How to make neutral particles "feel" a magnetic field?

Quic Lecture, 28/05/2020

Quantum simulation

Problem: Quantum mechanical systems are usually hard to simulate using classical computers due to the exponentially large size of the Hilbert space.

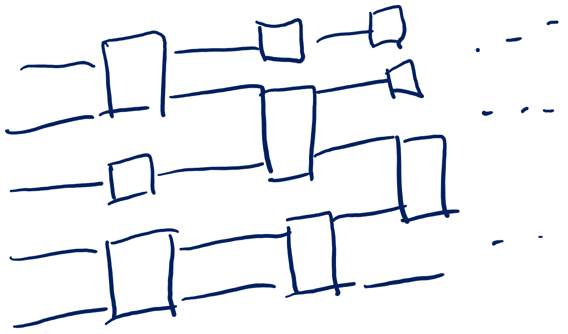
Feynman (82): "Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws."

Idea: Use a controllable quantum system to simulate the quantum system we would like to study

$\rightarrow \begin{cases} \text{digital quantum simulator} \\ \text{analog} \quad " \quad\quad " \end{cases}$

Digital Quantum Simulator

Quantum circuit



$$e^{-iHt} \simeq \prod_{j=1}^{M=\text{poly}(n)} U_j$$

time-evolution of system we want to simulate

→ quantum gates

Simplest approach: Lloyd '96, "Universal quantum simulators"

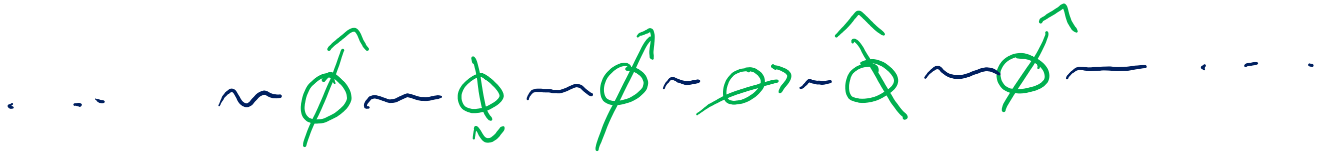
$$H = \sum_{l=1}^{L} H_l \,, \quad e^{-iHt} \simeq \left(e^{-iH_1 t/s} e^{-iH_2 t/s} \cdots e^{-iH_L t/s} \right)^s$$

terms acting on a few qubits

for large s

Example: Heisenberg model

$$H = \sum_j J\left(\sigma_x^{(j)}\sigma_x^{(j+1)} + \sigma_y^{(j)}\sigma_y^{(j+1)} + \sigma_z^{(j)}\sigma_z^{(j+1)}\right) + \sum_j h_j \sigma_z^{(j)}$$



type of gates needed for a digital Q. sim.:

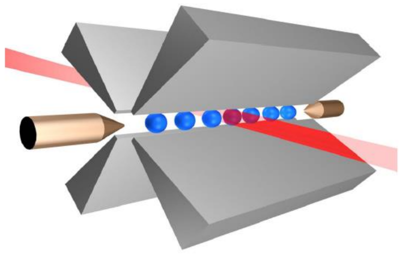
- $e^{-i\tau\sigma_\alpha^{(j)}\sigma_\alpha^{(j+1)}}$, $\alpha \in \{x, y, z\}$
- $e^{-i\tau h_j \sigma_z^{(j)}}$

Analog Quantum Simulator

Quantum system described by a Hamiltonian $H(t)$ with tunable parameters.

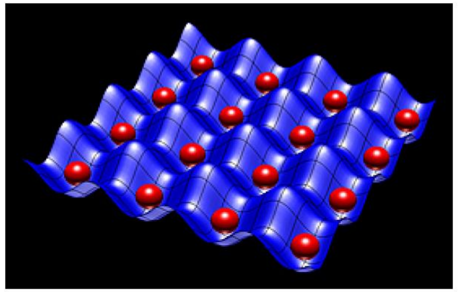
Examples:

