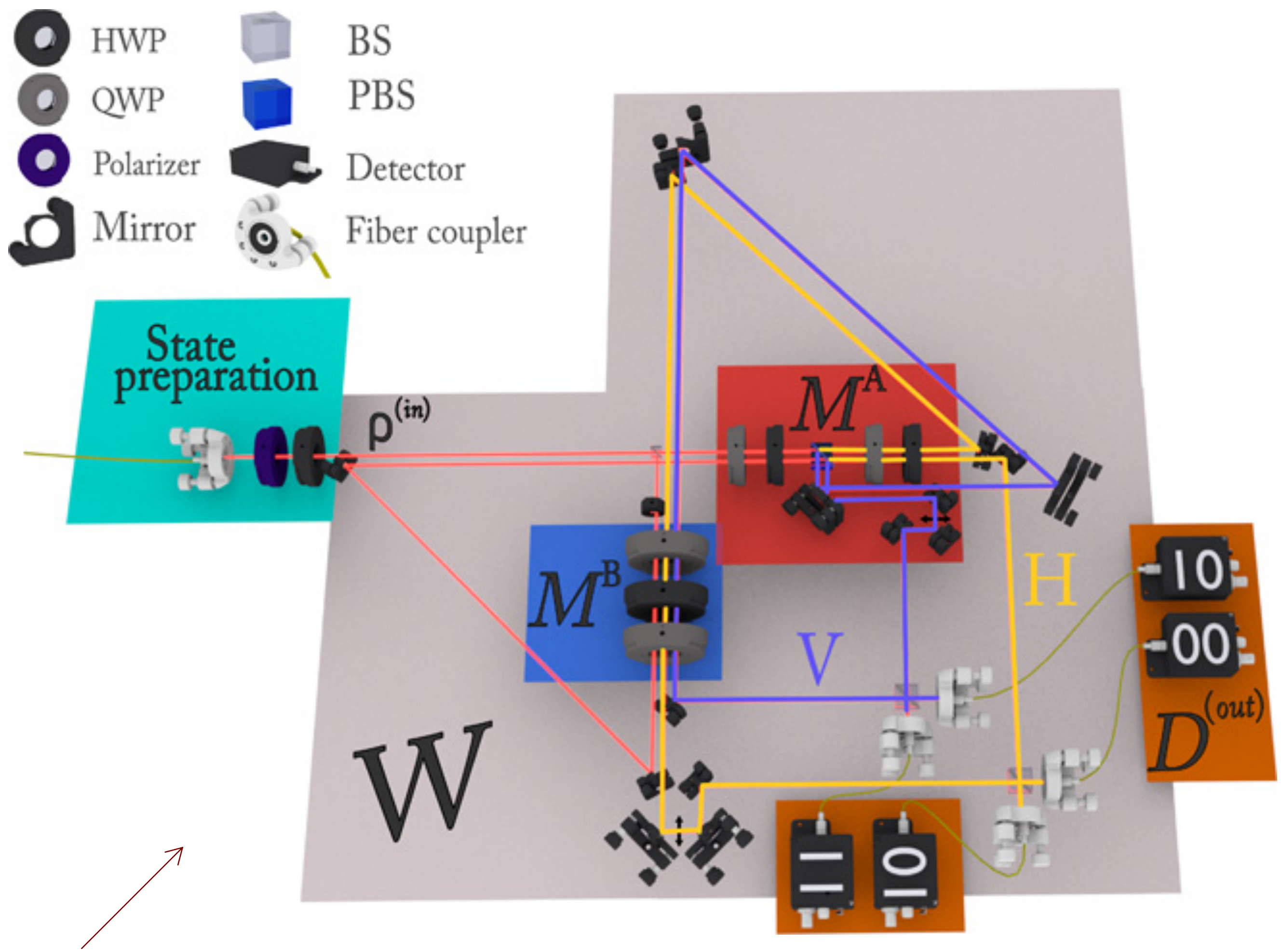


a) Multipartite case.

b) Bipartite case.

Oreshkov and Giarmatzi, NJP 18, 093020 (2016)

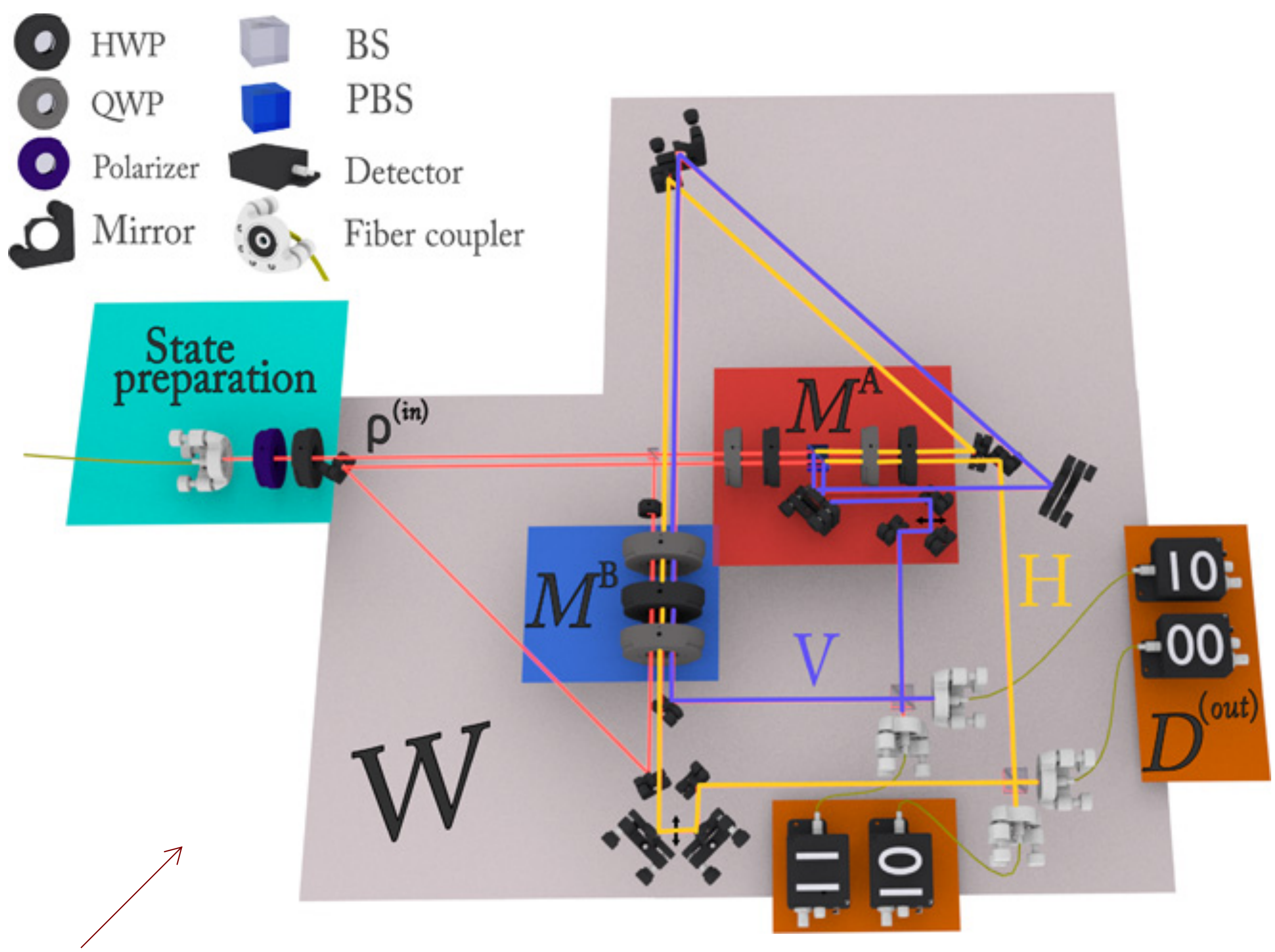
Experimental realizations of the quantum SWITCH



Rubio et al., Sci. Adv. (2017)

Procopio et al., Nat. Commun. (2015)

Realizations or simulations?