Ion traps



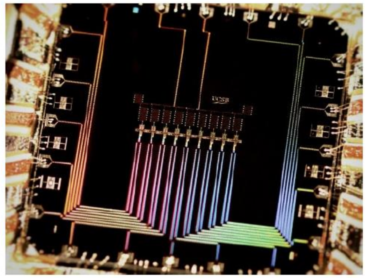
Credit: ITP, Uni. Innsbruck

Optical lattices



Credit: NIST

Superconducting qubits



Credit: UCSB/Google

Approach: Tune the system so it simulates the system under study

	Pros	Cons
Digital Q Sim	→ more versatile	→ requires large depth circuits → probably needs QEC
Analog Q Sim	→ more particles (e.g. 10^3 in BECs) → longer evolution times before system decoheres (?)	→ more limited (special purpose)

Charged particle in an EM-field

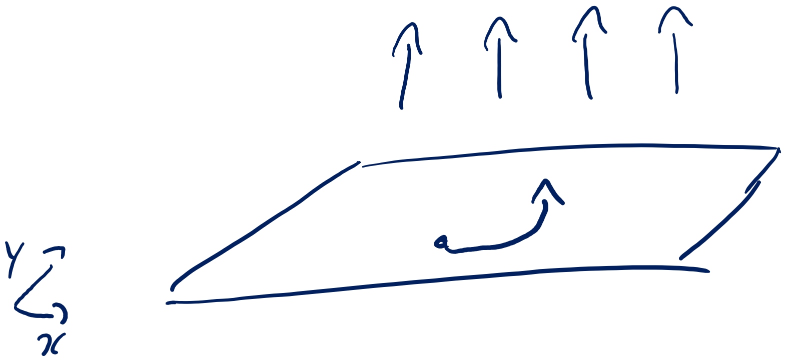
Classical

Lorentz force:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Example:

$$\vec{B} = B\,\vec{e_z}$$



Quantum

$$\hat{P} \longrightarrow \hat{P} - q\frac{\hat{A}}{c}$$

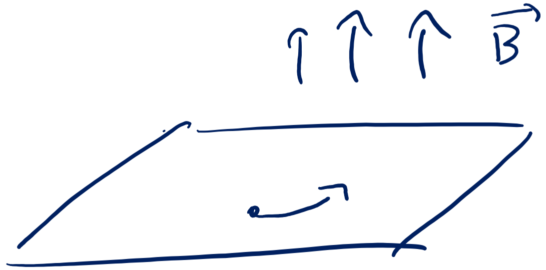
$$\hat{H} = \frac{1}{2m}\left(\hat{P} - q\frac{\hat{A}}{c}\right)^2 + q\varphi$$

$$\vec{E} = -\nabla\varphi - \frac{\partial A}{\partial t}\,, \quad \vec{B} = \nabla \times \vec{A}$$

$$\vec{A} \xrightarrow[\text{gauge freedom}]{} \vec{A} + \nabla\phi_0$$

$\hookrightarrow$ scalar function

Landau Levels



$$\vec{B} = B\,\vec{e}_z$$

Landau gauge: $\vec{A} = (0, Bx, 0)$

$$H = \frac{\hat{p}_x^{\,2}}{2m} + \frac{1}{2m}\left(\hat{p}_y - \frac{qB\hat{x}}{c}\right)^2$$

Assuming $|\psi(x,y)\rangle = |p_y\rangle\,|\psi(x)\rangle$

$$\hat{H} = \underbrace{\frac{\hat{p}_x^{\,2}}{2m} + \frac{1}{2} m \omega_c^2 \left(\hat{x} - \frac{\hbar k_y}{m\omega_c}\right)^2}_{\text{shifted Harm. Osc.}}$$

$$\boxed{\omega_c = \frac{qB}{mc}}$$

Eigenenergies:

$$E_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

Eigenstates:

$$\psi(x,y) = e^{ik_y y}\,\phi_n(x - x_0)$$

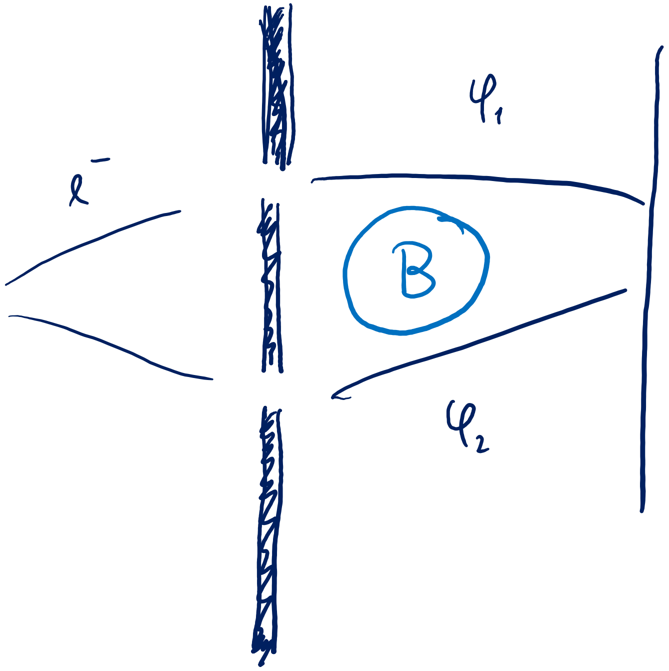
To see effect of Landau levels

$$k_B T \ll \hbar\omega_c$$

low T

high B

Aharonov-Bohm effect



$\vec{B} = 0$ outside sollenoid

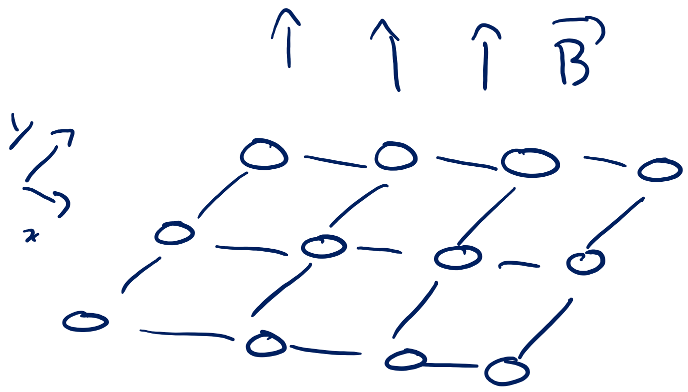
$\vec{A} \neq 0$ 〃 〃

Wave-function picks a phase:

$$\varphi_{1/2} = \frac{q}{\hbar} \int_{P_{1/2}} \vec{A} \cdot d\vec{x}$$

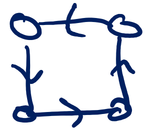
$\Delta\varphi$ measurable by interference experiment

Harper-Hofstadter model



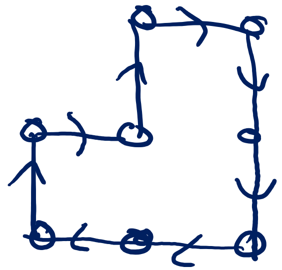
$$H = \sum_{m,n} J \left(e^{i\phi m} a^\dagger_{m,n+1} a_{m,n} + a^\dagger_{m+1,n} a_{m,n} + h.c. \right)$$