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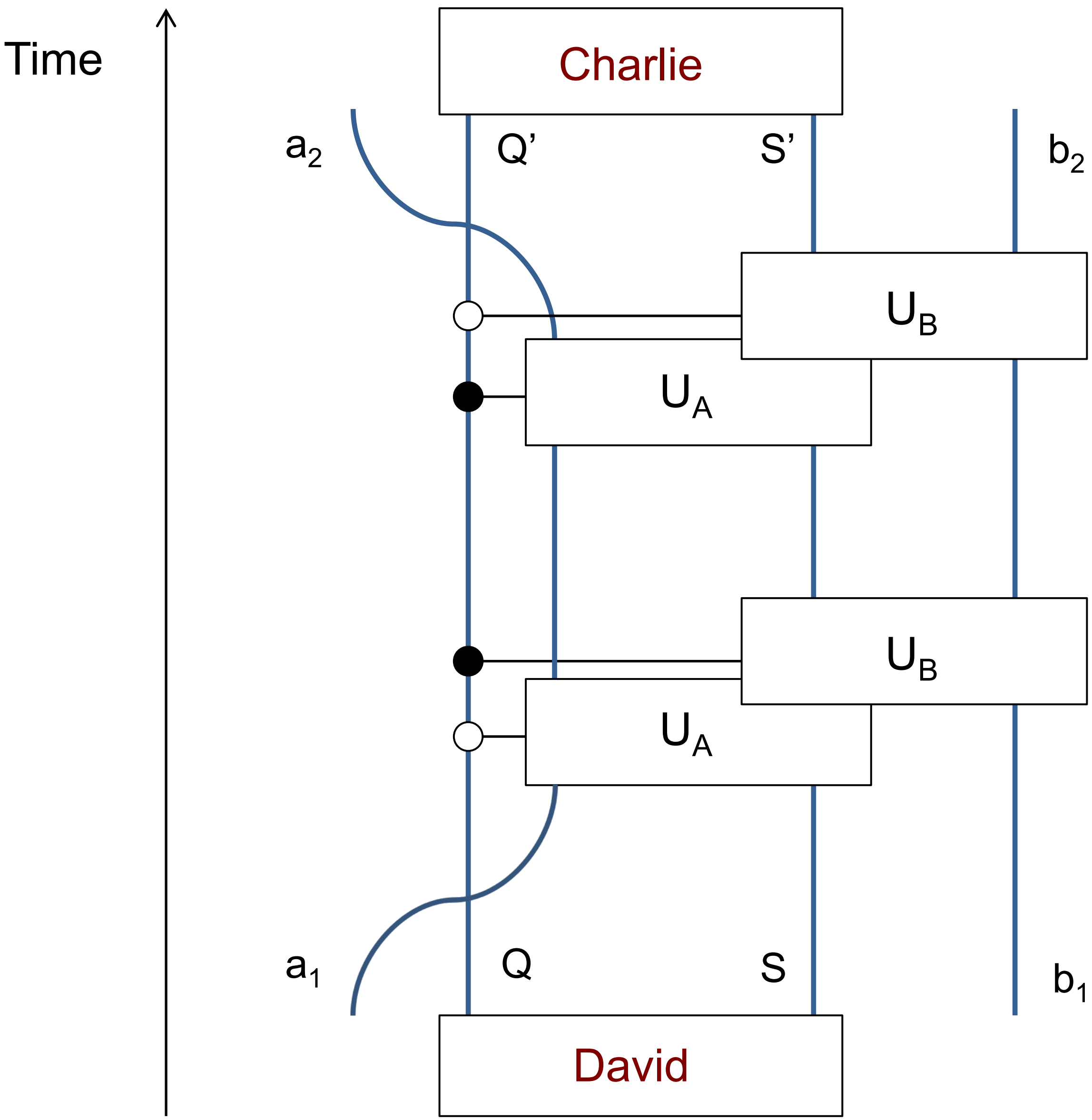
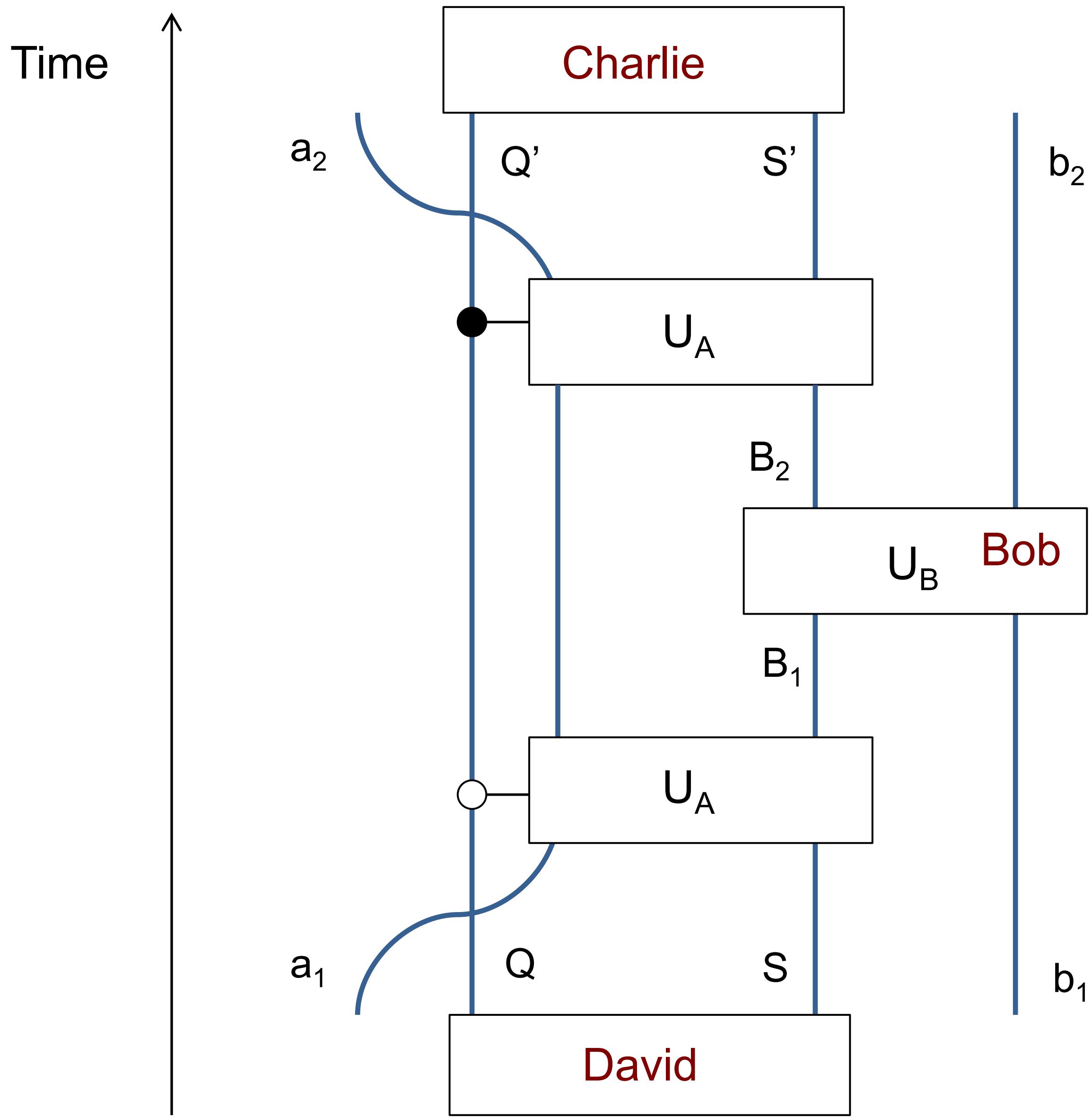


Temporal description

Realizations or simulations?

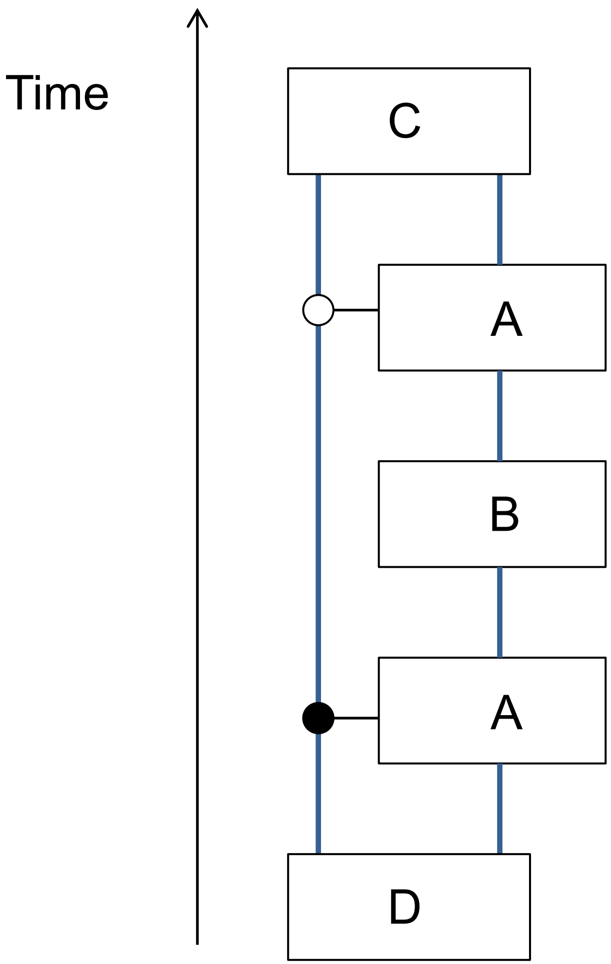


Temporal description (simple version)

Bob at a fixed time

Realizations or simulations?

WARNING: ignoring ancillas for simplicity



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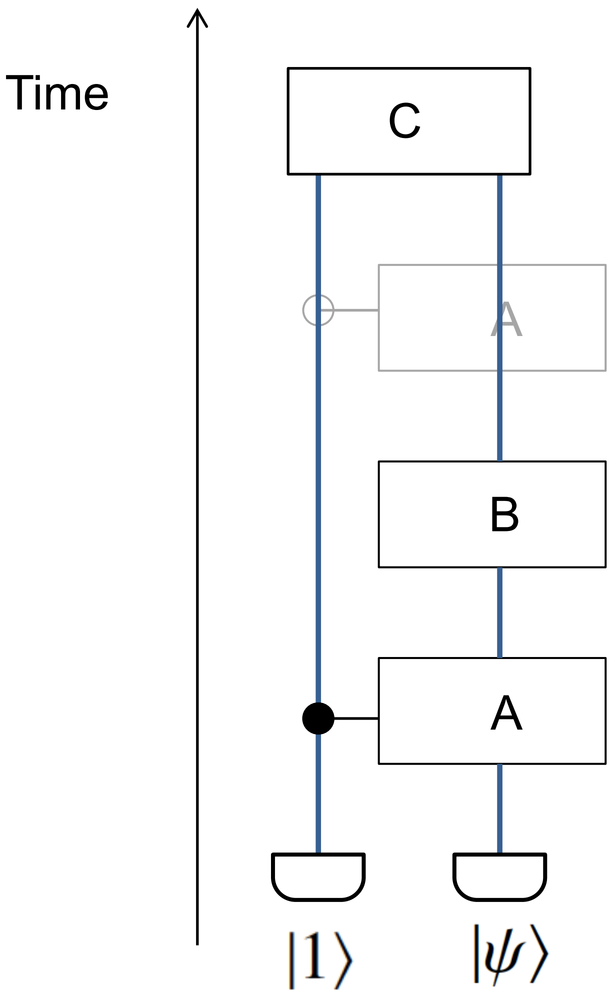

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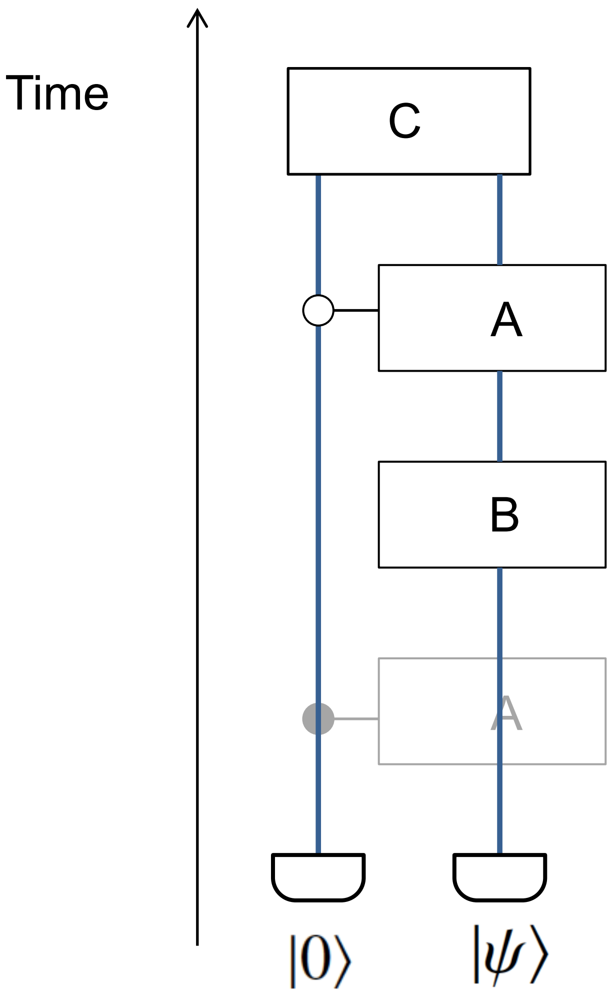