$$\vec{A} = (0, Bx, 0)$$



$e^{i\phi}$

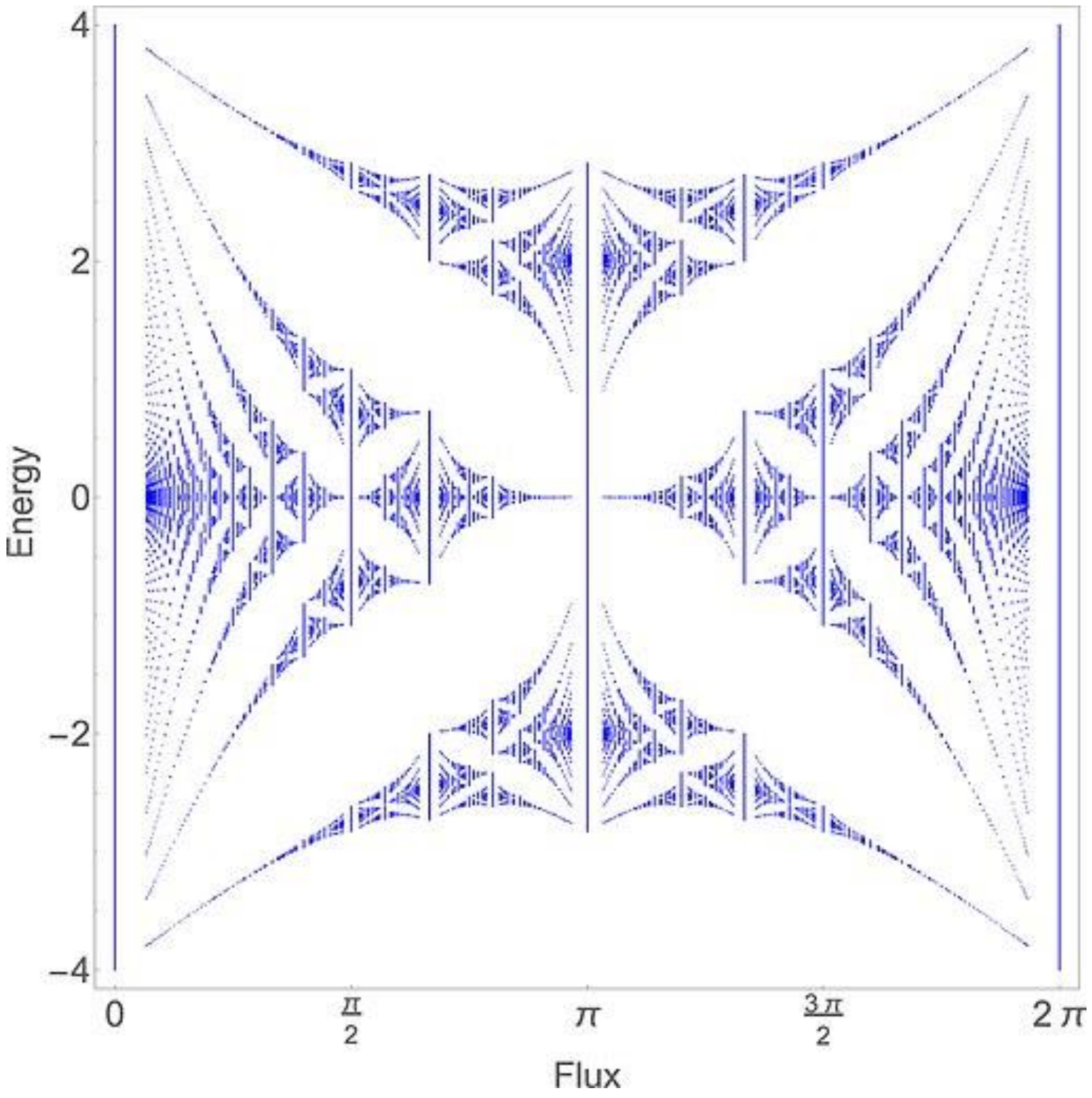
Peierls substitution:



$e^{-i3\phi}$

$$\phi_{(m,n)\to(m,n+1)} = \frac{q}{\hbar}\int_{\vec{r}_{m,n}}^{\vec{r}_{m,n+1}} \vec{A}\cdot d\vec{r} = \frac{m q B d^2}{\hbar} = m\phi$$