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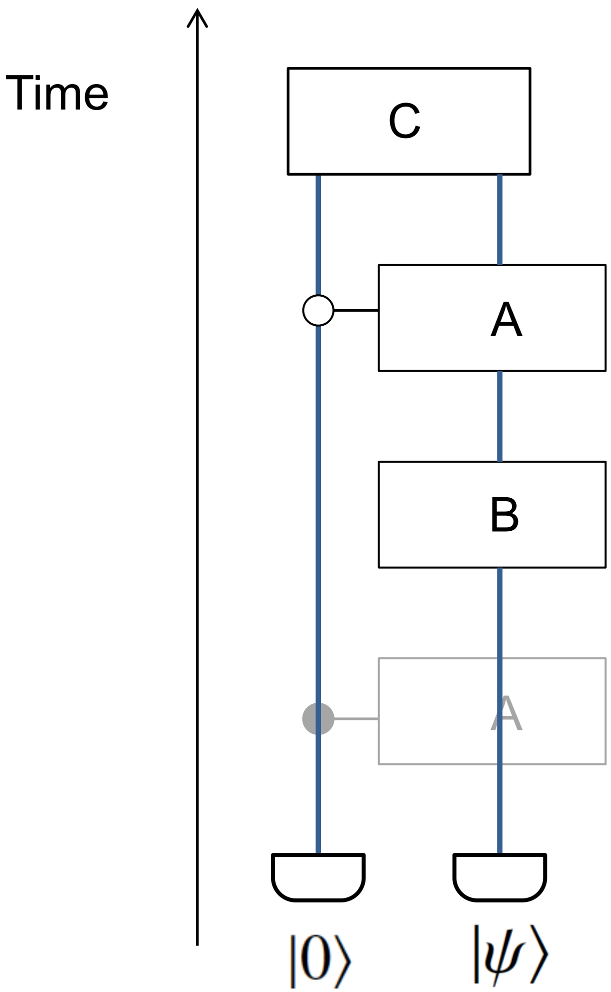
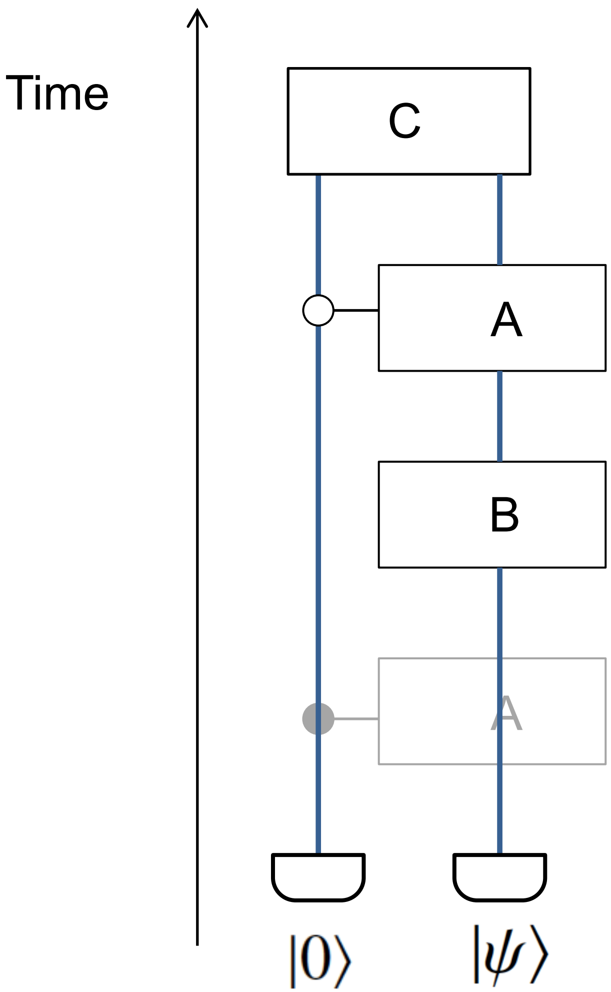


In each of these extreme cases, we can say that the operation of Alice takes place once on the target system: this can be verified through tomography.

Realizations or simulations?



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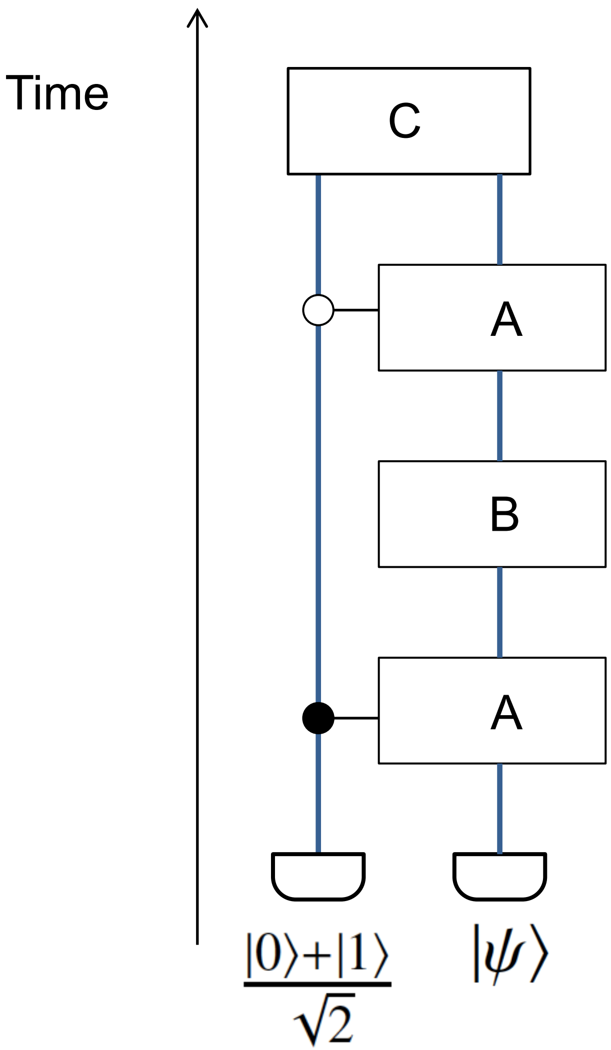
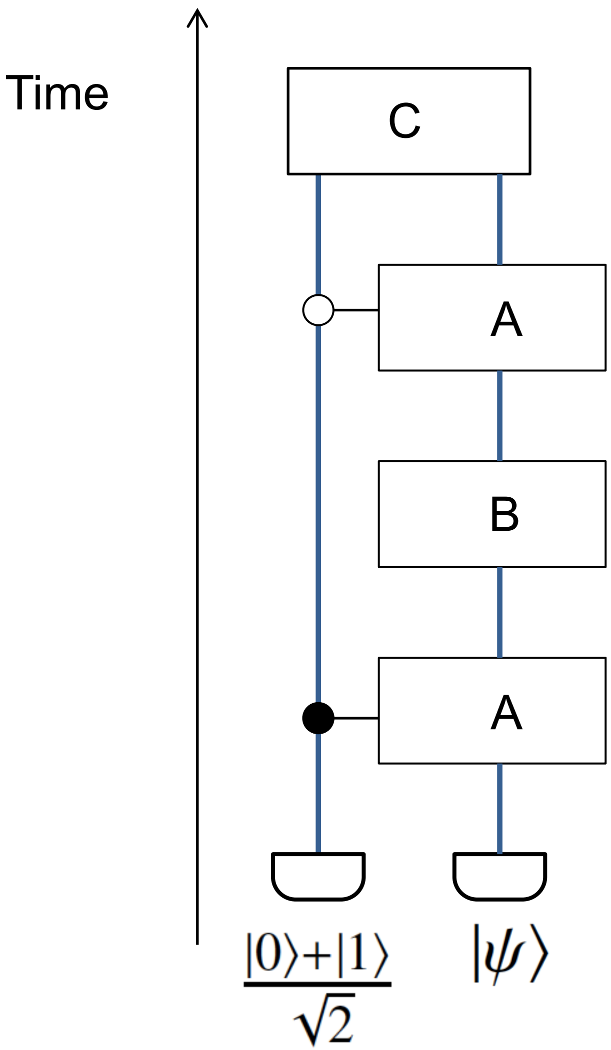
Note that the operation happens in a different place in the circuit in each case!

Realizations or simulations?



On what grounds can we claim that the correct operation happens when the control qubit is prepared in some different state?

Realizations or simulations?



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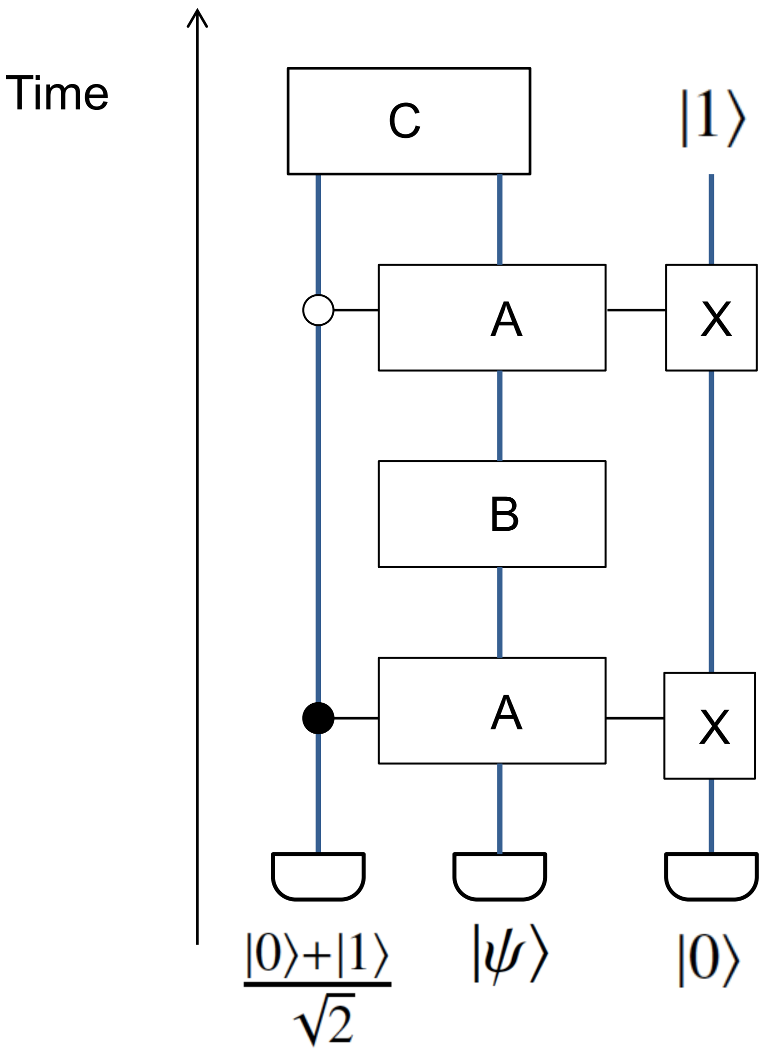
The heuristic ‘argument’: since we can claim this in each of the extreme cases, we should be able to claim it in the case of superpositions.

Realizations or simulations?



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Can (artificially) add a 'counter' which could be regarded as evidence that the operation happened once.

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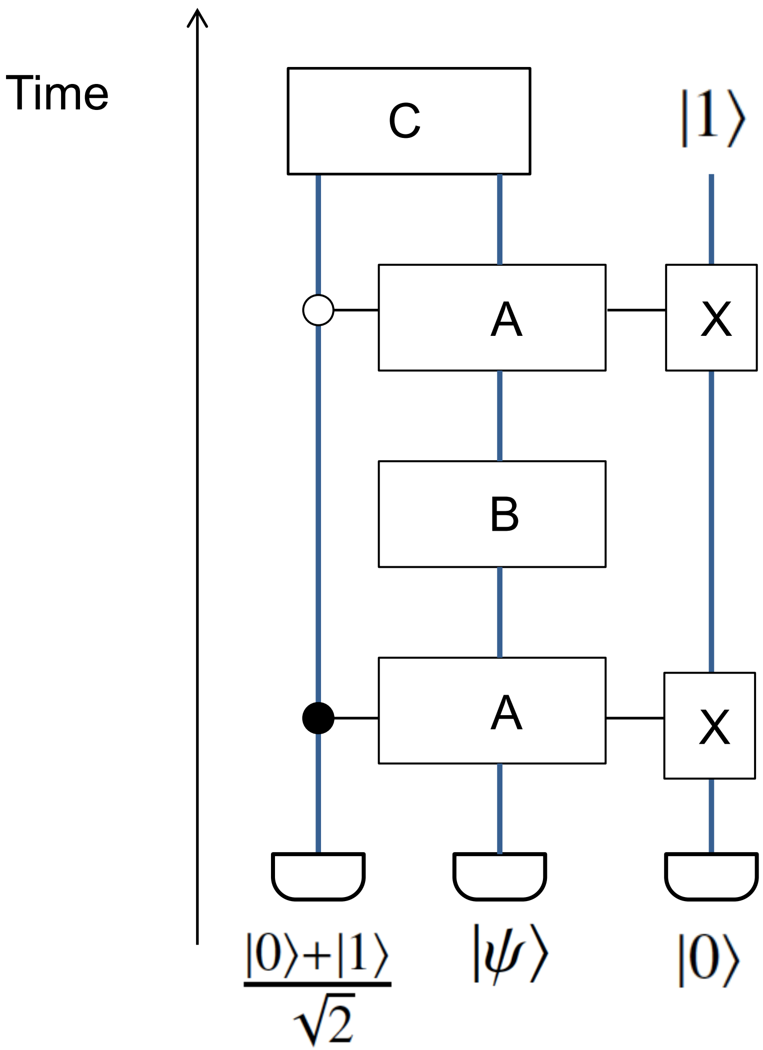

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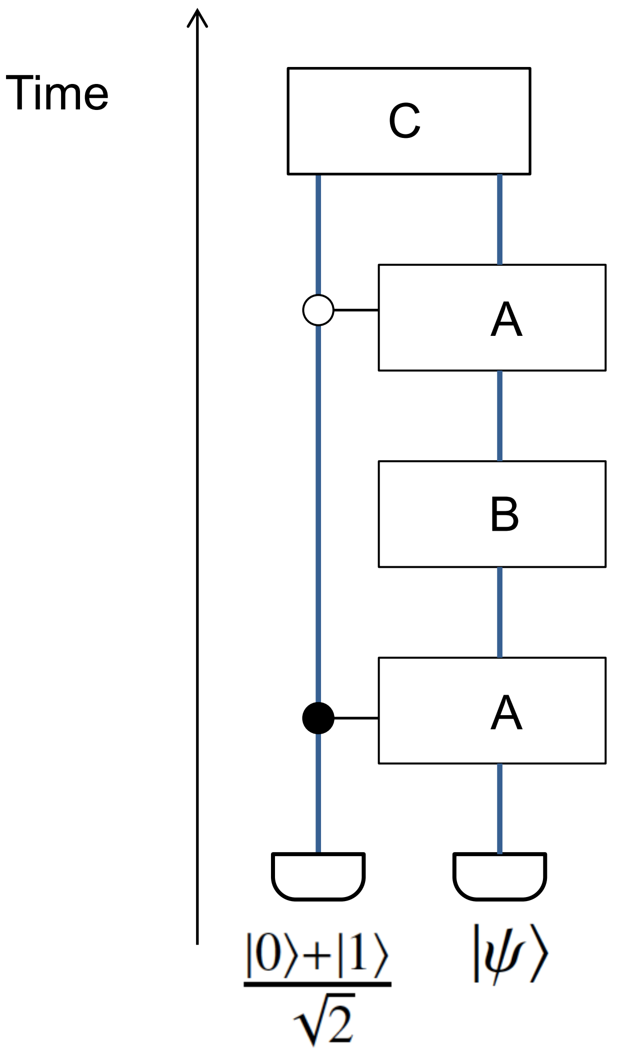
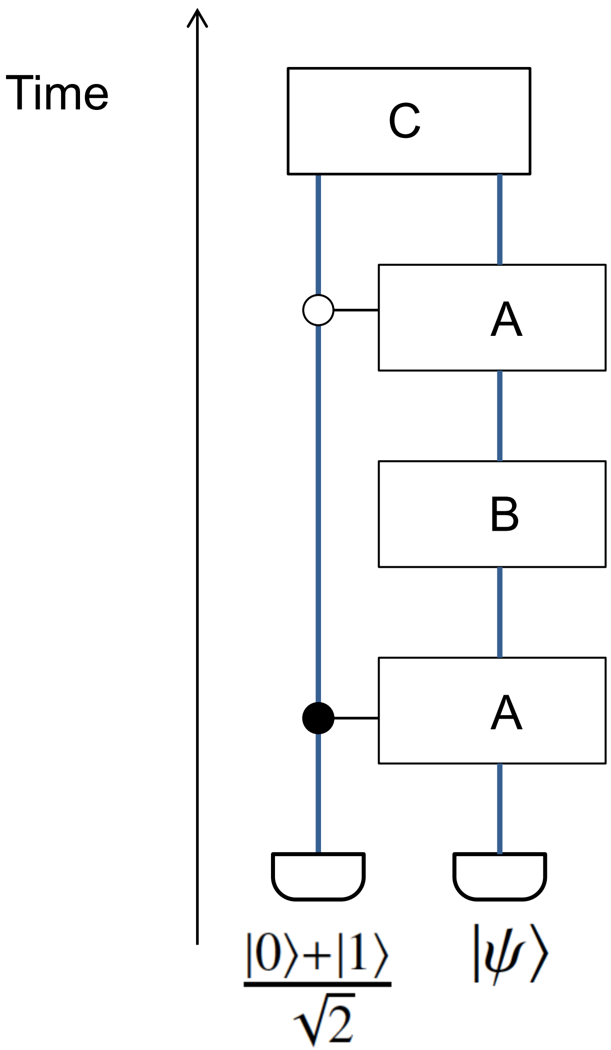
Problem: the counter is only verified to work as evidence in the extreme cases.

Realizations or simulations?



It is tempting to ***postulate*** it.

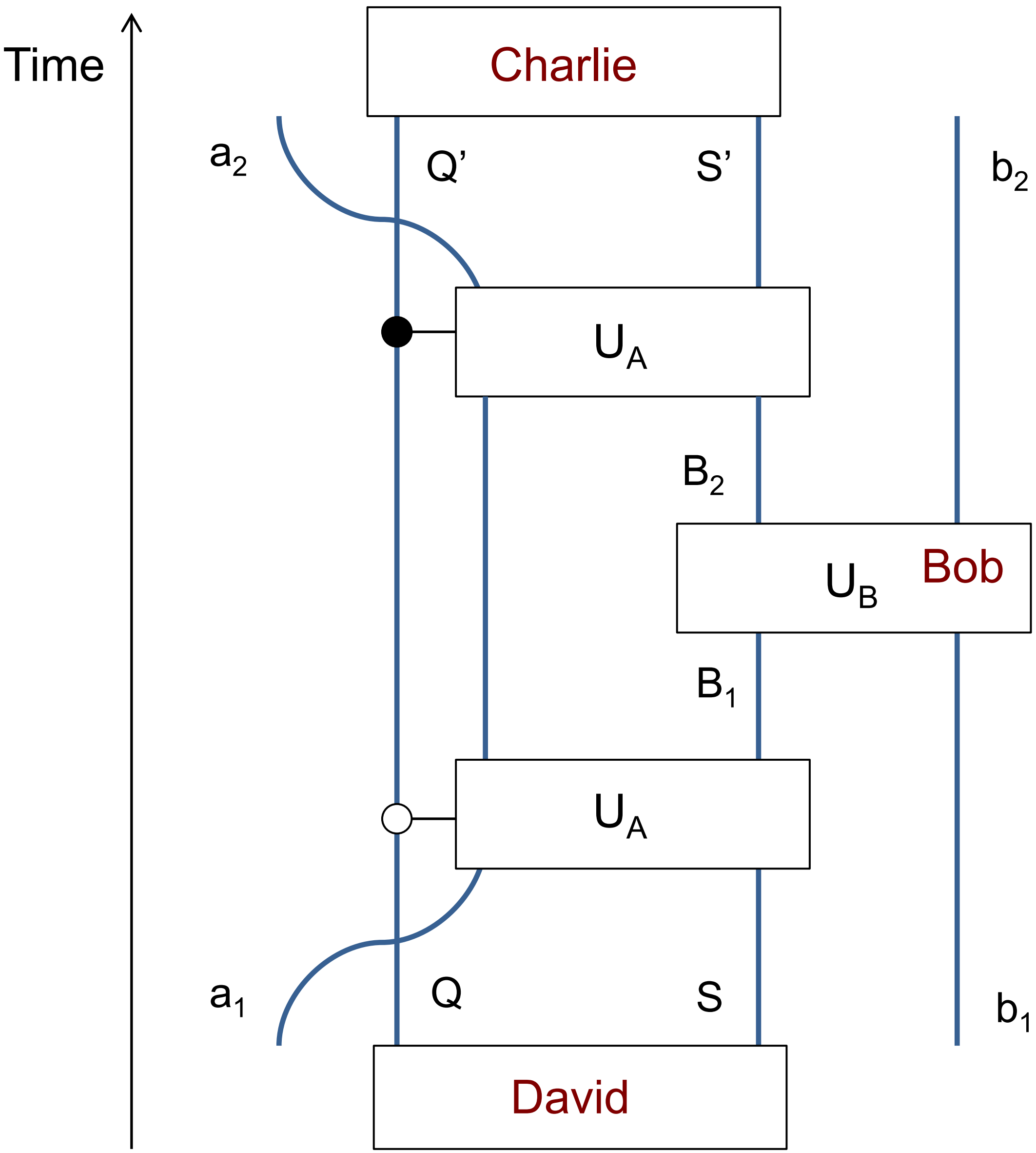
Realizations or simulations?



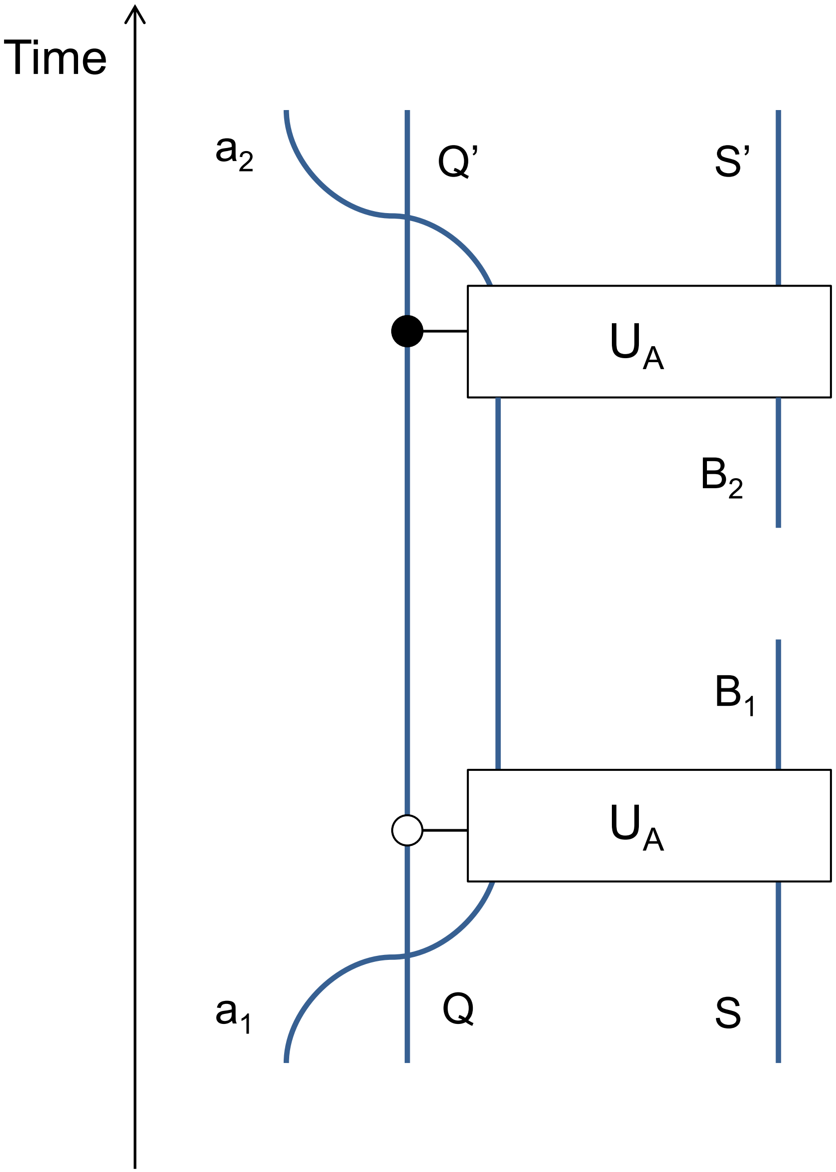
It is tempting to ***postulate*** it.

But this is empty unless supported by a theory that says where the operation takes place (what are its input and output spaces) and offers a means of **testing** that claim.