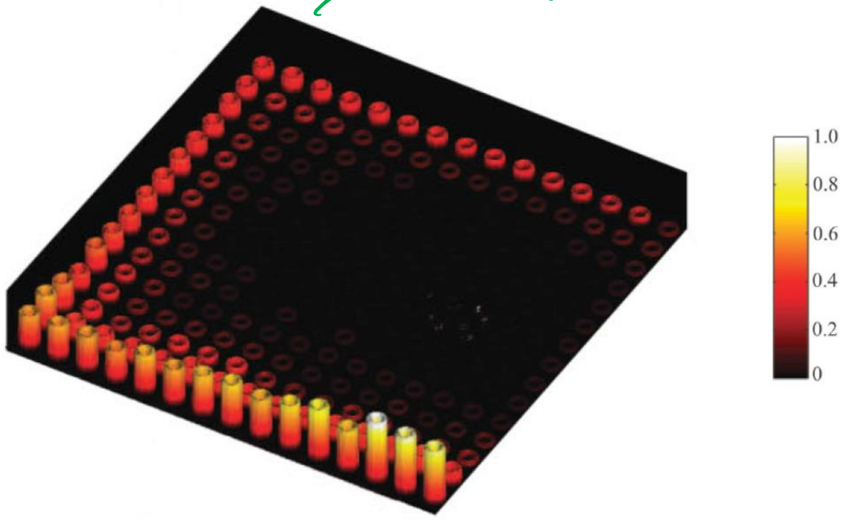
Hofstadter butterfly



Hofstadter's butterfly in quantum geometry, *New J. Phys.* **18**, 103023 (2016) [doi:10.1088/1367-2630/18/10/103023](https://doi.org/10.1088/1367-2630/18/10/103023)

Chiral edge states

transport along edges robust to disorder



Topologically Robust Transport of Photons in a Synthetic Gauge Field, S. Mittal et al., PRL 113, 087403 (2014)

Simulating magnetic fields with neutral particles

What we want: $H = \sum_{m,n} J \left(e^{i\phi m} a^{+}_{m,n+1} a_{m,n} + a^{+}_{m+1,n} a_{m,n} + h.c. \right)$

What we have: $H(t) = \sum J \left(a^{+}_{m,n+1} a_{m,n} + a^{+}_{m+1,n} a_{m,n} + h.c. \right) + \sum \varepsilon_j(t) \hat{n}_j$

$$+ \mu \sum_m n_m (n_m - 1)$$

Idea: Use fast periodic potentials $\varepsilon_j(t+T) = \varepsilon_j(t)$

Floquets Theorem: If $H(t+T) = H(t)$, $U(t_1, t_2) = K(t_1) e^{-iH_F (t_2 - t_1)} K(t_2)$

How to compute H_{eff} ?

$\to$ Fast-driving limit : $\Omega = \frac{2\pi}{T} \gg$ Energy scale of $H(t)$

$$\max_{0 \le t \le T} |E_{max}(t) - E_{min}(t)| = \|H_{max}\|$$

$\to$ Time-evolution after n periods $U(nT) = [U(T)]^n$

where $U(T) = \mathcal{T}\left\{ \exp\left(-i \int_0^T H(t)\, dt\right)\right\} \equiv e^{-i H_{eff} T}$

Magnus expansion : $H_{eff} = \sum_{j=0}^{\infty} H_{eff}^{(j)}$

Magnus expansion

$$U(T) = e^{-iH_{eff}T}, \quad H_{eff} = \sum_{j=0}^{\infty} H_{eff}^{(j)}$$

$$H_{eff}^{(0)} = \frac{1}{T}\int_0^T H(t)\, dT = \langle H(t) \rangle_T$$

$$H_{eff}^{(1)} = \frac{1}{2iT}\int_0^T \int_0^{t_1} dt_1\, dt_2 \, [H(t_1), H(t_2)]$$

$$\| H_{eff}^{(1)} \| = \mathcal{O}\left(\| H_{max} \|^2 T \right)$$

$$\sum_{j=1}^{\infty} \| H_{eff}^{(j)} \|$$

$$\| U(nT) - e^{-iH_{eff}^{(0)} nT} \| = \varepsilon$$

$$\Rightarrow T = \mathcal{O}\left(\frac{\varepsilon}{\sqrt{n}\, \| H_{max} \|} \right)$$

guarantees bounded error

Sufficient condition for convergence

$$\int_0^T \| H(t) \| \, dt < \pi$$

Simulating magnetic fields with neutral particles

What we want: $H = \sum_{m,n} J\left(e^{i\phi m} a^\dagger_{m,n+1} a_{m,n} + a^\dagger_{m+1,n} a_{m,n} + h.c.\right)$

What we have: $H(t) = \sum J\left(a^\dagger_{m,n+1} a_{m,n} + a^\dagger_{m+1,n} a_{m,n} + h.c.\right) + \sum \varepsilon_j(t)\,\hat{n}_j$

It turns out we need large values for $|\varepsilon_j|$, in fact $|\varepsilon_j| T > 1$

$\Rightarrow$ Magnus expansion does not converge!