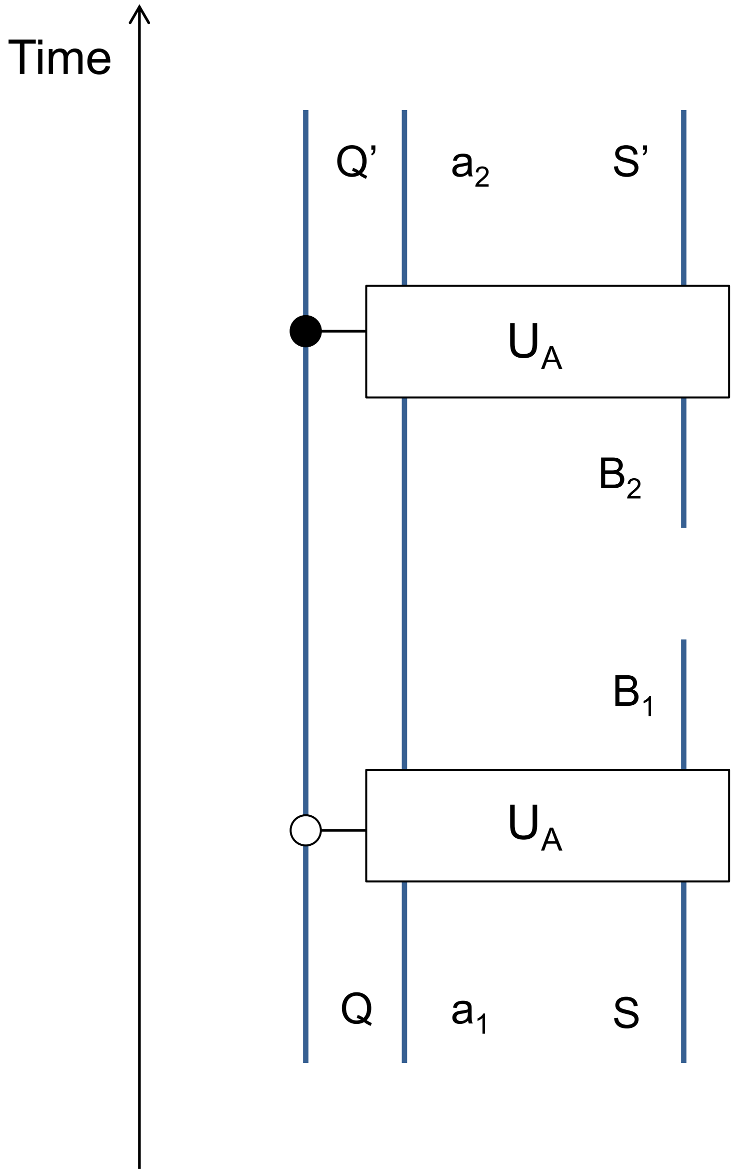
Identifying Alice's operation



Identifying Alice's operation



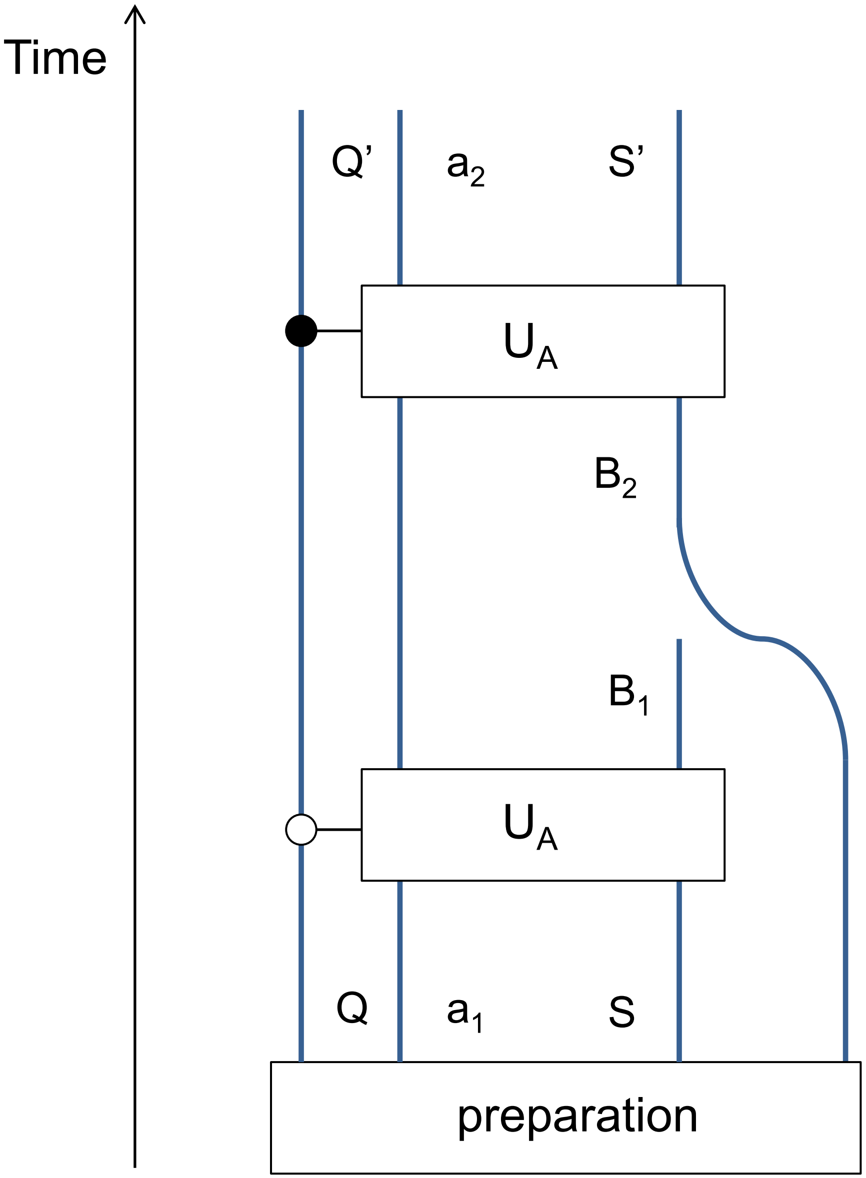
Identifying Alice’s operation



This fragment of a circuit is a quantum operation from a_1QSB_2 to $a_2B_1Q'S'$ (*a quantum comb*).

Chiribella, D’Ariano, and Perinotti, PRA 80, 022339 (2009).

Identifying Alice's operation



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It can be tested by tomography.

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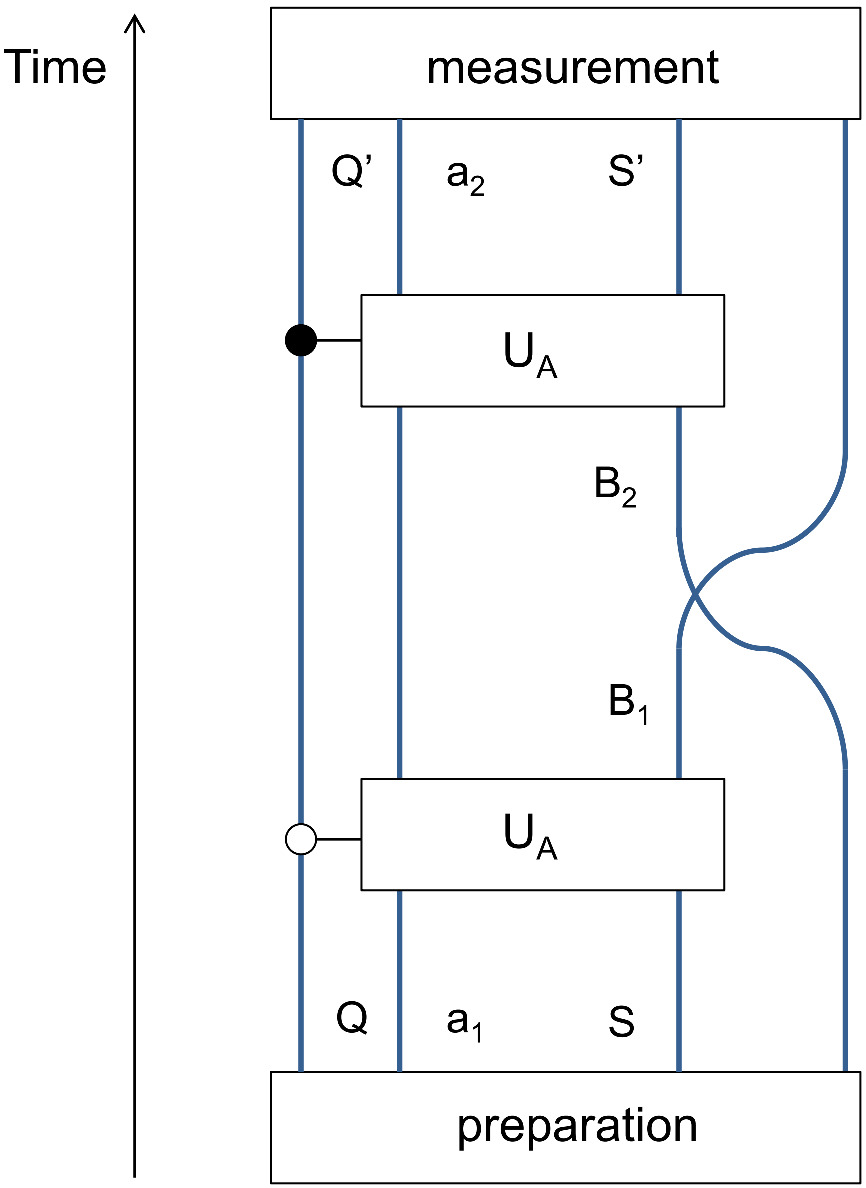

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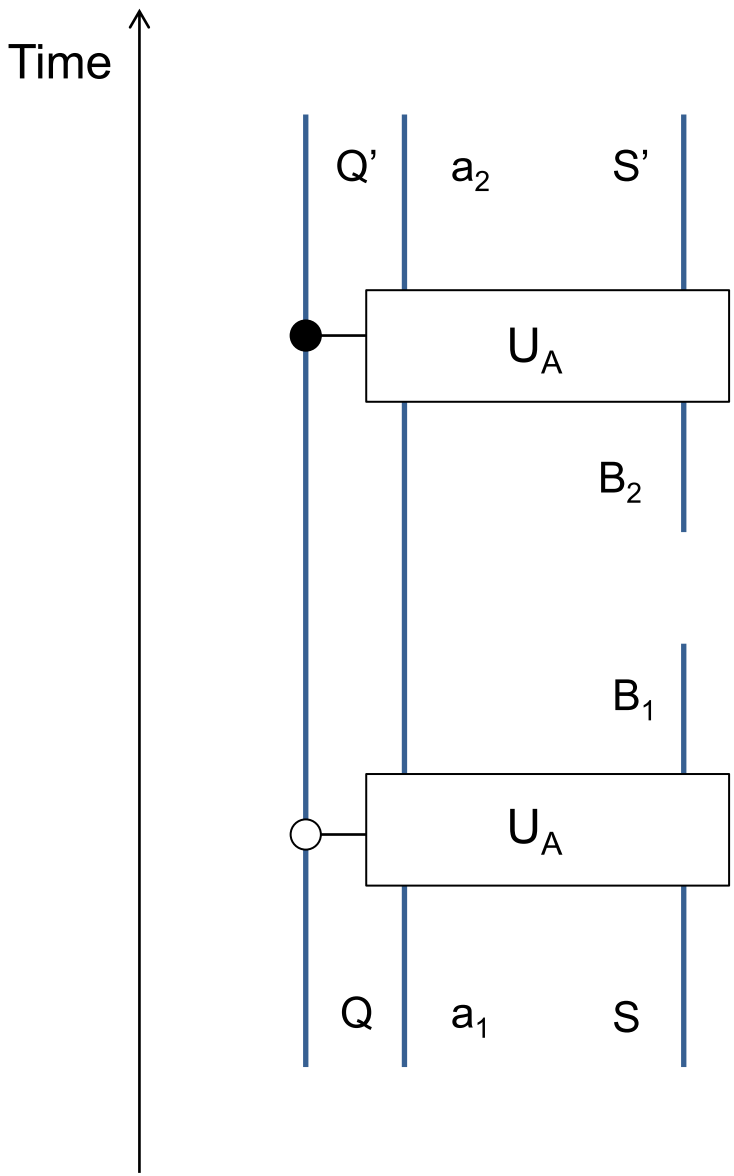


This fragment of a circuit is a quantum operation from a_1QSB_2 to $a_2B_1Q'S'$ (*a quantum comb*).

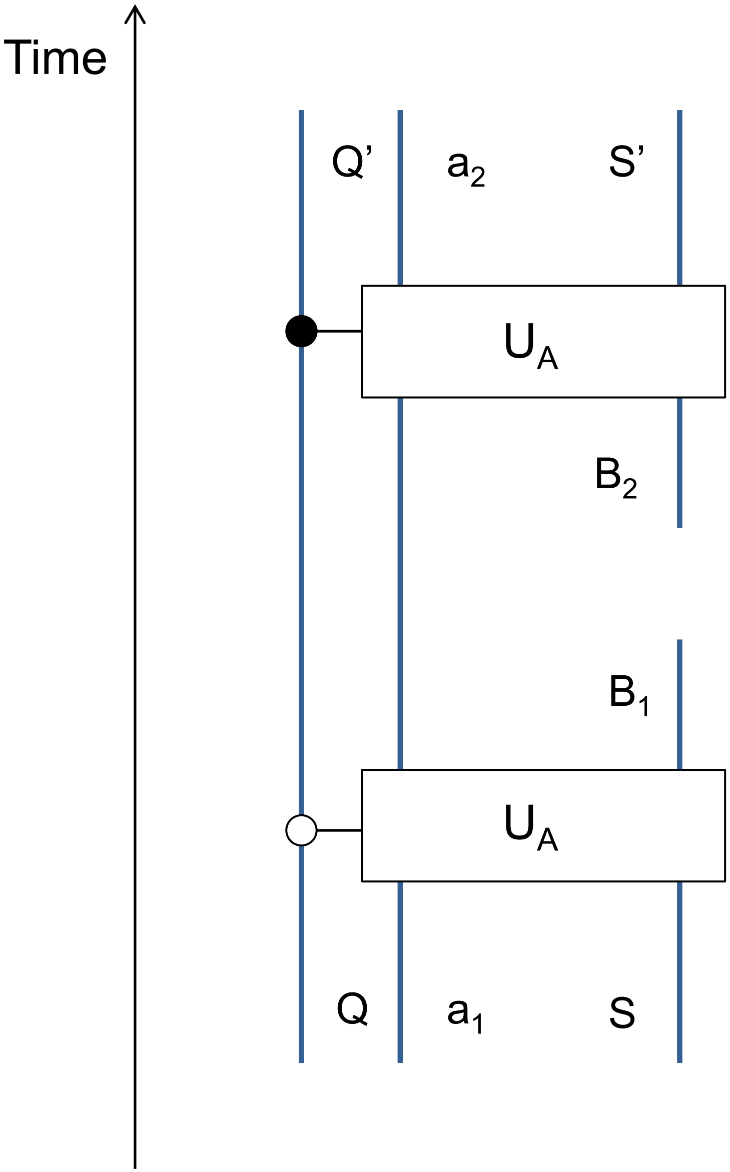
It is described by the following unitary:

$$U^{QS\,B_2a_1\to Q'S'B_1a_2} =$$

$$|0\rangle^{Q'}\langle 0|^{Q}\otimes U_A^{B_2a_1\to S'a_2}\otimes \mathbb{1}^{S\to B_1}$$

$$+|1\rangle^{Q'}\langle 1|^{Q}\otimes U_A^{S\,a_1\to B_1a_2}\otimes \mathbb{1}^{B_2\to S'}$$

Identifying Alice's operation



Claim:

$$U^{QS\,B_2 a_1 \to Q'S'B_1 a_2} = U_A^{A_1 a_1 \to A_2 a_2} \otimes \mathbb{1}^{\overline{A_1} \to \overline{A_2}}$$

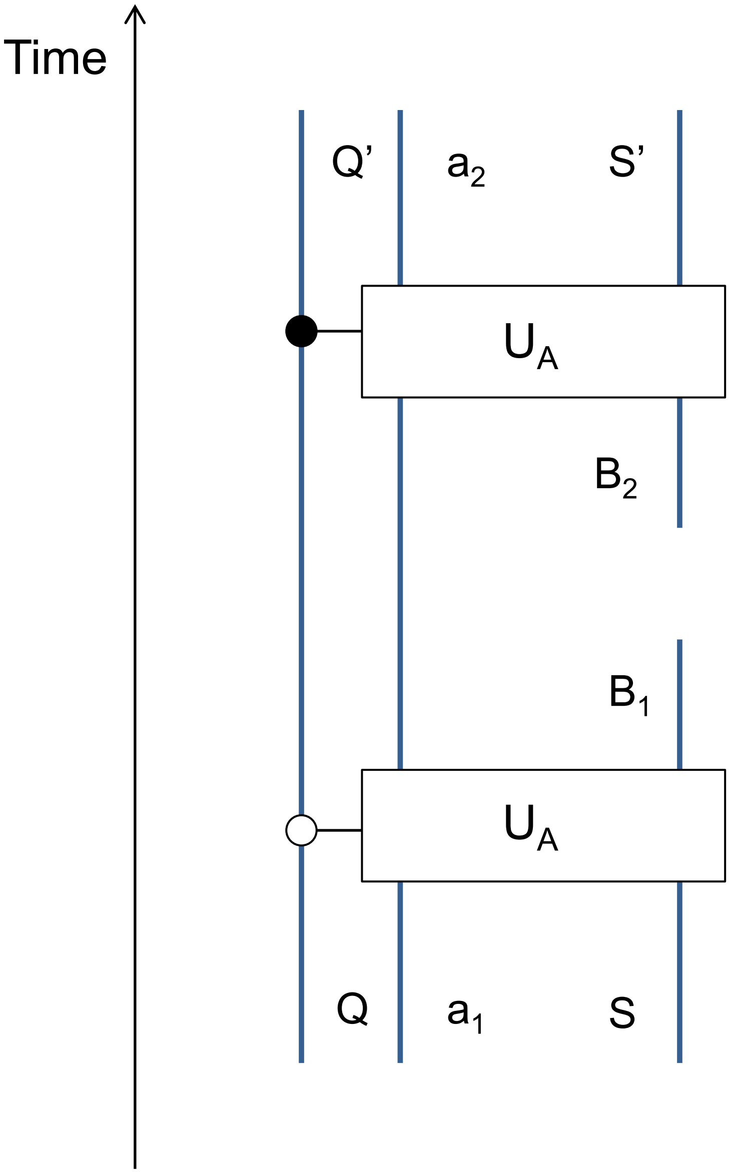
where A_1 is a nontrivial *subsystem* of QSB_2, defined by the algebra of operators

$$O^{A_1} \equiv |0\rangle\langle 0|^Q \otimes O^{B_2} \otimes \mathbb{1}^S + |1\rangle\langle 1|^Q \otimes \mathbb{1}^{B_2} \otimes O^S ,$$

and A_2 is a nontrivial *subsystem* of Q'S'B1, defined by the algebra of operators

$$O^{A_2} \equiv |0\rangle\langle 0|^{Q'} \otimes \mathbb{1}^{B_1} \otimes O^{S'} + |1\rangle\langle 1|^{Q'} \otimes O^{B_1} \otimes \mathbb{1}^{S'} .$$

Identifying Alice's operation



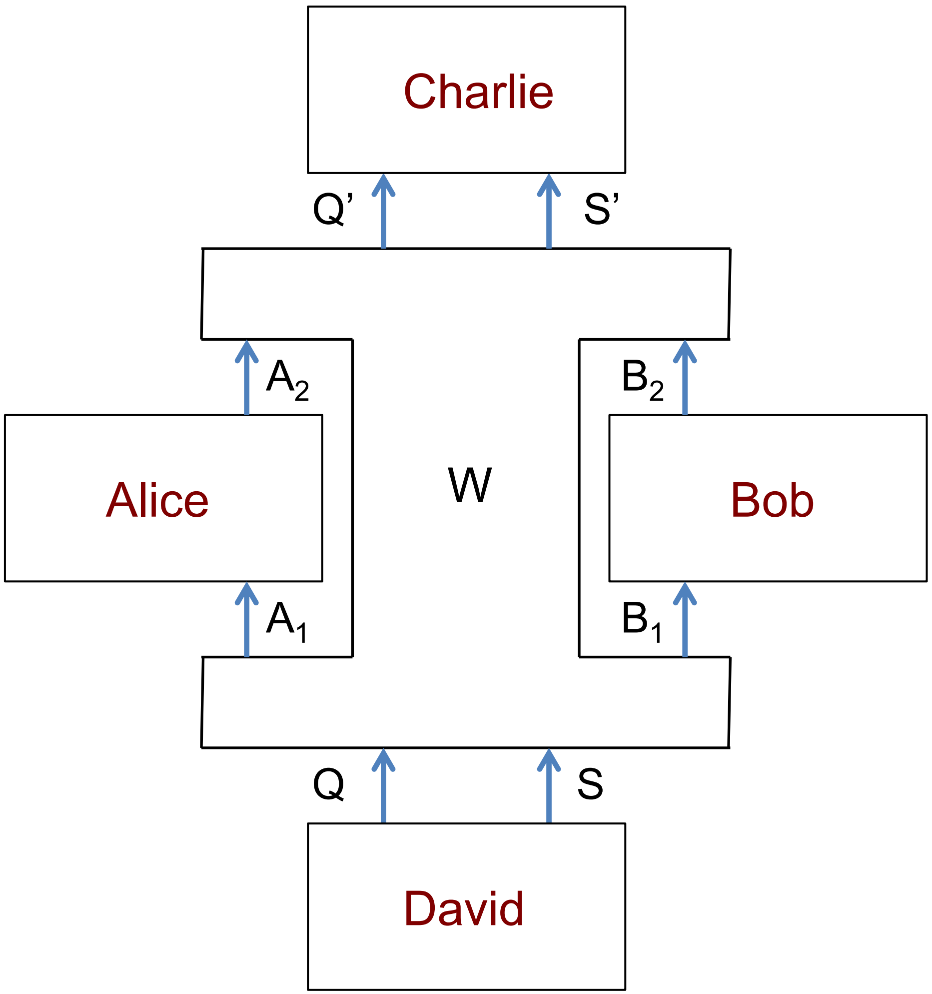
In other words, the supposed operation U^A indeed take place (*but not solely on the target system!*).

Its input and output systems A_1 and A_2 are specific time-delocalized systems, which are nontrivial subsystems of Hilbert spaces that involve both the target system and the control qubit at different times.

We can verify U^A experimentally through tomography just as we can verify the full comb.

Identifying Alice's operation

With respect to A_1 and A_2, the experiment has the structure of a circuit with a cycle.