Trick: Go to rotated frame (RF) $|\psi'(t)\rangle = V(t)\,|\psi(t)\rangle$

Lab frame

$$i\,|\dot{\psi}(t)\rangle = H(t)\,|\psi(t)\rangle$$

$$H(t) = \sum_j \varepsilon_j(t)\, n_j + \sum_{j,k} J\left(a_j^\dagger a_k + h.c.\right)$$

Rotated frame ($|\psi'(t)\rangle = V(t)|\psi(t)\rangle$)

$$i\,|\dot{\psi}'(t)\rangle = \underbrace{\left(V(t)H(t)V^\dagger(t) + i\dot{V}(t)V^\dagger(t)\right)}_{H'(t)}|\psi'(t)\rangle$$

Choose $V(t) = e^{i\sum_j W_j(t)\, n_j}$

with $W_j(t) = \int_0^t \varepsilon_j(t')\, dt'$

$$i\,\dot{V}(t)\,V^\dagger(t) = -\sum_j \varepsilon_j(t)\, n_j$$ → "eliminates" potential

$$H'(t) = V(t)\sum_{j,k} J\left(a_j^\dagger a_k + h.c.\right)V^\dagger(t) =$$

$$= \sum_{j,k} J\left(e^{i(W_j(t) - W_k(t))}\, a_j^\dagger a_k + h.c.\right)$$

RF:

$$H'(t) = \sum_{j,k} J\left(e^{i(W_j(t) - W_k(t))} a_j^\dagger a_k + h.c. \right), \quad W_j(t) = \int_0^t \varepsilon_j(t')\,dt'$$

For small enough T

$$U'(T) \simeq e^{-i H_{eff}'^{(0)} T}, \quad \text{with } H_{eff}'^{(0)} = \langle H(t) \rangle_T$$

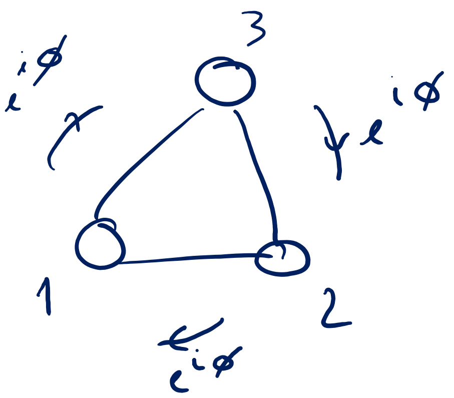
Effective Hamiltonian:

$$\langle H'(t) \rangle_T = \sum_{j,k} J \left[\langle e^{i(W_j(t) - W_k(t))} \rangle_T a_j^\dagger a_k + h.c. \right]$$

How to choose these functions so that we get the complex phases we want?

Simple example: Triangle

Struck et. al, PRL 108, 225304 (2012)

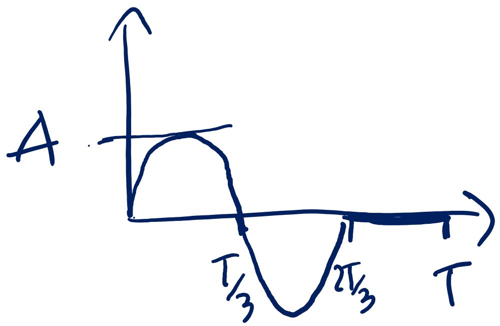


Real model:

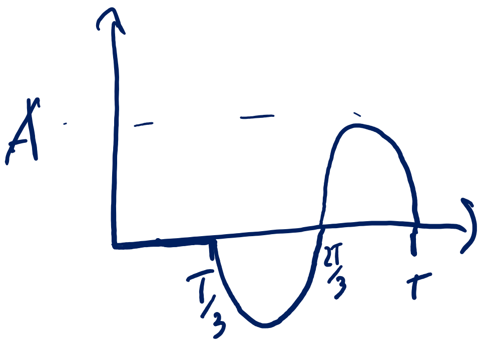
$$H(t) = \sum \varepsilon_j(t)\, n_j + \sum_{\langle j,k\rangle} J\,(a_j^\dagger a_k + h.c)$$