Identifying Alice's operation

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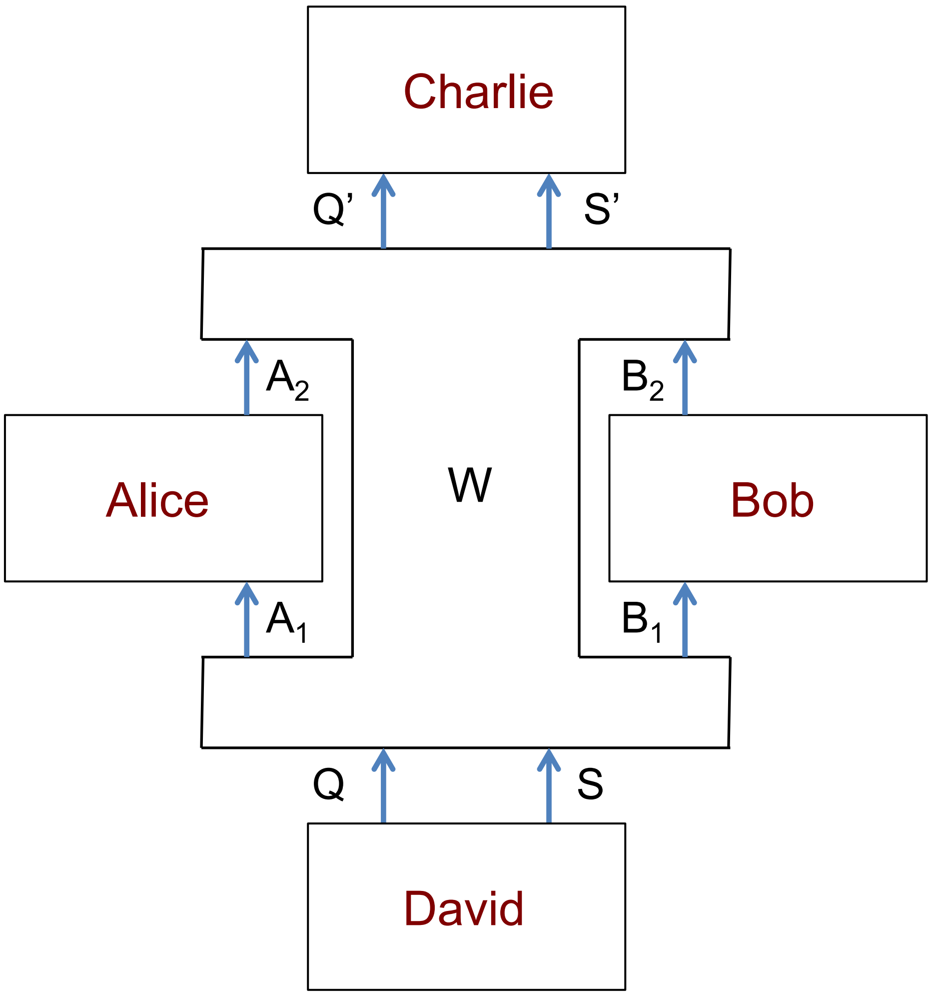
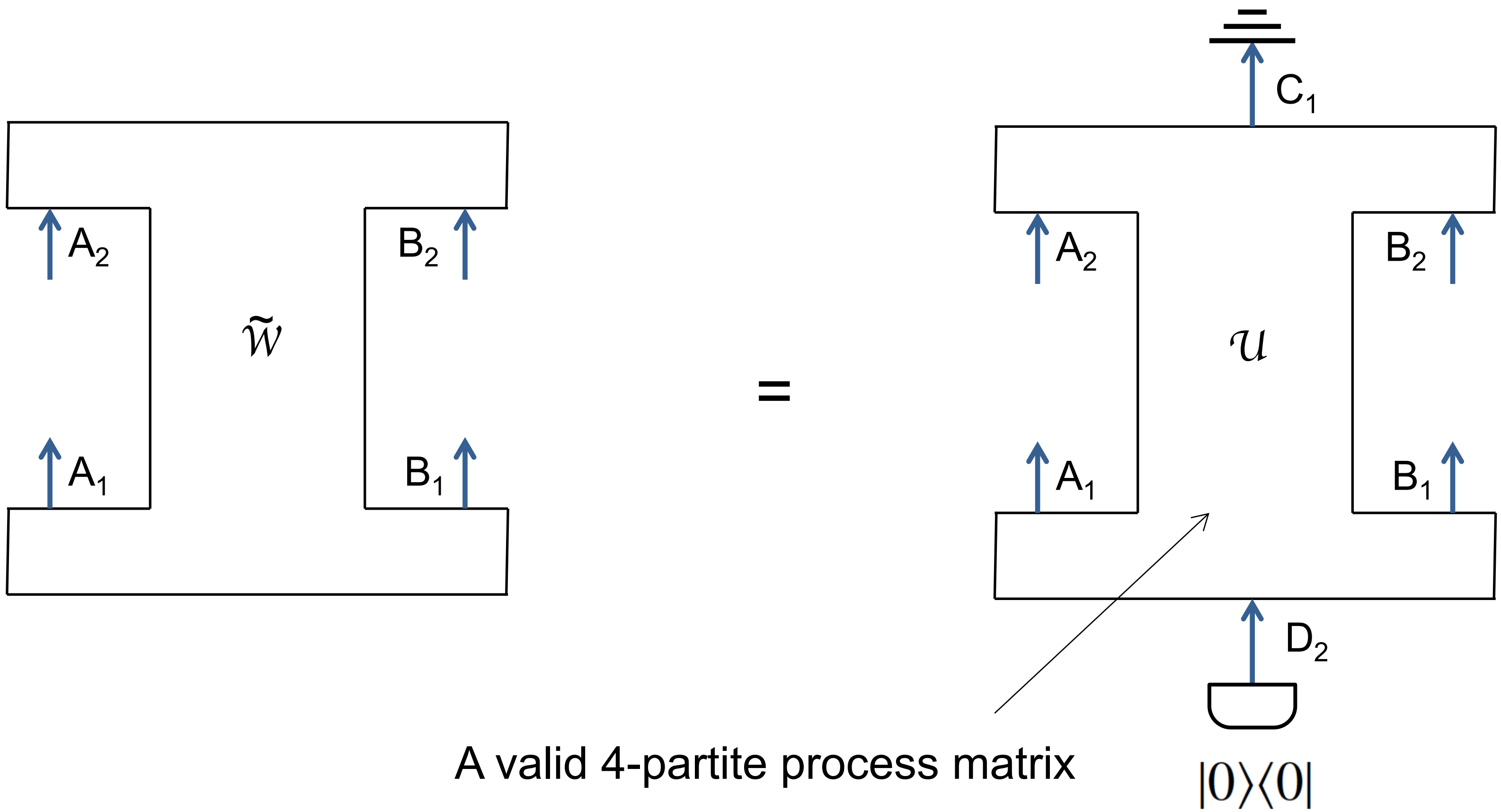


Causal nonseparability of W means that this cyclic circuit cannot be reduced to a finer-grained acyclic circuit or a (dynamical) probabilistic mixture of acyclic circuits!

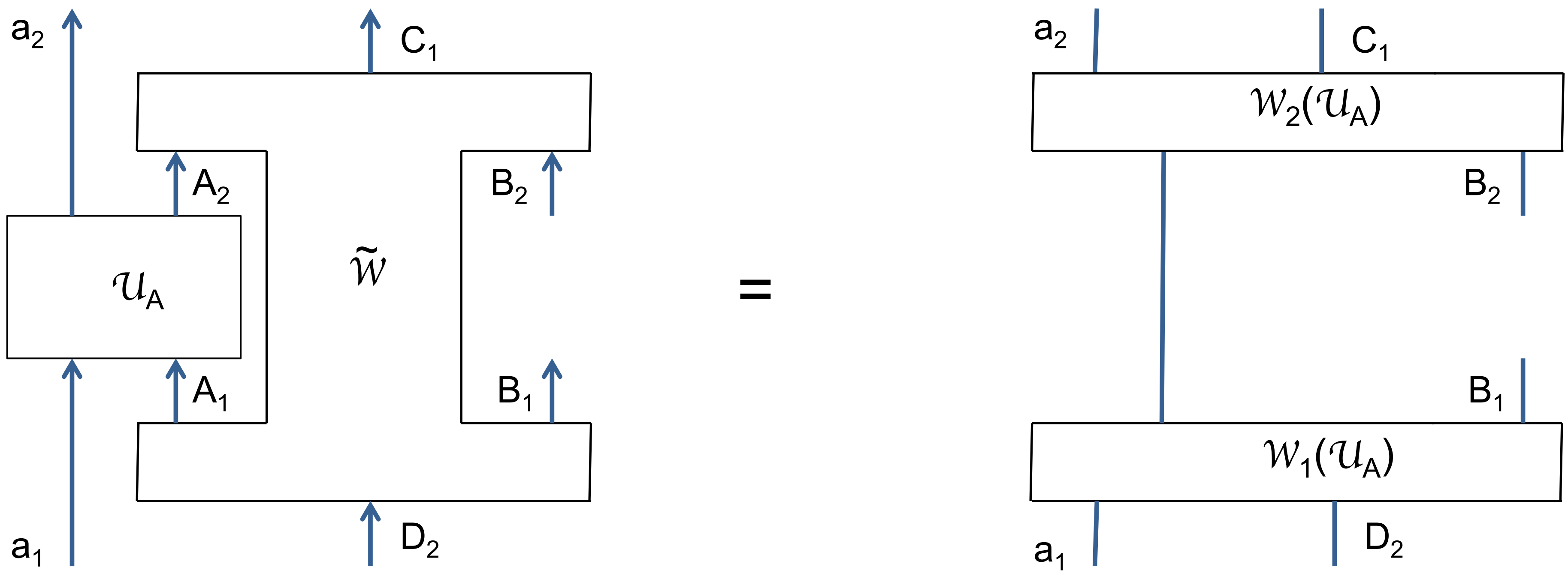
Could any process matrix have a realization with suitable time-delocalized subsystems?

Unitarily extensible bipartite processes

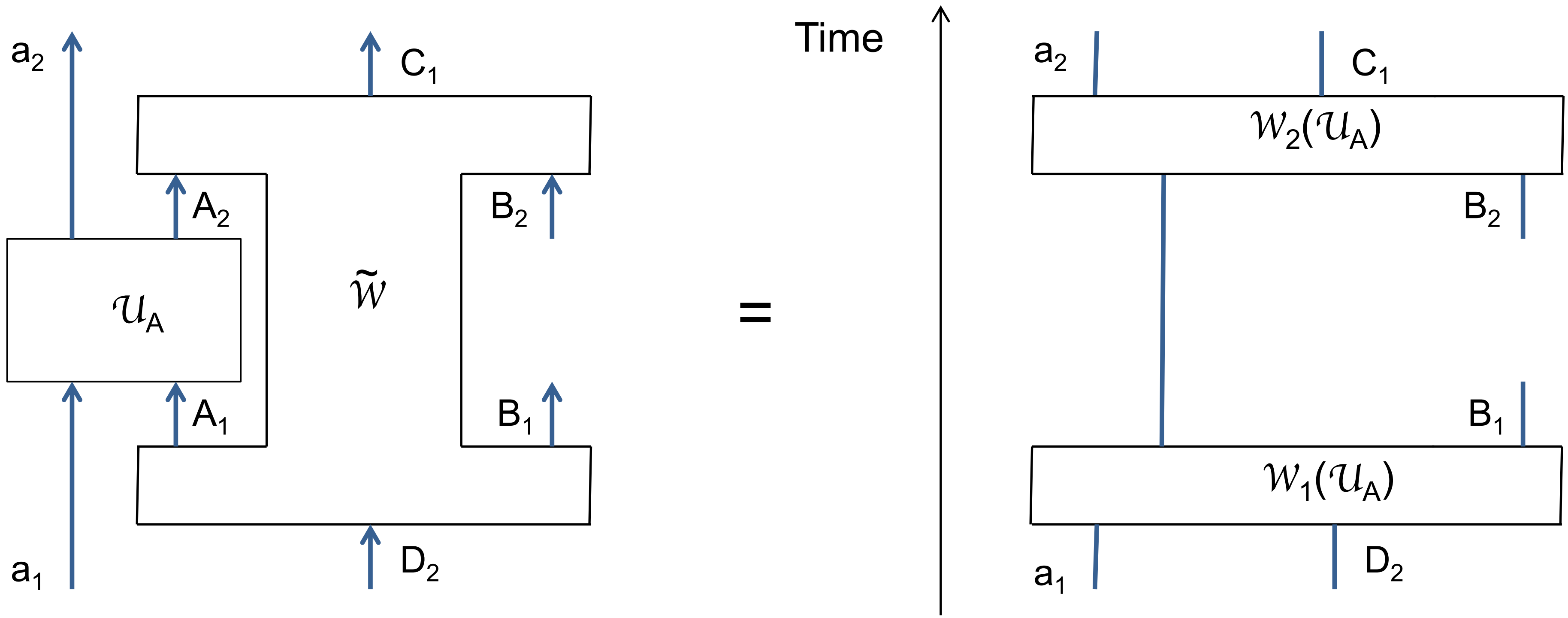


Araujo, Feix, Navascues, Brukner, Quantum 1, 10 (2017)

The following holds (because the left-hand-side is a supermap on Bob's operation):

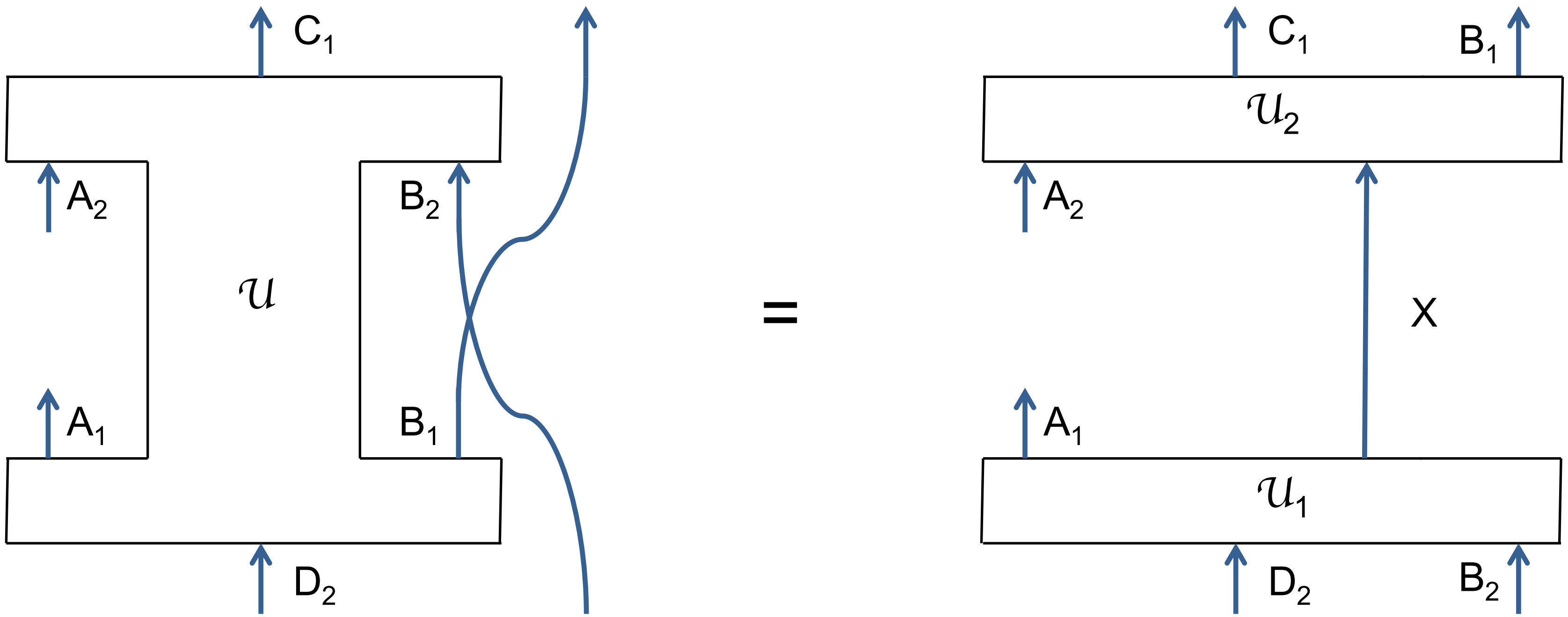


Chiribella, D'Ariano, and Perinotti, EPL 83, 30004 (2008)



→ Seek implementation where David, Bob, and Charlie are at definite times.

For a unitary process we have:

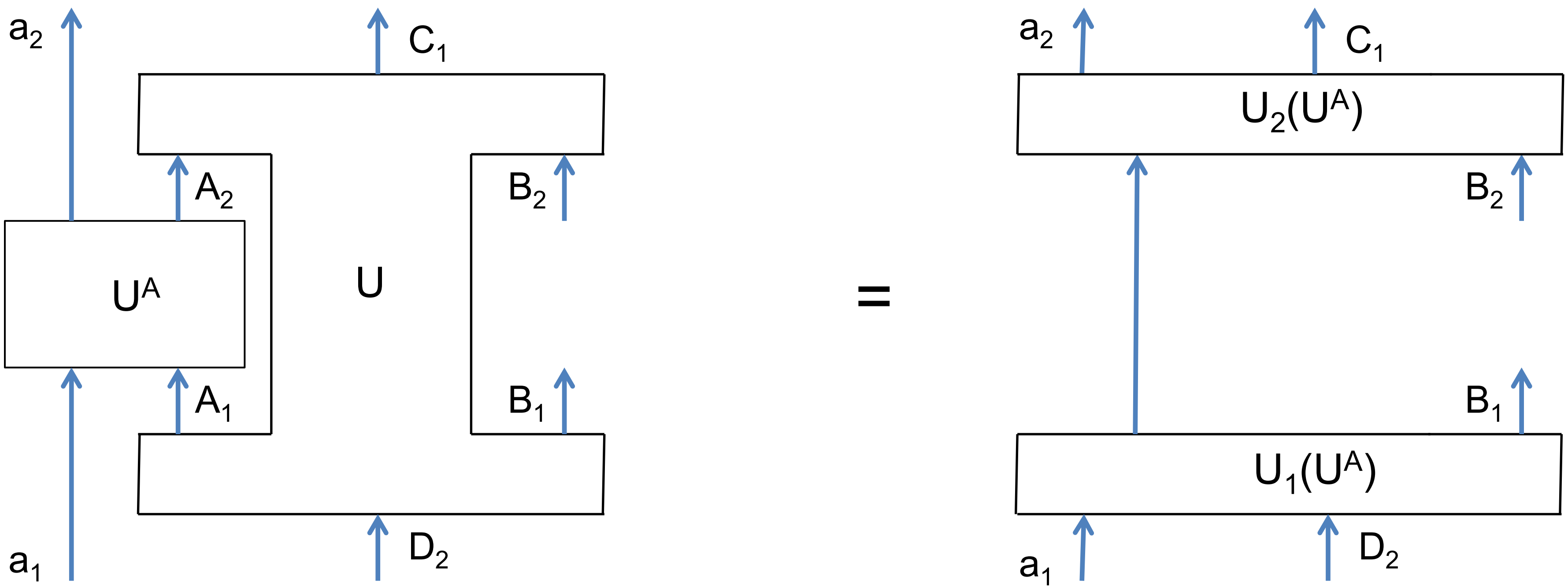


A_2 is mapped via $\mathcal{U}_2$ onto a subsystem $\tilde{A}_2$ of C_1B_1.

A_1 is mapped via the inverse of $\mathcal{U}_1$ onto a subsystem $\tilde{A}_1$ of D_2B_2.

We identify the abstract systems A_I and A_O with the physical subsystems $\tilde{A}_I$ and $\tilde{A}_O$.

Remember:



The fact that Alice's operation U^A happens on the subsystems $\tilde{A}_1$ and $\tilde{A}_2$ as part of the quantum comb on the right-hand side is guaranteed:

$$U^{a_1 D_2 B_2 \to a_2 B_1 C_1}(U_A) = U_A^{a_1 \tilde{A}_1 \to a_2 \tilde{A}_2} \otimes \mathbb{1}^{\bar{\tilde{A}}_1 \to \bar{\tilde{A}}_2}$$

What about more general processes?

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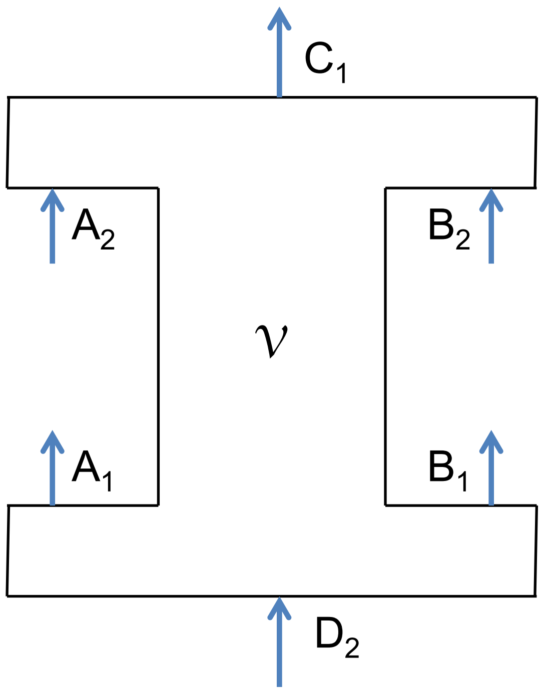
Can show a similar thing for the following class of processes:



$\mathcal{V}$ is an isometric channel that maps a subsystem of D_2B_2 onto A_1.

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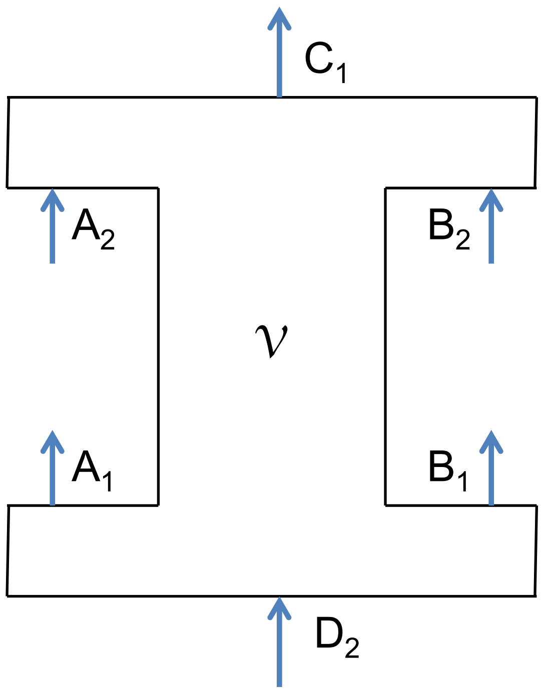

$\mathcal{V}$ is an isometric channel that maps a subsystem of D_2B_2 onto A_1.

How big is this class? It is certainly at least as big as the unitary class.

Is it strictly larger? Could it be that all bipartite processes can be purified in this way?

Potential objections/doubts:

- There are two (or more) uses of controlled operations (or operations acting on a larger space including the vacuum).

Answer: These are NOT the input operations of the process of interest. The correct input operations happen exactly once.

- There is a no-go theorem (Chiribellat et al.) showing that if we can do the SWITCH, we can realize deterministic time travel.

Answer: This theorem assumes that the operations of Alice and Bob could be placed in a circuit such that one is in the past of the other. This is not the case here.

- ???

Summary and questions

- Time-delocalized subsystems are subsystems that can be probed just like regular (fixed-time) subsystems.
- There exist processes that have nonseparable cyclic structures with respect to such subsystems.
- Could it be that all mathematically possible processes have realizations on time-delocalized subsystems?
- Can this perspective inform useful applications?
- Is there a notion of space-time reference frame with respect to which Alice's operation is seen as a standard operation? (Links to the gravitational quantum SWITCH by Zych et al.)