Choose: $\varepsilon_1(t) = 0$

$\varepsilon_2(t)$



$\varepsilon_3(t)$

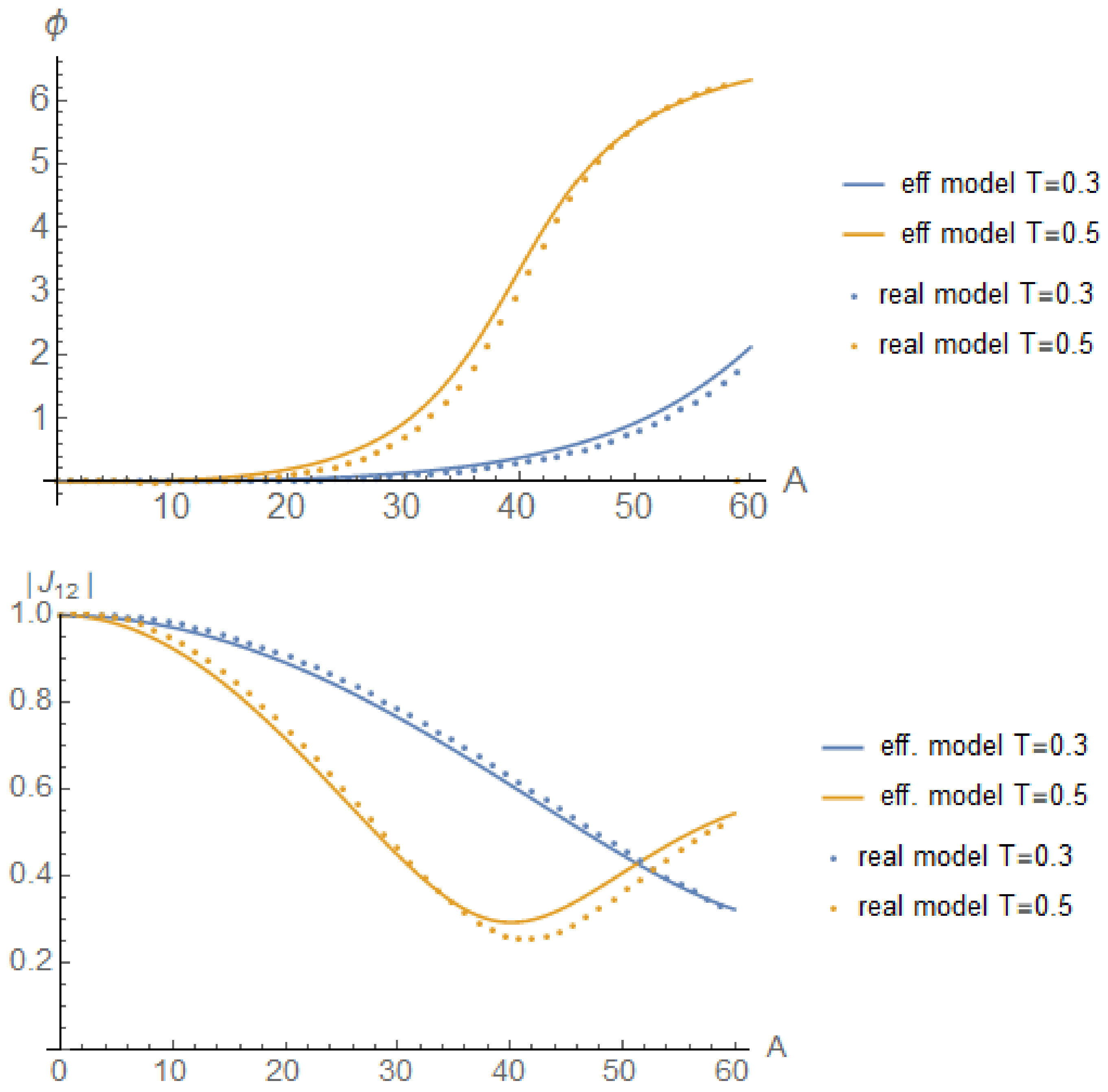


Effective model

$$J\langle e^{i(W_j(t) - W_k(t))}\rangle =$$

$$= J\left[\frac{2}{3}\mathcal{J}_0\left(\frac{AT}{3\pi}\right)e^{i\frac{AT}{9\pi}} + \frac{1}{3}e^{i\frac{2AT}{3\pi}}\right]$$

$$H_{eff} = \sum_{\langle jk\rangle} |J_{eff}|\left(e^{i\phi_{eff}}\, a_j^\dagger a_k + h.c.\right)$$



ϕ
6
5
4
3
2
1
10
20
30
40
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60
A
eff model T=0.3
eff model T=0.5
real model T=0.3
real model T=0.5
$|J_{12}|$
1.0
0.8
0.6
0.4
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0
10
20
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40
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60
A
eff. model T=0.3
eff. model T=0.5
real model T=0.3
real model T=0.5

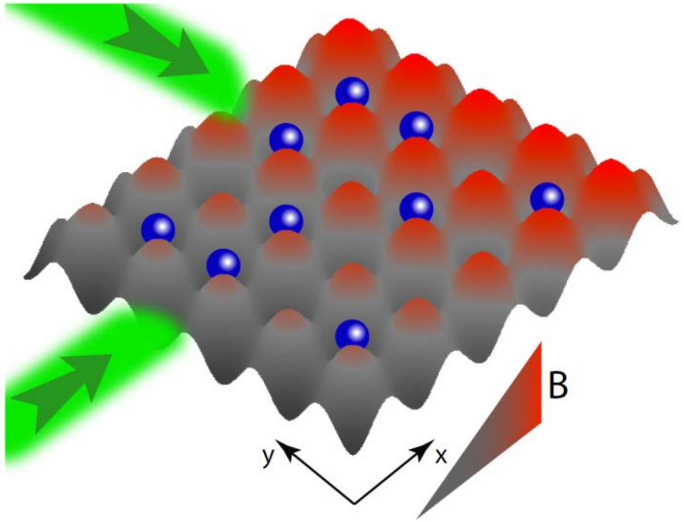
Realization of the Harper-Hofstadter model

$$H(t) = H_0 + H_1(t)$$

$$H_0 = \sum_{m,n} \left[J_x \left(a^\dagger_{m+1,n} a_{m,n} + h.c. \right) + J_y \left(a^\dagger_{m,n+1} a_{m,n} + h.c. \right) \right] + H_{Int.}$$

$$H_1(t) = \Omega \sum_{m,n} \left[\frac{\lambda}{2} \sin\left(\Omega t - \phi_{mn} + \frac{\Phi}{2} \right) + \Omega m \right] n_{mn}$$

$$\phi_{mn} = \Phi (m+n)$$

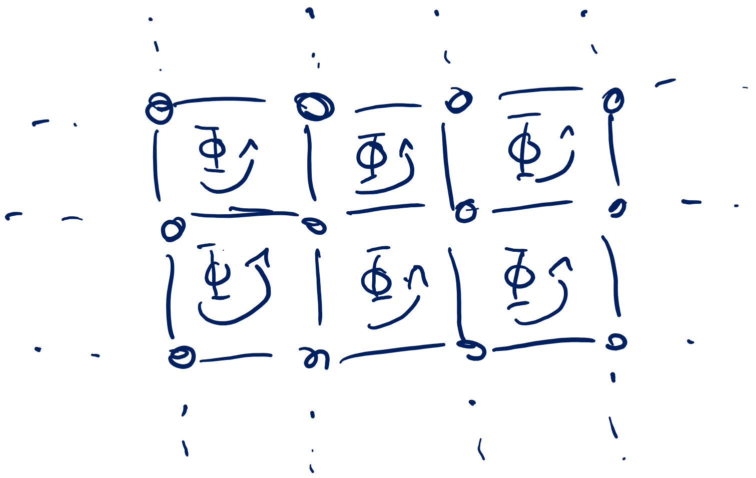


Simulated model

$$H_{eff} = -K \sum_{m,n} \left(e^{i\phi_{mn}} a^\dagger_{m+1,n} a_{m,n} + h.c. \right)$$

$$- J \sum_{m,n} \left(a^\dagger_{m,n+1} a_{m,n} + h.c. \right)$$

$$K = J_x \mathcal{J}_1(\varphi) \; ; \; J = J_y \mathcal{J}_0(\varphi), \; \varphi = J \sin\left(\frac{\Phi}{2}\right)$$



Review Bukov et al

arXiv: 1407.4803